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International Migration and Economic Growth: A Source Country Perspective

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ABSTRACT

This study analyzes the impact of international migration on economic growth of a source country in a stochastic setting. The model accounts for endogenous fertility decisions, and distinguishes between public and private schooling systems. We find that economic growth crucially depends on the international migration since the migration possibility will affect fertility decisions and school expenditures. Relaxation of restrictions on the emigration of high-skilled workers will damage the economic growth of a source country in the long run although a 'brain gain' may happen in the short run. Furthermore, the growth rate of a source country under a private education regime will be more sensitive to the probability of migration than a country under a public education regime.

Keywords: Migration; Brain drain; Economic growth.

JEL Classification: F22, J24, O15.

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1. **INTRODUCTION**

There has been considerable recent debate on the pros and cons of international migration from developing countries. The traditional view of openness to migration in a developing country demonstrates that such openness would induce the emigration of high-skilled workers and create a 'brain drain' problem.[1] Miyagiwa (1991) developed a theoretical model with scale economies in advanced education to analyze human capital formation for both host and source countries, and concluded that a 'brain drain' will impact upon the availability of intermediate-skilled workers in the source country. Conversely, Stark, et al. (1998) and Stark and Wang (2002) argued that migration raises the return on human capital which will in turn raise the average level of human capital in the source country.

Following the prior literature, we investigate the impact of migration on economic growth through the role of human capital in this paper. However, there are three features to distinguish this paper from previous works. First, it is well-known that fertility and education are interdependent decisions for parents. Becker et al. (1990) developed a model to study how the joint decisions of fertility and education affect economic growth. de la Croix and Doepke (2003) explored the linkage between growth and inequality when differential fertility matters. However, studies in migration tend to assume constant population and do not take fertility decision into account. Rodriguez (1975) and Mountford (1997) presented a dynamic model to study the issues of migration and economic growth; however, population was taken as an exogenous variable and was assumed to grow at a constant rate. In this paper, we allow parents to make fertility and education decisions and to have an opportunity to migrate to a foreign country. Internalizing fertility decisions will induce a trade-off between quality and quantity for parents when the possibility of migration changes. The quality-quantity trade-off of children will then have an effect on economic growth because it affects children's human capital accumulation. Furthermore, when considering an economy with heterogeneous agents, fertility matters since it will affect the structure of the labor force.

Second, a stochastic model of migration is developed. The uncertainty of migration was considered by Beine et al. (2001) to differentiate an ex ante 'brain effect' and an ex post 'brain effect'. We adopt the stochastic model developed by Kalemli-Ozcan (2003) to stress the random property of migration and to internalize fertility decisions within the model. For parents, the uncertainty of migration will induce the need of insurance against failed migration of children. Hence, there will be a 'precautionary demand' of children for parents.

Third, the previous theoretical literature of migration only focuses on the accumulation of human capital under a private education regime. As pointed out by Glomm (1997) that when considering secondary schools, for most developing countries, the public school enrollment rate is higher than the private school enrollment rate. Under a private education regime, parents decide the level of educational investment that they will make for their children and the school expenditure is heterogeneous.[2] Under a public education regime, public schools are financed by tax revenue and adults vote for the tax rate. Hence, public school expenditure is provided by the government and is homogenous. Economic performance will be different under different school regimes because the school expenditure will affect the accumulation of human capital. Therefore, a theoretical approach is required in order to study the impacts of migration on human capital accumulation under these two different education systems. Glomm and Ravikumar (1992) found that private schooling generates a higher growth rate, whilst income inequality is lower under public schooling. de la Croix and Doepke (2004) incorporated education and fertility decisions and compared the implications for economic performance within private and public schooling. In this paper, we study the impact of international migration on economic growth under two different education regimes when a source country is open to migration, arguing that the type of education regime matters.

We first analyze an economy with homogenous agents, then extend the model to an economy with heterogeneous agents when there is a change of migration possibility. The impacts of an increase in the probability of migration on fertility and educational expenditure depend on the wage ratio of the home country to a foreign country, parents' preference and the migration probability. Assuming that high- and low-skilled workers have different probabilities of migration, we show that if the wage ratio of the home country to a foreign country is large, or if high-skilled parents strongly prefer their children to emigrate to a foreign country, or if the probability of emigration for high-skilled workers is high enough, high-skilled parents will have fewer children and a greater level of educational expenditure per child under a private education regime. Under a public education regime, high-skilled parents will similarly prefer to have fewer children. Hence, parents will have more time for work because they spend less time to raise children and their earnings will increase. The educational expenditure per student in public schools will increase due to the higher tax base. Similar results can be also acquired for low-skilled parents.

Allowing more high-skilled workers to emigrate will damage the economic growth of a source country in the long run although a ‘brain gain’ may happen in the short run. But relaxing migration restrictions on low-skilled workers will increase or decrease the economic growth, depending on the economic conditions. In a comparison of the implications of migration under private and public schooling, we find that fertility is more sensitive to the possibility of migration when education is not free. Hence, the per capita income growth rate will be more sensitive to the probability of migration under private schooling than under public schooling.

The remainder of this paper is organized as follows. In the next section we describe the setting of the model, introducing a stochastic model of migration and analyzing economic performance under private and public schooling. This is followed by an examination of an economy with heterogeneous agents. The numerical experiments are given in the penultimate section, followed by the conclusions drawn from this study in the final section.

2. THE MODEL

We consider an infinite-horizon, discrete time overlapping generations model where agents with identical preferences live for two periods. Each period covers approximately 30 years, corresponding to childhood (young agents) and adulthood (old agents).

We assume that adults can migrate to a foreign country (country B) with probability $p \in (0,1)$ or stay in the home country (country A) with probability $(1-p)$. Let w_A and w_B represent the real wage per unit of human capital in country A and B, respectively. In order to reflect the motivation for migration, we assume that w_B is higher than w_A. Adult earnings are equal to their level of human capital, h_t, multiplied by the real wage per unit of human capital of the country in which they live ($w_j h_t$, $j = A, B$).

Education in the source country could be under a private regime (denoted by r) or a public regime (denoted by u). Let i represent the school type. Individuals born in period

$t-1$ need to decide their adult consumption c_{it} and the optimum number of children n_{it}. A proportion of these children, pn_{it}, will migrate to country B and earn $w_B h_{it+1}$, where h_{it+1} is the human capital of children educated under a certain type of education system. The remainder, $(1-p)n_{it}$ will stay in country A and earn $w_A h_{it+1}$. Define $N_{it} = pn_{it}$ to represent the mean number of migrants; thus we define the utility function as:

$$u_{it} = \log(c_{it}) + \beta \log(N_{it} w_B h_{it+1} + a(n_{it} - N_{it}) w_A h_{it+1}), \ i = r, u . \quad (1)$$

The parameter β >0 reflects the degree of altruism amongst parents. Agents care about their adult's consumption, the expectation of total income earned by their children in a foreign country and the total income earned by their children staying in the home country. The parameter a measures how much income earned by children in a foreign country that will provide the same utility as one unit of income earned by children in the home country.T[3] Equation (1) can be rewritten as

$$u_{it} = \log(c_{it}) + \beta(\log w + \log n_{it} + \log h_{it+1}), \quad (1')$$

where $w = pw_B + a(1-p)w_A$.

Suppose that e_{it} represents school expenditure. The human capital accumulation function depends on both school expenditure e_{it}, and parental human capital h_{it}, and is given by:

$$h_{it+1} = \lambda e_{it}^{\gamma} h_{it}^{1-\gamma}, \quad i = r, u . \quad (2)$$

where λ is a positive constant and $\gamma \in (0,1)$. The parameters γ and $1-\gamma$ represent the elasticity of human capital amongst children in terms of their respective school expenditure and parental human capital.

A Private Education Regime

Each adult is endowed with one unit of time which they need to allocate between working and raising children. We assume that each child consumes a fraction ($\phi \in (0,1)$) of his/her parent's unit of time. Hence, the budget constraint for an adult staying in country A is:

$$c_{rt} + n_{rt} e_{rt} = (1-\phi n_{rt}) w_A h_{rt} . \quad (3)$$

Adults need to make decisions on fertility, education and have a chance to migrate to country B. If they stay in country A, they will maximize Equation (1') subject to Equations (2) and (3) under private schooling. The optimal choices of n_{rt} and e_{rt} are:

$$n_{rt} = \frac{\beta(1-\gamma)}{\phi(1+\beta)}, \tag{4}$$

$$e_{rt} = \frac{\gamma\phi}{1-\gamma} w_A h_{rt}. \tag{5}$$

Given the parameter values, n_{rt} is constant whereas e_{rt} is a linear function of parental human capital. Both n_{rt} and e_{rt} are independent of p. Hence, without considering the random property of migration, changes in p will not affect n_{rt} and e_{rt}.

A Public Education Regime

Under a public education regime, education is provided free. We assume that adults need to pay income tax and we use τ_t to represent the tax rate. The government uses tax revenue to support public schools and runs a balanced budget. Let n_{ut} and e_{ut} represent the respective fertility and school expenditure under a public education regime. School expenditure (e_{ut}) under a public education regime is:

$$e_{ut} = \tau_t (1-\phi n_{ut}) w_A H_{ut}, \tag{6}$$

where H_{ut} is the average human capital under a public education regime. Therefore, the budget constraint for adults becomes:

$$c_{ut} = (1-\tau_t)(1-\phi n_{ut}) w_A h_{ut}. \tag{7}$$

Agents who stay in country A will maximize Equation (1') subject to Equations (2), (6) and (7) by choosing fertility and tax rate. Since the school expenditure is determined by the public policy, parents do not need to decide the educational investments for their children. The optimal choices of fertility (n_{ut}) is:

$$n_{ut} = \frac{\beta}{\phi(1+\beta)}. \tag{8}$$

Substituting Equation (8) into Equation (6), we can get the public school expenditure as

$$e_{ut} = \frac{1}{1+\beta}\tau_t w_A H_{ut} . \qquad (6')$$

Using Equations (6') and (8), the indirect utility function under public schooling is

$$\log(\frac{1-\tau_t}{1+\beta} w_A h_{ut}) + \beta[\log w + \log\frac{\beta}{\phi(1+\beta)} + \log(\lambda(\frac{1}{1+\beta}\tau_t w_A H_{ut})^{\gamma} h_{ut}^{1-\gamma})] .$$

Hence, the optimal choice of the tax rate is

$$\tau_t = \frac{\beta\gamma}{1+\beta\gamma} = \tau \in (0,1) . \qquad (9)$$

Equations (8) and (9) show that both the tax rate and fertility are constant. We can then derive, from Equation (6), that school expenditure is a fraction of average human capital. Proposition 1 summarizes the impact of p on fertility and school expenditure under private and public schooling.

Proposition 1. Without considering the random nature of migration, any variation in the probability of migration will not affect fertility and school expenditure per student under both education regimes. Furthermore, fertility will be higher when education is provided free.

In order to compare fertility under the two different education systems, note that:

$$n_{ut} = \frac{\beta}{\phi(1+\beta)} > \frac{\beta(1-\gamma)}{\phi(1+\beta)} = n_{rt} .$$

Hence, fertility is higher under a public education regime than under a private education regime because when education is provided free, the cost of having children is lower. Let L_t represent the total population and π_t represent the population growth rate, Proposition 2 reflects the implications of p on population growth.

Proposition 2. Without considering the random nature of migration, an increase in the probability of migration will reduce the population growth rate under both education regimes.

Proof: The population growth rate is:

$$\pi_{it}(p) = \frac{L_{it+1}}{L_{it}} - 1 = (1-p)n_{it} - 1, \;\; i = r, u \,.$$

Without incorporating the random nature of migration, fertility is positively constant under both education regimes. Hence:

$$\pi_{it}'(p) = -n_{it} < 0 \,, \;\; i = r, u \,.$$

QED.

A 'brain drain' problem occurs if changes in the probability of migration deplete the accumulation of human capital. On the other hand, changes in the probability of emigration will cause a 'brain gain' if they raise the accumulation of human capital. Propositions 1 and 2 demonstrate that an increase in the probability of migration will reduce only the population growth rate, and will not affect fertility or school expenditure. This implies that an increase in p will neither cause a 'brain drain' nor a 'brain gain'. Hence, we need to adopt a model to incorporate the random nature of migration.

The Stochastic Model

In order to embody the random feature of adult migration, a stochastic model of migration is constructed based on Kalemli-Ozcan (2003).[4] If we assume that the number of migrants, N_{it} , is a random variable drawn from a binomial distribution, then the expected utility for agents can be written as:

$$\sum_{N_{it}=0}^{n_{it}} \{\log c_{it} + \beta \log(N_{it} w_B h_{it+1} + a(n_{it} - N_{it}) w_A h_{it+1})\} \binom{n_{it}}{N_{it}} p^{N_{it}} (1-p)^{n_{it}-N_{it}} \;. \qquad (10)$$

Appendix 1 shows that using Taylor series expansions to approximate the utility function around the mean, the household maximization problem can be written as:

$$\max_{e_{it} n_{it}} \{\log((1-\phi n_{it}) w_A h_{it} - n_{it} e_{it}) + \beta(\log w + \log n_{it} + \log h_{it+1} - \frac{p(1-p)d^2}{2w^2 n_{it}})\}$$

where $d = w_B - a w_A$.

When the random feature of migration is present, agents maximize Equation (10), subject to Equations (2) and (3), by choosing fertility and educational investment under a private education regime. The first-order conditions of the maximization problem are:

$$\frac{\beta\gamma+\phi n_{rt}}{1-\phi n_{rt}}=\beta[1+\frac{p(1-p)d^2}{2w^2 n_{rt}}], \tag{11}$$

$$e_{rt}=\frac{\beta\gamma(1-\phi n_{rt})w_A h_{rt}}{(1+\beta\gamma)n_{rt}}. \tag{12}$$

Using Equations (11) and (12) to substitute fertility and educational investment into Equation (2), we can derive the law of motion of human capital under private schooling.

Under a public education regime, agents maximize Equation (10), subject to Equations (2), (6) and (7), by choosing fertility and tax rate. Optimization with respect to n_{ut} implies that:

$$\frac{\phi}{1-\phi n_{ut}}=\beta[\frac{1}{n_{ut}}+\frac{p(1-p)d^2}{2w^2 n_{ut}^2}]. \tag{13}$$

Although we are not able to solve for the analytical solution of n_{ut} from equation (13), the tax rate is the same as in Equation (9) because of the property of the logarithm utility function.

Substituting the tax rate and fertility in Equations (9) and (13) into Equation (2) gives us the law of motion of human capital under public schooling; from the human capital accumulation functions under private and public education regimes, we can derive the constant growth rate of average human capital under both education regimes.

Proposition 3. Given the probability of migration, the growth rates of average human capital (g_i^H, $i=r,u$) are constant under both education regimes when agents are homogeneous.

Proof: See Appendix 2.

Implications of Migration

Equations (11) and (13) show that with the random property of migration, fertility depends on the probability of migration under both education systems. This is because the random nature of migration will cause a 'precautionary demand' of children for parents.[5] Furthermore, combining Equations (11) and (12) shows that educational investment under a private education regime also depends on the probability of migration.

Under a public education regime, a change in the probability of migration will affect both fertility and the time spent by parents on work. This will in turn change the tax base and affect public school expenditure per student.

Proposition 4. With the random nature of migration, an increase in the probability of migration will lead to a trade-off between quality and quantity. If $\frac{w_B}{w_A} > \frac{a(1-p)}{p}$, this will reduce fertility and increase the level of school expenditure per student under both education regimes. The situation will be reversed if $\frac{w_B}{w_A} < \frac{a(1-p)}{p}$.

Proof: See Appendix 3.

We define p^* such that $\frac{a(1-p^*)}{p^*} = \frac{w_B}{w_A}$. First notice that a decrease in a or an increase in p will lower the value of $\frac{a(1-p)}{p}$. Hence, if the ratio of the domestic wage rate to the foreign wage rate is sufficiently large, or if parents strongly favor their children migrating to a foreign country (a is sufficiently small) or if the probability of migration is sufficiently high, a rise in p will lower fertility because it decreases the need for an insurance against failed migration and it reduces the 'precautionary demand' of children. Moreover, it also implies that more children will be able to migrate to a foreign country to earn higher wage rates per unit of human capital, and thus, there will be an increase in the return on human capital. Under private schooling, parents will prefer to have fewer children, but they will spend more on each child's education. Under public schooling, with an increase in p, fertility will decline; having fewer children means that parents can spend more time at work and this will increase the tax revenue/school expenditure per student. Conversely, if the ratio of the domestic wage rate to the foreign wage rate is sufficiently small, or if a is sufficiently large or if p is sufficiently small, an increase in p will augment fertility because it increases the expected income earned by children. Thus, school expenditure per student will be lower. The influence of an increase of p under two education regimes are summarized in Table 1.

Table 1 Impacts of an increase of p under private and public schooling

	Private/ Public schooling		
	n_{it}	e_{it}	h_{it+1}
$p > p^*$	↓	↑	↑
$p < p^*$	↑	↓	↓

As regards population growth, if $p > p^*$, an increase in p will not only reduce fertility, but will also mean that more adults will migrate to a foreign country. However, if $p < p^*$, fertility will increase with an increase in p and the impact of p on population growth is uncertain. Thus:

Proposition 5. With the random property of migration, if $p > p^*$, an increase in the probability of migration will reduce population growth rate under both education regimes. However, if $p < p^*$, the impact of changes in p on population growth is uncertain under both education regimes.

Proof: The population growth rate is:

$$\pi_{it}(p) = \frac{L_{it+1}}{L_{it}} - 1 = (1-p)n_{it} - 1, \quad i = r, u \ .$$

Therefore, $\pi_{it}'(p) = -n_{it}(p) + (1-p)n_{it}'(p)$, $i = r, u$.

From Proposition 4, if $p > p^*$, then $n_{it}'(p) < 0$ and $\pi_{it}'(p) < 0$ for $i = r, u$. However, if $p < p^*$, then $n_{it}'(p) > 0$ and the sign of $\pi_{it}'(p)$ is uncertain under both education regimes.

QED.

In order to study the 'brain drain' or 'brain gain' problem, we analyze the impact of p on human capital accumulation.

Proposition 6. If $p > p*$, with the random feature of migration and homogeneous agents, an increase in the probability of migration will create a 'brain gain' under both education regimes. However, if $p < p*$, an increase in the probability of migration will cause a 'brain drain' under both education regimes.

Proof: Proposition 4 shows that $e_{it}'(p) \gtrless 0$ if $p \gtrless p*$, for $i = r, u$, then:

$$\frac{\partial H_{it+1}}{\partial p} = \frac{\partial h_{it+1}}{\partial p} = \frac{\partial h_{it+1}}{\partial e_{it}} e_{it}'(p) \begin{matrix} > 0 \\ < 0 \end{matrix} \text{ if } p \gtrless p*.$$

QED.

Proposition 6 demonstrates that as the probability of migration increases, both 'brain gain' and 'brain drain' could happen. When the wage ratio is quite large between the home country and a foreign country, or when parents favor children to migrate or when the probability of migration is high enough, an increase in p will create a 'brain gain' and reduce the population growth. On the other hand, if the wage ratio of the home country to a foreign country is small, or if parents favor children staying in the home country, or if the probability of migration is low, an increase in p will create a 'brain drain' and its impact on the population growth is uncertain.

3. HETEROGENEOUS AGENTS

In the previous section, we assumed that everyone in the economy was homogeneous with the same probability of migration. However, it is well known that high-skilled workers are more welcome within a host country than low-skilled workers.[6] Hence, in this section, we extend our model to an economy with heterogeneous agents.

Without loss of generality, we assume that there are two types of workers: workers with low human capital, h_{it}^L (these are referred to as low-skilled workers) and workers with high human capital, h_{it}^H (these are referred to as high-skilled workers), $i = r, u$. The respective ratios of low-skilled workers and high-skilled workers to the total adult population in period t are θ_{it}^L and θ_{it}^H. Hence, $\theta_{it}^L + \theta_{it}^H = 1$. The respective probabilities of migration to country B for low- and high-skilled workers are p^L and p^H. We assume that $p^L < p^H$ in order to reflect the fact that it is easier for high-skilled workers to migrate to a foreign country.[7] Let n_{it}^L and n_{it}^H represent respective fertility for low- and high-skilled workers. The respective ratios of low- and high-skilled workers to the total population in period $t+1$ become:

$$\theta_{it+1}^L = \frac{(1-p^L)\theta_{it}^L n_{it}^L}{(1-p^L)\theta_{it}^L n_{it}^L + (1-p^H)\theta_{it}^H n_{it}^H} \text{ and } \theta_{it+1}^H = \frac{(1-p^H)\theta_{it}^H n_{it}^H}{(1-p^L)\theta_{it}^L n_{it}^L + (1-p^H)\theta_{it}^H n_{it}^H}.$$

Therefore, average human capital in period $t+1$ becomes $H_{t+1} = \theta_{it+1}^L h_{t+1}^L + \theta_{it+1}^H h_{t+1}^H$.

To discuss the impacts of the probability of migration, we need to consider three cases: (1) $p^H > p^L > p*$, (2) $p^H > p* > p^L$ and (3) $p* > p^H > p^L$. Using parameter values calibrated in the following section, Figures 1 and 2 describe the decisions made by low-skilled parents when facing the migration probability of p^L. Figure 1 presents decisions on fertility and educational investments under private schooling while Figure 2 shows fertility decision under public schooling.

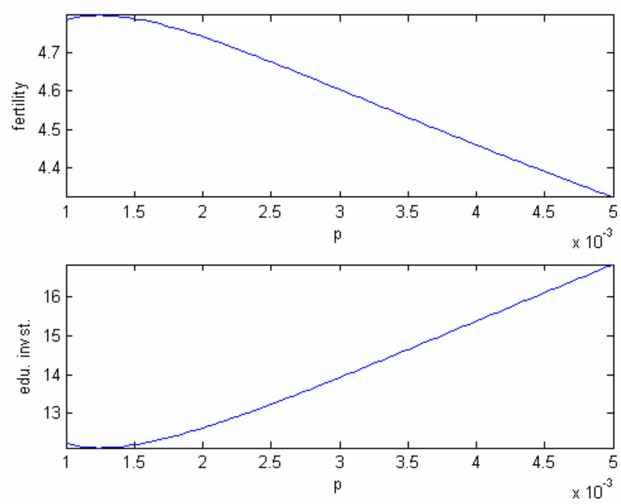

Figure 1 Impacts of p^L on fertility and educational investment for low-skilled workers under private schooling

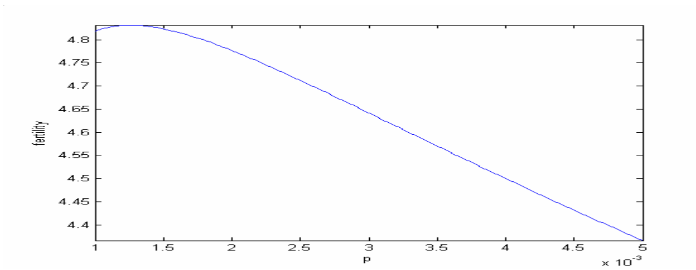

Figure 2 Impacts of p^L *on fertility for low-skilled workers under public schooling*

We use e_{rt}^L and e_{rt}^H to represent respective educational investment for low- and high-skilled workers under a private education regime. Equations (11) and (12) show that the educational investment under private schooling depends on the migration probability and parental human capital. From Proposition 4, we know that if $p^H > p^L > p*$, high-skilled parents will have fewer children than low-skilled workers and spend more on each child's education ($n_{rt}^H < n_{rt}^L$, $e_{rt}^H > e_{rt}^L$). Hence, according to Equation (2), the children of high-skilled (low-skilled) parents will be high-skilled (low-skilled) workers in the next period because of high (low) parental human capital and high (low) educational expenditure ($h_{rt+1}^L < h_{rt+1}^H$). Under a public education regime, the amount of educational expenditure per student is $e_{ut} = \theta_{ut}^L \tau (1-\phi n_{ut}^L) w_A h_{ut}^L + \theta_{ut}^H \tau (1-\phi n_{ut}^H) w_A h_{ut}^H$ and is the same for every student. Hence, $h_{ut+1}^L < h_{ut+1}^H$ is due to lower parental human capital for the children of low-skilled parents.[8]

However, in the cases (2) and (3), e_{rt}^L can be larger than e_{rt}^H if h_{rt}^H is not sufficiently larger than h_{rt}^L. Hence, it is possible that h_{rt+1}^L will be higher than h_{rt+1}^H under private schooling.[9] But under public schooling, the school expenditure is provided by the government, $h_{ut+1}^L < h_{ut+1}^H$ because parental human capital is the crucial determinant of children's human capital. Before carrying out the computational procedure, Propositions 7 and 8 consider the impacts of migration probability for high-skilled workers and for low-skilled workers on the economic growth, respectively.

Proposition 7. Under the condition that $p^H > p*$, then an increase in the probability of migration for high-skilled workers will create a 'brain gain' if an increase in p^H does not cause a large increase in θ^L_{it+1}. Conversely, if an increase in p^H induces a large increase in θ^L_{it+1}, then an increase in high-skilled emigrants will create a 'brain drain'.

Proof. See Appendix 4.

Under a private education regime, with an increase in p^H, high-skilled parents will decide to have fewer children and increase their educational investment for each child; hence, the human capital accumulation for children of high-skilled workers will increase. If an increase in p^H will not cause a large increase in θ^L_{rt+1}, an increase in h^H_{rt+1} will compensate the loss of h^H_{rt+1} due to the emigration of some high-skilled workers and the economy will end up with a 'brain gain'. Under a public education regime, decisions by high-skilled workers to have fewer children when there is an increase in p^H will contribute to an increase in public school expenditure; thus, the human capital accumulation for all children will increase. If an increase in p^H does not induce a large increase in θ^L_{ut+1}, an increase in h^H_{ut+1} and h^L_{ut+1} will compensate the loss of h^H_{ut+1} due to emigration of some high-skilled workers and the economy will end up with a 'brain gain'. Conversely, if an increase in p^H induces a large increase in θ^L_{it+1}, the emigration of high-skilled workers will create a 'brain drain', since the economy will be occupied by the low-skilled workers and the loss will exceed the gain.

Since when $p^L > p*$, an increase in p^L will increase the human capital accumulation of children belonging to low-skilled parents under both education regimes, it will cause a 'brain gain'. This result is demonstrated in Proposition 8.

Proposition 8. Under the condition that $p^L > p*$, then an increase in the probability of migration for low-skilled workers will create a 'brain gain'.

Proof. See Appendix 5.

In order to study and to quantify the influence of migration on economic performance both in the short run and in the long run, we simulate our model in the next section.

4. NUMERICAL EXPERIMENTS

Before proceeding with our computational work, we need to calibrate the parameters used in the model. We begin by calibrating the parameter values of the human capital accumulation function. The results of the empirical study by Johnson and Stafford (1973) showed that income elasticity for education expenditure was 0.198, whilst the figure used by Fernandez and Rogerson (1997), based on the estimates of Card and Kreuger (1992), was 0.2. Since the figures provided by these studies are virtually identical, we also set γ as being equal to 0.2.[10] The study of Haveman and Wolfe (1995) demonstrated that parents spend around 15 per cent of their time raising children; hence, we also assign a value of 0.15 to ϕ.

We calibrate the remaining parameter values according to 1985 data, choosing the United States as the foreign country and the Philippines as the source country.[11] Respective per capita GDP levels for 1985 for the United States and the Philippines were $17,267 and $2165.1.[12] Normalizing w_A as 100, a value of 797.5 is assigned to w_B.[13]

In order to consider the case of heterogeneous agents, we need to calibrate the initial distribution of human capital for the source country- h_1^L, h_1^H, θ_1^L and θ_1^H. In 1985, the ratio of the share of income in the Philippines between the top and bottom quintiles was 10.019; hence, we assume an extreme case and calibrate h_1^H to be 10.019 times h_1^L.[14] Setting h_1^L to 87 and h_1^H to 10.019* h_1^L = 871.653 allows us to roughly match the per capita GDP level of the Philippines.[15] The values of θ_1^L and θ_1^H are calibrated to match the Gini coefficient of 46.08 per cent in the first period.[16] This gives $\theta_1^L = 87.73\%$ and $\theta_1^H = 12.27\%$.

Given that, for the Philippines, the enrollment rate in 1985 (as a proportion of the total enrollment in secondary schools) was higher for public schools than for private schools, we use an economy under a public education regime as our baseline model.[17] Since it will necessarily take many years for educational investment to contribute to economic growth, we calculate the average annual growth rate from 1985 to 1994 for the Philippines. This gives us an average annual growth rate in real GDP of 2.28 per cent.

Hence, λ is set at 2.27 so that the growth rate over ten periods under a public education regime will roughly match the average economic growth rate from 1985 to 1994.

Since only tax revenue is used for education in the model, the tax rate is calibrated according to public expenditure on education. Public spending on education accounted for 1.4 per cent of GNP in 1985.[18] Therefore, under our model setting for a public education regime, public spending on education is equal to 2.373 per cent of GNP.[19] Setting $\tau = 2.373\%$, the value of β is calibrated as 0.122.

We assume that low-skilled workers have only a very small chance to migrate to country B and set $p^L = 0.004$. The fertility rate in the Philippines (births per woman) was 4.4 in 1985.[20] The probability of migration for high-skilled workers is calibrated as 0.06 to match fertility under a public education regime. We consider a situation in which parents strongly favor their children to migrate to a foreign country and assign $a = 0.01$. All the parameter values calibrated above are referred to as baseline model parameter values, and given that our main purpose is to study the influence of the probability of migration, we will also test the sensitivity of p^L and p^H.

4.1 Results

Using the parameter values we just calibrated, $p*$ equals 0.00125. In the following, we consider the impacts of migration in three possible cases.

4.1.1 High probabilities of migration for high- and low-skilled workers

We start our analysis by considering the case (1). Note that our baseline model describes the case (1) since $p^H = 0.06 > p^L = 0.004 > p*$. When carrying out our computational process, we analyze the impacts of the probability of migration, first fixing p^L at 0.004 and examining the effects of two different measures of p^H (p^H = 0.06 and 0.1) under two different education regimes. Keeping p^H at 0.06, we then study the impacts of p^L (p^L = 0.004 and 0.01) under both education regimes. Table 2 presents the short-run (1st period) and long-run (5th period) impacts of migration on fertility, labor structure, the logarithm of per capita income and Gini coefficients under a private and a public education regime.

Table 2 Impacts of migration probability in case (1)

	Private Education Regime			Public Education Regime		
p^H	0.0600	0.1000	0.0600	0.0600	0.1000	0.0600
p^L	0.0040	0.0040	0.0100	0.0040	0.0040	0.0100
n^H	2.2185	1.8358	2.2185	2.2884	1.9118	2.2884
n^L	4.4600	4.4600	3.7947	4.5002	4.5002	3.8442
Fertility 1	4.1849	4.1830	3.6015	4.2288	4.1826	3.6533
θ_1^H (%)	6.1612	4.9448	7.2039	6.2900	5.0995	7.3260
$\log(Y_1)$	7.5303	7.6009	7.7255	7.5053	7.5758	7.6980
Gini 1(%)	40.3705	37.1861	41.3862	23.4929	20.2375	25.9869
Fertility 5	4.4449	4.4530	3.7743	4.4839	4.4924	3.8223
θ_5^H (%)	0.3179	0.0995	0.7315	0.3548	0.1665	0.8004
$\log(Y_5)$	8.5077	8.3612	9.2562	9.7632	9.7290	10.3328
Gini 5(%)	11.1454	4.9383	15.6300	0.3972	0.1310	0.8875

Note: Definitions of variables are: n^H - fertility of high-skilled parents; n^L - fertility of low-skilled parents; Fertility 1 and Fertility 5 – average fertility in the 1st and 5th period; θ_1^H and θ_5^H - the ratio of (high-skilled workers)/(labor force) in the 1st and 5th period; $\log(Y_1)$ and $\log(Y_5)$ - the logarithm of per capita income at the end of the 1st and 5th period; Gini 1 and Gini 5- Gini coefficient at the end of the 1st and 5th period.

To study the impacts of migration under different education regimes, we compare column 2 with column 5. The transitions of the logarithm of per capita income, the Gini coefficient and θ^H over 5 periods are presented in Figure 3. Our computational results show that under a public education regime, fertility is higher. Per capita income is lower under a public education regime in the short run. However, in the long run, per capita income is higher under a public education regime than under a private education regime due to the higher ratio of high-skilled workers to the labor force and the tax rate calibrated in the previous section; in addition, the Gini coefficient is lower under a public education regime than under a private education regime both in the short run and in the long run since under a public schooling, the school expenditure is the same for every student. Similar results can be also obtained if we compare column 3(4) with column 6(7).

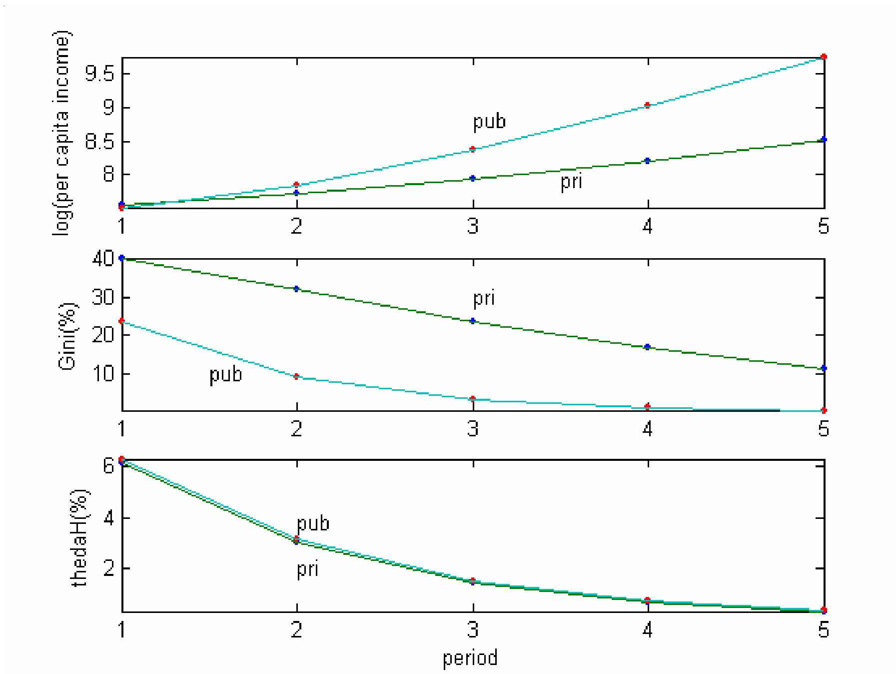

Figure 3 Comparison between private and public schooling

Columns 2, 3, 5 and 6 show the situation with an increase in p^H from 0.06 to 0.1 under the two different education regimes. For both education regimes, an increase in p^H will initially lower fertility and raise per capita income as a result of the trade-off between quality and quantity for high-skilled workers. However, with the migration of more high-skilled workers, there will be an increase in fertility as θ^H decreases and, in the long run, a higher p^H will result in higher fertility, but lower per capita income and income inequality. Hence, allowing higher probability of migration for high-skilled workers will cause a 'brain gain' the short run, but it will induce a 'brain drain' in the long run. Moreover, in the long run, the logarithm of per capita income will fall by 1.722 per cent under a private education regime and by 0.35 per cent under a public education regime; thus, in the long run, an increase in p^H will have a more detrimental effect on economic growth under a private education regime.

Columns 2, 4, 5 and 7 display the effects when p^L increases from 0.004 to 0.01 under the two different education regimes. For both education regimes, an increase in p^L will reduce fertility whilst also increasing both per capita income and inequality. In the 5th period, the logarithm of per capita income will increase by 8.798 per cent under a private education regime, and by 5.834 per cent under a public education regime. Since fertility is more sensitive to the probability of migration when education is not free, the growth rate of per capita income is more susceptible to the probability of migration under a private education regime than under a public education regime, which is consistent with the results shown in Table 2. Although an increase in p^L would stimulate economic growth under both education regimes, it would also cause high income inequality in the long run.

4.1.2 High probability of migration for high-skilled workers and low probability of migration for low-skilled workers

The difference between case (2) and case (1) is that p^L is lower than the critical value $p*$. The impacts of p^H are demonstrated in Proposition 7. However, an increase in p^L will affect the future average human capital in two ways. First, it increases fertility and lowers the educational expenditure for low-skilled parents. Second, higher p^L means that more low-skilled workers will migrate to country B in the next period.

The simulation results under both education regimes with low p^L are presented in Table 3. The two values of p^H (0.06 and 0.1) we consider are the same as in Table 2. The statistics in columns 2, 3, 5 and 6 illustrate that the influence of p^H under private and public schooling. The results are similar to those we obtain from Table 2. Columns 2, 4, 5 and 7 in Table 3 exhibit the effects when p^L increases from 0.0001 to 0.0005 under the two different education regimes. Note that these two values are all smaller than $p*$. It shows that under both education regimes, an increase in p^L will raise fertility (hence, lower θ_1^H and θ_5^H) and decrease per capita income both in the short run and in the long run.

In the long run, when p^L increases from 0.0001 to 0.0005, fertility will increase by 34.996 per cent and the logarithm of per capita income will decrease by 13.571 per cent under a private education regime while fertility increase by 33.992 per cent and per capita income will decrease by 9.297 per cent under a public education regime. Therefore, both fertility and per capita income are more sensitive to the probability of migration under a private education regime than under a public education regime.

Table 3 Impacts of migration probability in case (2)

	Private Education Regime			Public Education Regime		
p^H	0.0600	0.1000	0.0600	0.0600	0.100	0.0600
p^L	0.0001	0.0001	0.0005	0.0001	0.0001	0.0005
n^H	2.2185	1.8358	2.2185	2.2884	1.9118	2.2884
n^L	3.4084	3.4084	4.5844	3.4626	3.4626	4.6228
Fertility 1	3.2624	3.2154	4.2941	3.3185	3.2723	4.3364
θ_1^H (%)	7.8834	6.3499	5.9843	7.9948	6.4989	6.1132
$\log(Y_1)$	7.8436	7.9097	7.4946	7.8143	7.8801	7.4701
Gini 1(%)	41.9748	39.0599	40.2015	27.4517	24.0192	23.0380
Fertility 5	3.3855	3.3963	4.5703	3.4387	3.4495	4.6076
θ_5^H (%)	1.1855	0.3732	0.2723	1.2782	0.4222	0.3049
$\log(Y_5)$	9.6726	9.5159	8.3599	10.6349	10.5882	9.6462
Gini 5(%)	18.9496	8.9896	10.5036	1.4030	0.4719	0.3417

Note: Definitions of variables: see Table 2.

4.1.3 Low probabilities of migration for high- and low-skilled workers

In this case, both p^H and p^L are lower than $p*$. The impacts of p^H and p^L are presented in Table 4. Columns 2, 3, 5 and 6 illustrate the situation with an increase in p^H from 0.0007 to 0.001 while columns 2, 4, 5 and 7 show the effects when p^L increases from 0.0001 to 0.0005 under the two different education regimes. Note that increasing the probability of migration will raise fertility and decrease educational expenditure for high- (low-) skilled parents. Hence, high-skilled parents will have higher fertility than low-skilled parents. Increasing p^H (p^L) will cause a 'brain drain' under private and public schooling both in the short run and in the long run. The fertility and per capita income are also more volatile to the changes of the probability of migration under private schooling than under public schooling. When p^H goes up from 0.0007 to 0.001, fertility increases by 1.128 per cent and per capita income is lowered by 0.433 per cent under private schooling and fertility increases by 1.07 per cent and per capita income decreases by 0.275 per cent under public schooling. When p^L goes up from 0.0001 to 0.0005, fertility increases by 19.595 per cent and per capita income is reduced by 10.327 per cent under private schooling and fertility increases by 19.372 per cent and per capita income decreases by 8.15 per cent under public schooling.

Table 4 Impacts of migration probability in case (3)

	Private Education Regime			Public Education Regime		
p^H	0.0007	0.0010	0.0007	0.0007	0.0010	0.0007
p^L	0.0001	0.0001	0.0005	0.0001	0.0001	0.0005
n^H	4.7104	4.7834	4.7104	4.7469	4.8188	4.7469
n^L	3.4084	3.4084	4.5844	3.4626	3.4626	4.6228
Fertility 1	3.5681	3.5771	4.5999	3.6202	3.6290	4.6380
θ_1^H (%)	16.1898	16.3955	12.5626	16.0807	16.2805	12.5559
$\log(Y_1)$	7.3156	7.2965	6.8938	7.2866	7.2674	6.8683
Gini 1(%)	45.9050	45.8028	46.0114	38.6885	38.8534	35.0154
Fertility 5	3.8475	3.8909	4.6014	3.8866	3.9282	4.6395
θ_5^H (%)	41.2795	43.1180	13.7933	40.3055	42.0897	13.7568
$\log(Y_5)$	9.7295	9.6874	8.7247	10.5758	10.5467	9.7139
Gini 5(%)	34.0891	32.6563	45.6520	18.6562	18.6418	11.5843

Note: Definitions of variables: see Table 2.

Finally, notice that Tables 2, 3 and 4 all demonstrate that an increase in p^H will cause a ‘brain drain’ in the long run. However, the stories behind these results are different. In the case that $p^H > p*$ (Tables 2 and 3), although relaxation of restrictions on the emigration of high-skilled workers would increase school expenditure which will contribute to the economic growth in the short run, it will hurt the economic growth in the long run since the labor market will be dominated by low-skilled workers. But in the case that $p^H < p*$ (Table 4), a ‘brain drain’ will happen in the long run when increasing the probability of migration for high-skilled workers because it increases fertility for high-skilled parents while school expenditure is lowered.

In addition, if $p^L > p*$ (Table 2), with more low-skilled workers emigrating to foreign countries, there would be an increase in domestic economic growth because of the increase of school expenditure and the reduction in the proportion of low-skilled workers in the labor market. But if $p^L < p*$ (Tables 3 and 4), an increase in p^L will reduce economic growth in the long run because fertility for low-skilled parents will be higher while school expenditure will be lower. These results show that fertility matters when considering the ‘brain drain’ or ‘brain gain’ problem since the economic growth depends on the structure change of the labor force.

5. CONCLUSIONS

This paper proposes a stochastic dynamic model to study the implications of migration on economic growth from a source country perspective. In contrast to the existing literature, in this study, adults need to make fertility and education decisions and are possible to migrate to a foreign country. We find that with the uncertainty of migration, there is a “precautionary demand” of children for parents and it will induce a trade-off between quality and quantity for parents when the migration probability changes. And this trade-off between quality and quantity will in turn affect the economic growth and income distribution in the short run and in the long run.

Our work also has some interesting policy implications for the debate on restrictions to migration. Hence, according to our simulation results, the government of a source country whose goal is to increase economic growth should aim to place some restrictions on the emigration of high-skilled workers. On the other hand, allowing more low-skilled workers to emigrate to a foreign country will increase the economic growth if $p^L > p*$ and will reduce the economic growth if $p^L < p*$.

Migration also has different effects when an economy is under a private or a public education regime. Our model predicts that economic growth is more sensitive to the probability of migration under a private education regime than under a public education regime. Therefore, a source country under a private education regime should be more careful when formulating any policy on migration. Furthermore, the long-run economic growth under a public education regime would dominate that of a private education regime for a source country with a higher tax rate.

The estimation by Beine et al. (2001) showed that the growth rate is negatively correlated with the migration flow and positively correlated with the shared of educated people for source countries. However, they did not distinguish countries under private schooling with countries under public schooling. Furthermore, they have discussed the difficulties either of the data collection or of the econometric techniques that researchers need to deal with when conducting empirical researches in migration. It would be a challenging work in the future to empirically test the implications drawn from a migration model.

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Appendix 1

In this appendix, we omit the time index t explicitly to make our derivations easier to read. We should first of all note that the first and second order partial derivatives of the utility function with respect to N are:

$$u_N = \frac{\beta(w_B - aw_A)}{Nw_B + a(n-N)w_A} = \frac{\beta d}{Nw_B + a(n-N)w_A}, \tag{A1.1}$$

$$u_{NN} = \frac{-\beta d^2}{[Nw_B + a(n-N)w_A]^2}, \tag{A1.2}$$

where $d = w_B - aw_A$.

The second-degree Taylor series expansions of the utility function around the mean $N = pn$ is:

$$u \approx u(pn) + (N - pn)u_N(pn) + \frac{(N-pn)^2}{2!}u_{NN}(pn). \tag{A1.3}$$

Substituting Equations (A1.1) and (A1.2) into Equation (A1.3) and using the statistical results that $E(N - pn) = 0$ and $E(N - pn)^2 = np(1-p)$, we can derive the expectation of the utility function as:

$$E(u) = u(pn) + \frac{\beta np(1-p)}{2!}\{-\frac{d^2}{[(pn)w_B + an(1-p)w_A]^2}\}$$

$$= u(pn) - \frac{p(1-p)\beta d^2}{2nw^2}. \tag{A1.4}$$

where $w = pw_B + a(1-p)w_A$.

Appendix 2

Proof of Proposition 3

We first analyze an economy under a private education regime and then go on to study an economy under a public education regime.

Under a Private Education Regime

Given all parameter values, Equation (11) implies that the number of children is constant. Hence, from Equation (12), e_{rt} is a linear function of h_{rt} and can be expressed as $e_{rt} = \varepsilon h_{rt}$ where ε is a positive number. Equation (2) tells us that human capital in the next period will be $h_{rt+1} = \lambda(\varepsilon h_{rt})^{\gamma} h_{rt}^{1-\gamma} = \lambda\varepsilon^{\gamma} h_{rt}$. With homogeneous agents, $H_{rt} = h_{rt}$ for all t. Therefore, the growth rate of average human capital under a private education regime is:

$$g_r^H = \frac{H_{rt+1}}{H_{rt}} - 1 = \frac{h_{rt+1}}{h_{rt}} - 1 = \varepsilon^{\gamma} - 1. \qquad \text{(A2.1)}$$

Equation (A2.1) implies that g_r^H is constant.

Under a Public Education Regime

Under a public education regime, the human capital accumulation function becomes:

$$h_{ut+1} = \lambda e_{ut}^{\gamma} h_{ut}^{1-\gamma} = \lambda[\tau_t (1-\phi n_{ut}) w_A H_{ut}]^{\gamma} h_{ut}^{1-\gamma}. \qquad \text{(A2.2)}$$

When agents are homogeneous, $H_{ut} = h_{ut}$ for all t. This implies that the growth rate under a public education is:

$$g_u^H = \frac{H_{ut+1}}{H_{ut}} - 1 = \frac{h_{ut+1}}{h_{ut}} - 1 = \lambda[\tau_t (1-\phi n_{ut}) w_A]^{\gamma} - 1. \qquad \text{(A2.3)}$$

Because Equations (9) and (13) show that given the probability of migration, the tax rate and fertility are constant, Equation (A2.3) implies that g_u^H is constant.

QED.

Appendix 3

Proof of Proposition 4

We first consider an economy under a private education regime. Then we study an economy under a public education regime.

Under a Private Education Regime

The left-hand side of Equation (11) is a function of n_{rt} and can be expressed by $\xi(n_{rt})$. The right-hand side depends on n_{rt} and p and can be represented by $\mu(n_{rt}, p)$. Hence, we can rewrite Equation (11) as

$$\xi(n_{rt}) = \mu(n_{rt}, p) . \qquad \text{(A3.1)}$$

Taking the derivative of both sides of Equation (A3.1) with respect to p, we get

$$\left(\frac{d\xi}{dn_{rt}} - \frac{\partial\mu}{\partial n_{rt}}\right) n_{rt}'(p) = \frac{\partial\mu}{\partial p} . \qquad \text{(A3.2)}$$

Note firstly that
$$\frac{d\xi}{dn_{rt}} = \frac{\phi(1+\beta\gamma)}{(1-\phi n_{rt})^2} > 0, \qquad \text{(A3.3)}$$

secondly that
$$\frac{\partial\mu}{\partial n_{rt}} = -\frac{\beta p(1-p)d^2}{2w^2 n_{rt}^2} < 0, \qquad \text{(A3.4)}$$

and thirdly that
$$\frac{\partial\mu}{\partial p} = \frac{\beta d^2}{2n_{rt}} \frac{(1-2p)w - 2p(1-p)d}{w^3} . \qquad \text{(A3.5)}$$

Substituting the definitions of w and d into the numerator of Equation (A3.5), we can get

$$\frac{\partial\mu}{\partial p} = \frac{\beta d^2}{2n_{rt}} \frac{[-pw_B + a(1-p)w_A]}{w^3} .$$

Define p^* such that $\frac{a(1-p^*)}{p^*} = \frac{w_B}{w_A}$. Hence, $\frac{\partial\mu}{\partial p} < 0$ if $p > p^*$. Then one must have $n_{rt}'(p) < 0$ if $p > p^*$ for Equation (A3.2) to hold. The situation will be reversed ($\frac{\partial\mu}{\partial p} > 0$ and $n_{rt}'(p) > 0$) if $p < p^*$.

From Equation (12), the derivative of e_{rt} with respect to n_{rt} is:

$$\frac{de_{rt}}{dn_{rt}} = -\frac{\beta\gamma w_A h_{rt}}{(1+\beta\gamma)n_{rt}^2} < 0. \tag{A3.6}$$

Using the implicit differentiation of e_{rt}, we can get that $e_{rt}'(p) = \frac{de_{rt}}{dn_{rt}} n_{rt}'(p) > 0$ if $p > p^*$ and vice versa.

Under a Public Education Regime

The left-hand side of Equation (13) is a function of n_{ut} and can be expressed by $\xi(n_{ut})$. The right-hand side depends on n_{ut} and p and can be represented by $\mu(n_{ut}, p)$. Hence, we can rewrite Equation (13) as:

$$\xi(n_{ut}) = \mu(n_{ut}, p). \tag{A3.7}$$

Taking the derivative of both sides of Equation (A3.7) with respect to p, we get

$$\left(\frac{d\xi}{dn_{ut}} - \frac{\partial\mu}{\partial n_{ut}}\right) n_{ut}'(p) = \frac{\partial\mu}{\partial p}. \tag{A3.8}$$

Note firstly that $$\frac{d\xi}{dn_{ut}} = \frac{\phi^2}{(1-\phi n_{ut})^2} > 0, \tag{A3.9}$$

secondly that $$\frac{\partial\mu}{\partial n_{ut}} = -\beta\left[\frac{1}{n_{ut}^2} + \frac{p(1-p)d^2}{w^2 n_{ut}^3}\right] < 0, \tag{A3.10}$$

and thirdly that $$\frac{\partial\mu}{\partial p} = \frac{\beta d^2}{2n_{ut}^2} \frac{[-pw_B + a(1-p)w_A]}{w^3}. \tag{A3.11}$$

Hence, if $p > p^*$, $\frac{\partial\mu}{\partial p} < 0$. Then from Equation (A3.8), we know that $n_{ut}'(p) < 0$. The situation will be reversed ($\frac{\partial\mu}{\partial p} > 0$ and $n_{ut}'(p) > 0$) if $p < p^*$.

From Equation (6), the derivative of e_{ut} with respect to n_{ut} is

$$\frac{de_{ut}}{dn_{ut}} = -\tau\phi w_A H_{ut} = -\frac{\beta\gamma\phi}{1+\beta\gamma} w_A H_{ut} < 0. \tag{A3.12}$$

Using the implicit differentiation of e_{ut}, we can get that $e_{ut}'(p) = \frac{de_{ut}}{dn_{ut}} n_{ut}'(p) > 0$ if $p > p^*$ and vice versa.

QED.

Appendix 4

Proof of Proposition 7

We should first of all note that:

$$\frac{\partial \theta_{it+1}^{L}}{\partial p^{H}} = -\frac{\partial \theta_{it+1}^{H}}{\partial p^{H}} = \frac{(1-p^{L})\theta_{i}^{L} n_{it}^{L}[\theta_{i}^{H} n_{it}^{H} - (1-p^{H})\theta_{i}^{H} n_{it}^{H}{}'(p^{H})]}{[(1-p^{L})\theta_{i}^{L} n_{it}^{L} + (1-p^{H})\theta_{i}^{H} n_{it}^{H}]^{2}} > 0,\ i = r, u. \quad \text{(A4.1)}$$

Under a private education regime, the partial differentiation of H_{rt+1} with respect to p^H can be written as:

$$\frac{\partial H_{rt+1}}{\partial p^{H}} = \frac{\partial \theta_{rt+1}^{L}}{\partial p^{H}}(h_{rt+1}^{L} - h_{rt+1}^{H}) + \theta_{rt+1}^{L}\frac{\partial h_{rt+1}^{L}}{\partial e_{rt}^{L}} e_{rt}^{L}{}'(p^{H}) + \theta_{rt+1}^{H}\frac{\partial h_{rt+1}^{H}}{\partial e_{rt}^{H}} e_{rt}^{H}{}'(p^{H}). \quad \text{(A4.2)}$$

Since $e_{rt}^{L}{}'(p^{H}) = 0$, Equation (A4.2) can be expressed as:

$$\frac{\partial H_{rt+1}}{\partial p^{H}} = \frac{\partial \theta_{rt+1}^{L}}{\partial p^{H}}(h_{rt+1}^{L} - h_{rt+1}^{H}) + \theta_{rt+1}^{H}\frac{\partial h_{rt+1}^{H}}{\partial e_{rt}^{H}} e_{rt}^{H}{}'(p^{H}). \quad \text{(A4.3)}$$

Because $\frac{\partial \theta_{rt+1}^{L}}{\partial p^{H}} > 0$ and $\theta_{rt+1}^{H}\frac{\partial h_{rt+1}^{H}}{\partial e_{rt}^{H}} e_{rt}^{H}{}'(p^{H}) > 0$ if $p^{H} > p^{*}$, a sufficient condition for $\frac{\partial H_{rt+1}}{\partial p^{H}} > 0$ under a private education regime is that $\left|\frac{\partial \theta_{rt+1}^{L}}{\partial p^{H}}\right|$ is not too large (that is, an increase in p^H will not cause a large increase in θ_{rt+1}^{L}).

Under a public education regime with heterogeneous agents, public school expenditure is:

$$e_{ut} = \theta_{ut}^{L}\tau(1-\phi n_{ut}^{L}) w_{A} h_{ut}^{L} + \theta_{ut}^{H}\tau(1-\phi n_{ut}^{H}) w_{A} h_{ut}^{H}. \quad \text{(A4.4)}$$

From Equation (A4.4), we can derive that $e_{ut}{}'(p^{H}) = \frac{\partial e_{ut}}{\partial n_{ut}^{H}} n_{ut}{}'(p^{H}) > 0$.

The partial differentiation of H_{ut+1} with respect to p^H can be written as:

$$\frac{\partial H_{ut+1}}{\partial p^H} = \frac{\partial \theta^L_{ut+1}}{\partial p^H}(h^L_{ut+1} - h^H_{ut+1}) + [\theta^L_{ut+1}\frac{\partial h^L_{ut+1}}{\partial e_{ut}} + \theta^H_{ut+1}\frac{\partial h^H_{ut+1}}{\partial e_{ut}}]e_{ut}'(p^H)\,. \qquad \text{(A4.5)}$$

Because $\frac{\partial \theta^L_{ut+1}}{\partial p^H} > 0$, $\frac{\partial h^L_{ut+1}}{\partial e_{ut}} > 0$, $\frac{\partial h^H_{ut+1}}{\partial e_{ut}} > 0$ and $e^H_{ut}'(p^H) > 0$ if $p^H > p*$, a sufficient condition for $\frac{\partial H_{ut+1}}{\partial p^H} > 0$ under a public education regime is that $\left|\frac{\partial \theta^L_{ut+1}}{\partial p^H}\right|$ is not too large (that is, an increase in p^H will not cause a large increase in θ^L_{ut+1}).

QED.

Appendix 5

Proof of Proposition 8

We should first of all note that:

$$\frac{\partial \theta_{it+1}^{L}}{\partial p^{L}} = -\frac{\partial \theta_{it+1}^{H}}{\partial p^{L}} = \frac{-(1-p^{H})\theta_{i}^{H} n_{it}^{H}[\theta_{i}^{H} n_{it}^{H} - (1-p^{L})\theta_{i}^{L} n_{i}^{L}{}'(p^{L})]}{[(1-p^{L})\theta_{i}^{L} n_{it}^{L} + (1-p^{H})\theta_{i}^{H} n_{it}^{H}]^{2}} < 0, i = r, u . \text{(A5.1)}$$

Under a private education regime, the partial differentiation of H_{rt+1} with respect to p^L can be written as:

$$\frac{\partial H_{rt+1}}{\partial p^{L}} = \frac{\partial \theta_{rt+1}^{L}}{\partial p^{L}}(h_{rt+1}^{L} - h_{rt+1}^{H}) + \theta_{rt+1}^{L}\frac{\partial h_{rt+1}^{L}}{\partial e_{rt}^{L}} e_{rt}^{L}{}'(p^{L}) + \theta_{rt+1}^{H}\frac{\partial h_{rt+1}^{H}}{\partial e_{rt}^{H}} e_{rt}^{H}{}'(p^{L}) . \qquad \text{(A5.2)}$$

Since $e_{rt}^{H}{}'(p^{L}) = 0$, Equation (A5.2) can be expressed as:

$$\frac{\partial H_{rt+1}}{\partial p^{L}} = \frac{\partial \theta_{rt+1}^{L}}{\partial p^{L}}(h_{rt+1}^{L} - h_{rt+1}^{H}) + \theta_{rt+1}^{L}\frac{\partial h_{rt+1}^{L}}{\partial e_{rt}^{L}} e_{rt}^{L}{}'(p^{L}) . \qquad \text{(A5.3)}$$

Because $\frac{\partial \theta_{rt+1}^{L}}{\partial p^{L}} < 0$ and $\theta_{rt+1}^{L}\frac{\partial h_{rt+1}^{L}}{\partial e_{rt}^{L}} e_{rt}^{L}{}'(p^{L}) > 0$ if $p^{L} > p*$. From Equation (A5.3), we have $\frac{\partial H_{rt+1}}{\partial p^{L}} > 0$.

Under a public education regime, from Equation (A4.4), we can derive that

$$e_{ut}{}'(p^{L}) = \frac{\partial e_{ut}}{\partial n_{ut}^{L}} n_{ut}{}'(p^{L}) > 0 .$$

The partial differentiation of H_{ut+1} with respect to p^L can be written as:

$$\frac{\partial H_{ut+1}}{\partial p^{L}} = \frac{\partial \theta_{ut+1}^{L}}{\partial p^{L}}(h_{ut+1}^{L} - h_{ut+1}^{H}) + [\theta_{ut+1}^{L}\frac{\partial h_{ut+1}^{L}}{\partial e_{ut}} + \theta_{ut+1}^{H}\frac{\partial h_{ut+1}^{H}}{\partial e_{ut}}] e_{ut}{}'(p^{L}) . \qquad \text{(A5.5)}$$

Because $\frac{\partial \theta_{ut+1}^{L}}{\partial p^{L}} < 0$, $\frac{\partial h_{ut+1}^{L}}{\partial e_{ut}} > 0$, $\frac{\partial h_{ut+1}^{H}}{\partial e_{ut}} > 0$ and $e_{ut}^{H}{}'(p^{L}) > 0$ if $p^{L} > p^{*}$, we can get that $\frac{\partial H_{ut+1}}{\partial p^{L}} > 0$ from Equation (A5.5).

QED.

Endnotes

[1] See Bhagwati and Rodriguez (1975) for a literature survey of earlier works on this issue.
[2] For the model setting of human capital accumulation under a private education regime, see Uzawa (1965) and Lucas (1988).
[3] Parents would care domestic and foreign children differently due to several reasons. For example, parents may care less about migrating children since they see them less often. On the other hand, parents may care more about migrating children because of a direct monetary reason such as remittances.
[4] However, several modifications have been made because the intention of Kalemli-Ozcan (2003) was to study the implications of mortality.
[5] Kalemli-Ozcan referred this as the 'insurance effect' since with the uncertainty of mortality of children, a self-insurance strategy for parents is to overshoot fertility.
[6] The probability of migration for both low- and high-skilled workers will be determined by the policies adopted by the governments from both the source and home countries. However, in this paper, we assume that the probability of migration is exogenous, and we do not study the migration issue from a host country's perspective.
[7] The model can be easily extended to allow for the endogenous choice of migration by including the cost of migration and the innate ability into the human capital accumulation function. The results would be that young agents with high parental human capital and high innate ability (agents with high human capital accumulation) will emigrate. Hence, similar results can be obtained by assuming that high-skilled workers with higher probability to emigrate than low-skilled workers.
[8] Notice that there is no intergroup mobility in our model. Similar model setting can be found in de la Croix and Doepke (2004). One possible way to allow for the intergoup mobility is to incorporate the innate ability into the human capital accumulation function and assume that the probability of migration depends on each agent's human capital accumulation. However, this will complicate the model without changing our main results about the impacts of migration probability on the economic growth.
[9] However, our computational results in the next section show that through the 5 periods (which is approximately 150 years), e_{rt}^{L} (h_{rt}^{L}) is always lower than e_{rt}^{H} (h_{rt}^{H}) in every period.
[10] Given the exogenous real wage per unit of human capital (w_j), the elasticity of human capital for children with respect to school expenditure equals the income elasticity with respect to school expenditure.
[11] We choose the Philippines as our source country because, based on 1990 data, Carrington and Detragiache (1999) demonstrated that highly-educated migrants from the Asia-Pacific region were the second largest group of immigrants to the United States, and of this particular group, the Philippines was shown to be the major source country.
[12] Source: per capita GDP, PPP (constant 1987 international dollar), World Development Index, World Bank.
[13] Given that per capita GDP in the US is 7.975 times the level of per capita GDP in the Philippines, w_B is calibrated as 7.975 w_A.
[14] The dataset composed by Deininger and Squire (1996) shows that the share of income of the bottom quintile was 5.2 per cent, whilst the share of income of the top quintile was 52.1 per cent.
[15] Using these calibrated numbers along with other calibrated parameter values for fertility gives us a per capita GDP level equal to $2522.6 at the end of the first period.
[16] Source for income share and Gini coefficient: Deininger and Squire (1996), World Bank.
[17] Public school enrollment was 59 per cent of total enrollment in secondary schools for the Philippines in 1985.
[18] Data source for public school enrollment rate and public spending on education: UNESCO, United Nations.
[19] In the baseline model, everyone attends a public school. Hence, the proportion of public school expenditure to GNP becomes 0.014/0.59 = 2.373 per cent.
[20] Data source for economic growth rate and population growth rate: World Development Index, World Bank.