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# Environmental Tax Policy, Habit Persistence and Complex Dynamics

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## Abstract

In this paper, we consider habit formation of environmental quality and environmental tax policy in an environment-growth model: the John-Pecchenino (1994) model. We show that the economic transition can be represented by a difference equation of order one or two, depending on no habit formation or persistent habits of environmental quality. In both cases (without and with persistent habits of environmental quality), we show that entropic chaos might present under certain conditions and the economy moves from complex dynamics to simple dynamics as the tax rate increases. This result implies that environmental tax policy is a useful tool to affect the complexity of the dynamic behavior of an economy. Furthermore, in the presence of habit formation of environmental quality, an increase in the degree of habit persistence lowers the possibility of complex dynamics.

*Keywords:* Chaos; Environmental policy; Persistent habit.

*JEL Classification:* D90; H20; Q20.

## 1 Introduction

The controversy of the interplay between environmental quality and economic growth causes great debate on the pros and cons of environmental taxation. Supporters of environmental taxation demonstrate that the enforcement of environmental taxation can prevent environmental quality from deterioration and is beneficial to economic growth. On the other hand, opponents of environmental taxation argue that it increases the production cost which will hurt economic growth.

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In the literature of environment and natural resource, there have been many studies analyzing the link between environmental quality and economic development.<sup>1</sup> In these studies, the economic transition can be represented by a state variable called environmental quality. Among them, the model developed by John and Pecchenino (1994) is the first one where an environment-growth model was developed based on an overlapping generations setting. This highly admitted model has been adopted in many studies. Wendner (2000a, 2000b) and Ono (2003) considered (environmental) policy implications of this model. Gutierrez (2005) modified the model by assuming that households do not care directly about the environment quality, but the deterioration of environmental quality makes them incur health costs when they become old.

It has been shown by many studies that complex dynamics might emerge in an overlapping generations model (see Grandmont, 1985; Michel and de la Croix, 2000; Yokoo, 2000). Hence, with the unsatisfactory that most studies of environment-growth model only focus on the local property around the steady state, Zhang (1999) showed that the dynamics of the John-Pecchenino (1994) model can be represented by a difference equation of one order and analyzed the global property of the model. He argued that although environmental quality is deteriorated by households' consumption activities, it can be improved by households' investments in environmental enhancement. Thus, cycles and chaotic motion in the sense of Li and Yorke (1975) may present under such circumstance. A future study left by Zhang (1999) is how to extend an environmental-growth model to a difference equation of high order.

In this paper, we follow the research line of Zhang (1999) and propose a natural way to extend the John-Pecchenino (1994) model to a difference equation of high order. By assuming that households have persistent habits of environmental quality, we show that the economic transition will be driven by a difference equation of order two. The literature of habit formation usually assume that households compare their current consumption with the average (external habit) or individual (internal habit) consumption in the past.<sup>2</sup> Unlike traditional assumption about habit formation, we assume that households have persistent habits of environmental quality.<sup>3</sup> The other contribution of this paper is that not only households make private investments to improve environmental quality, the government can also commit itself to the environment preservation. We assume that government levies consumption tax and uses the tax revenue as public investments for environmental improvement.

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<sup>1</sup>See Beltratti et al. (1993), Tahvonen and Kuuluvainene (1993), John and Pecchenino (1994), John et al. (1995), Bovenberg and Smulders (1995, 1996), Bovenberg and de Mooij (1997), Fisher and van Marrewijk (1998), Bovenberg and Heijdra (1998) and Grimaud (1999) for environment-growth model

<sup>2</sup>Studies of complex dynamics in an overlapping generations models with bequeathed tastes of consumption can be found in de la Croix (1996) and Nishimura and Shimomura (1999).

<sup>3</sup>Studies considering habit formation of consumption in the John-Pecchenino (1994) model can be found in Wendner (2000a, 2000b). However, the analysis only concentrates on the local property around the steady state.

We construct an overlapping generations model where households concern about their consumptions and environmental quality. Furthermore, we assume that households get used to the environment while they grew up and will compare environmental quality in their old age with the one when they were young. Households allocate their income between consumptions and investments for environment improvement. However, for each unit of consumption, households need to pay consumption tax to the government. The government will use the tax revenue to enhance environment quality (the environmental tax).

The focus of this paper is to study the impact of habit formation of environmental quality and consumption tax. We find that when there is no persistent habit of environmental quality, the economic dynamics can be represented by a nonlinear first-order difference equation which has been widely explored in the literature of economic dynamics. However, when there are persistent habits of environmental quality, the dynamics of the economy becomes two-dimensional. Because of the difficulties that economists may encounter when dealing with an economic model represented by a difference equation of high order, there are very few studies constructing economic models which generate difference equations of order greater than one.<sup>4</sup> In the literature of economic dynamics and chaos, economists tend to verify the existence of Li-Yorke chaos in economic models because of its convenience for verification,<sup>5</sup> but recent studies started to explore the presence of other types of chaos.<sup>6</sup> Besides, the result of Li and Yorke is not for a high-dimensional dynamical system and hence, it is not suitable to study the presence of Li-Yorke chaos when there is habit formation of environmental quality. In order for our analysis to be consistent in both cases, we follow Mitra (2001) to study the possibility of entropic chaos. In both cases (without and with persistent habits of environmental quality), we show that entropic chaos might emerge under certain conditions.

We also find that environmental taxation and the degree of habit persistence are important determinants to the dynamic behavior of the economy. In the presence of habit formation of environmental quality, an increase in the degree of habit persistence lowers the possibility of chaotic dynamics. The economy moves from complex to simple dynamics as the tax rate increases no matter habit formation of environmental quality persists or not. This result implies that the government can use environmental tax policies to affect the complexity of the dynamic transition of the economy. Furthermore, the required consumption tax rate to avoid complex

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<sup>4</sup>A two-dimensional overlapping generations model which generates complex dynamics are studied by Medio and Negroni (1996) and Yokoo (2000). In order to produce endogenous fluctuations, Medio and Negroni (1996) incorporated Leontief and the CES production function and Yokoo (2000) took the accumulation of government debt into consideration.

<sup>5</sup>According to Li and Yorke (1975), period three implies chaos. Studies concerned about the emergence of Li-Yorke chaos can be found in Day (1982), Boldrin et al. (2001) and Auray et al. (2002).

<sup>6</sup>For example, Boldrin et al. (2001) and Mitra (2001) studied the presence of ergodic chaos and entropic chaos, respectively.

dynamics in an economy with habit formation of environmental quality is lower than then the one in an economy without habit formation of environmental quality.

The remainder of the paper is organized as follows. In the next section, we develop an environment-growth model with persistent habits of environmental quality and environmental taxation. The dynamic property of the economy and policy implications are analyzed in section 3. The final section concludes.

## 2 The Model

In this section, we extend the John-Pecchenino (1994) model by including habit formation of environmental quality and environmental taxation. We consider an infinite-horizon, discrete-time overlapping generations model where agents live for two periods, corresponding to young and old age. Each old agent (a parent) gives birth to a young agent (a child). Hence, there is no population growth and we normalize the population size to 1.

We use variable  $E_t$  to represent the index of environmental quality in period  $t$ . This index includes all concerns about environmental problems, such as the inverse of the diffusion of toxic compounds in the soil, atmosphere and water, the inverse of the greenhouse gases (such as carbon dioxide, methane, nitrous oxide and chlorofluorocarbons emissions), the inverse of dispersion of radioactivity and so on. Agents have identical preferences and derive utility from their old age. Old agents care about their consumptions ( $c_{t+1}$ ) and environmental quality ( $E_{t+1}$ ). Furthermore, environmental quality when agents were young has an externality impact on the preference in their old age because agents have persistent habits of the lifestyle characterized by environmental quality. That is, old agents have persistent habits of environmental quality and will compare current environment quality with the one when they were young.<sup>7</sup> The utility function is represented as

$$u(c_{t+1}, E_{t+1}, E_t) = \log c_{t+1} + \eta \log(E_{t+1} - \phi E_t), \quad (1)$$

where  $\eta$  is the substitution between consumption and environment quality and  $\phi$  measures the degree of habit persistence of environment quality.

Each agent is endowed with 1 unit of time. Young agents devote all of the time for work to earn a real wage rate ( $w_t$ ) and old agents use the time for leisure. Young agents decide how to allocate  $w_t$  between savings ( $s_t$ ) for the old-age consumptions and investments ( $m_t$ ) to promote environmental quality. The budget constraint for young agents is

$$s_t + m_t = w_t. \quad (2)$$

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<sup>7</sup>Note that this assumption about the habit persistence differs from most of studies in the literature of the habit formation which is composed by the comparison of the current (individual) consumption with the past (individual or aggregate) consumption.

When old agents consume, they need to pay the consumption tax with the rate of  $\tau$  for each unit of consumptions. Using  $R_{t+1} = 1 + r_{t+1}$  to represent the gross rate of returns for savings, the budget constraint for old agents is

$$(1 + \tau)c_{t+1} = R_{t+1}s_t. \quad (3)$$

We assume that government runs a balanced budget and uses the tax revenue ( $T_t$ ) to improve environmental quality (environmental taxation). The budget constraint for the government is

$$T_t = \tau c_t. \quad (4)$$

The evolution of environmental quality follows

$$\begin{aligned} E_{t+1} &= (1 - b)E_t - \beta c_t + \gamma(m_t + T_t) \\ &= (1 - b)E_t - (\beta - \gamma\tau)c_t + \gamma m_t, \end{aligned} \quad (5)$$

where  $b \in (0, 1)$  is the autonomous evolution of environmental quality. This indicates that when there is no economic activity, environmental quality depreciates by the rate of  $b$  in every period. With economic activity, parameter  $\beta > 0$  measures the deterioration of environmental quality caused by consumption. However, households and the government can devote some resources to enhance (or to maintain) environmental quality and  $\gamma > 0$  measures the maintenance efficiency. Equation (5) implies that there are two sources of investments to improve environmental quality: one comes from private investment ( $m_t$ ) which is the optimal decision by households and the other one comes from public investment ( $T_t = \tau c_t$ ).

Net output per capita ( $y_t$ ) is produced by a constant-returns-to-scale production function  $y_t = f(k_t) - \delta k_t = Ak_t^\alpha - \delta k_t$ , where  $k_t > 0$  is the capital per capita,  $\alpha \in (0, 1)$  is the capital share of output,  $A > 0$  is the total factor productivity and  $\delta \in (0, 1)$  is the depreciation rate of capital. The competitive behavior of firms will equalize the factor prices of labor and capital to their respective marginal product:

$$w_t = f(k_t) - k_t f'(k_t) = (1 - \alpha)f(k_t), \quad (6)$$

$$r_t = f'(k_t) - \delta. \quad (7)$$

Given the initial condition  $\{E_1, E_2\}$  and the tax rate  $\tau$ , a perfect foresight equilibrium comprises the sequences of individuals' decisions  $\{c_t, s_t, m_t\}_{t=1}^\infty$ , the stock of capital per capita  $\{k_t\}_{t=1}^\infty$ , the factor prices  $\{w_t, r_t\}_{t=1}^\infty$  and the quality of the environment  $\{E_t\}_{t=1}^\infty$  such that: (i) The household maximization problem will be solved by maximizing the utility function subject to Equations (2), (3) and (5) and the non-negativity constraints of  $c_t$  and  $m_t$ ; (ii) The factor prices satisfy Equations

(6) and (7); (iii) The capital market clears,  $k_{t+1} = s_t$ ; (iv) The evolution of environmental quality follows Equation (5); and (v) The government maintains a balanced budget.<sup>8</sup>

Solving for the optimization problem for households, we can get

$$c_{t+1} = \frac{R_{t+1}}{\eta\gamma(1+\tau)}(E_{t+1} - \phi E_t). \quad (8)$$

Substituting Equation (8) into Equation (3) and applying the clearing condition of capital market, the result implies that

$$k_{t+1} = s_t = \frac{E_{t+1} - \phi E_t}{\eta\gamma(1+\tau)}. \quad (9)$$

Combining Equations (2), (5) and (8), the evolution of environmental quality becomes

$$E_{t+1} = (1-b)E_t - (\beta - \gamma\tau) \frac{R_t}{\eta\gamma(1+\tau)}(E_t - \phi E_{t-1}) + \gamma[w_t - \frac{E_{t+1} - \phi E_t}{\eta\gamma}]. \quad (10)$$

After substituting factor prices Equations (6) and (7) and some calculations, Equation (10) can be written as

$$\begin{aligned} E_{t+1} = & \frac{\eta}{1+\eta} \left\{ (1-b)E_t - \frac{(\beta - \gamma\tau)}{\eta\gamma(1+\tau)}(E_t - \phi E_{t-1})(1 - \delta + f'(k_t)) \right. \\ & \left. + \gamma(1 - \alpha)f(k_t) + \frac{\phi E_t}{\eta} \right\}. \end{aligned} \quad (11)$$

### 3 Chaotic Dynamics

In this section, we study the dynamic behavior of the economy. Combining Equations (9) and (11), the transitional dynamics of environment quality follows:

$$\begin{aligned} E_{t+1} = & \frac{\eta}{1+\eta} \left\{ \left[ 1 - b - \frac{(\beta - \gamma\tau)(1 - \delta)}{\eta\gamma(1+\tau)} + \frac{\phi}{\eta} \right] E_t \right. \\ & \left. + \frac{\phi(\beta - \gamma\tau)(1 - \delta)}{\eta\gamma(1+\tau)} E_{t-1} + \frac{A[\gamma(1 - \alpha) - \frac{\alpha(\beta - \gamma\tau)}{1+\tau}]}{(\eta\gamma)^\alpha} (E_t - \phi E_{t-1})^\alpha \right\} \\ = & a_0 E_t + a_1 E_{t-1} + a_2 (E_t - \phi E_{t-1})^\alpha, \end{aligned} \quad (12)$$

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<sup>8</sup>As we will see in the following section, we need the initial condition  $\{E_1, E_2\}$  when the habit formation exists. However, when persistent habits of environmental quality does not present, the initial condition we need is only  $\{E_1\}$ .

where

$$\begin{aligned}
a_0 &= a_0(\phi, \tau) = \frac{\eta}{1 + \eta} \left[ 1 - b - \frac{(\beta - \gamma\tau)(1 - \delta)}{\eta\gamma(1 + \tau)} + \frac{\phi}{\eta} \right], \\
a_1 &= a_1(\phi, \tau) = \frac{\phi(\beta - \gamma\tau)(1 - \delta)}{(1 + \eta)\gamma(1 + \tau)}, \quad \text{and} \\
a_2 &= a_2(\phi, \tau) = \frac{\eta^{1-\alpha}}{1 + \eta} \frac{A[\gamma(1 - \alpha) - \frac{\alpha(\beta - \gamma\tau)}{1 + \tau}]}{\gamma^\alpha}.
\end{aligned}$$

Equation (12) shows that the dynamical transition of the economy can be represented by a difference equation of order two in environmental quality. For simplicity, we denote the identity function by  $f^0$ , and inductively define  $f^n = f \circ f^{n-1}$  for a positive integer  $n$ .

### 3.1 No Persistent Habits of Environmental Quality

We first study a simple case where agents do not have persistent habits of environmental quality ( $\phi = 0$ ) and the government does not levy any tax on consumption ( $\tau = 0$ ). Under this circumstance, Equation (12) is reduced to a one-dimensional dynamical system  $E_{t+1} = f(E_t)$  considered by Zhang (1999), where

$$f(E_t) = a_0(0, 0)E_t + a_2(0, 0)E_t^\alpha. \quad (13)$$

To distinguish this paper from Zhang (1999), we focus our study on the role of tax rate and habit formation of environmental quality. Figure 1 gives a numerical example of the economic dynamics for a given initial value. In Figure 1, the first 500 iterations of a random initial point under  $f$  with  $b = 0.9$ ,  $\delta = 0.5$ ,  $\beta = 4.285$ ,  $\gamma = 0.7$ ,  $\alpha = 1/9$ , and  $A = 100$  are shown and it indicates that complex dynamics may occur under this parameter setting.<sup>9</sup>

Traditionally, researchers tend to concentrate on the presence of chaos in the sense of Li and Yorke when analyzing a one-dimensional dynamical system because of its easiness to verify. Zhang (1999) has proved that when consumers prefer greener preferences and the maintenance efficiency relative to degradation is not sufficiently high, the chaotic motion in the sense of Li and Yorke will present. However, the Li-Yorke chaos is only defined in a one-dimensional dynamical system and does not exist in a high-dimensional dynamical system. In order for our analysis to be consistent in both one- and two-dimensional dynamical systems, we need to use other types of chaos which can be examined in dynamical systems with different orders. Following Mitra (2001), we choose to examine the possibility of entropic chaos for the dynamical system represented by Equation (12).

The definition of topological entropy and entropic chaos is given as following:

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<sup>9</sup>We will present the other three diagrams in the following discussion. For all numerical examples given in this paper, we use the same parameter setting with  $b = 0.9$ ,  $\delta = 0.5$ ,  $\beta = 4.285$ ,  $\gamma = 0.7$ ,  $\alpha = 1/9$ , and  $A = 100$ .



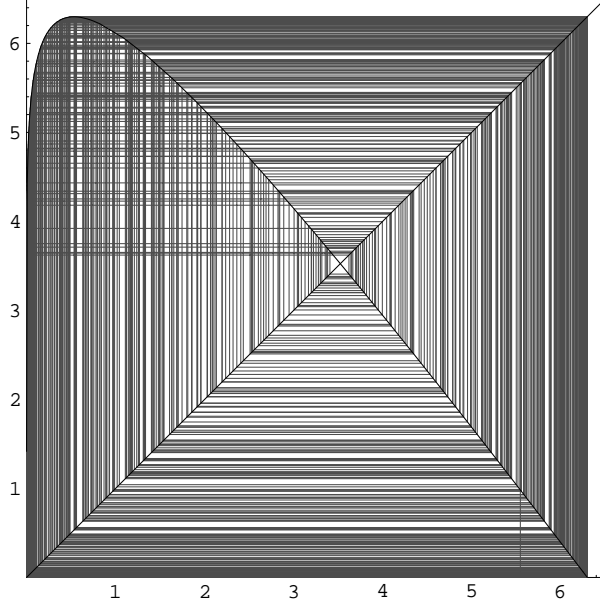


Figure 1: The graphs of  $f$  with  $b = 0.9, \delta = 0.5, \beta = 4.285, \gamma = 0.7, \alpha = 1/9$ , and  $A = 100$ , the diagonal line, and the first five hundred iterations of the critical point of a random initial point.

**Definition 1.** Let  $g : X \rightarrow X$  be a continuous map on the space  $X$  with metric  $d$ . For  $n \in \mathbb{N}$  and  $\epsilon > 0$ , a set  $S \subset X$  is called an  $(n, \epsilon)$ -separated set for  $g$  if for every pair of points  $x, y \in S$  with  $x \neq y$ , there exists an integer  $k$  with  $0 \leq k < n$  such that  $d(g^k(x), g^k(y)) > \epsilon$ . The topological entropy of  $g$  is defined to be

$$h_{\text{top}}(g|X) = \lim_{\epsilon \rightarrow 0, \epsilon > 0} \limsup_{n \rightarrow \infty} \frac{\log(\max\{\#(S) : S \subset X \text{ is an } (n, \epsilon)\text{-separated set for } g\})}{n},$$

where  $\#(S)$  is the cardinality of elements of  $S$ .

We say that  $g$  exhibits entropic chaos on  $X$  if  $h_{\text{top}}(g|X) > 0$ .

Topological entropy describes the total exponential complexity of the orbit structure with a single number in a rough but expressive way. The topological entropy is positive for chaotic systems and is zero for non-chaotic systems.

It is easy to examine the possibility of entropic chaos induced by Equation (13) by utilizing the results of Mitra (2001) for a one-dimensional dynamical system. Note that Equation (13) has one critical point  $\bar{E}$  and one nontrivial fixed point  $E^*$  which satisfy  $f^2(\bar{E}) < \bar{E} < f(\bar{E})$  and  $f^3(\bar{E}) < E^*$ . Therefore, according to the Proposition 2.3 of Mitra (2001), Equation (13) may exhibit entropic chaos.

We then extend the model of Zhang (1999) by considering environmental taxation in the economy. Then the economic dynamics can be represented by the following

one-dimensional dynamical system:

$$E_{t+1} = a_0(0, \tau)E_t + a_2(0, \tau)E_t^\alpha, \quad \tau \in [0, 1]. \quad (14)$$

Figure 2 presents the bifurcation diagram with varying  $\tau$  under the same parameter setting (except  $\tau$ ) as in Figure 1. In order to give a clear picture about the impact of  $\tau$  on the economic dynamics, we give two panels with different scales of  $\tau$  on the x-axis in Figure 2. We see from Figure 2 that the economy undergoes from complex to simple dynamics as  $\tau$  increases. The left panel of Figure 2 shows that the economic dynamics is quite complex when  $\tau$  is small enough and chaotic dynamics may present for  $\tau \in [0, 0.007]$ . It also shows that the dynamic motion experiences from chaotic to cyclical dynamics as  $\tau$  increases from 0 to 0.1. The right panel of Figure 2 shows that the dynamic motion experiences from cycles to simple dynamics as  $\tau$  increases from 0.09 to 1.

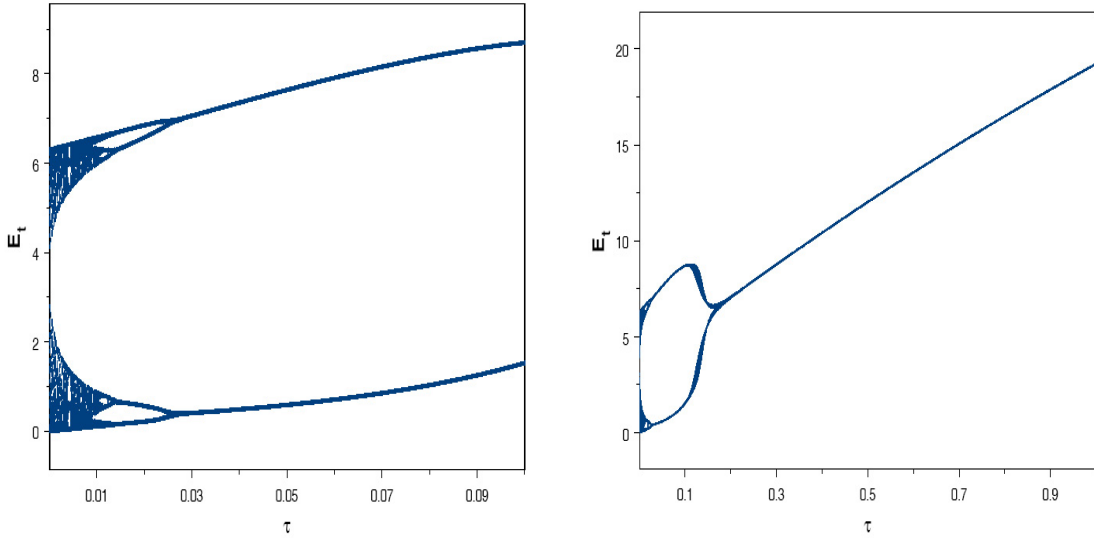


Figure 2: Bifurcation diagram in  $\tau$  with  $b = 0.9, \delta = 0.5, \beta = 4.285, \gamma = 0.7, \alpha = 1/9, A = 100$ , and  $\phi = 0$ .

In the following theorem, we give a sufficient condition for the presence of entropic chaos of Equation (14) from the perspective of consumption tax rate.

**Theorem 1.** *There exists a value  $\bar{a} \in (\frac{1+\alpha}{1-\alpha}, \frac{\alpha^{-\alpha/(1-\alpha)}}{1-\alpha})$  such that if  $\phi = 0$  and  $a_0(0, 0) < -\bar{a}$  then for any  $\tau$  close to zero, Equation (12) exhibits entropic chaos.*

*Proof.* Let  $\phi = 0$  and  $\tau = 0$ . Then Equation (12) becomes  $E_{t+1} = f(E_t)$ , where  $f$  is given in Equation (13). By Proposition 3 of Zhang (1999), there exists  $\bar{a} \in (\frac{1+\alpha}{1-\alpha}, \frac{\alpha^{-\alpha/(1-\alpha)}}{1-\alpha})$  such that if  $a_0(0, 0) < -\bar{a}$  then  $f$  has a period-3 cycle and hence the topological entropy of  $f$ ,  $h_{\text{top}}(f)$ , is positive. Allowing  $\tau > 0$ , we get that

Equation (12) becomes  $E_{t+1} = f_\tau(E_t)$ , where  $f_\tau(E_t) = a_0(0, \tau)E_t + a_2(0, \tau)E_t^\alpha$ . Then the family of functions  $f_\tau$  is continuous in  $\tau$ , each  $f_\tau$  is an unimodal map whose critical point is nondegenerate, and  $f_0 = f$ . By the continuity of topological entropy (e.g., Theorem 9.1 in Chapter II of de Melo and van Strien (1993)), we have that  $h_{\text{top}}(f_\tau) > 0$  for all small  $\tau$ .  $\square$

Figure 2 shows that when the tax rate is sufficiently high, the economic dynamics will be driven by simple dynamics. That is, when the tax rate is high enough, for a given initial condition of environmental quality, the economy will converge to a steady state an analysis around the steady state as most studies did in the literature do is appropriate. In the previous studies of environmental-growth models, this steady state is referred to as the sustainable development level. However, as the tax rate decreases, cycles start to emerge. When the tax rate is low enough, Theorem 1 elucidates that complicated dynamics and chaotic motion are more likely to happen and a study of local property around the steady state is not enough.

### 3.2 Persistent Habits of Environmental Quality

We now turn to study a more complicated case when households have persistent habits of environmental quality ( $\phi > 0$ ). In this case, Equation (12) exhibits a two-dimensional dynamical system. The analysis of a dynamical system with a high order is much more complicated than the study of a one-dimensional dynamical system and we use the technique recently developed by Juang et al. (2005) to study the possibility of entropic chaos. We first assume that there is no tax ( $\tau = 0$ ) and study the impact of  $\phi$  on the global property of the economic dynamics. The economic transition of the economy can be represented by the following difference equation of order two:

$$E_{t+1} = a_0(\phi, 0)E_t + a_1(\phi, 0)E_{t-1} + a_2(\phi, 0)(E_t - \phi E_{t-1})^\alpha. \quad (15)$$

Figure 3 presents the bifurcation diagram with varying  $\phi$  when there is no environmental taxation. It shows that the economy undergoes from complex dynamics, through cycles and to simple dynamics as  $\phi$  increases.

In the following theorem, we give a sufficient condition for the presence of entropic chaos induced by Equation (15) from the perspective of the degree of habit persistence.

**Theorem 2.** *Let  $\bar{a}$  be the same as in Theorem 1. Then if  $\tau = 0$  and  $a_0(0, 0) < -\bar{a}$ , then for any  $\phi$  close to zero, Equation (12) exhibits entropic chaos.*

*Proof.* In the proof of Theorem 1, we have known that if  $a_0(0, 0) < -\bar{a}$ , then  $h_{\text{top}}(f) > 0$ , where  $f$  is given in Equation (13). Let  $\tau = 0$ . Then Equation (12) becomes  $E_{t+1} = g_\phi(E_{t-1}, E_t)$ , where  $g_\phi(E_{t-1}, E_t) = a_0(\phi, 0)E_t + a_1(\phi, 0)E_{t-1} + a_2(\phi, 0)(E_t - \phi E_{t-1})^\alpha$ . Since  $a_1(0, 0) = 0$ ,  $g_0(E_{t-1}, E_t) = f(E_t)$ .

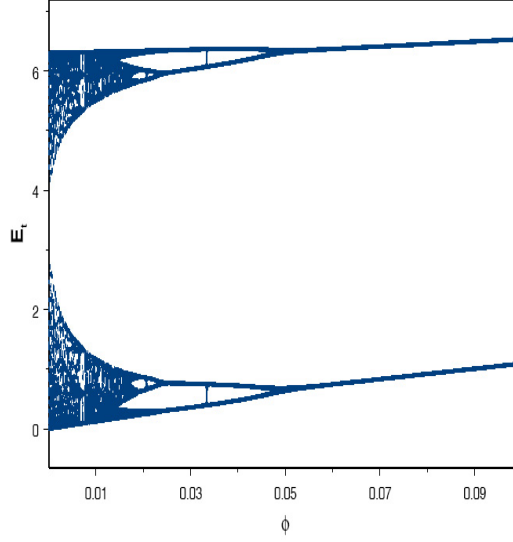


Figure 3: Bifurcation diagram in  $\phi$  with  $b = 0.9, \delta = 0.5, \beta = 4.285, \gamma = 0.7, \alpha = 1/9, A = 100,$  and  $\tau = 0$ .

Let  $a_0(0, 0) < -\bar{a}$ . Then there exists a unique  $\bar{E} > 0$  such that  $f'(\bar{E}) = 0$ . Let  $B = f(f(\bar{E}))$  and  $C = f(\bar{E})$ . Then  $0 < B < C$ . For  $\eta < 1$ , define  $\Phi_\phi : [B, C]^3 \rightarrow \mathbb{R}$  by

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = E_{t+1} - g_\phi(E_{t-1}, E_t).$$

Let  $Y_\phi$  be the set of solutions of the difference equation

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = 0, \tag{16}$$

i.e., the set of bisequences  $\underline{E} = (E_t) = (\dots, E_{-1}, E_0, E_1, \dots)$  such that for any  $t \in \mathbb{Z}$ ,

1.  $E_t \in [B, C]$ ; and
2. three consecutive components  $E_{t-1}, E_t, E_{t+1}$  of  $\underline{E}$  satisfy Equation (16).

Let  $\sigma$  be the shift map on  $Y_\phi$ , i.e.,  $(\sigma(\underline{E}))_t = E_{t+1}$  for all  $t \in \mathbb{Z}$ .

The function  $\Phi_\phi$  is  $C^1$  on  $[B, C]^3$  for each  $\phi$ ,  $\phi \mapsto \Phi_\phi$  is continuous on  $[0, 1]$ , and for  $i = 1, 2, 3$  the function  $\phi \mapsto \partial_i \Phi_\phi$  is continuous on  $[0, 1]$ , where  $\partial_i \Phi_\phi$  is the partial derivative of  $\Phi_\phi$  with respect to the  $i$ th variable. Now letting  $\phi = 0$  we have the limit function  $\Phi_0(E_{t-1}, E_t, E_{t+1}) = E_{t+1} - f(E_t)$ , where  $f$  is given in Equation (13). By Theorem 4 in Appendix, for all  $\phi$  near 0, there is a closed  $\sigma$ -invariant subset  $\Gamma_\phi$  of  $Y_\phi$  in the product topology, such that  $h_{\text{top}}(\sigma|_{\Gamma_\phi}) > 0$ . Therefore, the dynamics of the economy system has entropic chaos.  $\square$

Theorem 2 demonstrates that chaotic equilibria will emerge when the degree of habit persistence is low enough.

Finally, we study the most complicated case where both persistent habits of environmental quality and environmental taxation exist. In this case, the dynamical behavior of the economy is represented by Equation (12). Figure 4 presents the bifurcation diagram with varying  $\tau$  when there are persistent habits of environmental quality with degree of  $\phi = 0.01$ . Figure 4 shows that the economy undergoes from complex to simple dynamics as  $\tau$  increases. The left panel of Figure 4 shows that in the presence of habit formation of environmental quality, the dynamic motion experiences from complex dynamics to cycles as  $\tau$  increases from 0 to 0.1 and chaotic dynamics may present for  $\tau \in [0, 0.003]$ . The right panel of Figure 4 shows that dynamic motion experiences from cycles to simple dynamics as  $\tau$  increases from 0.09 to 1.

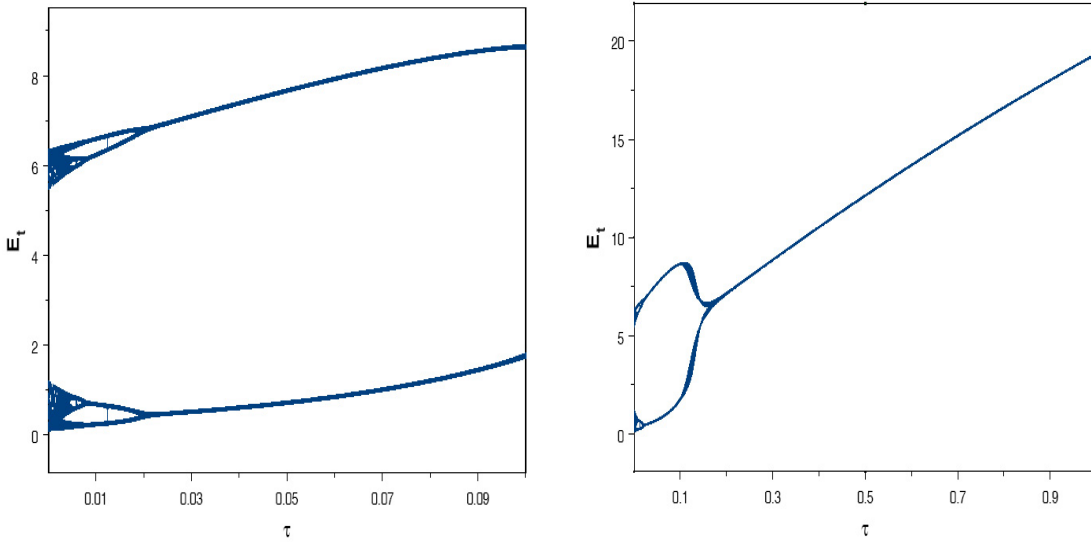


Figure 4: Bifurcation diagram in  $\tau$  with  $b = 0.9, \delta = 0.5, \beta = 4.285, \gamma = 0.7, \alpha = 1/9, A = 100$ , and  $\phi = 0.01$ .

**Theorem 3.** *Let  $\bar{a}$  be the same as in Theorem 1. Then if  $a(0, 0) < -\bar{a}$ , then for either any  $\tau$  or any  $\phi$  close to zero, Equation (12) exhibits entropic chaos.*

*Proof.* Similar to the proof of Theorem 2. □

Theorem 3 indicates that when both the degree of habit formation of environmental quality and consumption tax rate are low, the economic dynamics may be quite complex. Comparing Figure 4 with Figure 2, we find that the presence of persistent habits of environmental quality will reduce the possibility of chaotic motion (that is, the range of  $\tau$  for the emergence of chaos is smaller in the presence of

habit formation). This implies that it is easier for the government to use environmental policy (a lower consumption tax rate required) to avoid complex dynamics when habit formation of environmental quality presents. Theorems 1, 2 and 3 indicate that we cannot only focus our research on the local property when the degree of habit formation of environmental quality or the tax rate is sufficiently low in an environmental-growth model and a more careful study of the global property is needed.

The economic dynamics becomes periodic or simple when the degree of habit formation of environmental quality or the tax rate is sufficiently high. This is because there are two forces to affect the dynamic pattern of  $E_t$ . Equation (5) indicates that consumption deteriorates environmental quality while private and public environmental investments improve or maintain environmental quality. The dynamics of  $E_t$  is driven by the interaction between these two effects. An increase in the degree of habit formation of environmental quality or consumption tax rate will make the latter dominate the former and lower the possibility of chaotic dynamics.

## 4 Conclusion

In this paper, we develop an environment-growth model with habit persistent and consumption tax. This study considers habit formation from another point of view: the persistent habits of environmental quality. We show that economic transition can be represented by a two-dimensional dynamical system when habit formation of environmental quality presents. We examine the possibility of entropic chaos and give sufficient conditions for the presence of chaotic motion from the perspective of tax rate and the degree of habit persistence. Our results indicate that environmental tax can be a useful tool to affect the dynamic pattern in an environment-growth model. In the presence of habit formation of environmental quality, we find that the economy will undergo from chaotic to simple dynamics, through periodic dynamics, as the degree of habit formation increases. Furthermore, the presence of habit formation affects the impact of environmental policies on the dynamic behavior of the economy. In an economy with habit formation of environmental quality, the required consumption tax rate enforced by the government to prevent the economy from trapping in complex dynamics is lower than in an economy without habit formation.

Previous studies of nonlinear dynamics in economic models used to examine the possibility Li-Yorke chaos in a one-dimensional dynamical system. With more and more studies interesting in a high-dimensional dynamical system, our study shows that entropic chaos would be a good tool to examine the possibility of chaotic motion in difference equations of order one or higher.

Our results illustrate that habit formation of environmental quality plays an important role in determining the dynamic property of the economy. Hence, it is worthy in the future research to empirically measure the degree of habit persistence

of environmental quality. The model elucidates that complex dynamics or cycles may arise in an environmental-growth model and we also show that this model can be easily extended to generate a difference equation of order two. It would be interesting to extend this model from different aspects to produce a higher-order dynamical system and we believe that this research line deserves further exploration.

## Appendix

The following theorem is a simple version of Theorem 3 by Juang et al. (2005), which is an extension of Li and Malkin (2006).

**Theorem 4.** *Consider a difference equation of order two in the form*

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = 0, \quad t \geq 1, \quad (17)$$

where  $\phi \in [0, 1]$  is a parameter and the real-valued function  $\Phi_\phi$  is defined on a cube  $[B, C]^3 \subset \mathbb{R}^3$  with constants  $0 < B < C$ . Assume that (i)  $\Phi_\phi$  is  $C^1$  on  $[B, C]^3$  for each  $\phi \in [0, 1]$ ; (ii) the function  $\phi \mapsto \Phi_\phi$  is continuous on  $[0, 1]$ ; and (iii) for  $i = 1, 2, 3$ , the function  $\phi \mapsto \partial_i \Phi_\phi$  is continuous on  $[0, 1]$ , where  $\partial_i \Phi_\phi$  is the partial derivatives of  $\Phi_\phi$  with respect to the  $i$ th variable. Suppose that for  $\phi = 0$ , the difference equation (17) reduces to a difference equation of order one in the form  $E_{t+1} - \varphi(E_t)$ ,  $t \geq 0$ , where  $\varphi : [B, C] \rightarrow \mathbb{R}$  is a  $C^2$  function with positive topological entropy. Let  $Y_\phi$  be the set of solutions for (17), i.e. the set of sequences  $\underline{E} = (E_0, E_1, E_2, \dots)$  such that for any  $t \geq 1$ ,

1.  $E_t \in [B, C]$ ; and
2. three consecutive components  $E_{t-1}, E_t, E_{t+1}$  of  $\underline{E}$  satisfy (17).

Let  $\theta$  be the shift map on  $Y_\phi$ , i.e.,  $\theta(\underline{E}) = \underline{E}'$ , where  $E'_t = E_{t+1}$  for all  $t \geq 0$ .

Then there exists  $\epsilon > 0$  such that for any  $0 < \phi < \epsilon$ , there is a closed  $\theta$ -invariant subset  $\Gamma_\phi$  of  $Y_\phi$  in the product topology such that  $h_{top}(\theta|_{\Gamma_\phi}) > 0$ .

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## 赴國外出差或研習心得報告

計畫編號	95-2415-H-002-018
計畫名稱	環境偏好持續性與綠色租稅
出國人員姓名 服務機關及職稱	陳虹如 臺灣大學經濟系 副教授
出國時間及地點	時間：June 9-16, 2007 地點：日本
研習機構	Department of Mathematics, Kyoto University of Education

### 一、內心得報告：

Prof. Shin Kiriki of the Department of Mathematics at Kyoto University of Education invited me to visit his department for a week from June 9, 2007 to June 16, 2007. The main interest of Prof. Kiriki's research is dynamical system and its applications. During this visit, we discussed about how to apply theoretical results in dynamical systems to the study of dynamical behavior in economic models.

I did benefit a lot from discussing those issues related to the application of dynamic behavior in economic models with Prof. Kiriki and his colleagues. This could be very helpful for polishing the paper based on this project and for my future research. All the suggestions and comments about the paper (either for the model or for its implications) will contribute to the improvement of the paper. I also learned a lot from other people's work. Besides, I also talked about the possibility of future collaboration with Prof. Kiriki and his colleagues.

Overall, I think it is very worthy of visiting the Kyoto University of Education. Besides receiving many feedbacks about my current work, this visit also stimulated my thoughts for future studies.