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Subsistence Consumption and Rising Saving Rate

Abstract

This paper investigates the implications of the permanent income hypothesis with subsistence consumption being the driving force of a rising saving rate during economic development. To remedy the problem that subsistence consumption is unavailable, we use cointegrating regressions to generate the unobservable, subsistence-level adjusted saving rate series, and then test the orthogonal conditions imposed by the model using the generated series. The implications of the model cannot be rejected for Taiwan statistically, but receive weak support for Japan.

JEL classification: E21

Keywords: Saving rate; Subsistence consumption; Permanent income hypothesis

1. Introduction

The saving rate rose in the post-war period for many countries with a noticeable exception being the U.S., and there exists substantial cross-country disparity in the slopes of rising saving rates.[1] This paper studies the long-run behavior of the saving rate in the framework of the permanent income model, and the subsistence level of consumption is the key driving force. When consumption is near the subsistence level, an agent's major concern is meeting the subsistence requirement, and the elasticity of intertemporal substitution in consumption is close to zero. When the economy grows at a sustained rate, agents become more willing to substitute current consumption for future consumption. That is, the subsistence requirement can generate a rising saving rate. Once the importance of the subsistence requirement in consumption decisions declines, the saving rate behaves more like a stationary variable.[2]

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[1] These stylized facts were documented in Maddison (1992).

[2] An alternative approach in modeling the positive relationship between growth and saving would be the life cycle model of consumption. However, recent cross-country regressions did not find population-related variables such as the population growth rate, measures of the age structure, and mortality rates to be significant factors in accounting for the country-difference in the saving rate (Rebelo, 1992).

The idea that subsistence consumption induces a time-varying saving rate is not new. Zellner (1960) showed that subsistence consumption can make the ratio of optimal consumption to permanent income depend on the level of permanent income. Christiano (1989) introduced the subsistence level of consumption in the representative agent's utility function. Given certain parameterizations, the model was able to account for the hump-shaped pattern of Japan's saving rate in the post-war period. Rebelo (1992) also used the utility function with the subsistence level of consumption to explain the cross-country difference in the growth rate of consumption with a perfect international capital market. Chatterjee (1994) argued that if the economy grows toward the steady state and there exists a minimum consumption level requirement, then the average saving propensity is positively related to wealth. Recently, Ogaki, Ostry and Reinhart (1996) estimated a model that emphasizes the role of subsistence consumption and found strong empirical support for the hypothesis that the saving rate and its sensitivity to the interest rate are a rising function of income.

According to the permanent income hypothesis, when agents save only for consumption smoothing, changes in savings reflect anticipated changes in future incomes. In addition, there is a tilting motive related to any discrepancy between the subjective discount factor and the market discount factor. With a perfect international capital market, most growth models with standard preferences imply that consumption and output growths would equalize across countries. The model predicts that the saving rate contains no upward trend as the U.S. observations over the past one hundred years. However, during the process of economic development the saving rate rises in many countries. A rising saving rate indicates that the growth of income outpaces the growth of consumption. Therefore, imperfect international capital mobility is necessary to have cross-country disparity, or the real interest rate must be implausibly high in order to generate a rising saving rate in low-income countries. In this paper, we try to provide an alternative explanation for a rising saving rate while maintaining the assumption of perfect capital mobility.

The subsistence level of consumption imposes testable restrictions on the trend and cyclical properties of saving rate and labor income growth. We show that once the subsistence requirement is properly considered in measuring the saving rate, the adjusted saving rate becomes a stationary series. However, econometricians cannot observe the subsistence consumption and this causes difficulty in regular estimating. Ogaki, Ostry and Reinhart (1996) assessed a range of values of subsistence consumption around the value of US$123 (1985 prices) found for India by Atkeson and Ogaki (1993). Rather than assigning the value, this paper employs a two-step econometric procedure to remedy the problem that subsistence consumption is unavailable. We first use Park's (1992) canonical cointegrating regressions (CCR) to generate the unobservable, subsistence-level adjusted saving rate series. We then apply the Generalized Method of Moments (GMM) to test the orthogonal conditions imposed by the model using the adjusted saving rate and labor income growth rate series.

In an empirical investigation, we ask whether the saving rate series in Japan and Taiwan is consistent with the long-run predictions of our model. These two countries were near subsistence at the end of World War II, and managed to have sustained and impressive growth in the past forty years. They did not begin with a high saving rate, but experienced a dramatic increase during economic development. Hence, our focus is not on why these countries save a lot, but rather on why they have rising saving rates.

The remainder of the paper is as follows. The next section presents a permanent income model with the subsistence requirement. Section 3 uses the solution for the Euler equations and the intertemporal budget constraint to derive the long-run implications for the saving rate. Section 4 proposes a two-step econometric procedure for parameters estimating and hypothesis testing. Section 5 describes the time series data for Japan and Taiwan and presents the empirical results. The last section contains some conclusions.

2. A model with a subsistence level of consumption

The representative agent faces the problem of choosing a contingency plan of consumption $\{c_t, t \geq 0\}$, so as to maximize

$$E_0\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t U(c_t;\alpha_t)\right],$$

subject to $b_{t+1} = (1+r)(b_t + y_t - c_t)$, with b_0 given. Here y_t is the private agent's labor income at time t, b_t is the non-human wealth valued in units of consumption good held at the beginning of period t, and r is the constant real rate of return on non-human wealth. Function $U(c_t;\alpha_t)$ is the time separable utility function of consumption at time t with $U'(c_t;\alpha_t) > 0$ and $U''(c_t;\alpha_t) < 0$. Term ρ is the positive rate of time preference, and E_t denotes the mathematical expectation conditioned on the information set available at the beginning of time t, Ω_t. Term α_t captures the existence of subsistence consumption, because marginal utility of consumption shoots off to infinity as consumption decreases toward α_t:

$$\lim_{c_t \to \alpha_t^-} U'(c_t;\alpha_t) = \infty .$$

To model the subsistence level of consumption, we use the following log version of the HARA (hyperbolic absolute risk aversion) utility function:[3]

$$U(c_t; \alpha_t) = \ln(c_t - \alpha_t).$$

Here α_t is assumed to evolve according to $E_t \alpha_{t+1} = (1+m)\alpha_t$ for all t in which m is a constant. Two specifications of α_t were used in previous studies. First, the subsistence level of consumption is constant over time ($\alpha_t = \alpha_0$) in Rebelo (1992) and Atkeson and Ogaki (1993). Second, the subsistence level of consumption has a linear time trend: $\alpha_t = (1+m)^t \alpha_0$ with $m > 0$ in Christiano (1989). The Euler equation for the representative agent's problem is

$$\left[\frac{(1+r)(c_t - \alpha_t)}{(1+\rho)(c_{t+1} - \alpha_{t+1})}\right] = 1 + \varepsilon_{t+1}, \tag{1}$$

in which ε_{t+1} is the least square projection error that satisfies $E[\varepsilon_{t+1} | \Omega_t] = 0$ and reflects news about future income.[4] It is clear from (1) that marginal utility is convex in $c_t - \alpha_t$ under the logarithm-form utility function. An increase in uncertainty raises the expected marginal utility of consumption. To maintain the equality in the Euler equation, the expected future consumption must increase relative to current consumption. Furthermore, we assume that $E[\varepsilon_{t+1}^2 | \Omega_t] = \sigma^2$ for all t.

Assuming that the subsistence level of consumption does not grow, equation (1)

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[3] The widely used quadratic utility function has two undesired features for the purpose of our study. First, it implies the finite marginal utility for any admissible level of consumption. Second, the intertemporal elasticity of substitution falls as the level of consumption increases.

[4] As is common in empirical research, we take conditional expectations to be linear least-square projections on the information set.

yields the following expression:

$$\frac{E_t c_{t+1}}{c_t} - 1 = \phi_t \left(r - \rho + \frac{\sigma^2}{2} \right), \tag{2}$$

in which $\phi_t \equiv 1 - \alpha_0 / c_t$ is the intertemporal elasticity of substitution. Term ϕ_t is no longer constant and it converges to one when consumption grows at a sustained rate. The larger the value of ϕ_t is, the easier it is for private agents, in terms of utility, to forego current consumption for future consumption, and thus the higher the expected consumption growth rate is. The positive relationship between ϕ_t and c_t implies that agents in the high-income economy are more willing to substitute current consumption for future consumption than those in the low-income economy. Hence, the fluctuations of consumption in the high-income economy could be higher than those in the low-income counterpart.

The expected consumption growth rate is determined by two equally contributing factors. First, $\phi_t(r-\rho)$ characterizes the optimal response of consumption decisions to the real rate of return on the non-human wealth. When the real rate of return exceeds the rate of time preference, there is an incentive for private agents to increase consumption in the future. The higher real rate of return causes the intertemporal consumption path to tilt toward future periods. Second, $\phi_t \sigma^2 / 2$ represents the effect of uncertainty in future events on the consumption profile, since savings provide a reserve against any bad outcome from future events. Private agents prefer to defer current consumption for higher future consumption in the face of uncertainty. We call saving induced by the above two factors the estate saving and precautionary saving, respectively.

Without the subsistence consumption ($\phi_t = 1$), equation (2) predicts that any cross-country difference in consumption growth must be attributed to the cross-country difference in ρ or σ^2 or preferences in a world with a perfect international capital market. The rising intertemporal elasticity of substitution apparently offers an explanation for the cross-country difference in consumption growth. Another approach assumes that the rate of time preference falls as an economy become richer.[5] Using the Indian villages' panel data, Atkeson and Ogaki (1993) found that the rate of time preference is constant across poor and rich households, while the intertemporal elasticity of substitution is higher for rich households than it is for poor households.

3. Saving rate and the subsistence consumption

The Euler equation is only one of the necessary conditions for optimization so that the implied saving rate could be inconsistent with the budget constraint. In this

[5] Gersovitz (1983) emphasized the importance of nutrition and health in the determination of saving behavior. In his model, the probability of survival depends on an agent's consumption in the previous period and the rate of time preference decreases as the previous period's consumption increases.

section we use the Euler equation and the budget constraint to yield a tighter characterization for the saving rate.

When consumption grows at a sustained rate and $c_t > \alpha_t$, for all t, equation (1) can be written as

$$E_t[c_{t+1} - \alpha_{t+1}] = (1 + r - \rho + \frac{\sigma^2}{2})[c_t - \alpha_t]. \qquad (3)$$

Recursively substituting (3) into the budget constraint under the assumptions that $r > m$ and $\rho > \sigma^2/2$ gives the solution of c_t:

$$c_t = \beta y_t^p + \psi_t, \qquad (4)$$

in which $\beta = (\rho - \sigma^2/2)/r$, y_t^p is the permanent income at time t:

$$y_t^p = \frac{r}{1+r}\left(E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i y_{t+i}\right] + b_t\right),$$

and $_t$ is the permanent subsistence consumption at time t:

$$_t = \alpha_t\left[1 - \frac{\rho - \frac{\sigma^2}{2}}{r - m}\right] = \alpha_t\left[1 - \frac{\beta}{1 - m/r}\right].$$

The marginal propensity to consume out of permanent income (β) stays constant over time, but the consumption-permanent income ratio will change with the level of permanent income. The three factors r, ρ and σ^2 determine the value of β. If r exceeds ρ, agents have incentives to increase future consumption. Hence, the intertemporal pattern of consumption tilts toward future periods and β is a decreasing function of r. On the other hand, when future events become more uncertain, agents tend to save more of their permanent income as the reserve against bad future outcomes and β is a decreasing function of σ^2. Unlike the quadratic utility function, when $\rho = r$ the value of β is still less than one due to precautionary savings.

Agents are more willing to substitute current consumption for future consumption only when current and future subsistence consumption becomes less of a concern in consumption decisions. It is unlikely that c_t / y_t^p stays constant during the period of economic development. The permanent subsistence consumption appears to be a source of trend for the saving rate. However, the role of subsistence consumption in the time profile of consumption depends on the sign of $m - k$, where $k \equiv r - \rho + \sigma^2 / 2$ is the expected growth rate of consumption when there is no subsistence requirement.

Consider that the expected growth rate of α_t (m) is less than the slope of the intertemporal consumption path (k) and $_t > 0$ for all t. Since meeting future subsistence consumption is not a major concern for agents, agents tend to save less than what they would save without the subsistence consumption. The expected consumption growth rate is less than k. For $m = k$, the additional future income generated by estate saving and precautionary saving exactly offset the expenditure required for the future increase in the permanent subsistence consumption. The negative effect of an increase in c_t on the marginal utility is exactly offset by the positive effect of an increase in α_t on the marginal utility. The expected marginal rate of substitution between current consumption and future consumption stays constant over time. That is, the subsistence level of consumption has no effect on consumption decisions. For $m > k$, we expect the subsistence consumption to grow faster than consumption. Once agents worry about the fast-increasing permanent subsistence consumption in their decisions, they must save more to meet future subsistence consumption. The consumption path must tilt toward the future period so that consumption stays above its subsistence level in every future period. Agents save to survive. Since the assumptions of $r > m$ and $\rho > \sigma^2/2$ required in the derivation of (4) are not sufficient to pin down the sign of $m-k$, we assume that $k-m>0$ hereafter.

To precisely characterize the relationship between consumption growth and income growth, consider the following transformation of equation (4):

$$\frac{c_t}{y_t} = \frac{\beta r}{1+r}\left\{E_t\left[1+\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \prod_{j=1}^{i}(1+\lambda_{t+j})\right]+\frac{b_t}{y_t}\right\}+\frac{\psi_t}{y_t}, \qquad (5)$$

in which λ_t is the growth rate of labor income at time t: $y_t = (1+\lambda_t)y_{t-1}$. The derivation of (4) and (5) do not require any explicit relationship among b_t, y_t and α_t, except that their expected growth rates are all less than r, i.e., $r > \lambda$ and $r > m$.

Suppose that the growth rate of labor income is expected to be greater than that of α_t (m). When agents were near subsistence, they had to spend a significant portion of their income in consumption simply for survival. Equation (5) predicts that both

$_t / y_t$ and c_t / y_t are high in the early stage of economic development. When their consumption moves away from the subsistence level, agents are more willing to substitute current consumption for future consumption so that the intertemporal consumption path tilts toward future periods.

The ratio $_t / y_t$ here is the driving force for the declining consumption-labor income ratio during economic development. However, the effect on c_t / y_t is weaker as the level of labor income increases. On the other hand, if the growth rate of labor income is expected to be less than *m*, then $_t / y_t$ becomes an increasingly dominant driving force for the rising consumption-labor income ratio. The permanent subsistence consumption exerts stronger influence on agents' consumption decisions as the economy grows. Since this is an undesirable feature of the model, such a possibility is ruled out on the grounds that the consumption-labor income cannot rise without an upper bound.

To derive savings, let $\hat{y}_t$ denote the total income, which is the sum of labor income (y_t) and capital income ($rb_t /(1+r)$):

$$\hat{y}_t = \frac{r}{1+r} b_t + y_t ,$$

and the conventional saving is defined to be $\hat{y}_t - c_t$. Generally, equation (5) cannot be transformed in terms of this saving. Define a new variable: $\hat{s}_t \equiv \beta\hat{y}_t - c_t$ such that

$\hat{s}_t$ is the conventional saving when $\beta = 1$. For simplicity, $\hat{s}_t$ will be referred to as saving throughout the remainder of the paper.[6]

A standard manipulation of equation (5) produces:

$$\frac{\hat{s}_t}{y_t} = \frac{\beta}{1+r} - \frac{\beta r}{1+r} E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \prod_{j=1}^{i}(1+\lambda_{t+j})\right] - \frac{\psi_t}{y_t}. \qquad (6)$$

The permanent subsistence consumption ($_t / y_t$) is clearly a driving force for the rising saving rate. Imagine that two countries have the same time profile of λ_t and α_t. Equation (6) implies that the country in the early stage of development should have a lower saving rate than the country in the later stage of development. This is because agents in the early stage of development have to spend most of their income simply for survival. After an economy achieves a sufficiently high level of income, the importance of the permanent subsistence consumption ($_t / y_t$) declines for driving the saving rate.

Equation (6) can be used to examine the effects of changes in the labor income growth on the saving rate, too. To do this we take the first-order Taylor's expansion of $\sum_{i=1}^{\infty}(1+r)^{-i}\prod_{j=1}^{i}(1+\lambda_{t+j})$ around λ to obtain

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[6] Campbell (1987) measures saving as the discrepancy between disposable income and consumption divided by the marginal propensity to consume out of permanent income. Thus, the saving defined in our paper is proportional to the one in Campbell's with the proportionality factor β.

$$\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^{i}\prod_{j=1}^{i}(1+\lambda_{t+j})=\frac{1+\lambda}{r-\lambda}+\frac{1}{r-\lambda}\sum_{i=0}^{\infty}\left(\frac{1}{1+\tau}\right)^{i}(\lambda_{t+1+i}-\lambda),$$

in which λ is the mean value of the labor income growth rate, and $1+\tau=(1+r)/(1+\lambda)$ with $r>\lambda$. The absolute summability of $\{(1+\tau)^{-i}\}$ implies that the first and second moments of $\sum_{i=0}^{\infty}(1+\tau)^{-i}(\lambda_{t+1+i}-\lambda)$ are well defined. Equation (6) can then be transformed to

$$\frac{s_t}{y_t}=-\frac{\beta\lambda}{r-\lambda}-\frac{\beta r}{(1+r)(r-\lambda)}E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+\tau}\right)^{i}(\lambda_{t+1+i}-\lambda)\right], \tag{7}$$

in which $s_t\equiv\hat{s}_t+\psi_t$ is the level of saving adjusted for the permanent subsistence consumption.

If λ_t is a stationary variable, then the present value of a future deviation from its mean is stationary as well. Therefore, equation (7) implies that s_t/y_t is stationary. Suppose labor income growth is expected to temporarily slow down in the next several periods. To keep consumption on the planned path, private agents must increase their saving so that higher future capital income can offset the decline in future labor incomes caused by the temporary slowdown. Hence, cyclical fluctuations of λ_t only induce fluctuations in saving. As mentioned above, the declining pattern of $_t/y_t$ in economic development induces an upward trend in $\hat{s}_t/y_t$. Only the proper consideration of the permanent subsistence consumption in measuring the saving rate can eliminate the trend component of the saving rate.

4. A two-step econometric procedure

As shown in equation (6), the permanent subsistence consumption ($_t$) is the driving force for the rising saving rate. One potential problem in estimating equation (6) is that econometricians cannot observe $_t$. In this section we propose a two-step econometric procedure, which uses the information contained in the trend and cyclical properties of the saving rate, to remedy this problem.

From equation (6), it follows that

$$\frac{s_t}{y_{t-1}} - \beta\lambda_t - (1+r)\frac{s_{t-1}}{y_{t-1}} = \omega_t ,$$

in which

$$\omega_t = -\beta r\left\{(E_t - E_{t-1})\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \prod_{j=1}^{i}(1+\lambda_{t-1+j})\right]\right\}$$

is the unpredictable revision from $t-1$ to t in the expected value of human wealth. When s_t/y_t, s_t/y_{t-1} and λ_t are stationary variables, a linear combination of the three stationary variables eliminates all serial correlation and is completely unpredictable with respect to the past information set. The above equation implies a set of orthogonality restrictions (or the moment conditions):

$$E_{t-1}\left[\left(\frac{1}{1+r}\frac{s_t}{y_{t-1}}-\frac{\beta}{1+r}\lambda_t-\frac{s_{t-1}}{y_{t-1}}\right)z_{t-1}\right]=0, \tag{8}$$

for any random vector $z_{t-1}\in\Omega_{t-1}$.

The GMM procedure appears to be suitable for estimating parameters and testing the model by fitting equation (8). Since the unobserved series of the subsistence level of consumption (α_t) can be completely characterized by α_0 and m under our specification, α_0 and m are treated as parameters to be estimated. However, one important assumption under which large sample properties of GMM estimators are derived is the stationarity of variables. When α_0 and m are parameters to be estimated, not all of the variables in equation (8) can be transformed into stationary ones. This rules out the direct application of the GMM procedure to equation (8). For this reason, we propose a two-step econometric procedure.

In the first step we impose the stationarity of s_t / y_t as a restriction in estimation, and then generate a sequence of s_t / y_t using the estimates of cointegration estimation. By the definition of s_t, the stationarity of s_t / y_t implies that there exists at least a 3×1 vector, e, for $\hat{y}_t / y_t$, c_t / y_t, and α_t / y_t such that the three variables are cointegrated. The cointegration vector implied by the model is

$$e=\left[\frac{\rho-\frac{1}{2}\sigma^2}{r},\quad -1,\quad 1-\frac{\rho-\frac{1}{2}\sigma^2}{r-m}\right]=\left[\frac{r-k}{r},\quad -1,\quad 1-\frac{r-k}{r-m}\right].$$

To obtain the generated series of s_t / y_t, consider the following static regression:

$$\frac{c_t}{y_t} = e_0 + e_1 \frac{\hat{y}_t}{y_t} + e_2 \frac{\alpha_t}{y_t} + v_t$$
$$= e_0 + e_1 \frac{\hat{y}_t}{y_t} + e_2' \frac{(1+m)^t}{y_t} + v_t,$$

under the assumption that $\hat{y}_t / y_t$, c_t / y_t, and α_t / y_t are cointegrated. The theory predicts $e_1 = 1 - k / r = \beta$, $e_2' = e_2 \alpha_0 = [1 - (r-k)/(r-m)]\alpha_0$, and $s_t / y_t \equiv -e_0 - v_t$. Using the estimated value of e_0 and estimated residuals ($\hat{v}_t$) obtained in the above regression can generate the unobservable s_t / y_t series.

One straightforward approach is to apply OLS to the static regression. However, as argued in Campbell and Perron (1991), when either the error term v_t is serially correlated or the innovations in c_t / y_t Granger cause either innovations in $\hat{y}_t / y_t$ or innovations in α_t / y_t, the OLS estimate of *e* is not asymptotically optimal and may not have good finite sample properties. The aim here is not only testing the cointegration but also finding the estimate of *e*, we thus apply Park's (1992) CCR procedure to the static regression and use the Park's (1992) H(*p*,*q*) statistics for testing the cointegration restriction.

Once the s_t / y_t and s_t / y_{t-1} series are constructed, the second step is to apply the GMM procedure to estimate relevant parameters and to conduct the hypothesis testing for the orthogonality restrictions using the generated series of s_t / y_t and

s_t / y_{t-1}. The GMM estimators are instrumental variable estimators exploiting the moment conditions: $E[\omega_t z_t] = 0$. When there are over-identifying restrictions, the χ^2 statistic can be used to test the validity of the orthogonality conditions. This two-step procedure does not alter the asymptotic distributions of the GMM estimators and test statistics, because our cointegrating regression estimator is super consistent and converges faster than the GMM estimators.

5. Data and empirical results

In this section, we explain the data used in estimation and test results to examine the empirical validity of the subsistence level of consumption as a source of generating a rising saving rate.

The post-war annual data on real per capita consumption, disposable income, and compensation of employees are obtained from Japan and Taiwan. Since the compensation of employees is not comparable with the concept of labor income in the model, we follow Blinder and Deaton's (1985) procedure to decompose proprietors' income (or income from private unincorporated enterprises) into the labor and capital components according to their overall shares. We then convert nominal, aggregate magnitudes to a real, per capita basis by dividing by the available general price index and total population. Finally, two specifications for the growth rate of the subsistence level of consumption are considered: $m = 0$ (the constant subsistence level of consumption) and $m = 0.3\%$ (the subsistence level of consumption grows at the annual rate of 0.3%).

Japanese data are from *Historical Statistics of Japan* and *Japan Statistical Yearbook*. The sample period of the data series for Japan is 1952-1995. For Taiwan, we use two different measures of labor income. Data from *Taiwan National Income Account* begins earlier, but ceases to report the separate proprietor's income series since 1987. Thus, our first Taiwan data set (TWI) is over the periods 1952 to 1987. Data from *Survey of Family Incomes and Expenditure in Taiwan* begins later and the series in 1965, 1967, and 1969 are interpolated due to lack of data. The second Taiwan data set (TWII) is from 1964 to 1996. Because the proprietor's income sequences in two Taiwan data sets are not compatibly constructed, they cannot be combined to one.

Figure 1 plots the personal saving rate for the two countries.[7] The high saving rate for Japan and Taiwan in the post-World War II period has attracted lots of attention in empirical studies. The two countries did not begin with a high saving rate, but both

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[7] Here the personal saving is obtained by subtracting private consumption from disposable income.

experienced dramatic increases in the saving rate during economic development.[8] As for Japan's personal saving rate, it displays a pronounced hump-shaped pattern. Initially it was very low, reached its first peak in 1961 and a higher peak around 1974-1976, and then steadily fell to its current stationary rate. Like Japan, Taiwan's personal saving rate also exhibited an upward trend and then reached a peak around 1986 (TWI) or 1993 (TWII). However, it is too early to conclude that Taiwan's saving rate will fluctuate around the current stationary rate. The two countries experienced rising saving rates in the process of economic development. As their economies matured, the saving rates in both Japan and Taiwan began to decline toward the end of 1970s and 1980s, respectively.

For the three data sets (one for Japan, two for Taiwan), the ratio using the constructed labor income exhibits a smoother pattern than that using wage compensation, which indicates high mobility between employees and owners of private unincorporated enterprises for private agents in responding to external shocks. More importantly, the proprietor's income, which is the major component of household

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[8] On the other hand, based on 15 OECD countries for the period 1960-1985, Carroll and Summers (1991) presented low frequency evidence that there was a nearly perfect equality between consumption growth and income growth. The saving rate is stationary.

income in the early stage of development, usually exhibits a different trend from the wage compensation. The disposable income-labor income ratios using constructed labor income are plotted in *Figure* 2.

Tests for difference stationarity

To test the difference stationarity (or unit root non-stationarity) for each component of s_t / y_t (c_t / y_t, $\hat{y}_t / y_t$, and α_t / y_t), we apply Park and Choi's (1988) J(1,*q*) test and Augmented Dickey-Fuller (ADF) test. Given the known growth rates of α_t and y_t, an arbitrary choice of the initial value of the α_t / y_t series will not change its nonstationarity. We simply assume that the initial subsistence consumption is ninety percent of actual consumption here. This artificial subsistence consumption ratio sequence, denoted as α_t^a / y_t, is used to substitute the unobservable α_t / y_t series in the unit-root tests.

The null of difference stationary is rejected when the J(*p*,*q*) statistic is smaller than the critical values tabulated in Park and Choi (1988). The parameter *p* in the J(*p*,*q*) test is the order of the time polynomial in the null (unit-root) hypothesis, while the *q* denotes the order of the time polynomial in the fitted regression. The *J*(*p*,*q*) test does not require the estimation of the long-run variance and has an advantage over the Phillips and Perron's test and ADF test in that neither the bandwidth parameter nor the order of autoregression needs to be chosen. The Monte Carlo experiments also show that the J(p,q) test has a stable size and is not dominated by the ADF test in small samples in terms of powers.

Table 1 displays J(1,q) test results for the null of the difference stationarity for the three data sets. We expect the difference stationarity for each component of s_t / y_t. The J(1,q) test with $q = 2, \cdots 5$ cannot reject the null at the 5% significance level for c_t / y_t, $\hat{y}_t / y_t$ and α_t^a / y_t ($m = 0$ and $m = 0.3\%$) as expected. Furthermore, after differencing all of these series, the J(1,q) test does reject the unit-root null except for the difference of c_t / y_t in TWII and for the difference of α_t^a / y_t in Japan.

We next report the ADF test for the null of difference stationarity with a time trend in *Table* 2 because it was widely used in the literature. Since considerable evidence exists that data dependent methods for selecting the value of the lag order in the ADF regressions are superior than choosing a fixed order a priori, we follow the recursive t-statistic procedure suggested by Campbell and Perron (1991). Results of the ADF test for the three data sets are generally consistent with the J(1,q) tests. There are two exceptions: c_t / y_t series in TWI and the difference of c_t / y_t series in TWII; their ADF tests reject the unit-root hypothesis firmly. Even though the above results should not be viewed as conclusive evidence on the difference stationarity, it is acceptable to assume that there exists unit roots in the individual components of s_t / y_t.

Empirical results of cointegration regression

In the light of the results obtained in tests for difference stationarity, we adopt the following specification for the rest of the paper: c_t / y_t, $\hat{y}_t / y_t$, and α_t / y_t are assumed to have a unit root around a linear time trend. If the growth rates of labor income are stationary without a time trend, then equation (7) implies that the three components of s_t / y_t are deterministically cointegrated. Thus, we apply the CCR procedure[9] to

$$\frac{c_t}{y_t} = e_0 + e_1 \frac{\hat{y}_t}{y_t} + e_2^t \frac{(1+m)^t}{y_t} + \sum_{i=1}^{q} \eta_i (\text{time})^i + \varepsilon_t^c .$$

An important property of the CCR procedure is that linear restrictions can be tested by χ^2 tests, which are free from nuisance parameters. Let H(p,q) statistic denote the Wald statistic to test the hypothesis $\eta_{p+1} = \eta_{p+2} = \cdots = \eta_q = 0$. Park (1990) showed that the H(p,q) statistic converges in distribution to a $\chi^2(q-p)$ random variable under the null of cointegration. According to Ogaki and Park's (1997) definition, to say a set of difference stationary process is deterministic cointegrated requires that their cointegrating vector eliminate both the stochastic and deterministic trends. Stochastic

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[9] The CCR procedure requires an estimate of nuisance parameters such as the long-run covariance of the disturbances in the system. We use Park and Ogaki's (1991) VAR prewhitening method with Andrews's (1991) QS kernel and his automatic band width parameter estimator to estimate the nuisance parameters. The VAR of order one was used for prewhitening.

cointegration requires only that the stochastic trend components of the series are cointegrated. Therefore, the H(0,1) statistic tests the hypothesis of the deterministic cointegrating restriction while the H(1,q) statistic tests stochastic cointegration.

Table 3 reports the H(0,1) test results for the null of the deterministic cointegration among the three components of s_t / y_t with and without dummy variables.[10] The H(0,1) tests are more favorable for the specification with the dummy variables. Among the test statistics reported without a dummy variable, only one case is insignificant at the 5% level (TWII with m = 0.3%). While including dummy variables in regression, none of the Taiwanese cases is significantly rejected, but the results for Japan improve only marginally.

The results for the rejection of deterministic cointegration appear to suggest that the full set of stationarity restrictions imposed by the stationarity of the growth rate of labor income and thus s_t / y_t is too strong for Japan. The ADF test shows that whether the unit root null for the growth rate of Japanese labor income fails or not depends on the trend specification. The statistic favors the time trend hypothesis

[10] There are two events that may be crucial for the saving behaviors of open economies like Japan or Taiwan to change, the oil shock in 1974 and the G5 Plaza Accord in September 1985. We estimate a crash model with two structural breaking points, 1974 and 1986, respectively.

against the unit root null. To take into account the decreasing growth rates of labor income in the transition of development and to identify the possible causes of the rejection, we drop the cotrending restriction among the individual components of s_t / y_t, and include a linear trend in the static regression. The H(1,q) test is appropriate for testing the null of stochastic cointegration among three components of s_t / y_t. For Japan, the results of the H(1,q) test with q = 2, 3, 4 (not reported here) favor the null of the stochastic cointegration among these variables now.[11] These results, together with the result for the deterministic cointegration in *Table* 3, confirm that the measure of saving rates adjusted for the permanent subsistence consumption can eliminate the trend component once the growth rate of labor income is stationary.

Table 3 also reports coefficient estimates of e_1 and e_2' in the static regression using Park's CCR procedure. Different data sets yield very different coefficient estimates. For example, when applying Japan and TWII without a dummy variable, the CCR estimates of e_1 are all greater than one. Note that $e_1 = \beta$ and $\beta = (r - k) / r$ when the theoretical model is true. According to our model, the real rate of return on the non-human wealth must be well below zero and/or the expected

[11] Based on the size and power properties, Han and Ogaki (1991) recommend the H(1,q) test with small values of q when the sample size is small.

growth rate of consumption without a subsistence requirement is negative in order for β to be above one, which is inconsistent with the empirical regularities of fast-growing developing countries. On the other hand, when estimating the deterministic cointegration regression with dummy variables, only the TWII estimates of e_1 are greater than one. Since a reasonable β should be positive and less than one, our following discussion will focus on the cases where the estimate of e_1 is less than one. That is, we use the data set of Japan and TWI and their estimates with dummy variables included in the CCR regression for the rest of the paper.

We now use the CCR estimate of e_0 and the estimated residuals ($\hat{v}_t$) in the static regression to generate the s_t / y_t series. According to the model, when financing the permanent subsistence consumption is the major concern in agents' intertemporal decisions of consumption in the early course of economic development, the relatively high value of ψ_t / y_t makes the value of s_t / y_t significantly higher than the $\hat{s}_t / y_t$ counterpart. As the economy grows, the importance of ψ_t / y_t in the determination of saving behavior declines and ψ_t / y_t is expected to exert decreasing effects on s_t / y_t over time. Eventually, s_t / y_t and $\hat{s}_t / y_t$ will converge, and $\hat{s}_t / y_t$ becomes a stationary process. That is, the permanent subsistence consumption consideration is capable of inducing the stationarity of s_t / y_t.

Figures 3 and 4 plot those generated series for Japan and Taiwan. Unlike the hump- shaped personal saving rate in *Figure* 1, Japan's $\hat{s}_t / y_t$ series now exhibits two deeper troughs around 1967 and 1985 and three mild peaks during the whole sample period. The difference between s_t / y_t and $\hat{s}_t / y_t$ contracts until the first oil shock, but after that it has no tendency for convergence. The subsistence consumption requirement appears to be helpful in explaining the rising saving rates in the 1950s and 60s only. On the other hand, plots in *Figures* 4 clearly indicate that the subsistence level of consumption effectively induces the stationarity of Taiwanese s_t / y_t. When conducting the formal unit root test, the null of the difference stationarity without time trend for Taiwanese s_t / y_t can be rejected firmly under the ADF test and the J(p,q) test. However, for Japanese s_t / y_t the same null can be rejected under the J(p,q) test only. The weak evidence for Japan is consistent with the less satisfactory results of the previous H(p,q) test.

Empirical results and tests for orthogonality restrictions

In the second step we estimate r and conduct the χ^2 test for the orthogonality restrictions based upon the CCR estimate of β and the generated s_t / y_t series. Since $s_t(1+\lambda_t)/y_t = s_t / y_{t-1}$, s_{t-1} / y_{t-1} does not contain extra information once s_t / y_{t-1} and λ_t have been chosen as instruments. The instrumental variables chosen for the benchmark GMM procedure are $z_t \equiv \{s_{t-k} / y_{t-k-1}, \lambda_{t-k}, 2 \leq k \leq q\}$. The lag length of the instrument variables (q) is from 2 to 4.[12] As q increases, more orthogonality conditions are employed in the estimation.

Table 4 reports the GMM estimate of r with $m = 0$ and $m = 0.3\%$, and the corresponding χ^2 test statistic. The model's orthogonality restrictions cannot be rejected and the estimates of r are significantly positive in all cases. Although the null hypothesis is satisfied, the GMM estimates of r are too high for Japan to support the theoretical model except for one case: when $m = 0.3\%$ and when the lag length of the instrument variables is four. In other cases, the estimates of r for Japan fall in the 38.4%-53.0% range. Excluding these cases from consideration, the GMM estimates of r for Taiwan range from 8.9% to 13.4%. The estimation results are very plausible from the viewpoint of the permanent income hypothesis. The values of estimated r are also close to those obtained in Hayashi (1982).[13]

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[12] To take account for the problem of measurement errors and time aggregation, we introduce a first-order moving average process in the error term and select instruments that lag two periods or more. See, e.g., Hall (1988), Heaton (1995), and Ostry and Reinhart (1992).

[13] Using post-war U.S. time series data, Hayashi obtained the estimates of r ranging from 13.2% without any restriction on the size of the liquidity-constrained households in the population to 17.3% without the equality restriction between r and ρ.

subsistence consumption is the driving force of a rising saving rate during economic development.

Even though the model completely abstracts from the demographic considerations of the life-cycle hypothesis, we have found substantial evidence for the permanent income hypothesis with subsistence consumption in Japan and Taiwan after WWII. The cointegration and orthogonality restrictions imposed by the model cannot be rejected for Taiwan statistically, and all of the estimates of parameters are significant and with the correct sign. On the other hand, we find weak support for Japan. The subsistence consumption requirement is helpful to explain the rising saving rates in the 1950s and 60s, but it appears not an important factor for the changing of Japanese saving rates since the oil shock.

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Keep in mind that $e_1 = \beta$ and $e_2' = \alpha_0[1 - \beta/(1 - m/r)]$ when the model is true. We now have the estimates of e_1, e_2' and r, and therefore can recover the unobservable subsistence level of consumption. It is confirmed that the consumption expenditure is above its subsistence counterpart during the whole sample period for Japan and Taiwan. In addition, a sensitivity analysis is conducted with different growth rates about the subsistence consumption. Given the presumption that the growth rate of the subsistence consumption cannot be higher than the steady-state growth rate of output, we raise m to 0.5% and 1% per year and repeat all of the estimations and tests above. In general, the empirical results are robust to the choice of m. However, when m is as high as 1%, the CCR estimation implies a slow rate of convergence and a less stationary s_t / y_t series.

6. Concluding Remarks

As pointed out by Rebelo (1992) and others, a subsistence consumption requirement induces a rising saving rate. However, unobservable subsistence consumption causes difficulty in empirical study. This paper employs a two-step econometric procedure to remedy this problem. We test the implications of the permanent income hypothesis with subsistence consumption that a permanent

Table 1. J(*p*, *q*) statistic

			J(1, 2)	J(1, 3)	J(1, 4)	J(1, 5)
Japan	c_t / y_t		0.368	0.398	0.957	1.218
	$\hat{y}_t / y_t$		0.606	0.641	2.527	2.659
	α_t^a / y_t	$m = 0$	9.518	73.246	108.663	181.783
		$m = 0.3\%$	9.636	59.515	96.074	160.471
TWI	c_t / y_t		0.075	0.175	0.391	0.396
	$\hat{y}_t / y_t$		0.023	0.184	0.465	0.544
	α_t^a / y_t	$m = 0$	3.236	5.621	8.721	9.538
		$m = 0.3\%$	2.396	4.633	7.253	8.011
TWII	c_t / y_t		0.799	1.775	2.181	4.808
	$\hat{y}_t / y_t$		2.547	2.972	3.043	5.115
	α_t^a / y_t	$m = 0$	10.536	10.981	14.541	21.904
		$m = 0.3\%$	9.249	9.426	12.528	19.029
Japan	$\Delta(c_t / y_t)$		0.001*	0.058*	0.060*	0.074*
	$\Delta(\hat{y}_t / y_t)$		0.014	0.048*	0.055*	0.253*
	$\Delta\alpha_t^a / y_t$	$m = 0$	0.434	0.456	0.515	0.567
		$m = 0.3\%$	0.368	0.400	0.462	0.521
TWI	$\Delta(c_t / y_t)$		0.016	0.042*	0.048*	0.050*
	$\Delta(\hat{y}_t / y_t)$		0.038	0.045*	0.062*	0.085*
	$\Delta\alpha_t^a / y_t$	$m = 0$	0.037	0.069	0.109*	0.163*
		$m = 0.3\%$	0.045	0.078	0.121*	0.174*
TWII	$\Delta(c_t / y_t)$		0.209	0.210	0.248	0.404
	$\Delta(\hat{y}_t / y_t)$		0.031	0.034*	0.047*	0.048*
	$\Delta\alpha_t^a / y_t$	$m = 0$	0.000*	0.013*	0.028*	0.114*
		$m = 0.3\%$	0.000*	0.013*	0.029*	0.117*

Note: * means rejection of the null at a 5% level.

Table 2. ADF test

			lag	statistics
Japan	c_t / y_t		0	-1.655
	$\hat{y}_t / y_t$		0	-2.193
	α_t^a / y_t	$m = 0$	4	-2.422
		$m = 0.3\%$	4	-2.431
TWI	c_t / y_t		0	-4.071*
	$\hat{y}_t / y_t$		0	-3.152
	α_t^a / y_t	$m = 0$	0	-0.122
		$m = 0.3\%$	0	-0.174
TWII	c_t / y_t		5	0.212
	$\hat{y}_t / y_t$		0	-0.888
	α_t^a / y_t	$m = 0$	3	-0.552
		$m = 0.3\%$	3	-0.453
Japan	$\Delta(c_t / y_t)$		0	-5.631*
	$\Delta(\hat{y}_t / y_t)$		0	-5.746*
	$\Delta\alpha_t^a / y_t$	$m = 0$	3	-0.954
		$m = 0.3\%$	3	-0.983
TWI	$\Delta(c_t / y_t)$		3	-5.226*
	$\Delta(\hat{y}_t / y_t)$		0	-6.481*
	$\Delta\alpha_t^a / y_t$	$m = 0$	4	-3.479*
		$m = 0.3\%$	4	-3.405^a
TWII	$\Delta(c_t / y_t)$		4	-4.594*
	$\Delta(\hat{y}_t / y_t)$		1	-5.395*
	$\Delta\alpha_t^a / y_t$	$m = 0$	2	-4.655*
		$m = 0.3\%$	2	-4.601*

Note: * means rejection of the null at a 5% level, *a* means rejection of the null at a 10% level.

Table 3. Canonical Cointergrating Regression Results

			$\hat{e}_1$	$\hat{e}'_2$	H(0,1)
without dummy	$m = 0$	Japan	1.040*	0.025	17.223*
		TWI	0.319*	5.270*	14.052*
		TWII	1.295*	0.024*	3.954*
	$m = 0.3\%$	Japan	1.042*	0.026	17.202*
		TWI	0.272*	5.398*	13.565*
		TWII	1.257*	0.026*	2.610
with dummy	$m = 0$	Japan	0.740*	0.029*	11.265*
		TWI	0.362*	4.796*	1.168
		TWII	1.423*	0.003	1.720
	$m = 0.3\%$	Japan	0.730*	0.029*	10.685*
		TWI	0.336*	4.921*	1.221
		TWII	1.373*	0.005	1.021

Note: * means significant at a 5% level

Table 4. Generalized Method of Moments Results

	Japan		Taiwan (TWI)	
$m=0$				
q	r	χ^2	r	χ^2
2	0.437*	3.402	0.109*	2.228
3	0.530*	6.948	0.134*	5.784
4	0.484*	11.804	0.092*	9.880
$m=0.3$ %				
q	r	χ^2	r	χ^2
2	0.384*	3.318	0.106*	2.544
3	0.475*	6.596	0.126*	5.920
4	0.153*	12.085	0.089*	8.508

Note: * means significant at a 5% level.