行政院國家科學委員會專題研究計畫　成果報告

平滑轉變的波動模型(II-I)

計畫類別： 個別型計畫
計畫編號： NSC91-2415-H-002-007-
執行期間： 91 年 08 月 01 日至 92 年 07 月 31 日
執行單位： 國立臺灣大學經濟學系暨研究所

計畫主持人： 林建甫

報告類型： 精簡報告

處理方式： 本計畫可公開查詢

中　華　民　國　92 年 11 月 10 日

行政院國家科學委員會補助專題研究計畫 ▧ 成 果 報 告
□期中進度報告

平滑轉變的波動模型

計畫類別：▧ 個別型計畫 □ 整合型計畫
計畫編號：NSC 91-2415-H-002-007
執行期間： 91 年 8 月 1 日至 92 年 7 月 31 日

計畫主持人：林建甫
共同主持人：
計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：▧精簡報告 □完整報告

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢
□涉及專利或其他智慧財產權，□一年□二年後可公開查詢

執行單位：台灣大學經濟學系

中 華 民 國 92 年 10 月 31 日

平滑轉變的波動模型

台灣大學經濟學系
林建甫

In this project, we incorporate the smooth transition model into the GARCH volatility model and invent a brand new smooth transition GARCH (STAR-GARCH) volatility model. The smooth transition model is a hot research topic over the past fifteen years. In recent years, it evolves more, for example, the multi-regimes smooth switching, the time varying switching, the vector smooth switching. This makes the mode even useful to approximate more nonlinear phenomena. The STAR-GARCH volatility model estimates the conditional heteroscedasticity and allows the unconditional variance to be constant. The model helps us to predict the volatility in the future. Therefore when we incorporate the smooth transition model into the volatility model can solve many remaining problems left. We allow the variables of time, lag conditional variance, and other exogenous variables in the transition function. The model is hence rich to cover different nonlinear transformation. The model can also avoid the weak point that many volatility models can fit well in the past and cannot have good out sample forecast by regime switching. To keep away from the data mining, we also provide the test method to diagnosis the data and model-specification.

Key words: nonlinear model, smooth transition model, volatility model, transition function, GARCH Model

本計畫的目的在將平滑轉換模型的觀念引入GARCH 波動性模型中，以得到新的平滑轉變GARCH 波動模型(STAR-GARCH volatility model)。平滑轉換模型是最近相當熱門的時間數列研究焦點，它本身具有逼近相當多的非線性轉換模型，近三年又發展出多狀態轉換、因時而異的平滑轉換、多變量的狀態轉換，因此使得模型的應用力又相當的增加。波動性模型假定非條件變異數 (unconditional variance) 是常數，可是條件變異數不但是可以變動，而且未來的變異數也可以做預測的。因此將平滑轉換模型的觀念引入波動性模型中可以探討過去無法解決的問題。我們允許轉換函數是時間、過去條件變異數的函數或其他變數的函數，因此可以逼進不同的非線性轉換。而因為允許非條件變異數方程式可以有所轉變，這個模型也可以解決一般波動性模型中估計很好但樣本外預測不好的窘境。我們也提出檢定及模型設定的方法，以避免一般直接進入非線性模型的資料深耕。

關鍵詞：非線性模型、平滑轉換模型、波動性、轉換函數、GARCH 模型

I. Introduction

Modeling nonlinear time series is a new developed topic and has recently received considerable attention. Many of these series are high volatile, that is, they consist of weekly, daily or even intra-daily observations and the conditional variance of the process is not constant over time. Just like many high frequency series show little or no linear dependence, the focus has been on modeling the conditional variance. The idea of conditional autoregressive heteroskedasticity (ARCH; Engle, 1982) has led to a large number of extensions of the original model and applications. Bollerslev, Engle and Nelson (1994) and Palm (1996) are examples of recent surveys of the area. Another variant of ARCH, the so called Stochastic Volatility model (Taylor, 1986, p 73fi) has gained popularity recently, and the latest developments were surveyed in Ghysels, Harvey and Renault (1996).

The main empirical results are that regime-switching GARCH resolves the problemthat standard GARCH forecasts are significantly too high in volatile periods and that regime-switching GARCH forecasts significantly outperform GARCH forecasts in terms of mean squared error. These results hold out-of-sample and for both forecast horizons we examine, namely the one-day and ten-day horizons. This provides evidence for the conjecture raised by West and Cho (1995) that it will be productive to explore models that explicitly account for movement in the variance generating process, for instance, by regime switches.

It has been argued, however, that despite the absence of linear dependence there may be nonlinear dependence in the conditional mean. This should then be appropriately modeled in order to avoid misspecification of the conditional variance. Tong (1990, p.116) suggested combining the self exciting threshold autoregressive (SETAR) model for the conditional mean with an ARCH model for the conditional variance. Li and Lam (1995) followed this suggestion and also devised a specification strategy for building SETAR-ARCH models. They applied their model to the daily Hong Kong Hang-Seng stock index and reported nonlinearities in the conditional mean. The original ARCH model and its most important extension, the generalized ARCH (GARCH), are symmetric: while the size of the shock matters, the sign does not. Many authors have argued that shocks may have asymmetric effects to volatility: the dynamic response to a positive shock is not necessarily the mirror image of the response to a negative shock of the same size. Pagan (1996), in his survey of developments in financial econometrics, provided a useful review of models that can handle this type of asymmetry.

One possibility to allow for periods with different unconditional variances is, of course, by introducing deterministic shifts into the variance process, but this is rather ad hoc. A popular approach to endogenize changes in the data generating process is the Markov regime-switching model. Hamilton (1989) introduces this model to describe the U.S. business cycle, which is characterized by periodic shifts from recessions to expansions and vice versa. In our context of exchange rate volatility, a Markov process can be used to govern the switches between regimes with different variances. See Kaufmann and Scheicher (1996) for a survey on Markov-switching models.

Another natural idea would be to combine such a parameterization of the conditional variance with a nonlinear model for the conditional mean. Li and Li (1996) did exactly that by defining a double threshold autoregressive heteroskedastic (DTARCH) time series model. A DTARCH model has a SETAR-type conditional mean The conditional variance is parameterized similarly, and the authors called their specification the threshold ARCH (TARCH) model Note, however, that it differs from the TARCH model of Zakolan (1994) in that the latter is a parameterization of the conditional standard deviation. Li and Li (1996) also provided a comprehensive modeling strategy for DTARCH models. It was based on the idea of ordered autoregressions which Tsay (1989) successfully applied to the specification of SETAR models. The authors fitted their DTARCH specification to the daily Hong Kong Hang Seng stock index Recently, Lee and Li (1998) generalized the DTARCH model by allowing the transition of the first and second regime to be smooth. They called this model the Smooth transition Double Threshold model.

In order to overcome these drawbacks, we explore a new approach in looking at volatility. This paper we follow Lee and Li (1998) by adopting the idea of smooth transition in the conditional mean which first appeared in Bacon and Watts (1971). Our paper may be seen as an extension of Lee and Li's work in the sense that we simultaneously allow a rather flexible specification for the conditional variance as well. Besides, misspecification testing will receive plenty of attention in this paper. The conditional mean is thus specified as a Smooth transition Autoregressive (STAR) model; see, for example, Chan and Tong (1986), Granger and Terasvirta (1993) and Terasvirta(l994) The conditional variance is specified as a Smooth Transition GARCH (STGARCH) model; see Hagerud (1997) and GonzBlezRivera (1998). Our STAR-GARCH model is a generalization of the GJR-GARCH model (Glosten, Jagannathan and Runkle, 1993) and the generalized quadratic ARCH model of Sentana (1995). It allows plenty of scope for explaining asymmetries in volatility. Our aim is to construct a complete modeling cycle for our STAR-GARCH family of models, consisting of three stages: specification, estimation and evaluation.

II. The model

We apply the smooth transition idea into GARCH(p,q) model to describe the change of conditional variance. The GARCH(p,q) model with the mean equation can be written as the following,

(1)
$$\begin{aligned} \varepsilon_t &= y_t - x_t' b \\ \varepsilon_t | \Omega_{t-1} &\sim N(0, h_t) \\ h_t &= z_t' w \end{aligned}$$

where, $z_t' = (1, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q}^2, h_{t-1}, \ldots, h_{t-p})$, $w_t' = (\alpha_0, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p)$, $p \geq 0$, $q > 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \ldots, q$, $\beta_i \geq 0, i = 1, \ldots, p$, and Ω_t is the information set. Assume that all the possible parameters $(b', w')'$ belongs to a compact set in the Euclidian space, and the ε_t has a finite second moment. To provide a wider possibility change in the model and let parameters w can have a rich formation, we use the following general setup.

(2) $$h_t = z_t' w_1 + z_t' w_2 F(z)$$

where $F(z)$ is a transition function. Its purpose is to let the transition part transform from one regime to the other regime. The first regime can be written as

(3) $$h_t = z_t' w_1$$

and the second regime is

(4) $$h_t = z_t'(w_1 + w_2).$$

A discrete change of the transition function can be modeled as $F(z) = 0, t < t_0$, and $F(z) = 1, t >= t_0$, where t_0 is the change point of the time. The smooth transition part can be motivated by time or any other variable. For example, in Farley, Hinich and Mcguire (1975), they think the parameter change results from the linear function of time, $F(z) = t$ as a special example. Terasvirta (1994) applies the smooth transition autoregressive model. Similarly, Lin and Terasvirta (1994) use a smooth transition regression model. They both separate the transition function into odd function and even function. A logistic function which has monotonic and non-monotonic possibilities can represent the odd function in the transition. The even function uses exponential function in the transition part to model a cyclic change. This pattern represent a transition which is exponential function and allows the pattern to change back and forth.

For example, if we assume $F(z)$ is a logistic distribution function, in particular as the following

(5) $$F(z) = \{1 + \exp[-z(t)]\}^{-1}.$$

But $z(t)$ can be specified as a time polynomial in Lin (1992),

(6) $$z(t) = r(t^k + c_1 t^{k-1} + \ldots + c_{k-1} t + c_k).$$

Because this logistic distribution function with higher order time polynomial can approximate many different type of structural change without using exponential function in the transition. When $k = 1$, $z(t)$ can be rewritten as $z(t) = r(t - c)$. This is a monotonic transform case. In this case, c is a location parameter, which point out the location where the speed is the fastest, r is the transition speed. When r approach infinite, the function approximate Heaviside function and make the discrete change. When r is moderate, the model become smooth transition. And when r approach zero, no sign of change at all. The model with $k = 2$ covers the even function that generate back and forth behavior, whereas $k = 3$, the model wraps non-monotonic change and make the transition very complicated.

III. Testing conditional volatility change

To test a conditional variance structural change in GARCH model, we use $w_2 = 0$ or $F(z) = 0$ in (2) to discuss. If the null hypothesis in the former is true, $c_1, \ldots, c_k$ in $F(z)$ can be anything. Instead, if the null hypothesis in the later is true, w_2 cannot be identified. These create the problem of nuisance parameters. Terasvirta (1994), Lin and Terasvirta (1994) start with the later case. In order to derive the test statistics, without losing the general condition, we use $\tilde{F}(z) = F(z) - \frac{1}{2}$ to replace $F(z)$. This makes $\tilde{F}(0) = 0$ and let $H_0\!:r = 0$ in (2) becomes a nature assumption. Then to solve the problem of nuisance parameters, we apply the first order Taylor approximation of $\tilde{F}(z)$ and get

(7) $$T_1(z) = \tilde{F}(0) + \tilde{F}'(0)z = a_1 z.$$

Using (7) to replace $\tilde{F}(z)$ and plug into (2), we arrive

(8) $$\begin{aligned} h_t &= z_t' w_1 + z_t' w_2 (a_1 z) \\ &= z_t' w_1 + z_t' w_2 (a_1 r)(t^k + c_1 t^{k-1} + \ldots + c_{k-1} t + c_k). \end{aligned}$$

Next, we rearrange and collect the parameters in the same variables. A new parameterized form is formed.

(9) $$h_t = z_t' \varphi_1 + (s_t \otimes z_t)' \varphi_2,$$

where $\otimes$ is the Kronecker product, and other new variables here are $s_t = (t, t^2, \ldots, t^k)'$, $z_t = (1, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q}^2, h_{t-1}, \ldots, h_{t-p})'$, $\varphi_2 = (\phi_1', \phi_2', \ldots, \phi_k')'$, $\varphi_1 = \phi_0$. Here $\phi_i = (\phi_{i0}, \ldots, \phi_{iq}, \phi_{iq+1}, \ldots, \phi_{iq+p+1})'$, *i=0,1,....,M, M =*$k \times (p + q + 1)$. The old null hypothesis $H_0\!:r = 0$ becomes

(10) $$H_0\!: \varphi_2 = 0$$

One important remark, to conquer the problem of nuisance parameters, we give up information in (8) and adopt (9). Now, when $H_0\!: \varphi_2 = 0$ is true, applying the theorem 1 in Lin and Terasvirta (1994), we have the asymptotic distribution of $\varphi = (\varphi_1, \varphi_2')'$, and the test statistic in (9) under the linear constraint of (10) have asymptotic χ^2 distribution with degree of freedom *M*.

In this moment, to derive the *LM* test statistics in the null hypothesis $H_0\!: \varphi_2 = 0$, we need one more assumption that $\theta = (b', \varphi')'$ to simplify all the notations. Then the conditional log-likelihood function in the sample size n is the following,

(11) $$\begin{aligned} L(\theta) &= \frac{1}{n}\sum_{t=1}^{n} l_t(\theta) \\ l_t(\theta) &= -\frac{1}{2}\ln h_t - \frac{1}{2}\varepsilon_t^2 h_t^{-1}. \end{aligned}$$

Under this circumstance, the information matrix is block diagonal, that is, the parameters in the mean equation and variance equation form divided block and can be separated in the next process. Therefore, we consider the parameters in the variance equation directly. Under $H_0\!: \varphi_2 = 0$, the parameter estimated value can be plugged into the first derivative of likelihood function with respect to variance parameters and reach

(12) $$\frac{\partial l_t}{\partial \varphi} = \frac{1}{2}\tilde{h}_t^{-1}(\frac{\partial \tilde{h}_t}{\partial \varphi})(\frac{\tilde{\varepsilon}_t^2}{\tilde{h}_t} - 1),$$

where, $\frac{\partial \tilde{h}_t}{\partial \phi} = t^j \tilde{z}_t + \sum_{i=1}^{p} \tilde{\beta}_i \frac{\partial \tilde{h}_{t-1}}{\partial \phi}$ and $\tilde{h}_t = z_t' \tilde{w}_1$. Here "~" represents a consistent estimator under the null hypothesis. The second derivatives get

(13) $$\frac{\partial^2 l_t}{\partial \varphi \partial \varphi'} = (\frac{\varepsilon_t^2}{h_t} - 1)\frac{\partial}{\partial \varphi'}(\frac{1}{2}h_t^{-1}\frac{\partial h_t}{\partial \varphi}) - \frac{1}{2}h_t^{-2}\frac{\partial h_t}{\partial \varphi}\frac{\partial h_t}{\partial \varphi'}\frac{\varepsilon_t^2}{h_t}.$$

Now, the hull hypothesis of fixed parameters can be written as $H_0 : R\varphi = [\tilde{0}, \mathrm{I}_M][\varphi_1', \varphi_2']' = 0$, and the alternative hypothesis is $H_1 : \varphi_2 \neq 0$. The *LM* test statistics only need to calculate the value under the null hypothesis. Following Aitheson and Silvey (1959), the *LM* test statistics under the constraint can be

(14) $$LM = n(\frac{\partial L}{\partial \varphi})' \tilde{A}_n^{-1} R'[R\tilde{C}_n R']^{-1} R\tilde{A}_n^{-1}(\frac{\partial L}{\partial \varphi}),$$

where $\tilde{A}_n$ is a consistent estimator of $A_0 = \lim_{n\to\infty} E(\frac{\partial^2 L}{\partial \varphi \partial \varphi'})$ and $\tilde{B}_n$ is a consistent estimator of $B_0 = \lim_{n\to\infty} Var(n^{1/2}\frac{\partial L}{\partial \varphi})$, and $\tilde{C}_n$ is the consistent estimator of $A_0^{-1} B_0 A_0^{-1}$. It can also be written as the form of product of Lagrange multipliers.

(15) $$LM = n\tilde{\lambda}_n' [R\tilde{A}_n^{-1} R'][R\tilde{C}_n R']^{-1}[R\tilde{A}_n^{-1} R']\tilde{\lambda}_n,$$

wher $\tilde{\lambda}_n$ is the Lagrange multipliers. And finally it has asymptotic χ^2 distribution.

III. Specification and estimation of a STAR-GARCH model

The nonlinear STAR-GARCH model is the most general parameterization considered in this paper. It is nevertheless possible that the time series under consideration may be adequately characterized by a sub-model of the general STAR-GARCH one For instance, the conditional mean may be linear or the conditional variance constant Furthermore, even if we eventually select a general model there are still choices to be made that have to be based on the data The delay d in the conditional mean usually has to specified from the data as well as the maximum lag length and the type of the transition function (n 1 or 2) We also have to select the lag length and the type of transition function (n~ 1 or 2) in the STGARCH specification of the conditional variance All this requires a coherent specification strategy such as, for example, in Box and Jenkins (1970), Li and Li (1996), Tsay (1989, 1998) and Terasvirta (1994)

Our general rule is to specify the conditional mean first, followed by the conditional variance The reason is that we may estimate the parameters of the conditional mean consistently even if the conditional variance is not specified, that is, even if it is assumed constant On the other hand, it is not possible to estimate the parameters of the conditional variance consistently if the conditional mean is mis-specified The specification of the STAR-GARCH model consists of the following stages:

1 Test linearity of conditional mean and, if rejected, choose d and n

2 Estimate the parameters of the conditional mean assuming that the conditional variance remains constant and test the null hypothesis of no linear ARCH against ARCH of a given order If the hypothesis of no ARCH is rejected, tentatively assume that the conditional variance follows a low order standard GARCH process

3 Estimate the parameters of the STAR-GARCH model and test the adequacy of the STAR (conditional mean) and the GARCH (conditional variance) specifications by various misspecification tests If rejected, specify a STAR-GARCH model

4 Estimate the parameters of the STAR-GARCH model and test the adequacy of both the conditional mean and the conditional variance of that specification by appropriate misspecification tests s If the model passes the tests tentatively accept it In the opposite case try another specification search or choose another family of models

It should be noted that by following the above modeling scheme we proceed from restricted models to more general ones This may be simpler than to start from the most general model and gradually reduce its size, but there is also a statistical rationale behind this choice of direction If the conditional mean is linear then no STAR specification is identified As for the conditional variance, the same is true for any STGARCH specification if the linear GARCH already is a valid parameterization The lack of identification leads to lack of consistency in the parameter estimation, which, in turn, is likely to create numerical difficulties in estimation See Hansen (1996) for a recent discussion of this problem To avoid estimating unidentified models we have to proceed from specific to general In the following we consider the specification stages in detail

IV. Estimation of the STAR-GARCH model

If the conditional variance is not constant the next step is to fit a STAR-GARCH model to the data The usual way of obtaining the estimates for the conditional mean and the conditional variance when the latter has a standard GARCH representation is to make use of the block diagonality of the information matrix The conditional mean model is estimated first This is followed by the estimation of the conditional variance model using the residuals from estimating the conditional mean This procedure yields consistent estimates, but in this paper all parameters are ultimately estimated simultaneously One advantage with simultaneous estimation is that it may lead to more parsimonious models, at least if the series are not very long The two step estimation has a tendency to yield over parameterized models because some effects due to the nonconstant conditional variance may at first be captured by the estimated conditional mean The autoregressive parameters turning out to be redundant are eliminated during joint estimation by applying the previous bachuard elimination algorithm The estimation is carried out using analytical second derivatives which gives numerically reliable estimates for the in formation matrix This is needed at the evaluation stage when the estimated model is tested for misspecification

However, the two step estimation is useful for obtaining initial values for the joint estimation We proceed as follows First estimate the STAR model for the conditional mean and then estimate a GARCH(1,1) model for the residuals As a first order GARCH model has very often been found to be adequate in practice, it is only expanded if necessary The decision to do that is based on a misspecification test of the functional form which together with other evaluation procedures is discussed in Section 4 To enforce the conditional variance generated by any higher order GARCH model to be nonnegative, the constraints in Nelson and Cao (1992) for parameters of such models are imposed The validity of restrictions constraining linear combinations of estimated parameters is verified after the estimation 3.4. Specification and estimation of the STAR-GARCH model

The STAR-GARCH model has to be subjected to misspecification tests. At this stage we assume that the linear GARCH specification is rejected in favour of STAR-GARCH and proceed to discuss STAR-GARCH models. We have to consider specification, estimation and evaluation of these models and begin by specification

When estimating STAR-GARCH models it is not certain that the conditional variance is symmetric with respect to the error terms (most often it is not) and therefore the assumption of block diagonality of the information matrix may not hold This implies that the two stage estimation algorithm does not yield asymptotically efficient estimates We maintain our previous strategy and ultimately estimate the conditional mean and the conditional variance jointly.

As discussed above, the validity of the assumptions used in the estimation of parameters must be investigated once the parameters of the STAR-GARCH model (or a sub-model) has been estimated These assumptions include:

1 The errors and the squared (and standardized) errors of the model are not serially correlated

2 The parameters of the model are constant

3 The squared (and standardized) errors of the model are independent and identically distributed

These assumptions are testable Furthermore, it is useful to find out whether or not there are any nonlinearities left in the process after fitting a STAR-GARCH model to the series under consideration In this paper that possibility is investigated by testing the hypothesis of no additive nonlinearity against this type of nonlinearity. As for the three testable assumptions, the first two may be tested following Eiteheim and Terasvirta (1996). These authors have also suggested a test of no additive nonlinearity for the conditional mean We only have to generalize these tests to the case where the conditional variance follows a STAR-GARCH process As to the independence hypothesis, the BDS test, see Brodr, Dechert, Scheinkman and LeBaron (1996), is applicable if the number of observations is sufficiently large.

V. Conclusions

More general specifications of volatility are required to capture the shocks to the short-term interest rates. In this paper, we explore the implications of STAR-GARCH model is intended to help us characterize the behavior of high volatility economic time series. Many modelers of such series tend to ignore the first moment, but in this paper the first and the second moment are modeled jointly. A coherent modeling strategy is a key to doing that in a systematic way, and such a strategy is designed and applied to data here. An advantage of the proposed strategy is that the specification and misspecification tests we use only require standard asymptotic theory and are easy to perform.

The main goal is to evaluate the performance of different GARCH models in terms of their ability to characterize and predict out-of-sample market volatility. Such out-of-sample comparison is carried out by comparing the one-step-ahead forecasts for the conditional variance obtained from each one of the models analyzed. The proxy for the true volatility is given by the demeaned squared returns. Even though this can be a lousy proxy of the real volatility, we argue that the most recent literature on stock market volatility shows that the results obtained with the so called realized volatility are not strikingly different from ours. The performances of each volatility model are measured by a set of six different loss functions in order to allow for a reasonably general comparison. Another interesting result highlighted in the paper is the ability of STAR-GARCH models to predict rather correctly the direction of the future changes in volatility. This result may be due to the excessive persistence of the standard GARCH models.

References

Agresti, A. (1984). Analysts of Ordinal Categorial Data, Wiley, New York.

Bacon, D. W. and Watts, D. C. (1971). Estimating the transition between two intersecting straight lines, Biometrika, 58: 525534.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, 31: 307-327

Bdlerslw, T., Engle, R F. and Nelson, D. B. (1994). ARCH models, in R F. Engle and D. L. McFadden (eds), HandBook of Econometrtcs, Vol. 4, Elsevier, Amsterdam, pp. 29593038.

Box, C. and Jenkins, C. M. (1970). Time Series Analysts, Forecasting and Control, Holden Day, San Francisco.

Breusch, T. S. and Pagan, A. R (1980). The Lagrange mulitiplier test and its applications to model specification in emnometrics. Review of Economic Studies, 17: 239253.

Brock, W. A., Dechert, W., Scheinkman, J. and LeBaron, B. (1996). A test for in dependence based on the correlation dimension, Econometric Reviews 15: 197

Chan, K. S. and Tong, H. (1986). On estimating thresholds in autoregressive models, Journal of Time Series Analysts 7: 17b190.

Chappell, D. and Peel, D. A. (1998). A note on some properties of the ESTAR model, Economics Letters 60: 311-315.

Eitrheim, O and Terasvirta, T. (1996). Testing the adequacy of smooth transition autoregressive models, Journal of Econometrics, 74: 5975.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom in8athn. Econometrica 50: 987-1007

Escribano, A. and JordB, 0. (1996). Improved testing and specification of smooth transition regression models. Universidad Carlos III de Madrid, Working Pa

Chysels, E., Harvey A. C. and Renault, E. (1996). Stochastic volatility, in C. S. Maddala and C. R Rao (eds), HandBook of Statistics, Vol. 14, Elsevier, Amsterdam, pp. 119191.

Closten, L., Jagannathan, R. and Runkle, D. (1993). On the relation between expected value and the volatility of the nominal excess return on stocks, Journal of Finance, 48: InltldOL

Cranger, C. W. J. and Newbold, P (1986). Forecasting Economic Time Series, Academic Press, London and San Diego.

Cranger, C. W. J. and Terasvilta, T. (1993). Modeling Nonlinear Economic Relationships, Oxford University Press, Oxford.
Hagerud, C. E. (1997). A New Nonlinear CARCH Model, EFI, Stockholm School of Economics.
Hansen, B. E. (1996). Inference when a nuisance parameter is not identified under the null hypothesis, Econometrica, 64: 413430.
He, C. and Terasvirta, T. (1999a). Fourth moment structure of the CARCH(p, q) process, Econometric Theory (forthcoming).
He, C. and Terasvilta, T. (1999b). Properties of the autocorrelation function of squared observations for second order CARCH processes under two sets of parameter constraints, Journal of Time Series Analysis (forthcoming) .
LeBaron, B. (1997). A fast algorithm for the BDS statistic, Studies in Nonlinear Dynamics and Econometrics 2: 5359.
Lee, Y. N. and Li, W. K. (1998). On smooth transition double threshold models. Department of Statistics, University of Hong Kong, Research Report No. 198.
Li, C. W. and Li, W. K. (1996). On a double threshold autoregressive heteroscedastic time series model, Journal of Applied Econometrics, 11: 253274.
Li, W. K. and Lam, K. (1995). Modelling asymmetry in stock returns by a threshold autoregressive conditional heteroscedastic model, The Stahshcian 44: 333341.
Li, W. K. and Mak, T. K. (1994). On the squared residual autocorrelations in non linear time series with conditional heteroskedasticity, Journal of Time Series Analysts Is: 627-636.
Lin, C.F. and Terasvirta, T. (1994). Testing the constancy of regression parameters against continuous structural change, Journal of Econometrics 62: 211228.
Lundbergh, S. and Terasvirta, T. (1998). Evaluating CARCH models. Stockholm School of Economics, Working Paper Series in Economics and Finance, No. 292.
McLeod, A. I. and Li, W. K. (1983). Diagnostic checking ARMA time series models using squared residual autocorrelations, Journal of Time Series Analysts 1: 269-273.
Mittnik, S. and Rachev, S. T. (1993). Modeling asset returns with alternative stable distributions, Econometric Reviews 12: 261-330.
Nelson, D. B. and Cao, C. 4. (1992). Inequality constraints in the univariate GARCH model, Journal of Business and Economic Statistics, 10: 22~235.
pagan, A. (1996). The econometrics of financial markets, Journal of Empirical Finance 3: 15102.
Palm, F. (1996). CARCH models of volatility, in C. Maddala and C. Rao (eds), HandBook of Stattsttcs, Vol. 14, Elsevier. Amsterdam, pp 220-240.
Sentana, E. (1995). 4uadratic ARCH models, Review of Economic Studies 62: 639-661.
Skalin, J. and Terasvirta, T. (1999). Another look at Swedish business cycles, 1861 1988, Journal of Applied Econometrics (forthcoming) .
Taylor, S. (1986). Modeling Financial Time Series, Wiley, Chichester.
Terasvilta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models, Journal of the American Statistical Association ,89: 208 21R.

Tong, H. (1990). Nonlinear Time Series. A Dynamical System Approach, Oxford University Press, Oxford.
Tsay, R. S. (1989). Testing and modeling threshold autoregressive processes, Journal of the American Statistical Association 84: 231240.
Tsay, R S. (1998). Testing and modeling multivariate threshold models, Journal of the American Statistical Association 93: 11881202.
Wong, C. S. and Li, W. K. (1997). Testing for threshold autoregression with conditional heteroscedasticity. Biometrika 84: 407-418.
Zakdlan, J.M. (1994). Threshold heteroskedastic models, Journal of Economic Dynamics and Control 18: 931955.