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外部性、財產稅、禁止開發的威脅
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行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□期中進度報告

外部性、財產稅、禁止開發的威脅

Externality, Property Taxes, and Threat of Development Prohibition

計畫類別：■ 個別型計畫　　□ 整合型計畫
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計畫參與人員　：黃弘毅

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一、中文摘要

本計劃的目的，乃在研究政府如何運用各項租稅及管制措施，來矯正因過度開發對社區居民造成的負面外部性效果。本計劃將假設房地產市場為一完全競爭或寡佔性市場，且土地開發邊際成本隨土地開發規模擴大而增加。由於政府限制土地用途，因此，土地一旦開發，其成本即全部無法回收。在土地未開發前，土地擁有者可得到土地開發後租金收益的某一固定百分比的現金流量。而過度開發所造成的負面外部性，則擬以房地產總開發規模擴大，導致購屋者對房屋建築所提供勞務的滿意程度下降，也因而只願意支付較低租金的效果來表示。土地擁有者同時選擇開發時機與規模，且無法確知開發成本及已開發土地租金收益的未來變動情形。和一個將外部效果內部化的中央規劃者相比，個別開發者會低估其開發行為對目前居民造成的不利影響，因而開發時機會較晚，而開發規模也會較大。

管制者可以採用各種財產稅（如土地稅、土地資本改良稅、或房屋稅）及禁止開發威脅的管制政策，以誘使土地開發者的投資行為和中央規劃者一致。如此一來，本研究計劃可提出下列假說以供實證研究測試：財產稅稅率及威脅程度隨著下列因素應增加或減少：（1）土地開發者越多，（2）租金收益或開發成本的不確定性增加，（3）空地所獲現金流量提升，（4）土地開發成本的預期成長率增加，（5）已開發房地產需求的預期成長率增加，及（6）過度開發的負面外部性效果變大。上述假說也可以提供政府擬定最適財產稅及最適禁止開發威脅管制措施的參考。

關鍵詞：外部性、財產稅、禁止開發的威脅

Abstract

This project combines the literature on real-options with the literature on urban economics to investigate the issue regarding the design of optimal threat of development prohibition and optimal property taxation (such as taxation on land, land improvements, and housing and constructions) when there exists consumption-production externality. This project considers a real estate industry which is perfectly competitive or oligopolistic. Each landowner employs production technology such that development on a larger scale is more costly. Each additional lot of housing generates an external diseconomy to anyone who has already rented a house. However, no one cares who is actually providing the additional lot. In order to characterize this aggregate consumption-production externality, we assume that each renter will pay a lower rent if the industry develops property at a higher density. Landowners choose the timing and the scale of property development at the same time. They are also uncertain about the costs of construction and the rent after development in the future. Each landowner will underestimate the adverse effect of his development decision on the welfare of the other renters, and will thus develop property at a higher density than will be socially optimal.

The regulator can impose property taxes and the threat of development prohibition to induce developers to mimic the behavior of the central planner. As a result, this project can propose the following hypotheses: How should the regulator change the rate of property taxes and the extent of the threat of development prohibition when the following factors are changed: (1) the number of property developers, (2) uncertainty in both the rent for developed property and the construction costs, (3) the return for undeveloped property, (4) the expected growth rate of the construction costs, (5) the expected growth rate of the demand for the developed properties, and (6) the consumption-production externality.

Keywords: Externality, Property Taxes, Threat of Development Prohibition

二、前言

政府在房地產市場所實施的各項租稅及管制措施，會影響建築商的開發意願及民眾的購屋意願。對建築商及一般民眾而言，由於房地產價值佔其總財富極高比重，因此，探討這些政策所造成的影響極為重要。

亨利．喬治（Henry Geroge）在「進步與貧困」（「Progress and Poverty」，1897）一書中認為，土地滋生的所得屬不勞而獲，而應用資金來改良土地，則有助於社會發展。因此，針對土地所課徵的稅，應較針對土地改良資本所課徵的稅高。在最理想的情況下，若對土地課重稅可支應政府支出，則不須再課徵其他任何租稅；這就是所謂土地「單一稅」（single tax）的主張。上述主張相當主觀，也因而頗具爭議性。本研究計劃則擬從傳統經濟學外部性理論的看法，來探討最適財產稅的問題。

三、研究目的

本計劃的目的，乃在研究政府如何運用各項租稅或管制措施，來矯正因過度開發對居民造成的負面外部性效果。本計劃將假設土地擁有者同時選擇開發時機及開發規模，且因政府限制土地的用途，導致建商一旦開發土地，則面臨土地開發決策不可逆轉的問題。當房地產市場的供給呈現非獨佔情況時，個別開發者會低估其開發行為對目前居民所造成的不利影響，因而和一個將外部效果內部化的中央規劃者相比，會選擇一個較晚的時機，來開發較大的規模。

管制者可以採用各種財產稅（如土地稅、土地資本改良稅、或房屋稅）及禁止開發威脅的管制政策，以誘使土地開發者的投資行為和中央規劃者一致。如此一來，本研究計劃可提出下列假說以供實證研究測試：財產稅稅率及威脅程度隨著下列因素應增加或減少：（1）土地開發者越多，（2）租金收益或開發成本的不確定性增加，（3）空地所獲現金流量提升，（4）土地開發成本的預期成長率增加，（5）已開發房地產需求的預期成長率增加，及（6）過度開發的負面外部性效果變大。上述假說也可以提供政府擬定最適財產稅及最適禁止開發威脅管制措施的參考。

四、文獻探討

和本計劃相關的文獻，依其是否考慮土地開發的收益或成本具不確定性的性質，而可歸類為「確定性模型」（certainty model）及實質選擇權模型（real options model）（見Turnbull, 2005a）。以下的討論，也依循此種分類方式。

確定性模型

確定性模型最早係由Shoup（1970）提出。Shoup 探討土地開發時機的選擇，並獲得土地最適開發時機是使得土地價值的成長率恰等於利率的結論。接著，Arnott and Lewis（1979）則同時討論開發時機及開發密度的選擇問題。Mills（1981, 1983）則開始將財產稅引入分析，並指出最適開發密度可能隨時間而改變，進而會影響開發後房屋建築的租金報酬。以上的分析模型為靜態性質，其後的研究，則多採動態分析。其中，Anderson（1986）關心土地開發前、後的財產稅稅率變動如何影響開發時機。Turnbull（1988b）則關心土地稅及土地資本改良稅如何影響開發時機及密度。Anderson（1999）及McFarlane（1999）則同時考慮土地開發前、後的財產稅率以及土地資本改良稅的影響。上述研究顯示，一旦引入開發密度的決策，則各種租稅政策對開發時機和開發密度是否造成正面或負面的影響，乃取決於土地開發後房屋建築的租金報酬是否與時俱增。通常分析的焦點是成長型社區，因此，多半採與時俱增的假設。在此假設下，通常發現（見Anderson, 2005），加重未開發土地的租稅稅率，會促使建商以較小的開發規模提前開發土地。而當土地資本改良稅或房屋稅增加，對開發時機與密度的影響皆不確定。惟若土地開發前、後稅率一致，則當此一齊一稅率增加，會促使建商提前投資，但無法確定對開發密度造成何種影響。

有關禁止開發管制的威脅係由Turnbull（2002）提出。該文發現，不管社區為何種成長型態，當此威脅程度增加，則建商會提前開發土地。而對一個成長型的社區而言，威脅程度增加，會促使建商採較低密集度的開發方式。有些都市經濟學（urban economics）的文獻，也曾討論和市中心的距離，如何影響土地開發的時機與規模，惟並未考慮到外部性的問題。例如，Turnbull（1988a, 1991, 2002, 2005a, b）。曾明顯將外部性引入分析的都市經濟學文獻只有Anderson（1993）一篇。Anderson 討論當空闊綠地對民眾造成外部性利益時，政府應如何補貼。由於認為正面外部性會使空地現金流量增加，因此，Anderson 採用一個隨該流量增加而遞增的函數來代表此外部性。Anderson 從而獲得對空地所課徵的稅率應較已開發土地低，且此差額（亦即補貼率）應等於外部性導致現金流量淨現值的加總，除以開發後房地產的價值。不過，Anderson 並無法確定補貼率是否必然應隨外部性程度增加而增加。Anderson 也討論土地開發後導致擁擠的負面外部性。此時，可透過對其課徵較空地更高的稅率來達成。惟此種稅率的決定因素，遠較對空地的補貼率複雜。Anderson 並未從個體的最適化行為，推導出外部性效果所衍生的影響。反觀本計劃則依循Lee and Jou（2007）

的分析方式，由個人追求效用極大化的行為，推導出過度開發所造成的外部性效果，會導致開發後房地產租金收益的私人價值及其社會價值出現歧異。

實質選擇權法

實質選擇權法強調在不確定性及投資不可逆轉性的交互作用下，廠商具有延後投資的選擇權價值。將實質選擇權研究運用在土地投資決策的文章首推Titman（1985）。Titman以未來土地投資收益呈現二項式（binomial）分配的兩期模型，顯示土地投資收益的不確定性，會使建商延後開發土地。後來陸續有許多文章以連續時間的分析模型，來討論土地投資時機的選擇問題。

Williams（1991）一文同時探討土地投資時機與開發密度的選擇問題。Williams 顯示，土地投資報酬不確定性增加，會使建商延後開發，但開發密度會較大。Williams（1997）一文，則延伸其先前的文章，來考慮土地投資再開發的選擇權，對開發時機與開發密度造成的影響。

Capozza and Li（1994）一文則探討土地開發前、後（如農地轉換為都市用地）所課徵的財產稅，如何影響開發資本密集度與開發時機的選擇。他們假設開發後的房地產規模取決於以土地及資本財為投入的Cobb-Douglas 型態。他們發現，提高開發後的財產稅稅率，會促使建商選取一個較小的資本密集度，並延後開發土地。而提高開發前財產稅的稅率或對開發前、後財產稅設定單一稅率，並提高此稅率，則會促使建商選取一個較小的資本密集度，並提前開發土地。Capozza and Li（2001, 2002）則延伸Capozza and Li（1994）一文，而允許資本及土地兩種生產要素間，呈現固定的替代彈性（ConstantElasticity of Substitution），並從事其理論模型的實證檢測。

禁止開發威脅管制政策的理論模型，由Riddiough（1997）提出。Riddiough 假設土地雖未開發，也可能有所謂的中間利用價值。未開發土地面臨開發權力被政府剝奪的潛在危機，而此威脅可用一個Poisson 過程來表示。一旦政府剝奪開發權利，則政府會適度補償，惟補償金額會小於已開發土地的價值。Riddiough 以數值分析方式顯示，在合理的參數範圍內，政府增加威脅程度，會促使建商提前開發土地。

其他曾探討土地開發時機與規模的選擇，惟並未考慮外部性問題的實質選擇權文獻還包括Clark and Reed（1988），Childs, Ott, and Riddiough（2002），Childs, Riddiough, and Triantis（1996），Cunningham（2004），Grenadier（1996, 2000, 2002），Jou and Lee（2006）。至於處理外部性方式較接近本計劃的為Lee and Jou（2007），惟該文著重的是最適開發密度（optimal development density）管制，而非課稅問題。在（三）研究方法、進行步驟及執行進度的小節，將會仔細介紹該文。

五、研究方法

本研究採用連續時間的不確定性模型，來研究政府如何制定最適的財產稅稅率及最適的禁止開發威脅政策。由於假設土地無法任意變更用途，因此，決策者面對一個單邊（one-sided）的最適控制（optimal control）問題（Harrison and Taksar, 1983）。這類的問題的解法已有一些實質選擇權文獻（Dixit and Pindyck, 1994）及最適控制理論的文獻（Harrison, 1985）可供參考。

六、結果與討論

本計劃衍生出兩篇文章，第一篇文章為「外部性與最適財產稅ー實質選擇權模型在不動產投資的應用」，（Jou and Lee, 2007b）其重要結論如下：

本論文建構一實質選擇權模型，探討政府如何針對房地產廠商開發土地「之前」及「之後」來課徵最適的財產稅。假設房地產產業內有固定家數廠商，在供需均具不確定性的情況下進行土地開發，且開發耗費成本完全無法回復。當廠商開發面積愈大，會減少綠地面積，因而對居民不利。然而，土地持有者會忽視此一負面外部性，因而其開發密度會高於社會最適水準。政府可在房地產廠商開發「之前」及「之後」分別課徵財產稅以矯正此狀況。本模型的結果顯示，雖然廠商開發土地後才造成負面外部性，但最適的「開發後」稅率不一定要高於「開發前」的稅率。

第二篇文章為「Taxation on Land Value and Development When There Are Negative Externalities from Development」（Jou and Lee, 2008），其主要結論如下：

This article employs a real options framework to investigate the design of taxation on both land value and development in a competitive real estate market. We assume that developed properties reduce open space, and thereby harm urban residents. However, ignoring this negative externality, landowners will develop properties sooner than is socially optimal. A regulator can correct this tendency by imposing a positive tax on development or a negative tax on land value. Alternatively, the regulator can implement both instruments simultaneously, in which case an increase in the tax rate on development will be accompanied by an increase in the tax rate on land value, and vice versa.

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八、計劃成果自評

本計劃原先建立寡佔模型，並分別討論（1）使用開發前、後的兩種財產稅；及（2）使用禁止開發的威脅及開發衝擊費兩種政策工具。（1）所用的兩種政策工具獲得「財務金融學刊」的青睞。然而，（2）所用的模型在投稿到JREFE時，卻飽受一名評審的批評。此評審論點有二：首先，房地產市場以假設完全競爭較佳；其次，禁止開發威脅很難數量化，且現實也不可行。因此，依據該評審意見修改後，最後也順利刊登在JREFE 2008年第一期。由於本計劃在短期順利衍生出TSSCI及SSCI期刊各一篇，因此，成果應屬優良。

接 受 函

周教授治邦道鑒：

恭禧台端之大作「外部性與最適財產稅－實質選擇權模型在不動產投資的應用」經二位匿名評審之審查並推薦後，本刊決定接受刊登。

請您就下列四點作最後修正。

1.論文格式請參照隨文附上財務金融學刊文稿格式，請依照格式修改（請您留意參考文獻之格式），並請於 2007 年 2 月 6 日前寄回兩份原稿、電腦磁片及版權移轉聲明書（中、英文稿請以 word 7.0 處理)，以便排版作業。

2.為求統一，中文字請以標楷體，英文、數字、公式以 Times New Roman 字型修改。並請書寫中、英文論文題目、關鍵字、中文（300 字）英文（700 字）摘要，註明 email 帳號，英文地址，電話以及傳真。

3.為求讀者之閱讀方便，請將圖表放置於文中（若磁片無法修改請標示適當位置，並附上清析圖表)。

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感謝您的投稿，希望日後能繼續看到您的大作。若有任何問題，請與本刊助理李雁雯、林韋君聯繫，E-mail: jfs@mba.ntu.edu.tw。謝謝！

敬　頌

研　安

財務金融學刊總編輯

李 存 修　　敬上

2007 年 2 月 6 日

外部性與最適財產稅－實質選擇權模型在不動產投資的應用*

Externality and Optimal Property Taxation: Application of the Real Options Model to Real Estate Investment

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元智大學

摘　要

本論文建構一實質選擇權模型，探討政府如何針對房地產廠商開發土地「之前」及「之後」來課徵最適的財產稅。假設房地產產業內有固定家數廠商，在供需均具不確定性的情況下進行土地開發，且開發耗費成本完全無法回復。當廠商開發面積愈大，會減少綠地面積，因而對居民不利。然而，土地持有者會忽視此一負面外部性，因而其開發密度會高於社會最適水準。政府可在房地產廠商開發「之前」及「之後」分別課徵財產稅以矯正此狀況。本模型的結果顯示，雖然廠商開發土地後才造成負面外部性，但最適的「開發後」稅率不一定要高於「開發前」的稅率。

關鍵詞：開發密度、負面外部性、財產稅、實質選擇權

Abstract

This article investigates the design of property taxation both before and after development in a real options framework where a fixed number of landowners irreversibly develop property in an uncertain environment. We assume that densely developed properties reduce open space, and thereby harm urban residents. However, landowners will ignore this negative externality, and will thus develop properties more densely than is socially optimal. The regulator can correct this tendency by imposing taxation on property both before and after development. It is, however, unclear whether the latter should be taxed at a higher rate than the former even though the negative externality arises only after the property is developed.

Keywords: Development Density, Negative Externality, Property Taxation, Real Options.

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Externality and Optimal Property Taxation

I. Introduction

The issue regarding the relationship between property taxation and choices over the timing and density of development has recently received widespread attention in studies on real estate economics. However, most of these studies investigate the positive aspect of property taxation, i.e., they examine how these two choices are affected by property taxation. Rarely have studies attempted to discuss the normative aspect of property taxation. Few studies, for instance, ask why the regulator uses property taxation in the first place? To our knowledge, the only theoretical article focusing on this issue is that by Anderson (1993).

Anderson constructs a model in which an owner of vacant land extends benefits to urban residents through the provision of open space. He models this external benefit as an increasing function of the cash inflow received by the owner. Anderson then shows that the regulator can correct the externality through the use of a Pigouvian subsidy to the owner, which should be provided at a rate equal to the ratio of the open space externality during the period of development to the value of the developed land in that period.

Like Anderson, this article also investigates the issue regarding the design of the Pigouvian taxes to correct market inefficiencies in the real estate market. This article, however, differs significantly from Anderson in the following two respects. First, this article explicitly derives the private and social values of the cash flow of the developed properties that exhibit negative externalities from congestion. Second, this article considers a landowner who simultaneously chooses the timing and scale of development. Consequently, to align these two choices of the landowner with those of the central planner, the regulator needs to impose two kinds of policy instruments, such as taxation both before and after development. By contrast, Anderson assumes that a landowner chooses only the timing of development, and thus he considers either the property subsidy before development (when an open space externality exists) or property taxation after development (when a congestion externality exists), but not both.

This article considers a real estate industry that consists of homogeneous landowners and renters. We assume that densely developed properties reduce open space, and thereby harm urban residents. However, no one cares who are actually providing these properties. In order to characterize this aggregate consumption–production externality (Tresch, 2002),[1] we assume that renters will pay lower rents if property is developed at a higher density. Landowners will ignore this externality, and will thus develop property at a higher density than is socially optimal. The regulator can correct this tendency by imposing taxation on property both before and after development. It is, however, unclear whether the latter should be taxed at a higher rate than the former even though the negative externality arises only after property is developed.

This article is closely related to the literature on real estate economics that investigates how

[1] See Laffont (1998, pp. 193-200) who provides an example regarding the relationship between the consumption-production externality and optimal taxation.

property taxation affects choices concerning the timing and scale of development in a framework where no uncertainty arises. Shoup (1970) is the first paper that investigates the optimal timing of land development. Arnott and Lewis (1979) later investigate the simultaneous choices regarding the timing and density of land development. Skouras (1978) provides an early analysis indicating that property taxation is non–neutral in its effect on the timing of land development. Mills (1983) further introduces property taxation in a model of competitive equilibrium and investigates the timing effects of taxation on land development. By contrast, Anderson (1986) focuses on how property taxation affects the timing decision of land development. Turnbull (1988b) and McFarlane (1999) further investigate how property taxation affects the choices regarding the timing and density of development. These effects, however, largely depend on the assumption regarding how the demanded density is increasing or decreasing over time (see, e.g., Turnbull, 1988a). Our results are in line with those of the case where the demanded density is increasing over time.

Our results are not, however, fully in line with those of Capozza and Li (1994) who also employ a real options model to investigate the timing and intensity decision of a landowner. Capozza and Li assume that the scale developed by a landowner is of a Cobb–Douglas functional form in land and capital, which implies the same cost functional form as ours. They also assume that the rent per unit of developed property is unrelated to the scale of developed property, which is a polar case of our framework. The key departure from our article is that they assume that the rent per unit of developed property follows an arithmetic Brownian motion, while we assume that it is driven by a geometric Brownian motion. With this departure, they find that an increase in the property–tax on undeveloped land accelerates development and reduces the capital intensity (and thus the development density), which is in line with our results. However, they find that an increase in the property–tax on developed land delays expected developed time and reduces capital intensity. The former is in line with our result, while the latter has just the opposite effect. With uniform tax rates on property both before and after development, they find that an increase in the tax rate accelerates development and reduces capital intensity. The former is not in line, while the latter is in line with our results.

The remainder of this article is organized as follows. Section II presents the basic model. Section III solves choices regarding the timing and density of development for both the decentralized and the centralized economy. We investigate how taxation before and after development affects these two choices in the case of the decentralized economy. We also provide the conditions under which the regulator is able to avoid the long run overbuilding problem when imposing these two kinds of taxation. Section IV shows the comparative statics results regarding how various exogenous forces affect the optimal taxation on property both before and after development. Section V concludes by offering testable implications.

II. The Model

Consider a real estate industry that is composed of N identical risk–neutral landowners. Suppose that at date $t = 0$ this industry has undeveloped land that is normalized at one unit. At any time, i.e. $t \geq 0$, landowner i ($i = 1, \cdots, N$) is able to develop property at a scale equal to q_i, and thus at a density equal to Nq_i, given that each landowner has $1/N$ units of undeveloped land. We also assume that the development cost for landowner i, which is fully irreversible, is equal to (see, e.g., Quigg, 1993; Williams, 1991)[2]

$$C(x_1(t), q_i) = x_1(t) q_i^{\eta}, \tag{1}$$

where $x_1(t)$ is a disturbance term that captures supply shocks, such as unexpected changes in weather or labor market conditions. We allow the housing production technology to be either increasing ($\eta < 1$), constant ($\eta = 1$), or decreasing ($\eta > 1$) returns to scale.

We assume that the rent per unit of developed property is given by

$$R(t) = x_2(t) Q^{b-1} S^{-a}, \quad 1 \geq b > a > 0, \tag{2}$$

where $x_2(t)$ denotes the macroeconomic shock from the demand side, Q is the aggregate demand for developed property, and $S = \sum_{i=1}^{N} q_i$ is the aggregate supply of developed property, which is also equal to the average density of development, given that the industry initially has one unit of undeveloped land. In equation (2), we assume that the rent per unit of developed property will be affected by two different measures of the scale of developed property. First, we assume a non-positive internal effect of Q on $R(t)$, i.e. the rent per unit of developed property is non-increasing with the scale of developed property. This captures the standard non-positive price-quantity relationship in a demand function. Second, we assume a negative external effect of S on $R(t)$ with a size measured by coefficient a; we call this effect external because the utility of a renter will be lower as the aggregate supply of developed property is higher.[3] However, no renters can have an appreciable effect on S, and therefore, all renters will take the external effect as exogenously given when deciding whether to rent developed property. We also assume that $b > a$ to ensure that landowner i's total rent, i.e. $q_i R(t)$, will be increasing with the scale of developed property, but at a decreasing rate.

Both the supply shock, $x_1(t)$, and the demand shock, $x_2(t)$, follow geometric Brownian motions given by

$$dx_i(t) = \alpha_i x_i(t) dt + \sigma_i x_i(t) d\Omega_i(t), \tag{3}$$

where $i = 1, 2$. Each variable $x_i(t)$ has a constant expected rate of growth α_i and a constant variance of the growth rate σ_i^2. Each $d\Omega_i(t)$ is an increment to a standard Wiener process,

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[2] McFarlane (1999) argues that investment on land development will be fully irreversible if demolition costs are extremely high. Similarly, Riddiough (1997) suggests that irreversibility is a reasonable assumption with real estate in which the physical asset is long-lived and switching costs to alternative uses are quite high. Turnbull (2005) argues that the irreversibility assumption may be not realistic, but provides analytically tractable solutions.

[3] The proof is available upon request.

with $E\{d\Omega_i(t)\}=0$, $E\{d\Omega_i(t)\}^2=dt$, and $E(d\Omega_1(t)d\Omega_2(t))=r_{12}\sigma_1\sigma_2 dt$, where $-1\le r_{12}\le 1$.

We assume that an individual landowner chooses the timing and scale of development, assuming that the others' decisions as exogenously determined. Consequently, the market outcome will be inefficient. The policy to be adopted to correct this includes density restrictions, or price controls such as property taxes, building fees, and entitlement fees. We focus on property taxes and abstract from the other instruments. We will assume that property taxes before and after development are denoted by τ_b and τ_a, respectively.[4] By following the literature that applies non–cooperative dynamic games to environmental management (see, e.g., Jou 2001, 2004), we model these two kinds of taxation as a hierarchical game. At the lower level of the game, landowners compete for the choice of the date and scale of land development in a Cournot-Nash environment. At the upper level is a Stackelberg game in which the regulator acts as the leader and a landowner acts as the follower. The regulator should anticipate the timing and scale chosen by the landowner, and then set these two kinds of taxation to induce the landowner to make these two choices at the socially optimal level accordingly.

We assume that the risk-less rate of interest ρ is constant per unit of time and that the undeveloped property per unit has a constant positive return given by net cash inflow per unit of time $\gamma x_2(t)$. We further assume that $\gamma>0$, an assumption implying that a landowner has no option value to abandon the undeveloped property. We also abstract from both the time-to-build problem that usually occurs in the real estate industry (see, e.g., Bar–Ilan and Strange, 1996; Grenadier, 2000), and the redevelopment problem addressed in Williams (1997). Consequently, in what follows, each landowner as well as the central planner will make his respective development decisions once and for all. Our simplified assumptions may be not so realistic, yet they help us gain insights regarding the determinants of optimal property taxation.

III. Choices of the Date and the Density of Development

Without risk of confusions, we use $x_1(t)=x_1$ and $x_2(t)=x_2$ in what follows. Consider any t after land is developed. Given that redevelopment is prohibited, the value of developed property is equal to the time t expected present value of the future cash flow given by

$$W_a(x_2,Q,q_i)=E_t\int_t^\infty [q_iR(s)-\tau_a W_a(x_2(s),Q,q_i)]e^{-\rho(s-t)}ds, \tag{4}$$

where $q_iR(s)$ is the cash inflow for landowner i at instant s, which is derived by multiplying the rent per unit of developed property, $R(s)$ in equation (2), by the scale of developed property he owns, q_i. Equation (4) can be rewritten as[5]

[4] Here and in what follows, we use subscripts "a" and "b" to represent "after" and "before" development, respectively.

[5] Here and in what follows, we assume that $\rho+\tau_a>\alpha_2$ and $\rho+\tau_b>\alpha_2$ so as to ensure that $W_a(\cdot)$ given by equation (4') and $W_b(\cdot)$ given by equation (5) are both finite.

$$W_a(x_2, Q, q_i) = E_t \int_t^{\infty} q_i R(s) e^{-(\rho+\tau_a)s} ds = \frac{q_i x_2 Q^{b-1} S^{-a}}{(\rho + \tau_a - \alpha_2)}. \tag{4'}$$

Denote T as the date at which vacant land is developed. Define $W_b(x_1, x_2, T, q_i)$ as the value of vacant land of a landowner who faces the tax rate from time t to T, which is thus given by

$$\begin{aligned} &W_b(x_1, x_2, T, q_i) \\ &= E_t\{\int_t^T (\frac{\gamma x_2(s)}{N} - \tau_b W_b(x_1(s), x_2(s), T, q_i)) e^{-\rho(s-t)} ds \\ &\quad + \int_T^{\infty} (q_i x_2(s) Q^{b-1} S^{-a} - \tau_a W_a(x_2(s), Q_a, q_i)) e^{-\rho(s-t)} ds - x_1(T) q_i^{\eta} e^{-\rho(T-t)}\}. \end{aligned} \tag{5}$$

Equation (5) indicates that the expected present value of returns to the vacant land is the sum of the after-tax expected present value of rents received until time T and the after–tax expected present value of land rent beginning at the time of development, less the expected present value of the development costs. Following Anderson (1993b), Capozza and Li (1994), and McFarlane (1999), a more tractable expression for the value of vacant land is given by

$$W_b(x_1, x_2, T, q_i) = E_t\{\int_t^T \frac{\gamma x_2(s)}{N} e^{-(\rho+\tau_b)} ds + e^{-(\rho+\tau_b)(T-t)}[\int_T^{\infty} q_i x_2(s) Q^{b-1} S^{-a} e^{-(\rho+\tau_a)(s-T)} ds - x_1(T) q_i^{\eta}]\}. \tag{6}$$

Equation (6) can be further written as

$$W_b(x_1, x_2, T, q_i) = W_b(x_1, x_2, \infty, q_i) + V_d(x_1, x_2, T, q_i), \tag{7}$$

where

$$W_b(x_1, x_2, \infty, q_i) = \frac{\gamma x_2}{N(\rho + \tau_b - \alpha_2)} \tag{8}$$

$$\begin{aligned} &V_d(x_1, x_2, T, q_i) \\ &= E_t\{e^{-(\rho+\tau_b)(T-t)}[\int_T^{\infty} q_i x_2(s) Q^{b-1} S^{-a} e^{-(\rho+\tau_a)(s-T)} ds - \int_T^{\infty} \frac{\gamma x_2(s)}{N} e^{-(\rho+\tau_b)(s-T)} ds - x_1(T) q_i^{\eta}]\}. \end{aligned} \tag{9}$$

In equation (7), the first term on the right–hand side is the value of vacant land if undeveloped forever, i.e., $W_b(x_1, x_2, \infty, q_i)$. Landowner i needs to choose an appropriate timing T and density q_i to maximize the value of the vacant land. This is defined as

$$Z_d(x_1, x_2) = \max_{T, q_i} V_d(x_1, x_2, T, q_i), \tag{10}$$

subject to the evolution of $x_1(t)$ and $x_2(t)$ defined in equation (3).[6] In equation (10), $Z_d(x_1, x_2)$ is the net value of a perpetual warrant to exchange the fixed q_i units of developed

[6] The problem of maximizing $W_b(\cdot)$ in equation (7) is equivalent to that of maximizing $V_d(\cdot)$ because the first term on the right-hand side of equation (7) is unrelated to either T or q_i. Furthermore, here and in what follows, we use subscript "d" to represent "the decentralized economy."

properties for $1/N$ units of vacant land.

Define $W_d(x_1, x_2)$ as the intrinsic value of the warrant if exercised at time t. Substituting $T = t$ into equation (9) yields its value as given by

$$W_d(x_1, x_2) = \max_{q_i} \left\{ W_a(x_2, Q, q_i) - W_b(x_1, x_2, \infty, q_i) - x_1 q_i^{\eta} \right\}, \tag{11}$$

where the term in braces are the value of developed properties, $W_a(x_2, Q, q_i)$, minus the opportunity costs of obtaining it, namely, the value of vacant land if undeveloped forever, $W_b(x_1, x_2, \infty, q_i)$, and the costs of development, $x_1 q_i^{\eta}$. To maximize the intrinsic value at time t, the optimal scale of development, denoted by q_d, must satisfy the first–order condition for q_i:

$$\frac{\partial \left\{ W_a(x_2, Q, q_i) - x_1 q_i^{\eta} \right\}}{\partial q_i} = 0, \tag{12}$$

which says that at the optimal scale of development, the expected marginal benefit of an additional scale of development must be equal to the marginal cost of developing it. Given that the intrinsic value of the warrant, if exercised at the optimal exercise date T, may be denoted by $W_d(x_1(T), x_2(T))$, we can rewrite equation (10) as

$$Z_d(x_1, x_2) = \max_{T} E_t \left\{ e^{-\rho(T-t)} W_d(x_1(T), x_2(T)) \right\}. \tag{13}$$

The solution for $Z_d(x_1, x_2)$ must satisfy the fundamental differential equation of optimal stopping given by

$$\begin{aligned} &\frac{1}{2}\sigma_1^2 {x_1}^2 \frac{\partial^2 Z_d(\cdot)}{\partial {x_1}^2} + r_{12}\sigma_1\sigma_2 x_1 x_2 \frac{\partial^2 Z_d(\cdot)}{\partial x_1 \partial x_2} + \frac{1}{2}\sigma_2^2 {x_2}^2 \frac{\partial^2 Z_d(\cdot)}{\partial {x_2}^2} \\ &+\alpha_1 x_1 \frac{\partial Z_d(\cdot)}{\partial x_1} + \alpha_2 x_2 \frac{\partial Z_d(\cdot)}{\partial x_2} - \rho Z_d(\cdot) = 0. \end{aligned} \tag{14}$$

The solution to equation (14) is given by

$$Z_d(x_1, x_2) = A_{1d} {x_2}^{\beta_{1d}} {x_1}^{1-\beta_{1d}} + A_{2d} {x_2}^{\beta_{2d}} {x_1}^{1-\beta_{2d}}, \tag{15}$$

where A_{1d} and A_{2d} are constants to be determined, and β_{1d}, β_{2d}, and the overall volatility, denoted by σ^2, are respectively equal to

$$\begin{aligned} \beta_{1d} &= \frac{1}{2} - \frac{(\alpha_2 - \alpha_1)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{(\alpha_2 - \alpha_1)}{\sigma^2}\right)^2 + \frac{2(\rho + \tau_b - \alpha_1)}{\sigma^2}}, \\ \beta_{2d} &= \frac{1}{2} - \frac{(\alpha_2 - \alpha_1)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{(\alpha_2 - \alpha_1)}{\sigma^2}\right)^2 + \frac{2(\rho + \tau_b - \alpha_1)}{\sigma^2}}, \\ \sigma^2 &= {\sigma_1}^2 - 2r_{12}\sigma_1\sigma_2 + {\sigma_2}^2. \end{aligned} \tag{16}$$

Landowner i simultaneously chooses the timing and scale of development. As indicated by Dixit and Pindyck (1994, p.139), when uncertainty arises, we are unable to determine a non-stochastic timing. Instead, the development rule takes the form where landowner i will not develop until the supply shock, x_1, declines to a certain level, denoted by x_{1d}, and the demand shock, x_2, rises to another level, denoted by x_{2d}. When these two trigger levels are reached, landowner i will develop vacant land at a scale denoted by q_d. The two critical levels, x_{1d} and x_{2d}, together with A_{1d} and A_{2d} in equation (15), are solved from the boundary conditions given by

$$\lim_{x_2 \to 0} Z_d(x_1, x_2) = 0, \tag{17}$$

$$Z_d(x_{1d}, x_{2d}) = W_d(x_{1d}, x_{2d}), \tag{18}$$

$$\frac{\partial Z_d(x_{1d}, x_{2d})}{\partial x_1} = \frac{\partial W_d(x_{1d}, x_{2d})}{\partial x_1}, \tag{19}$$

$$\frac{\partial Z_d(x_{1d}, x_{2d})}{\partial x_2} = \frac{\partial W_d(x_{1d}, x_{2d})}{\partial x_2}. \tag{20}$$

Equation (17) is the limit condition, which states that the option value of vacant land is worthless as the demand-shift factor approaches zero. Equation (18) is the value-matching condition which states that at the optimal timing of development, landowner i should be indifferent as to whether vacant land is developed or not. Equations (19) and (20) are the smooth-pasting conditions, which require that landowner i not obtain any arbitrage profits from deviating from the optimal timing of development.

Equations (17)-(20) are satisfied by the value function $Z_d(\cdot)$ that is linearly homogeneous in x_1 and x_2, and thus we can define $y = x_2 / x_1$, $z_d(y) = Z_d(x_1, x_2) / x_1$, $w_a(y, Q, q_i) = W_a(x_2, Q, q_i) / x_1$, and $w_d(y) = W_d(x_1, x_2) / x_1$ (Williams, 1991). Note that a higher value of y indicates that the state of nature is better because it comes from a larger value of x_2 and/or a smaller value of x_1, i.e., when demand for developed property is increased and/or the cost of developing land is reduced. Dividing both sides of equation (4') by x_1 yields

$$w_a(y, Q, q_i) = \frac{q_i y Q^{b-1} S^{-a}}{(\rho + \tau_a - \alpha_2)}, \tag{21}$$

while equations (17)-(20) can be rewritten as

$$\lim_{y \to 0} z_d(y) = 0, \tag{22}$$

$$z_d(y_d) = w_d(y_d), \tag{23}$$

$$\frac{\partial z_d(y_d)}{\partial y} = \frac{\partial w_d(y_d)}{\partial y}, \tag{24}$$

where y_d $(= x_{2d} / x_{1d})$ is the development timing chosen by landowner i. Define Q_d as the aggregate scale (density) of development chosen by all landowners as a whole. In Cournot-Nash equilibrium, all landowners will choose the same scale of development such that $Q = S = Nq_d = Q_d$. To solve a landowner's choice of development timing, we can first solve A_{1d} and A_{2d} from equations (22) and (24), respectively. We can then substitute these values and impose the Cournot-Nash equilibrium condition on equation (23). Referring to the result as $T_d^*(y_d, Q_d)$ yields

$$T_d^*(y_d, Q_d) = -(1-\frac{1}{\beta_{1d}})\frac{y_d}{N}[\frac{Q_d{}^{b-a}}{(\rho+\tau_a-\alpha_2)} - \frac{\gamma}{(\rho+\tau_b-\alpha_2)}] + (\frac{Q_d}{N})^{\eta} = 0, \tag{25}$$

where the notation $T_d^*(\cdot)$ represents the condition for the choice of development timing in a decentralized economy.

On the other hand, dividing equation (12) yields

$$\frac{\partial\{w_a(y_d, Q, q_d) - q_d{}^{\eta}\}}{\partial q_i} = 0\,. \tag{26}$$

Imposing the Cournot–Nash equilibrium condition on equation (26), and referring its result as $D_d^*(y_d, Q_d)$ yields

$$D_d^*(y_d, Q_d) = \frac{(N-1+b-a)}{N(\rho+\tau_a-\alpha_2)} y_d Q_d{}^{b-a-1} - \eta(\frac{Q_d}{N})^{\eta-1} = 0\,. \tag{27}$$

Solving equations (25) and (27) simultaneously yields

$$y_d = \frac{\eta(\rho+\tau_a-\alpha_2)}{(N-1+b-a)N^{(\eta-2)}} M_d^{\frac{\eta}{(b-a)}-1}, \tag{28}$$

$$Q_d = M_d^{\frac{1}{(b-a)}}, \tag{29}$$

where

$$M_d = \frac{\gamma(\rho+\tau_a-\alpha_2)}{(\rho+\tau_b-\alpha_2)}[1-\frac{(N-1+b-a)}{\eta N}(1-\frac{1}{\beta_{1d}})^{-1}]^{-1}.^{7} \tag{30}$$

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[7] It is required that the terms inside the brackets on the right-hand side of equation (30) be positive. We adopt this requirement here and in what follows, which is more likely to hold if either a, η, α_1, and ρ are higher or N, σ, and α_2 are lower.

We have obtained analytically tractable solutions for both the choice of date, y_d, and that of density, Q_d. However, to gain more insights regarding how the underlying exogenous forces affect y_d and Q_d, we will focus on both the condition for deriving y_d given by equation (25), and that for deriving Q_d given by equation (27). Equation (25) implicitly defines the positive dependence of y_d on Q_d, and equation (27) implicitly defines the positive dependence of Q_d on y_d. We derive these two relationships in equations (A1)-(A7) in Appendix C.

Proposition 1 stated below indicates how changes in the rates of taxation on property both before and after development affect a landowner's choices of timing and density of development.

Proposition 1: (*a*) *An increase in the property-tax on vacant land accelerates development and reduces development density.* (*b*) *An increase in the property–tax on developed land delays expected development time and raises development density.* (*c*) *With uniform tax rates before and after development, an increase in the tax rate reduces development density, while exhibiting an ambiguous effect on the choice of development timing.*

Proof: See Appendix B.

We explain the intuition behind Propositions 1(a) and 1(b) by using Figures 1 and 2, respectively. The reasoning behind Proposition 1(a) is as follows. Let us consider an increase in the property-tax rate on vacant land. Suppose that the initial equilibrium may be represented by point A, the intersection of lines $D_d D_d$ and $T_d T_d$ in Figure 1, where a landowner develops vacant land at the date y_d^1 and at the density level Q_d^1. As indicated in equation (B4), when the level of development density is fixed, an increase in this tax rate will be associated with less value from waiting, such that the landowner will develop earlier (as shown by the line $T_d T_d$ that shifts downward to line $T_d{}' T_d{}'$). This, in turn, will induce the landowner to develop less densely, as indicated by equation (A4). The equilibrium point thus moves from point A along line $D_d D_d$ to point B, the intersection of the lines $D_d D_d$ and $T_d{}' T_d{}'$. As compared to the initial equilibrium point A, at point B, the landowner develops vacant land at an earlier date y_d^2 ($< y_d^1$), and chooses a smaller density Q_d^2 ($< Q_d^1$) as indicated by equations (B1) and (B2), respectively. The statement in Proposition 1(a) will thus follow.

The reasoning behind Proposition 1(b) is as follows. As shown in Figure 2, suppose that the initial equilibrium is at point A, where a landowner chooses a density equal to Q_d^1, and a date of development equal to y_d^1. The effect of an increase in the property-tax rate on developed land on the timing of development y_d and the density of development Q_d combines the two effects stated below. First, when the level of development density is fixed, as this tax rate is increased, a landowner will develop later because the value of developed properties are decreased, as indicated by equation (B11) (This is shown by the line $T_d T_d$ that shifts upward to the line $T_d' T_d'$). This, in turn, induces the landowner to increase the developed density because the landowner will be bold as long as the state at which he develops land is more favorable, as indicated by equation (A4). The equilibrium point thus moves from point A along line $D_d D_d$ to point B, where the landowner chooses a density equal to Q_d^2 ($> Q_d^1$), and a date of development equal to y_d^2 ($> y_d^1$). Second, as shown by equation (B13), when the timing of development is fixed, a landowner perceives that the net marginal benefit from developing land will be reduced such that the landowner will develop less densely (This is shown by the line $D_d D_d$ that shifts leftward to the line $D_d' D_d'$). This, in turn, will induce the landowner to develop earlier because waiting will then be less valuable when he develops less densely, as indicated in equation (A1). The equilibrium point thus moves from point B along line $T_d' T_d'$ to point C. Summing these two effects may give rise to ambiguous results with regard to the choices of development timing and density. However, more precise calculations suggest that the landowner will develop later ($y_d^3 > y_d^1$) and at a larger density ($Q_d^3 > Q_d^1$), as indicated by equations (B8) and (B9), respectively, as compared to the initial equilibrium point A.

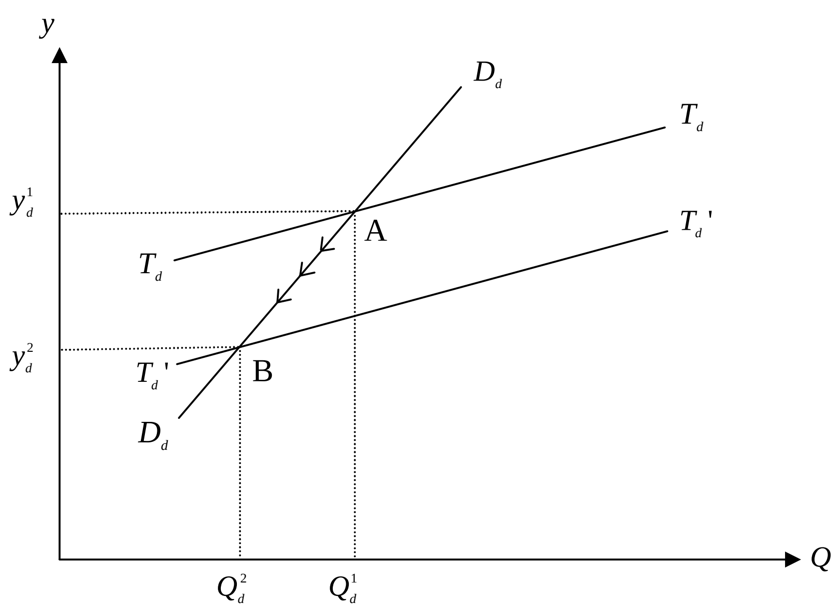

Figure 1: An increase in τ_b.

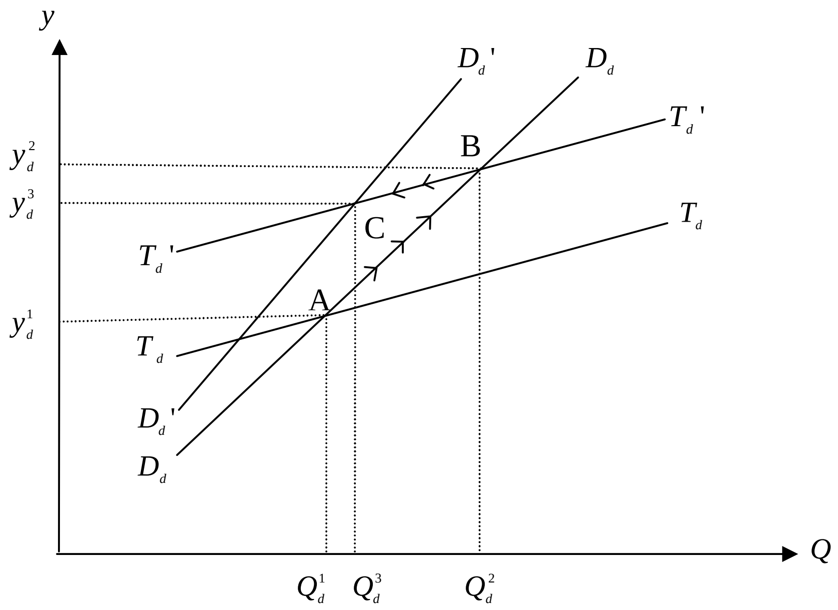

Figure 2: An increase in τ_a.

In Proposition 1(c), we also consider the case where uniform tax rates are imposed both before and after development. We then find that the impact of the property–tax rate on vacant land seems to dominate the choice of development density such that an increase in the tax rate will reduce development density, as indicated by equation (B15). However, the effect resulting from an increase in the tax rate on the choice of development timing is indefinite, as indicated by equation (B14).

We can compare our results with those of several studies that abstract from uncertainty and explore the relationship between taxation and land development. These studies include Anderson (1986), Turnbull (1988b) and Anderson (2005). All of them show that the effect of property taxation on choices regarding the timing and density of land development depends largely on the assumption as to whether the demanded density is increasing over time or not.[8] In particular, they reach the same conclusion as our result stated in Proposition 1(a) for the case where the demanded density is increasing over time, which is analogous to our analysis indicating that land will be developed more densely at a better state, as shown by equation (A4). However, unlike our results in regard to Propositions 1(b) and 1(c), their results with regard to an increase in the property–tax rate after development or an increase of uniform property–tax rate are all indefinite even for this same case. This divergence arises because we impose specific functional forms for both the construction costs and the rent derived from developed property (as shown by equations (1) and (2), respectively), while their results are based on more general functional forms.

[8] A through discussion of this issue can be found in Anderson (2005).

Our results stated in Proposition 1 are not fully in line with those of Capozza and Li (1994) who also employ a real–option model to investigate the timing and intensity decision of a landowner. Assuming that $N=1$, we can compare our results with theirs since they focus on a representative landowner. Capozza and Li assume that the scale developed by a landowner is of a Cobb-Douglas functional form in land and capital, which implies a cost functional form given by equation (1) in our framework. They also assume that the rent per unit of developed property is unrelated to the scale of developed property, which is analogous to imposing $a=0$ and $b=1$ in equation (2) in our framework. The key departure from our article is that they assume that the rent per unit of developed property follows an arithmetic Brownian motion, while we assume that it follows a geometric Brownian motion. With this departure, they find that an increase in the property–tax rate on undeveloped land accelerates development and reduces the capital intensity (and thus the development density), which is in line with our result stated in Proposition 1(a). However, they find that an increase in the property–tax rate on developed land delays that the expected development time and reduces the capital intensity. The former is in line with, while the latter is just the opposite of our results stated in Proposition 1(b). With uniform tax rates on property both before and after development, they find that an increase in the tax accelerates development and reduces capital intensity. The former is not in line, while the latter is in line with our results stated in Proposition 1(c).

This article can also investigate whether an increase in the tax rate before development and that after development ironically lead to a larger expected level of development density. This is stated in the following proposition.

Proposition 2: *Suppose that* $h=2(\alpha_2-\alpha_1)/\sigma^2-1$. (*a*) *An increase in the tax rate before development will result in a lower* (*higher*) *expected level of development density if* $h>(<)-1/(\eta-b+a)$. (*b*) *An increase in the tax rate after development will result in a higher* (*lower*) *expected level of development density if* $h>(<)-1/\eta$.

Proof: See Appendix C.

As shown in Proposition 2, when $h>-1/(\eta-b+a)$, the long-run overbuilding problem will not occur when the regulator raises the tax rate before development. This premise is more likely to hold if (i) developers expect the costs of developing land to decline rapidly (α_1 is relatively low), (ii) developers expect demand for developed property to grow rapidly (α_2 is relatively high), or (iii) the net return derived from land development is less volatile (σ is relatively low). This implies that the regulator is more likely to curb overbuilding under the following circumstances: development will eventually occur in the long–run regardless of whether this policy is implemented, including that future demand or supply conditions are favorable to developers, i.e., low α_1 or high α_2, or that the environment in the real estate market as a whole is stable for developers, i.e., low σ. Under the same circumstances mentioned above, however, an increase in the tax rate after development is more likely to lead to overbuilding in the long–run because the premise $h > -1/\eta$ is more likely to hold. We thus provide the conditions under which the regulator is able to avoid the long–run overbuilding problem when implementing taxation on property both before and after development.

2. The Centralized Economy

Consider the case of the centralized economy. A social planner will internalize the negative externality before choosing the development timing and density.[9] We can thus impose $Nq_i = S = Q$ and $\tau_a = \tau_b = 0$ on equations (1), (2), (4'), and (8), thus yielding the development cost function, the rent per unit of developed property, the value of property after development, and the value of vacant land if undeveloped forever perceived by the central planer, respectively, as given by:[10]

$$C_A(x_1, Q) = x_1(\frac{Q}{N})^{\eta}, \tag{1'}$$

$$R_A = x_2 Q^{b-a-1}, \tag{2'}$$

$$W_A(x_2, Q) = \frac{x_2 Q^{b-a}}{N(\rho - \alpha_2)}, \tag{4''}$$

$$W_B(x_2, \infty) = \frac{\gamma x_2}{N(\rho - \alpha_2)}. \tag{8'}$$

The social planner will choose an appropriate date and scale to maximize the expected present discounted value of the vacant land. This is defined as

$$V_c(x_1, x_2) = W_B(x_2, \infty) + Z_c(x_1, x_2), \tag{7'}$$

where

$$Z_c(x_1, x_2) = \max_{T,Q} E_t\{e^{-\rho(T-t)}[\int_T^{\infty} \frac{x_2(\tau)}{N}(Q^{b-a} - \gamma)d\tau - x_1(T)(\frac{Q}{N})^{\eta}]\}. \tag{10'}$$

In equation (10'), $Z_c(x_1, x_2)$ is the social value of a perpetual warrant to exchange the fixed Q units of developed properties for one unit of vacant land.

Define $W_c(x_1, x_2)$ as the intrinsic value of the warrant if exercised at time t, that is,

$$W_c(x_1, x_2) = \max_{Q} \{W_A(x_2, Q) - W_B(x_2, \infty) - C_A(x_1, Q)\}. \tag{11'}$$

---

[9] We assume that the social planner internalizes the externality, but does not rectify market inefficiencies associated with market power. Lee and Jou (2007) assume that the social planner rectifies market inefficiencies associated with both the negative externality and market power; however, they focus on the issue of the regulation of optimal development density.

[10] Here and in what follows, we use the subscript "c" to represent "centralized economy," while subscript "A" and "B" represent "after" and "before" development in a centralized economy.

To maximize the intrinsic value at time t, the optimal scale, denoted by Q_c, must satisfy the first-order condition for Q:

$$\frac{\partial\{W_A(x_2,Q)-C_A(x_1,Q)\}}{\partial Q}=0\,. \tag{12'}$$

Incorporating equation (11') into (10'), we have

$$Z_c(x_1,x_2)=\max_T E_t\left\{e^{-\rho(T-t)}W_c(x_1(T),x_2(T))\right\}. \tag{13'}$$

Following similar arguments to those for the case of decentralized economy yields

$$Z_c(x_1,x_2)=A_{1c}x_2^{\beta_1}x_1^{1-\beta_1}+A_{2c}x_2^{\beta_2}x_1^{1-\beta_2}, \tag{15'}$$

where A_{1c} and A_{2c} are constants to be determined, and β_{1c} and β_{2c} are respectively equal to

$$\beta_{1c}=\frac{1}{2}-\frac{(\alpha_2-\alpha_1)}{\sigma^2}+\sqrt{(\frac{1}{2}-\frac{(\alpha_2-\alpha_1)}{\sigma^2})^2+\frac{2(\rho-\alpha_1)}{\sigma^2}}, \tag{16'}$$

$$\beta_{2c}=\frac{1}{2}-\frac{(\alpha_2-\alpha_1)}{\sigma^2}-\sqrt{(\frac{1}{2}-\frac{(\alpha_2-\alpha_1)}{\sigma^2})^2+\frac{2(\rho-\alpha_1)}{\sigma^2}}.$$

Define y_c $(=x_{2c}/x_{1c})$ as the timing to develop vacant land and Q_c as the density chosen by the social planner, respectively. Applying similar conditions as shown by equations (22)-(24) and (26), and imposing the constraint $Q=Q_c$ on those conditions yields the counterparts of equations (25) and (27) as given by

$$T_c^*(y_c,Q_c)=-(1-\frac{1}{\beta_1})\frac{y_c}{N(\rho-\alpha_2)}[Q_c^{b-a}-\gamma]+(\frac{Q_c}{N})^{\eta}=0, \tag{25'}$$

$$D_c^*(y_c,Q_c)=\frac{1}{N(\rho-\alpha_2)}y_cQ_c^{b-a-1}-\eta N^{-\eta}Q_c^{\eta-1}=0. \tag{27'}$$

Solving equations (25') and (27') simultaneously yields

$$y_c=\frac{\eta(\rho-\alpha_2)}{(b-a)N^{(\eta-1)}}M_c^{\frac{\eta}{(b-a)}-1}, \tag{28'}$$

$$Q_c=M_c^{\frac{1}{(b-a)}}, \tag{29'}$$

where

$$M_c=\gamma[1-\frac{b-a}{\eta}(1-\frac{1}{\beta_1})^{-1}]^{-1}. \tag{30'}$$

Equation (25') implicitly defines the dependence of y_c on Q_c, while equation (27') implicitly defines the dependence of Q_c on y_c. We derive these two relationships in equations (D1)-(D7) in appendix D.

IV. Optimal Property Taxation

We will first assume that no property taxation is imposed, i.e. $\tau_a = \tau_b = 0$ so as to compare resource allocation under the decentralized economy with that under the centralized economy. Comparing equation (28) with equation (28'), and equation (29) with equation (29') yields $y_d = y_c$ and $Q_d = Q_c$ if both $N = 1$ and $\tau_a = \tau_b = 0$. The more interesting case, however, is where there is more than one landowner in which case we obtain the result stated below:

Proposition 3: *When the real estate market is not monopolized, a landowner will develop property later, but more densely, than a central planner who fully internalizes the consumption-production externality.*

Proof: Proposition 3 will follow given that $M_d > M_c$ for $N \geq 2$, and $\tau_a = \tau_b = 0$.

We use Figure 3 to explain the intuition behind Proposition 3. The same line TT depicts the dependence of the optimal date of development on the optimal density defined in both equation (25) given $\tau_a = \tau_b = 0$ and equation (25'). This is because without any taxation, the existence of consumption-production externalities will be irrelevant to the choice of development timing. This line has a positive slope, thus indicating that, as property in the real estate market becomes more densely developed, both a landowner and the central planner will be less eager to develop property since the rent per unit of developed land will then be lower, and thus waiting will be more valuable. On the other hand, the lines $D_d D_d$ and $D_c D_c$ depict the dependence of the optimal density on the optimal date of development defined in equation (27) given $\tau_a = \tau_b = 0$ and equation (27'), respectively. Both lines have a positive slope, which indicates that, at a better state, both a landowner and the central planner will develop property more densely.

Given that $N \geq 2$ and $\tau_a = \tau_b = 0$, line $D_c D_c$ will lie to the left of line $D_d D_d$. This indicates that, given the same timing of development, a central planner who internalizes the externality will develop property less densely than a landowner who ignores the same externality. The central planner will thus develop earlier than the landowner, as indicated by the positive slope of the line TT. This is shown by Point C that denotes the equilibrium for a central planner, which is where the lines TT and $D_c D_c$ intersect, for which the optimal density is Q_c and the optimal date of development is y_c. By contrast, Point A denotes the equilibrium for a landowner, which is where the lines TT and $D_d D_d$ intersect, and where the optimal density is Q_d^1 and the optimal date of development is y_d^1. Proposition 2 then follows because $Q_d^1 > Q_c$ and $y_d^1 > y_c$.

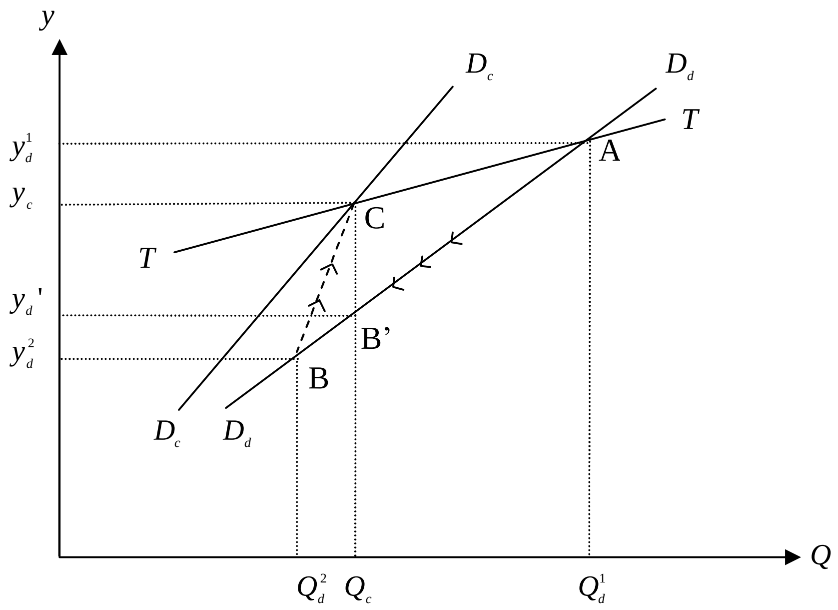

Figure 3: Difference between the centralized and the decentralized economy.

Proposition 3 indicates that the market outcome is inefficient for $N \geq 2$. Therefore, a social planner, who fully perceives the negative externality, can design a property taxation policy to correct this outcome. As mentioned before, our model presents a hierarchical game. At the lower level, a landowner competes with the other landowners, and chooses both a date of development equal to y_d and a density level equal to Q_d in a Cournot-Nash environment. At the upper level, the regulator acts as a leader and the landowner acts as a follower. The regulator anticipates that both the timing and density chosen by the landowner will be above the socially optimal level, y_c and Q_c, respectively. Consequently, the regulator needs to impose property taxation both before and after development so as to align the timing and density chosen by the landowner with those chosen by the central planner. Equating y_d in equation (28) with y_c in equation (28') and equating Q_d in equation (29) with Q_c in equation (29') yields the optimal tax rates after and before development, τ_a^* and τ_b^*, respectively, as given by

$$\tau_a^* = (\rho - \alpha_2)(1 - \frac{1}{N})(\frac{1}{b-a} - 1) > 0, \tag{31}$$

$$[1 + \frac{(\tau_a^* - \tau_b^*)}{(\rho + \tau_b^* - \alpha_2)}][1 - \frac{(b-a)}{\eta}(1 - \frac{1}{\beta_{1c}})^{-1}] = 1 - \frac{(N - 1 + b - a)}{\eta N}(1 - \frac{1}{\beta_{1d}})^{-1}. \tag{32}$$

The design of taxation before and after development can be explained by using Figure 3. As mentioned before, points A and C represent the equilibrium points for the decentralized and the centralized economy, respectively. As indicated by Proposition 1(a), when the regulator imposes a tax rate τ_b^* on property before development, a landowner will be induced to move downward along $D_d D_d$ until point B, where he develops property at a date equal to y_d^2 and a density equal to Q_d^2. As indicated by Proposition 1(b), when the regulator further imposes a tax rate τ_a^* on property after development, the landowner will be induced to move upward from point B to point C, the equilibrium point for the centralized economy. Note that if the central planner only imposes taxation before development, a landowner may be induced to develop on the same scale as the centralized economy, but earlier than will be socially optimal. For example, in Figure 3, a landowner will develop property at point B', where the development density is equal to Q_c, while the development timing is equal to y_d^3 $(< y_c)$.

Table 1 shows how changes in several exogenous variables affect optimal taxation before and after development, which we state in the following proposition:

Proposition 4: (*a*) *The regulator should increase property taxation both before* (τ_b^*) *and after* (τ_a^*) *development if there are more landowners in the real estate market* (N *is larger*). (*b*) *The regulator should increase property taxation after development, but may increase, reduce, or leave unchanged property taxation before development if* (*i*) *the negative externality becomes more significant* (a *is larger*), *or* (*ii*) *demand for developed property is expected to grow more slowly* (α_2 *is smaller*). (*c*) *The regulator should raise property taxation before development, while leaving taxation after development unchanged if* (*i*) *the development costs are expected to grow more rapidly* (α_1 *is larger*), *and* (*ii*) *uncertainty is reduced* (σ *is lower*).

Proof: See Appendix E.

We can use Figure 4 to explain the reason for Proposition 4(a). Suppose that the initial equilibrium is depicted by point A, which is the intersection of lines TT and DD, both of which are the common lines of the centralized economy and the decentralized economy with appropriate property taxation to correct market inefficiencies. When the scale of development is fixed, an increase in the number of landowners will reduce the expected development time by the same magnitude for both the centralized and the decentralized economy, as indicated by equation (E11)

(This is shown by a downward shift from line TT to $T'T'$). Furthermore, when the choice of development timing is fixed, an increase in the number of landowners will induce a landowner to increase the development density by a magnitude that outweighs the increase in the development density for a central planner because the landowner ignores the negative externality. This is indicated by equations (E12) and (E13), and is shown by an outward shift from line DD to lines $D_d'D_d'$ and $D_c'D_c'$ for the decentralized and the centralized economy, respectively. As indicated by Proposition 1, the regulator needs to raise taxation both before and after development because the new equilibrium for the centralized economy, point B, is on the south-western side of the new equilibrium for the decentralized economy, point C. The other results stated in Proposition 3 can also be derived using similar arguments.

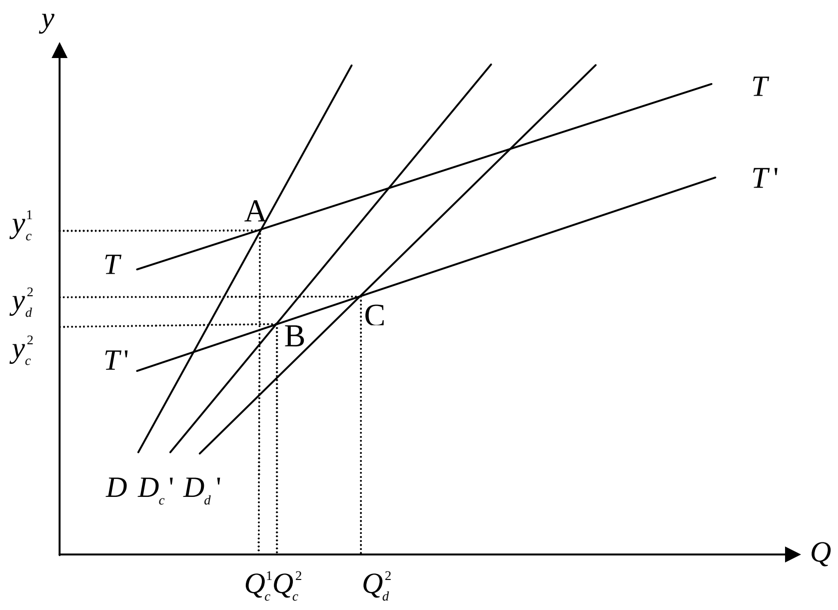

Figure 4: An increase in N.

The results of Proposition 3 (or Table 1) accord well with intuition. First, as the negative externality becomes more severe either by itself (a is larger) or results from an increase in the number of landowners (N is larger), the regulator should raise taxation both before and after development in response.[11] Second, as future demand for developed property is expected to grow more slowly (α_2 is lower), a developer will be induced to develop property earlier, but on a smaller scale as compared to a social planner. The regulator thus needs to raise taxation after development, as indicated by Proposition 1(b) (The impact on taxation before development, however, is indefinite). Third, as the costs of development are expected to grow more rapidly, the optimal condition for the choice of density of development for both a landowner (equation (25)) and a social planner (equation (25')) will not be affected. Consequently, the regulator only needs

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[11] Note that an increase in the magnitude of the externality has a positive impact on taxation before development, τ_b^*, only in the region where $\tau_a^* > \tau_b^*$.

to raise taxation before development so as to encourage a landowner to develop earlier, but on a smaller scale, while leaving taxation after development unchanged. Finally, the total instantaneous volatility (σ) will be greater as r_{12} is lower, i.e., as $x_1(t)$ and $x_2(t)$ move in the opposite directions. That is, uncertainty will be greater if more (less) advantageous supply conditions are associated with more (less) prospective demand conditions in the real estate market. The impact of greater uncertainty resembles that of a lower expected growth rate in relation to the development costs, and thus the regulator only needs to reduce taxation before development.

We assume that developed property exhibits a negative externality on urban residents, while vacant land does not exhibit any externality. To correct this externality, however, it is uncertain whether the rate of property taxation after development τ_a^* should be higher than that before development τ_b^*. We, however, can compare the order of τ_a^* and τ_b^* around the region where $\tau_a^* = \tau_b^*$, as stated in the Proposition below:

Table 1: Comparative-Statics Results

Variable	Property-tax rate before development τ_b^*	Property-tax rate after development τ_a^*
The number of landowners, N	+	+
The size of externality, a	+ if $\tau_a^* > \tau_b^*$	+
The expected growth rate of the rent of developed land, α_2	±	–
The expected growth rate of the costs of development, α_1	+	None
The total instantaneous volatility, σ	–	None

Proposition 5: *Starting from a coincidence of τ_a^* and τ_b^*, τ_a^* should be higher than τ_b^* if (a) the number of landowners increases; (b) the costs of development are expected to grow less rapidly, or the total instantaneous volatility is greater.*

Proof: See Appendix F.

Proposition 5(a) follows because as the number of landowners *increases*, both τ_a^* and τ_b^* will then be raised (as stated in Proposition 4(a)), but the latter will be raised by a smaller magnitudes than the former as indicated by equation (F1). Proposition 5(b) follows because as the costs of development are expected to grow less rapidly, or the total instantaneous volatility is greater (as stated in Proposition 4(c)), τ_b^* will then be lower, while τ_a^* will remain unchanged.

VI. Conclusion

This article investigates the design of property taxation both before and after development in a real options framework where a fixed number of landowners irreversibly develop property in an uncertain environment. We assume that densely developed properties reduce open space, and thereby harms urban residents. However, landowners will ignore this negative externality, and will thus develop properties more densely than is socially optimal. The regulator can correct this tendency by imposing taxation on property both before and after development. We then find that the tax on the former should be increased if the real estate market consists more landowners, the costs of development are expected to grow more rapidly, and uncertainty is less significant. In addition, taxation on the latter should be increased if the real estate market consists more landowners, the externality is more significant, or if demand for property after development is expected to grow less rapidly. Future studies may empirically test these theoretical predictions.

This article can be extended to investigate the issue discussed in Henry George's seminal book *Progress and Poverty* (1897), i.e., taxation on vacant land should be higher than taxation on land development. Brueckner (1986) has investigated how a shift to a graded tax system (where the tax rate is lowered and the land tax rate it raised) affects the level of development, the value of land and the price of housing in the long-run. Anderson (1993b) has extended the Brueckner's analysis by employing a perfect foresight model. His focus is how this shift affects choices regarding the timing and density of development. If we replace taxation on property after development by taxation on development, we not only can investigate the issue discussed in Anderson (1993b), but can also investigate whether there exists a tradeoff between land value taxation and land development taxation when these two instruments are employed to correct the externality associated with land development.

Appendix A:

Totally differentiating equation (25) with respect to Q_d, and using equations (28) -(30) yields

$$\frac{\partial y_d}{\partial Q_d} = \frac{\Delta_{12}}{-\Delta_{11}} > 0, \tag{A1}$$

where

$$\Delta_{11} = \frac{\partial T_d^*(y_d, Q_d)}{\partial y_d} = -\frac{1}{y_d}\left(\frac{Q_d}{N}\right)^{\eta} < 0, \tag{A2}$$

$$\Delta_{12} = \frac{\partial T_d^*(y_d, Q_d)}{\partial Q_d} = \left(1 - \frac{1}{\beta_{1d}}\right)\frac{y_d(\eta - b + a)}{N(\rho + \tau_a - \alpha_2)Q_d}\left(M_d - \frac{\eta\gamma}{(\eta - b + a)}\right) > 0. \tag{A3}$$

Totally differentiating equation (27) with respect to y_d, and using equations (28)-(30) yields

$$\frac{\partial Q_d}{\partial y_d} = \frac{\Delta_{21}}{-\Delta_{22}} > 0, \tag{A4}$$

where

$$\Delta_{22} = \frac{\partial D_d^*(y_d, Q_d)}{\partial Q_d} = -\eta(\eta - b + a)N^{1-\eta}{Q_d}^{\eta-2} < 0, \tag{A5}$$

$$\Delta_{21} = \frac{\partial D_d^*(y_d, Q_d)}{\partial y_d} = \frac{(N - 1 + b - a)}{N(\rho + \tau_a - \alpha_2)}{Q_d}^{b-a-1} > 0. \tag{A6}$$

The Jacobian condition also requires that

$$\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21} > 0. \tag{A7}$$

We depict the impact of Q_d on y_d in equation (A1), and that of y_d on Q_d in equation (A4) by line T_dT_d and line D_dD_d in Figure 1, respectively. Equation (A7) requires that the slope of D_dD_d be steeper than that of T_dT_d, and we find that this requirement is satisfied.

Appendix B:

Totally differentiating both y_d in equation (25) and Q_d in equation (27) with respect to τ_b yields

$$\frac{dy_d}{d\tau_b} = \underset{(-)}{\frac{\partial y_d}{\partial \tau_b}} + \underset{(+)}{\frac{\partial y_d}{\partial Q_d}}\underset{(0)}{\frac{\partial Q_d}{\partial \tau_b}} = \left(\frac{\eta}{(b-a)} - 1\right)\frac{y_d}{M_d}\frac{\partial M_d}{\partial \tau_b} < 0, \tag{B1}$$

$$\frac{dQ_d}{d\tau_b} = \frac{\partial Q_d}{\partial \tau_b} + \frac{\partial Q_d}{\partial y_d}\frac{\partial y_d}{\partial \tau_b} = \frac{Q_d}{(b-a)M_d}\frac{\partial M_d}{\partial \tau_b} < 0, \tag{B2}$$

(0) (+) (−)

where $\frac{\partial y_d}{\partial \tau_b} = \frac{\Delta_{13}}{-\Delta_{11}} < 0,$ (B3)

since

$$\Delta_{13} = \frac{\partial T_d^*(\cdot)}{\partial \tau_b} = -(1-\frac{1}{\beta_{1d}})\frac{y_d\gamma}{N(\rho+\tau_b-\alpha_2)^2} - \frac{y_d}{N}[\frac{Q_d^{b-a}}{(\rho+\tau_a-\alpha_2)} - \frac{\gamma}{(\rho+\tau_b-\alpha_2)}]\frac{1}{\beta_{1d}^2}\frac{d\beta_{1d}}{d\tau_b} < 0, \tag{B4}$$

where $\frac{d\beta_{1d}}{d\tau_b} = \frac{\partial\phi_d(\beta_{1d})/\partial\tau_b}{-\partial\phi_d(\beta_{1d})/\partial\beta} > 0,$ because $\partial\phi_d(\beta_{1d})/\partial\tau_b = 1,$

and $\partial\phi_d(\beta_{1d})/\partial\beta = -\frac{\sigma^2}{2}(2\beta_{1d}-1) - (\alpha_2-\alpha_1) < 0,$

$$\frac{\partial Q_d}{\partial \tau_b} = \frac{\Delta_{23}}{-\Delta_{22}} = 0, \tag{B5}$$

because $\Delta_{23} = \frac{\partial D_d(\cdot)}{\partial \tau_b} = 0,$ (B6)

and $\frac{\partial M_d}{\partial \tau_b} = -M_d[(\rho+\tau_b-\alpha_2)^{-1} + (1-\frac{1}{\eta N}(N-1+b-a)(1-\frac{1}{\beta_{1d}})^{-1})^{-1}B_1] < 0,$

where $B_1 = (N - \frac{(N-1+b-a)}{\eta N})(\sigma^2(\beta_{1d}-1)^2(\beta_{1d}-\frac{1}{2}))^{-1} > 0.$ (B7)

Totally differentiating y_d and Q_d with respect to τ_a yields

$$\frac{dy_d}{d\tau_a} = \frac{\partial y_d}{\partial \tau_a} + \frac{\partial y_d}{\partial Q_d}\frac{\partial Q_d}{\partial \tau_a} = \frac{\eta y_d}{(b-a)(\rho+\tau_a-\alpha_2)} > 0, \tag{B8}$$

(+) (+) (−)

$$\frac{dQ_d}{d\tau_a} = \frac{\partial Q_d}{\partial \tau_a} + \frac{\partial Q_d}{\partial y_d}\frac{\partial y_d}{\partial \tau_a} = \frac{Q_d}{(b-a)(\rho+\tau_a-\alpha_2)} > 0, \tag{B9}$$

(−) (+) (+)

where

$$\frac{\partial y_d}{\partial \tau_a} = \frac{\Delta_{14}}{-\Delta_{11}} > 0, \tag{B10}$$

since $\Delta_{14} = \frac{\partial T_d^*(y_d, Q_d)}{\partial \tau_a} = (1-\frac{1}{\beta_{1d}})\frac{y_d Q_d{}^{b-a}}{N(\rho+\tau_a-\alpha_2)^2} > 0,$ (B11) and $\dfrac{\partial Q_d}{\partial \tau_a} = \dfrac{\Delta_{24}}{-\Delta_{22}} < 0,$ (B12)

since $\Delta_{24} = \dfrac{\partial D_d^*(y_d, Q_d)}{\partial \tau_a} = \dfrac{-(N-1+b-a)}{N(\rho+\tau_a-\alpha_2)^2} y_d Q_d^{\,b-a-1} < 0.$ (B13)

Let $\tau_a = \tau_b = \tau$. Substituting this equality into y_d and Q_d, and then differentiating with respect to τ yields

$$\frac{\partial y_d}{\partial \tau} = \frac{\eta y_d}{(b-a)(\rho+\tau-\alpha_2)} - \left(\frac{\eta}{(b-a)} - 1\right)\frac{B_1 y_d M_d}{\gamma} \gtrless 0, \tag{B14}$$

$$\frac{\partial Q_d}{\partial \tau} = -\frac{B_1 Q_d M_d}{(b-a)\gamma} < 0, \tag{B15}$$

where B_1 is defined in equation (B7).

Appendix C:

Define $h = \dfrac{2(\alpha_2-\alpha_1)}{\sigma^2} - 1$. Following Riddiough (1997), given any $y < y_d$, the probability that y eventually hits y_d in the long run is given by

$$\begin{aligned} F_d(y) &= \quad 1 \quad, \text{ if } h \geq 0, \\ &= \left(\frac{y_d}{y}\right)^h, \text{ if } h < 0. \end{aligned} \tag{C1}$$

The long-run expected level of density is given by multiplying $F_d(y)$ by Q_d, thus yielding

$$\begin{aligned} F_d(y)Q_d &= Q_d, \text{ if } h \geq 0, \\ &= A_d^h Q_d^{1+h(\eta-b+a)} y^{-h}, \text{ if } h < 0, \end{aligned} \tag{C2}$$

where $A_d = \dfrac{\eta(1+u)(\rho-\alpha_2)}{(N-1+b-a)N^{\eta-2}}$.

Differentiating $F_d(y)Q_d$ with respect to τ_b yields

$$\frac{d(F_d(y)Q_d)}{d\tau_b} = \frac{dQ_d}{d\tau_b} < 0, \text{ if } h \geq 0, \tag{C3}$$

$$= (1+h(\eta-b+a))F_d(y)\frac{dQ_d}{d\tau_b} \lessgtr 0, \text{ if both } h < 0 \text{ and } h(\eta-b+a) \gtrless -1,$$

where $\dfrac{dQ_d}{d\tau_b} < 0$ by equation (B1). The statement in Proposition 2(a) will follow since $(\eta-b+a) > 0$. Differentiating $F_d(y)Q_d$ with respect to τ_a yields

$$\frac{d(F_d(y)Q_d)}{d\tau_a} = \frac{dQ_d}{d\tau_a} > 0, \text{ if } h \geq 0, \tag{C4}$$

$$= \frac{F_d(y)Q_d(1+h\eta)}{(\rho+\tau_a-\alpha_2)(b-a)} \frac{>}{<} 0, \text{ if } h \frac{<}{>} -1/\eta.$$

This completes the proof.

Appendix D:

Totally differentiating equation (25') with respect to Q_c and using equations (28')-(30') yields

$$\frac{\partial y_c}{\partial Q_c} = \frac{\Delta'_{12}}{-\Delta'_{11}} > 0, \tag{D1}$$

where

$$\Delta'_{11} = \frac{\partial T_c^*(y_c, Q_c)}{\partial y_c} = -(1-\frac{1}{\beta_{1c}})\frac{(M_c-\gamma)}{N(\rho-\alpha_2)} < 0, \tag{D2}$$

$$\Delta'_{12} = \frac{\partial T_c^*(y_c, Q_c)}{\partial Q_c} = (1-\frac{1}{\beta_{1c}})\frac{y_c(\eta-b+a)}{N(\rho-\alpha_2)Q_c}(M_c - \frac{\eta\gamma}{(\eta-b+a)}) > 0. \tag{D3}$$

Totally differentiating equation (27') with respect to y_c, and using equations (28')-(30') yields

$$\frac{\partial Q_c}{\partial y_c} = \frac{\Delta'_{21}}{-\Delta'_{22}} > 0, \tag{D4}$$

where

$$\Delta'_{22} = \frac{\partial D_c^*(y_c, Q_c)}{\partial Q_c} = -\eta(\eta-b+a)N^{-\eta}Q_c^{\eta-2} < 0, \tag{D5}$$

$$\Delta'_{21} = \frac{\partial D_c^*(y_c, Q_c)}{\partial y_c} = \frac{(b-a)}{(\rho-\alpha_2)N}Q_c^{b-a-1} > 0. \tag{D6}$$

The Jacobian condition also requires that

$$\Delta'_{11}\Delta'_{22} - \Delta'_{12}\Delta'_{21} > 0. \tag{D7}$$

We depict the impact of Q_c on y_c in equation (D1), and that of y_c on Q_c in equation (D4) by line TT and line D_cD_c in Figure 3, respectively. Equation (D7) requires that the slope of line D_cD_c be steeper than that of line TT in Figure 3, and we find that this condition is satisfied.

Appendix E:

Define respectively the left-hand and right-hand sides of equation (32) as $F(\tau_a^*, \tau_b^*; \theta)$ and $G(\tau_a^*, \tau_b^*, \theta)$, where $\theta = N$, a, α_2, α_1, or σ. Totally differentiating τ_a^* in equation (31) and both sides of equation (32) with respect to N yields

$$\frac{d\tau_a^*}{dN} = \frac{1}{N^2}(\rho - \alpha_2)[\frac{1}{(b-a)} - 1] > 0, \tag{E1}$$

$$\frac{d\tau_b^*}{dN} = (\frac{\partial F(\cdot)}{\partial \tau_a^*}\frac{\partial \tau_a^*}{\partial N} - \frac{\partial G(\cdot)}{\partial N})\Delta^{-1} > 0, \tag{E2}$$

since $\Delta = \frac{\partial G(\cdot)}{\partial \tau_b^*} - \frac{\partial F(\cdot)}{\partial \tau_b^*} > 0$

and $\partial G(\cdot)/\partial N < 0.$

Totally differentiating τ_a^* and both sides of equation (32) with respect to a yields

$$\frac{\partial \tau_a^*}{\partial a} = \frac{(\rho - \alpha_2)}{(b-a)^2}(1 - \frac{1}{N}) > 0, \tag{E3}$$

$$\frac{\partial \tau_b^*}{\partial a} = (\frac{\partial F(\cdot)}{\partial \tau_a^*}\frac{\partial \tau_a^*}{\partial a} + \frac{\partial F(\cdot)}{\partial a} - \frac{\partial G(\cdot)}{\partial a})\Delta^{-1} \underset{<}{\overset{>}{-}} 0, \tag{E4}$$

since $\partial F(\cdot)/\partial a > 0$ and $\partial G(\cdot)/\partial a > 0$.

Totally differentiating τ_a^* and both sides of equation (32) with respect to α_2 yields

$$\frac{\partial \tau_a^*}{\partial \alpha_2} = \frac{-1}{N^2}(\frac{1}{(b-a)} - 1) < 0, \tag{E5}$$

$$\frac{\partial \tau_b^*}{\partial \alpha_2} = (\frac{\partial F(\cdot)}{\partial \tau_a^*}\frac{\partial \tau_a^*}{\partial \alpha_2} + \frac{\partial F(\cdot)}{\partial \alpha_2} - \frac{\partial G(\cdot)}{\partial \alpha_2})\Delta^{-1} \underset{<}{\overset{>}{-}} 0, \tag{E6}$$

since $\partial F(\cdot)/\partial \alpha_2 > 0$ and $\partial G(\cdot)/\partial \alpha_2 > 0$.

Totally differentiating τ_a^* and both sides of equation (32) with respect to α_1 yields

$$\frac{\partial \tau_a^*}{\partial \alpha_1} = 0, \tag{E7}$$

$$\frac{\partial \tau_b^*}{\partial \alpha_1} = (\frac{\partial F(\cdot)}{\partial \alpha_1} - \frac{\partial G(\cdot)}{\partial \alpha_1})\Delta^{-1} > 0, \quad \text{since} \quad 0 > \frac{\partial F(\cdot)}{\partial \alpha_1} > \frac{\partial G(\cdot)}{\partial \alpha_1}. \tag{E8}$$

Totally differentiating τ_a^* and both sides of equation (32) with respect to σ yields

$$\frac{\partial \tau_a^*}{\partial \sigma} = 0, \tag{E9}$$

$$\frac{\partial \tau_b^*}{\partial \sigma} = (\frac{\partial F(\cdot)}{\partial \sigma} - \frac{\partial G(\cdot)}{\partial \sigma})\Delta^{-1} < 0, \quad \text{since} \quad \frac{\partial G(\cdot)}{\partial \sigma} > \frac{\partial F(\cdot)}{\partial \sigma} > 0. \tag{E10}$$

In order to explain the results in Proposition 3(a), we can start from an initial equilibrium where $y_d = y_c$, $Q_d = Q_c$. Partially differentiating $T_d^*(y_d, Q_d)$ and $T_c^*(y_c, Q_c)$ respectively with respect to N yields

$$\frac{\partial T_d^*(y_d, Q_d)}{\partial N} = \frac{\partial T_c^*(y_c, Q_c)}{\partial N} = \frac{-(\eta - 1)}{N}(\frac{Q_c}{N})^{\eta} < 0. \tag{E11}$$

Partially differentiating $D_d^*(y_d, Q_d)$ and $D_c^*(y_c, Q_c)$ with respect to N yields

$$\frac{\partial D_d^*(y_d, Q_d)}{\partial N} = \frac{(b-a)}{(\rho - \alpha_2)N^2} y_d Q_d^{b-a-1}[\frac{(1+a-b)N}{(N-1+b-a)} + (\eta - 1)N] > 0, \tag{E12}$$

$$\frac{\partial D_c^*(y_c, Q_c)}{\partial N} = \frac{(\eta - 1)(b-a)}{(\rho - \alpha_2)N^2} y_c Q_c^{b-a-1} > 0. \tag{E13}$$

This completes the proof.

Appendix F:

Totally differentiating equation (32) yields

$$\frac{d\tau_b^*}{d\tau_a^*} = \frac{\partial F(\cdot)/\partial \tau_a^*}{\Delta}, \tag{F1}$$

where the value of equation (F1) is between 0 and 1 and $\Delta > \partial F(\cdot)/\partial \tau_a > 0$ when evaluated at $\tau_a^* = \tau_b^*$. We thus obtain the following comparative-statics result: $0 < d\tau_b^* \big/ d\tau_a^* < 1$. This, together with Table 1, implies the results stated in Proposition 4.

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----- Original Message -----
From: "Kanak Patel" <kp10005@cam.ac.uk>
To: "JBJOU" <jbjou@ntu.edu.tw>
Sent: Monday, March 19, 2007 7:21 PM
Subject: Re: Resubmitting the manuscript

> Dear Jyh-bang,
>
> Thank you for your revised paper and reply to referees' reports. You have
> put in a great deal of effort in revising your paper. I am delighted to
> say that we have accepted your paper for publication in the Special Issue
> of JREFE. I will keep you informed about further development and will
> contact you for proof reading in due course.
>
> Yours sincerely,
>
> Kanak
>
> On Mar 19 2007, JBJOU wrote:
>
>>Dear Patel:
>>   Attached please find two WORD and trwo PDF files, which include two
>> reply letters and the main text and Table of the manuscript (co-author
>> with Tan Lee) "Taxation on Land Value and Development When There Are
>> Negative Externalities from Development," which I would like to resubmit
>> to the special issue of the Journal of Real Estate Finance and Economics.
>> I apologize for the delay of resubmitting this manuscript as I need to
>> wait for an English proofreader to finish editing it. Following the
>> suggestions of the two reviewers, I also change the title of the
>> manuscript. I am looking forward to receiving your response. Thank you
>> for dealing with this manuscript.
>>
>>   Sincerely yours,
>>
>>   Jyh-bang
>>

Taxation on Land Value and Development When There Are Negative Externalities from Development

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Abstract

This article employs a real options framework to investigate the design of taxation on both land value and development in a competitive real estate market. We assume that developed properties reduce open space, and thereby harm urban residents. However, ignoring this negative externality, landowners will develop properties sooner than is socially optimal. A regulator can correct this tendency by imposing a positive tax on development or a negative tax on land value. Alternatively, the regulator can implement both instruments simultaneously, in which case an increase in the tax rate on development will be accompanied by an increase in the tax rate on land value, and vice versa.

Keywords: Negative Externality, Real Options, Optimal Taxation

Introduction

Several recent articles investigate how taxation on the market value of vacant land and on the costs to develop such land affects choices regarding the timing of development. However, few studies discuss the normative aspect of these policy instruments, that is, why the regulator employs them in the first place. To the best of our knowledge, the only theoretical article that focuses on this issue is Anderson (1993a).[1]

Anderson constructs a non-stochastic model in which an owner of vacant land extends benefits to urban residents through the provision of open space. Modeling these benefits as an increasing function of the cash inflow received by the owner, he shows that the regulator can induce the landowner to delay developing property through the use of a Pigouvian subsidy. In this article we investigate an issue similar to Anderson (1993a), but different from Anderson, we derive the policy-maker's optimal property taxation in a stochastic environment.[2]

Specifically, we consider a perfectly competitive real estate industry that consists of homogeneous landowners.[3] We assume that developed properties reduce open space and thereby harm urban residents. Following Anderson (1993a), we assume that this external cost increases as landowners receive more cash flow. Landowners will ignore this externality, and will therefore develop properties sooner than is socially optimal. The regulator can correct

[1] Irwin and Bockstael (2004) also consider externalities involving open space. However, they focus on how a "smart growth" policy (such as a policy that preserves open space) associated with these externalities affects planned development timing rather than on the design of optimal property taxation.
[2] Anderson (1993a) suggests that land development imposes external costs associated with increased traffic congestion, higher density, and attendant problems.
[3] In this article we assume that landowners are also developers. We therefore use both terms interchangeably in what follows.

this tendency by implementing a single instrument such as a positive tax on land development or a negative tax (i.e., a subsidy) on land value. In the first case, the regulator should increase the tax rate on development if the negative externality rises; in the second case, the regulator should decrease the subsidy to land value if the negative externality falls, demand uncertainty increases, demand for developed properties is expected to grow at a higher rate, or the discount rate decreases. Alternatively, the regulator can simultaneously implement both policy instruments. In this case, if the regulator increases the tax rate on development, he should also increase the tax rate (i.e., decrease the subsidy rate) on land value, and vice versa.

Our article is closely related to the real options literature such as Dixit (1991) and especially Grenadier (1995) – while both of these papers abstract from the issue of externalities, they solve the competitive equilibrium associated with the maximization problem faced by a social planner.[4] Similar to Grenadier, we assume that land development requires an initial lump sum construction cost. At any point in time, landowners who have developed properties will rent their properties. The market clearing condition requires that the spot price adjusts so that current supply equals stochastic demand. If the industry's prospects become sufficiently favorable, new developers will find it optimal to develop properties. The decision to enter the real estate industry is analogous to the decision to exercise a call option on a real estate asset where the exercise price is the cost of development: when the value of entering the industry rises to the level of the development cost, new developers will enter

[4] See Capozza and Li (1994), who also employ real options analysis. However, they focus on a representative landowner and investigate how taxation on property values both before and after development affects the landowner's choices regarding development timing and capital intensity.

such that no excess profits will exist.

When property development exhibits negative externalities, however, the optimal development timing decision for the competitive real estate industry and that for the centralized economy are different because the former will ignore the negative externality, while the latter will internalize it. We assume that the decision rule for each is determined by a different kind of social planner: the “naive” social planner ignores the externality and solves for the competitive equilibrium, and the “sophisticated” social planner internalizes the externality and solves for the centralized economy. We then investigate how a regulator may employ a positive tax on development and/or a negative tax on land value to induce the naive social planner to develop properties in accordance with the timing that the sophisticated social planner would choose.

In addition to analyzing the design of optimal property taxation, we also investigate how property taxation affects a developer’s choice of development timing in the competitive equilibrium. We find that a developer will delay development if either the tax rate on land value decreases or the tax rate on development increases. Our results therefore provide further support to Anderson (1986), who abstracts from uncertainty. Given our article focuses only on the development timing choice, our results cannot be directly compared to those articles that allow for the simultaneous choices concerning both the timing and the density (or capital intensity) of land development, such as Arnott (2005), Arnott and Lewis (1979), McFarlane (1999), and Turnbull (1988a; 1988b).

The remainder of this article is organized as follows. “Basic Assumptions” section presents the assumptions in our model. “Optimal Development Timing Choices” section solves the development timing choice for both the competitive real estate industry and the centralized economy. In particular, we investigate how taxation on land value and development affects the timing choice for developers in the real estate industry. “Optimal Taxation on Land Value and Development” section reports the comparative-statics results on how various exogenous forces affect the optimal levels of these two taxation instruments. “Numerical Analysis” section employs plausible parameter values to do numerical analysis. The last section concludes with caveats and suggestions for future research.

Basic Assumptions

In this section we build a model that extends Grenadier (1995). Consider a competitive real estate industry with a large number of landowners, where each landowner owns one unit of vacant land.[5] We assume that the units are small enough and the number of landowners are large enough such that the current total supply of developed properties may be represented as a continuum whose mass at time t is $Q(t)$.

At each point in time, the rent per unit of developed property, $P_d(t)$, which evolves such that it clears the market, is of the constant-elasticity form

$$P_d(t) = X(t)Q(t)^{b-1}, \tag{1}$$

[5] The model will yield identical results if developers are permitted to own more than one unit of vacant land, provided no single developer yields significant market power.

where $1 > b > 0$ and $X(t)$ represents a multiplicative demand shock that evolves according to a geometric Brownian motion:

$$dX(t) = \alpha X(t)dt + \sigma X(t)d\Omega(t), \tag{2}$$

where α is the instantaneous expected percentage change in $X(t)$ per unit time, σ is the instantaneous standard deviation per unit time, and $d\Omega(t)$ is an increment of a standard Wiener process. Such a market is thus characterized by evolving uncertainty in the state of demand for the developed property. We assume that development, which reduces open space, harms urban residents. Similar to Anderson (1993a), we assume that the external cost increases as the cash flow from developed properties increases. Consequently, from the viewpoint of a "sophisticated" social planner who takes this externality into account, the marginal value of an additional developed property is reduced to

$$P_c(t) = (1-a)P_d(t),\ 0 < a < 1, \tag{1'}$$

where a denotes the size of the external effect.[6]

Landowners may add to the supply of developed properties by incurring a one-time construction cost of K that is proportional to the quantity of new units of developed property supplied. We assume that landowners can construct buildings instantly, and thus they earn an instantaneous profit per unit time of $P_d(t)$ per unit of developed property.[7] However, the construction costs, once incurred, are assumed to be irreversibly committed

[6] Here and in what follows, we use the subscripts "d" and "c" to represent the decentralized economy and the centralized economy, respectively.
[7] We do not consider the time-to-build problem. See Grenadier (2000) for a thorough discussion of this issue.

thereafter.

Like Grenadier (1995), we seek to construct a competitive equilibrium in which developers have rational expectations. Such an equilibrium involves the simultaneous determination of developers' rents and entry strategies, assuming that these developers ignore the negative externality from development. Development strategies, the rent process, and expectations are all interdependent and must be mutually consistent in equilibrium. Developers choose strategies so as to maximize the discounted present value of profits less development costs, given their expectations concerning the probability distribution governing the rent of developed property. These strategies, together with the exogenous demand shocks, determine the actual realization of the rent and supply in the real estate industry. If expectations turn out to be rational, then the rent process the developers use to form their strategies and the actual rent process will be the same function of the current state variables. Finally, to be a competitive equilibrium, the present discounted value of equilibrium profits at any point of entry must equal the cost of development at that time.

Such a rational expectations competitive equilibrium can be determined as the solution to a maximization problem. As in Lucas and Prescott (1971) or Dixit (1991), the equilibrium evolves as if to maximize the expected present discounted value of social welfare in the form of consumer surplus. That is, the equilibrium path of rents and quantities of developed properties can be derived from the perspective of a "naive" social planner who ignores the negative externality from development.

The problem for the naive social planner is to choose the path of new supply of developed properties so as to maximize the value of the flow of consumer surplus. This is precisely the same as choosing the path of $Q(t)$. The total flow rate of social surplus in the view of the naive social planner, $S_d[X(t), Q(t)]$, is equal to the area under the following demand curve:

$$S_d[X(t), Q(t)] = \int_0^{Q(t)} X(t) q^{b-1} dq = \frac{X(t) Q(t)^b}{b}. \quad (3)$$

The sophisticated social planner, in contrast, will internalize the externality before developing properties, and thus he takes the total flow rate of social surplus to be given by

$$S_c[X(t), Q(t)] = \frac{(1-a) X(t) Q(t)^b}{b}. \quad (3')$$

We assume that developers are risk neutral, and thus the appropriate discount rate is the risk-free interest rate, ρ. This seemingly restrictive assumption can be relaxed by adjusting the drift rate, α, by a risk premium in the way of Cox and Ross (1976).

When property development exhibits negative externalities as shown in equation (1'), then the market outcome will be inefficient. To correct this, a regulator can adopt policies that include density ceiling control (Lee and Jou, 2007) or Pigouvian taxes such as property taxes, building fees, and entitlement fees. We focus on two kinds of property taxation, a tax on the market value of vacant land and a tax on the development costs, but abstract from the other instruments. We assume that these two policy instruments are already set before vacant land is developed. Denote by τ the tax rate on land value and u the tax rate on development, both of which are constant over time. Note that τ can be either positive or negative (i.e., a subsidy), while u is always positive.

By following the literature that applies non-cooperative dynamic games to environmental management (see, e.g., Jou 2001, 2004), we model land use regulation as a hierarchical game. Developers compete to enter the real estate industry at the lower level of the game. At the upper level the game takes the form of a Stackelberg game in which a regulator acts as the leader and a developer acts as the follower. Anticipating the timing chosen by the developer, the regulator should set a tax on development and/or a negative tax on land value to induce the developer to choose the socially optimal level of development timing.

Optimal Development Timing Choices

In this section we investigate how to design optimal taxation on land value and development by sequentially analyzing the behavior of both the naive social planner and the sophisticated social planner. We show that these planners make different choices regarding the timing of development, because the former ignores the externality while the latter internalizes it.

The Choice of the Naive Social Planner

The maximization problem faced by the naive social planner has two state variables,

$X(t)$ and $Q(t)$. The former evolves exogenously, whereas the latter is subject to control. Suppose that we start at time 0 with $X(0) = X$ and $Q(0) = Q$ given. The problem for the naive social planner is to choose the path $Q(t)$ optimally. As in Pindyck (1988), we approach the solution to this problem by examining the incremental development decision faced by the naive social planner. The opportunity to invest in an additional unit of developed property is analogous to a perpetual American call option. The underlying asset is the value of an extra unit of developed property and the exercise price is the cost of constructing this unit, denoted by K.

Therefore, the solution to the naive social planner's development timing problem involves two steps. First, the value of an extra unit of developed property must be determined. Second, the value of the option to invest in this unit must be determined, together with the decision rule for exercising this option. This decision rule is then the solution to the problem of those developers waiting to enter the real estate industry.

The value of an additional developed property is the present value of the expected flow surplus from this unit, denoted by $\Delta G_d(X, Q)$, which is equal to

$$\Delta G_d(X, Q) = E\int_0^{\infty} e^{-\rho t} \frac{\partial S_d(X(t), Q)}{\partial Q} dt = \frac{XQ^{b-1}}{(\rho - \alpha)}. \tag{4}$$

Having valued this marginal unit, we can now value the option to invest in this unit, denoted by $\Delta F_d(X, Q)$. Using standard contingent-claim valuation methods, it is easy to show that

$\Delta F_d(X, Q)$ satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 \Delta F_d(\cdot)}{\partial X^2} + \alpha X \frac{\partial \Delta F_d(\cdot)}{\partial X} - (\rho + \tau)\Delta F_d(\cdot) = 0. \tag{5}$$

Equation (5) has an intuitive interpretation: if ΔF_d is an asset's value, then the normal return $\rho \Delta F_d$ equals the expected capital gain given by

$$\frac{E(d\Delta F_d)}{dt} = \alpha X \frac{\partial \Delta F_d}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 \Delta F_d}{\partial X^2}, \tag{6}$$

minus taxation on land value denoted by $\tau \Delta F_d$.

The solution to equation (5) is given by[8]

$$\Delta F_d(X, Q) = A_{1d}(Q) X^{\beta_{1d}} + A_{2d}(Q) X^{\beta_{2d}}, \tag{7}$$

where A_{1d} and A_{2d} are constants to be determined, and β_{1d} and β_{2d} are the large and small roots, respectively, of the quadratic equation given by

$$\phi(\beta) = -\frac{1}{2}\beta(\beta - 1) - \alpha\beta + \rho + \tau = 0. \tag{8}$$

Consequently, they are equal to

$$\begin{aligned} \beta_{1d} &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2(\rho + \tau)}{\sigma^2}}, \\ \beta_{2d} &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2(\rho + \tau)}{\sigma^2}}. \end{aligned} \tag{9}$$

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[8] Given this functional form for ΔF_d, we may derive the distribution of equilibrium rent, $P_d(t)$, in equation (2). We do not present the results here because they are quite messy, but they are available from the authors upon request. See also Grenadier (1995), who derives the distribution of the equilibrium rent in a model similar to ours.

As is well known in the real options literature (Dixit and Pindyck, 1994), when uncertainty arises, the development rule takes the form whereby landowners will not develop until the demand shock, X, rises to some threshold level, X_d. This critical level, together with $A_{1d}(Q)$ and $A_{2d}(Q)$ in equation (7), is solved from the boundary conditions given by

$$\lim_{X \to 0} \Delta F_d(X, Q) = 0, \tag{10}$$

$$\Delta F_d(X, Q) = \Delta G_d(X, Q) - (1+u)K, \tag{11}$$

$$\frac{\partial \Delta F_d(X, Q)}{\partial X} = \frac{\partial \Delta G_d(X, Q)}{\partial X}. \tag{12}$$

Equation (10) is the limit condition, which states that the option value of waiting is worthless as the demand shock approaches zero. Equation (11) is the value-matching condition, which states that at the optimal development timing choice, developers should be indifferent as to whether vacant land is developed or not. Equation (12) is the smooth-pasting condition, which requires that developers not obtain any arbitrage profits from deviating from the optimal development timing choice. Solving equations (10) through (12) simultaneously yields

$$A_{2d}(Q) = 0, \tag{13}$$

$$A_{1d}(Q) = \frac{(1+u)K}{(\beta_{1d}-1)}\left[\frac{\beta_{1d}(\rho-\alpha)(1+u)K}{(\beta_{1d}-1)}\right]^{-\beta_{1d}} Q^{-\beta_{1d}(1-b)}, \tag{14}$$

$$X_d(Q) = \frac{\beta_{1d}(1+u)(\rho-\alpha)K}{(\beta_{1d}-1)} Q^{1-b}. \tag{15}$$

Using the supply trigger equation, equation (15), both equilibrium supply, $Q^*(t)$, and equilibrium rent, $P_d^*(t)$, can be expressed as a function of the exogenous state variable:

$$Q^*(t) = \max[(\frac{(\beta_{1d} - 1)}{\beta_{1d}(1+u)(\rho - \alpha)K})^{\frac{1}{1-b}} \cdot \sup\{X(s)^{\frac{1}{1-b}}, 0 \le s \le t\},\ Q(0)], \tag{16}$$

$$P_d^*(t) = X(t)Q^*(t)^{b-1}. \tag{17}$$

Proposition 1, stated below, indicates how changes in both the tax rate on land value and the tax rate on development affect a landowner's choice regarding the timing of development in a competitive equilibrium.

Proposition 1: *A developer will delay development* (X_d *will increase*) *if* (*a*) *demand uncertainty is greater* (σ *increases*), (*b*) *the tax rate on land value decreases* (τ *decreases*), *and* (*c*) *the property-tax rate on development increases* (u *increases*). (*d*) *Given that the tax rate on land value is non-negative, a developer will develop sooner if demand for developed property is expected to grow at a higher rate* (α *increases*). (*e*) *A developer may develop earlier, later, or at the same schedule if the discount rate increases* (ρ *increases*).

Proof: See Appendix A.

The result of Proposition 1(a), i.e., uncertainty delays development, is consistent with the standard result of the real options literature (Dixit and Pindyck, 1994). The result of Proposition 1(b) follows because a decrease in the tax rate on land value raises the option value from waiting, and thus leads to a delay in development. This result is in line with

Anderson (1986), who abstracts from uncertainty but also focuses on only the development timing decision. The result of Proposition 1(c) follows because an increase in the tax rate on development decreases the net value from developing properties, and thus retards development. The results of Propositions 1(b) and 1(c) are not directly comparable with the results of articles such as Arnott (2005), Arnott and Lewis (1979), McFarlane (1999), and Turnbull (1988a; 1988b) because these articles allow landowners to choose the development timing and also the scale (or capital intensity) of development. However, these studies reach the same conclusion as we do in this article when they treat development scale as exogenously given.[9]

The rationale for Proposition 1(d) is as follows. If the regulator does not impose any taxation on land value, then an increase in the growth rate of the demand for developed properties accelerates development, which is the standard result of the real options literature (Dixit, 1991). When the regulator imposes a tax on land value, then we must add the effect stated in Proposition 1(a), i.e., a positive (negative) tax rate on land value accelerates (delays) development, thus reinforcing (offsetting) the effect mentioned above.

Our result stated in Proposition 1(e), which indicates that an increase in the discount rate has an indeterminate effect on the choice of development timing, resembles that of Bar-Ilan and Strange (1999). The ambiguous effect follows because of two mutually conflicting forces: first, the future rewards are discounted more, thus delaying development; second, the

[9] Whether their result is consistent with our result when landowners are allowed to vary the scale (or the capital intensity) of development depends on whether the demanded density is increasing over time.

development costs are also discounted more, thus accelerating development.

The Choice of the Sophisticated Social Planner

Consider the case of the centralized economy in which a sophisticated social planner internalizes the negative externality before choosing the development timing. Following similar arguments to those above for the naive social planner, we derive the sophisticated social planner's marginal value of an additional unit of developed property and value of the option to delay development of this unit as given, respectively, by

$$\Delta G_c(X,Q) = \frac{(1-a)XQ^{b-1}}{(\rho-\alpha)}, \tag{4'}$$

$$\Delta F_c(X,Q) = A_{1c}(Q)X^{\beta_{1c}} + A_{2c}(Q)X^{\beta_{2c}}, \tag{7'}$$

where $A_{1c}(Q)$ and $A_{2c}(Q)$ are constants to be determined, and β_{1c} and β_{2c} are equal to

$$\begin{aligned} \beta_{1c} &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}, \\ \beta_{2c} &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}. \end{aligned} \tag{9'}$$

The optimal development timing is therefore given by

$$X_c(Q) = \frac{\beta_{1c}(\rho-\alpha)K}{(\beta_{1c}-1)(1-a)} Q^{1-b}. \tag{15'}$$

Proposition 2 follows from equation (15').

Proposition 2: The *sophisticated social planner will delay development if* (a) *demand uncertainty is greater* (σ *increases*), (b) *the size of the negative externality is larger* (a *increases*), or (c) *demand for developed properties is expected to grow at a lower rate* (α *decreases*). (d) *The social planner may develop sooner, later, or remain at the same schedule as the discount rate* (ρ) *increases.*

Proof: See Appendix B.

The results of Proposition 2(a), 2(c), and 2(d) are similar to their counterparts in Proposition 1. The result of Proposition 2(b) follows because the sophisticated social planner will delay development so as to mitigate the harmful effect of property development on the welfare of urban residents when the size of the negative externality is greater.

Optimal Taxation on Land Value and Development

Substituting $\tau = u = 0$ into $X_d(Q)$ in equation (15) and comparing the result with $X_c(Q)$ in equation (15') yields $X_d(Q) < X_c(Q)$. This suggests that, in the absence of any taxation, landowners will develop properties sooner than is socially optimal. The regulator can employ a tax on land value and/or a tax on development to correct this tendency. The optimal taxation policy may be derived by equating $X_d(Q)$ in equation (15) and $X_c(Q)$ in equation (15'), and several comparative-statics results can be derived from this equality, as indicated by Proposition 3 stated below.

Proposition 3: (a) *Suppose that a regulator treats taxation on development as given. The regulator should decrease the optimal tax rate on land value, denoted by* τ^**, if the size of the external effect is greater* (a *increases*)*. If the regulator initially taxes* (*subsidizes*) *land value, then the regulator should decrease the optimal tax* (*subsidy*) *rate on land value if demand uncertainty is greater* (σ *increases*)*, future demand for developed properties is expected to grow at a higher rate* (α *increases*)*, and the discount rate is lower* (ρ *decreases*)*.* (b) *Suppose that the regulator treats taxation on land values as given; the impacts of* a, σ, α, *and* ρ *on the optimal tax rate on development, denoted by* u^**, will be just the opposite of their counterparts stated in* (a)*.* (c) *Suppose that a regulator simultaneously implement taxes on both land value and development; the regulator should increase the tax rate on land value if he increases the tax rate on development, and vice versa.*

Proof: See Appendix C.

The results of Proposition 3 are the main contribution of this article, and thus deserve a thorough explanation. First, as the negative externality becomes more severe (a increases), the regulator should either decrease the tax rate on land value or increase the tax rate on development. As a result, developers in the competitive real estate industry will be induced to delay development, as shown by Propositions 1(b) and 1(c), respectively. Second, when demand uncertainty is greater, both a landowner in the competitive real estate industry and the sophisticated social planner will delay development. If the regulator does not impose any tax on land value, then the impact will be the same for both. If the regulator initially taxes (subsidies) land value, then the incentive to delay development will be lower (higher) for the landowner than for the sophisticated social planner. Consequently, the regulator should either decrease the tax (subsidy) rate on land value or increase (decrease) the tax rate on development so as to induce the developer to delay (accelerate) development such that the timing decision matches the timing chosen by the sophisticated social planner. Similar reasoning can be used to explain why the regulator should implement the same policy if either future demand for developed properties is expected to grow at a higher rate or the discount rate declines.

The rationale for Proposition 3(c) is better explained by considering the polar case in which the regulator does not implement any tax on land value. In such a case, we find that the optimal tax rate on development is given by

$$u^* = \frac{1}{1-a} - 1. \tag{18}$$

If the regulator raises the tax rate on development from the level shown by equation (18), then developers in the real estate industry will develop later than is socially optimal, as shown by Proposition 1(c). To restore the social optimum, the regulator must tax land value so as to induce developers to develop sooner, as shown by Proposition 1(b). Consequently, the policy of taxing on land value is complementary to the policy of taxation on development.

In the other polar case, where the regulator does not implement any tax on development, then the optimal policy is to grant a subsidy rate to land value. As shown by Proposition 1(b), this policy will then induce landowners to delay development such that the timing schedule matches the timing chosen by the sophisticated social planner.

Numerical Analysis

We employ numerical analysis to make the theoretical results of the last section more vivid. We begin by establishing a set of benchmark parameter values. We assume that developers in a competitive real estate industry incur a cost of $1 to install one unit of capital stock ($K=1$). Additionally, developers face an industry demand function with a constant elasticity, $1/(1-b)$, equal to 3.33 (i.e., $b=0.7$). The demand-shift factor, X, is expected to grow 2% per year, $\alpha=0.02$, the volatility of this growth rate, σ, is equal to 20% per year, and the (risk-free) discount rate, ρ is equal to 5% per year. Due to the externality of developed properties, the rent per unit of developed property is reduced by 10% ($a=0.1$) from the point of view of a sophisticated social planner. Initially, the supply of developed properties is normalized to be equal to one unit, $Q=1$, and the regulator imposes a 5% tax rate on land value and a 20% tax rate on development ($\tau = 0.05,\ u = 0.2$).

Given these benchmark parameter values, Table 1 reports the results for several endogenous variables: for the decentralized economy, the option value of investing in an additional unit of developed property is equal to 0.6902 ($\Delta F_d = 0.9708$), the value of this additional unit is equal to 1.8902 ($\Delta G_d = 2.1708$), and the development trigger, X_d, is equal to 0.0651. Note that this trigger is lower than that for the centralized economy, X_c, which is equal to 0.0907. To induce landowners to delay development (until the timing chosen by the sophisticated social planner), the regulator should either increase the tax rate on development, u^*, to 67.1%, or decrease the tax rate on land value, τ^*, to 0.499%.

Table 1 also reports the effects of the size of the externality (a), the expected growth rate of the demand-shift factor (α), demand uncertainty (σ), the discount rate (ρ), the tax rate on land value (τ), and the tax rate on development (u) on the endogenous variables, holding all the other parameter values constant.[10] These results are consistent with those of Propositions 1, 2, and 3. First, consider an increase in the size of the externality. As the negative externality from development becomes greater (a increases), the sophisticated social planner will wait for a better state to develop (X_c increases). An increasing negative externality also leads the regulator to either increase the optimal tax rate on development (u^*) or reduce the optimal tax rate on land value (τ^*).

[10] Here we also impose the constraint $\beta_{1d} < 1/(1-b)$, which is required so that the option value to invest, $F_d(X,Q) = \int_0^\infty \Delta F_d(X(s),Q)ds$, will be positive.

Consider next an increase in the demand volatility (σ). For the decentralized economy, the option value of investing in an additional unit of developed property (ΔF_d) and the value of this additional unit (ΔG_d) are both increased. However, both a developer and the sophisticated social planner will be encouraged to develop later since both X_d and X_c increases as σ increases. Given that the regulator initially taxes land value, the impact on the sophisticated social planner is even stronger. Consequently, the regulator should either increase the optimal tax rate on development (u^*) or decrease the optimal tax rate on land value (τ^*) to induce the developer to delay development such that the developer's timing decision matches the timing schedule chosen by the sophisticated social planner.

Consider now an increase in the expected growth rate of the demand-shift factor (α). For the decentralized economy, the option value of investing in an additional unit of developed property (ΔF_d) and the value of this additional unit (ΔG_d) are both increased. A developer and the sophisticated social planner will both be encouraged to develop sooner since X_d and X_c decrease as α increases. Given that the regulator initially taxes land value, the impact on the developer, however, is even stronger. Consequently, the regulator should either increase the optimal tax rate on development or decrease the optimal tax rate on land value to induce the developer to delay development such that the developers' timing decision matches the timing schedule chosen by the sophisticated social planner.

Furthermore, we find that the effects of a decrease in the discount rate (ρ) on the endogenous variables resemble the effects of an increase in the expected growth rate on the demand for developed properties. We therefore do not repeat the discussion here.

Turning next to an increase in the tax rate on land value (τ), for the decentralized economy the option value of investing in an additional unit of developed property (ΔF_d) and the value of this additional unit (ΔG_d) are both decreased. However, a developer will be encouraged to develop sooner because the former will be reduced ever more than the latter, as shown in Table 1, which indicates that X_d decreases with an increase in τ. Therefore, the regulator should also increase the optimal tax rate on development (u^*) to induce the developer to choose the development timing selected by the sophisticated social planner. This also suggests that these two property taxation instruments are complementary to each other, and thus is consistent with the result of Proposition 3(c).

Finally, consider an increase in the tax rate on land development (u). In contrast to changes in the tax rate on land value, for the decentralized economy the option value of investing in an additional unit of developed property (ΔF_d) and the value of this additional unit (ΔG_d) are both increased. However, a developer will be encouraged to delay development because the former is increased by a magnitude larger than the latter. Naturally, the optimal tax rate on land value (τ^*) is also increased as a result of an increase in the tax rate on development.

Conclusion

This article employs a real options framework to investigate the design of taxation on both land value and development in a competitive real estate market. We assume that developed properties reduce open space, and thereby harm urban residents. However, ignoring this negative externality, landowners will develop properties sooner than is socially optimal. A regulator can correct this tendency by imposing a positive tax on development or a negative tax on land value. Alternatively, the regulator can implement these two instruments simultaneously, both of which will complement the other.

Our assumption that property development harms urban residents differs markedly from the idea of Henry George (1897), who advocates that the regulator should tax more on vacant land and less on land development. Based on Henry George's idea, Brueckner (1986) investigates how a shift to a graded tax system (where the tax rate on land development is decreased and the tax rate on land value is increased) affects the level of land development, the value of vacant land, and the price of housing in the long run. Anderson (1999) extends Brueckner's analysis to a perfect foresight model, but focuses on how this shift affects a landowner's choice of development timing. Both papers assume that a regulator collects the same amount of tax revenues when shifting the tax system. This contrasts with our analysis, as we allow tax revenues to vary when the regulator implements optimal property taxation policies. Future studies may investigate whether our theoretical results stated in Proposition 3 continue to hold when we impose constraints on tax revenues.

Appendix A

Differentiating $X_d(Q)$ in equation (15) with respect to σ, τ, u, α, and ρ yields

$$\frac{\partial X_d(Q)}{\partial \sigma} = \frac{-(1+u)(\rho-\alpha)KQ^{1-b}}{(\beta_{1d}-1)^2}\frac{\partial \beta_{1d}}{\partial \sigma} > 0, \tag{A1}$$

$$\frac{\partial X_d(Q)}{\partial \tau} = \frac{-(1+u)(\rho-\alpha)KQ^{1-b}}{(\beta_{1d}-1)^2}\frac{\partial \beta_{1d}}{\partial \tau} < 0, \tag{A2}$$

$$\frac{\partial X_d(Q)}{\partial u} = \frac{\beta_{1d}(\rho-\alpha)K}{(\beta_{1d}-1)}Q^{1-b} > 0, \tag{A3}$$

$$\begin{cases} \dfrac{\partial X_d(Q)}{\partial \alpha} = \dfrac{(\rho+\tau)(1+u)KQ^{1-b}B}{(\rho+\tau-\alpha)} < 0, \text{ if } \tau \geq 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \gtrless 0, \text{ if } \tau < 0, \end{cases} \tag{A4}$$

and

$$\frac{\partial X_d(Q)}{\partial \rho} = (1+u)KQ^{1-b}\left[\frac{\beta_{1d}}{(\beta_{1d}-1)} - \frac{(\rho-\alpha)}{(\beta_{1d}-1)^2}\frac{\partial \beta_{1d}}{\partial \rho}\right] \gtrless 0, \tag{A5}$$

where

$$B = \frac{(\rho-\alpha)}{\beta_{2d}^2}\frac{\partial \beta_{2d}}{\partial \alpha} - \left(1-\frac{1}{\beta_{2d}}\right)\frac{\tau}{(\rho+\tau-\alpha)} \quad , \quad \frac{\partial \beta_{1d}}{\partial \sigma} < 0, \quad \frac{\partial \beta_{1d}}{\partial \tau} > 0, \quad \frac{\partial \beta_{2d}}{\partial \alpha} < 0, \quad \text{and}$$

$\dfrac{\partial \beta_{1d}}{\partial \rho} < 0$.

QED.

Appendix B

Differentiating $X_c(Q)$ in equation (15’) with respect to σ, α, a, and ρ yields

$$\frac{\partial X_c(Q)}{\partial \sigma} = -\frac{(1-a)(\rho-\alpha)KQ^{1-b}}{(\beta_{1c}-1)} \frac{\partial \beta_{1c}}{\partial \sigma} > 0, \tag{B1}$$

$$\frac{\partial X_c(Q)}{\partial a} = \frac{-\beta_{1c}(\rho-\alpha)KQ^{1-b}}{(\beta_{1c}-1)} < 0, \tag{B2}$$

$$\frac{\partial X_c(Q)}{\partial \alpha} = \frac{(1-a)KQ^{1-b}}{\beta_{2c}{}^2} \frac{\partial \beta_{2c}}{\partial \alpha} < 0, \tag{B3}$$

and

$$\frac{\partial X_c(Q)}{\partial \rho} = (1-a)KQ^{1-b}\left[\frac{\beta_{1d}}{(\beta_{1d}-1)} - \frac{(\rho-\alpha)}{(\beta_{1d}-1)^2} \frac{\partial \beta_{1d}}{\partial \rho}\right] \underset{<}{\overset{>}{-}} 0, \tag{B4}$$

where

$$\frac{\partial \beta_{1c}}{\partial \sigma} > 0, \quad \frac{\partial \beta_{2c}}{\partial \alpha} < 0, \quad \text{and} \quad \frac{\partial \beta_{1c}}{\partial \rho} > 0.$$

QED.

Appendix C

Equating $X_d(Q)$ in equation (15) and $X_c(Q)$ in equation (15') yields

$$M(\tau, u) = (1-a)(1+u) - (1-\frac{1}{\beta_{1d}})(1-\frac{1}{\beta_{1c}})^{-1} = 0. \tag{C1}$$

Suppose that $\tau = \tau^*$ and $u = u^*$ satisfy equation (C1). Totally differentiating (C1) with respect to a, σ, α, ρ, and u yields

$$\frac{d\tau^*}{da} = \frac{\Delta_1}{-\Delta} < 0\,, \tag{C2}$$

$$\frac{d\tau^*}{d\sigma} = \frac{\Delta_2}{-\Delta} \underset{<}{\overset{>}{-}} 0, \tag{C3}$$

$$\frac{d\tau^*}{d\alpha} = \frac{\Delta_3}{-\Delta} \gtrless 0, \tag{C4}$$

$$\frac{d\tau^*}{d\rho} = \frac{\Delta_4}{-\Delta} \gtrless 0, \tag{C5}$$

and

$$\frac{d\tau^*}{du} = \frac{\Delta_5}{-\Delta} > 0, \tag{C6}$$

where

$$\Delta = \frac{\partial M}{\partial \tau} = \frac{-\beta_{1c}}{(\beta_{1c}-1)\beta_{1d}{}^2}\frac{\partial \beta_{1d}}{\partial \tau} < 0, \tag{C7}$$

$$\Delta_1 = -(1+u) < 0 \tag{C8}$$

$$\begin{aligned} \Delta_2 = \frac{\partial M}{\partial \sigma} = \frac{4\beta_{1c}(\beta_{1d}-1)(\beta_{1c}-\beta_{1d})}{\sigma\beta_{1d}(\beta_{1c}-1)(2\beta_{1c}-1)(2\beta_{1d}-1)} &< 0, \quad \text{if } \tau > 0, \\ &= 0, \quad \text{if } \tau = 0, \\ &> 0, \quad \text{if } \tau < 0, \end{aligned} \tag{C9}$$

$$\begin{aligned} \Delta_3 = \frac{\partial M}{\partial \alpha} = \frac{2(\beta_{1d}-1)\beta_{1c}}{(\beta_{1c}-1)\beta_{1d}\sigma^2}\left[\frac{1}{\beta_{1d}(\beta_{1d}-1)^2} - \frac{1}{\beta_{1c}(\beta_{1c}-1)^2}\right] &< 0, \quad \text{if } \tau > 0, \\ &= 0, \quad \text{if } \tau = 0, \\ &> 0, \quad \text{if } \tau < 0, \end{aligned} \tag{C10}$$

$$\begin{aligned} \Delta_4 &= \frac{\partial M}{\partial \rho} \\ &= \frac{2\beta_{1c}(\beta_{1d}-1)}{\sigma^2\beta_{1d}(\beta_{1c}-1)}\left[\frac{1}{\beta_{1c}(\beta_{1c}-1)(2\beta_{1c}-1)} - \frac{1}{\beta_{1d}(\beta_{1d}-1)(2\beta_{1d}-1)}\right] > 0, \quad \text{if } \tau > 0, \\ &\qquad = 0, \quad \text{if } \tau = 0, \\ &\qquad < 0, \quad \text{if } \tau < 0, \end{aligned} \tag{C11}$$

and

$$\Delta_5 = \frac{\partial M}{\partial u} = 1 - a > 0\,. \tag{C12}$$

It is easy to prove that the effects of a, σ, α, and ρ on u^* are just the opposite of those shown by equations (C2)-(C5).

QED.

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Table 1. Option Value of Waiting, Investment Value, Development Trigger, and Optimal Taxation on Both Land Value and Development

Benchmark Case: $b = 0.7$, $a = 0.1$, $\alpha = 0.02$, $\sigma = 0.2$, $\rho = 0.05$, $\tau = 0.05$, $u = 0.2$, $Q = 1$, $K = 1$, $\Delta F_d = 0.9708$, $\Delta G_d = 2.1708$, $X_d = 0.0651$, $X_c = 0.0907$, $\tau^* = 0.499\%$, $u^* = 67.1\%$.

	Variation in a				
	0	0.05	0.1	0.15	0.2
X_c	0.0816	0.0859	0.0907	0.0960	0.1020
u^*	50.4%	58.3%	67.1%	76.9%	88.0%
τ^*	1.402%	0.924%	0.499%	0.118%	-0.224%
	Variation in σ				
	0.1	0.15	0.2	0.25	0.3
ΔF_d	0.5413	0.7418	0.9708	1.2271	1.5111
ΔG_d	1.7413	1.9418	2.1708	2.4271	2.7111
X_d	0.0522	0.0583	0.0651	0.0728	0.0813
X_c	0.0667	0.0775	0.0907	0.1062	0.1239
u^*	53.1%	59.6%	67.1%	75.0%	82.9%
τ^*	0.623%	0.558%	0.499%	0.452%	0.415%
	Variation in α				
	0	0.01	0.02	0.03	0.04
ΔF_d	0.6099	0.8000	0.9708	1.2000	1.5165
ΔG_d	1.8699	2.0000	2.1708	2.4000	2.7165
X_d	0.0935	0.0800	0.0651	0.0480	0.0272
X_c	0.1035	0.0967	0.0907	0.0856	0.0813
u^*	32.9%	45.0%	67.1%	113.9%	259.1%
τ^*	1.56%	0.902%	0.499%	0.252%	0.098%
	Variation in ρ				
	0.03	0.04	0.05	0.06	0.07
ΔF_d	1.2000	1.0702	0.9708	0.8921	0.8279
ΔG_d	2.4000	2.2702	2.1708	2.0921	2.0279
X_d	0.0240	0.0454	0.0651	0.0837	0.1014
X_c	0.0605	0.0759	0.0907	0.1052	0.1194
u^*	202.7%	100.5%	67.1%	50.8%	41.3%
τ^*	0.110%	0.278%	0.499%	0.770%	1.088%

Table 1.(Continued) Option Value of Waiting, Investment Value, Development Trigger, and Optimal Taxation on Both Land Value and Development

Benchmark Case: $b = 0.7$, $a = 0.1$, $\alpha = 0.02$, $\sigma = 0.2$, $\rho = 0.05$, $\tau = 0.05$, $u = 0.2$, $Q = 1$, $K = 1$, $\Delta F_d = 0.9708$, $\Delta G_d = 2.1708$, $X_d = 0.0651$, $X_c = 0.0907$, $\tau^* = 0.499\%$, $u^* = 67.1\%$.

	Variation in τ				
	0	0.025	0.05	0.075	0.1
ΔF_d	2.0649	1.2814	0.9708	0.8000	0.6902
ΔG_d	3.2649	2.4814	2.1708	2.0000	1.8902
X_d	0.0979	0.0744	0.0651	0.0600	0.0567
u^*	11.1%	46.2%	67.1%	81.4%	91.9%
	Variation in u				
	0	0.1	0.2	0.3	0.4
ΔF_d	1.3660	1.5026	1.6392	1.7758	1.9124
ΔG_d	2.3660	2.6026	2.8392	3.0758	3.3124
X_d	0.0710	0.0781	0.0852	0.0923	0.0994
τ^*	-0.533%	-0.058%	0.499%	1.156%	1.938%

Note: We do not report values for endogenous variables if they are invariant to changes in exogenous variables, i.e., if they are equal to those of the benchmark case.

出席「Symposium on Risk Management & Property Value at Risk, Cambridge-UNCC」國際學術會議及至英、法參訪之報告

本人於 6 月 13 日 23:55 搭乘 BR87 班機由台北前往巴黎，再經由巴黎搭乘 TP425 班機，並於 6 月 14 日下午 13:40 抵達里斯本，最後於 17:00 抵達本次會議所在地 Pestana Cascais。總計經歷 24 小時，才由台北抵達 Cascais。

6 月 15 日下午 16:00 起，開始由四位來自實務界的人士，介紹 Basel II Accord 及 Property Value-at-Risk。當日下午 19:00，開始本次會議的晚宴。

6 月 16 日上午 8:45 開始本次研討會的正式議程。本次會議涵蓋的內容大致可分為下列三類，第一類為 Price Index，共四篇；第二類為 hedging 及 default risk，共四篇；最後一類為 real options，共三篇。第一場會議主席，Berkeley 經濟系教授 John Quigley 先請大家簡單的自我介紹。

本人在 6 月 16 日上午分別擔任報告人及評論人，所報告的論文為「Neutral Property Taxation Under Uncertainty」，由於先前準備充份，因此，報告起來頗為流暢。UNCC 教授 Steven H. Ott 擔任本文之評論人，他認為本文有下列兩點值得改進：(1)要考慮稅收在各個時間點所面臨的限制，而非只在開發那一刻；(2)數值分析要和美國財產稅現況吻合。在本人報告完畢，許多與會者皆認為本人報告極為成功。

本人接著又評論 Paul De Varies 等人所發表的「A House Price Index based on the SPAR method」一文。雖然該文作者認為，以荷蘭資料而言，SPAR 的 Price Index 比傳統的 Repeated Sales Index 及 Hedonic Index 都要好，本人則對作者建議，此三種 Price Index 不易互相比較，且其他兩種方法也有其優點，並不應任意抹煞之。

在會議結束後，由與會者從主辦單位建議的三篇論文中，投票選舉最佳論文。本人之論文有幸和 Yonghen Deng 及 John Quigley 所發表的論文共同獲得「最佳論文獎」，並各得 500 歐元的獎金。

由於此次會議採每篇文章一小時的密集討論方式，因此，各篇文章優劣與否顯而易見。本人此次有幸獲得最佳論文獎，證明會議前不斷修改論文以及不斷地口說練習是相當重要的。本次會議攜回論文集一本，這些論文也掛在劍橋大學土地經濟學系的 website 上，有興趣者可自行參閱。

於會議結束後，本人於 6 月 18 日至 6 月 26 日轉往英國劍橋大學參訪，並在這段期和 Kanak Patel 教授談論有關瑞士房地產市場價格上限管制所衍生的風險管理研究計劃的合作事宜。

本人於民國 96 年度，在社會政策暨管制政策中心曾提出「租金管制及房地產開發時機及規模決策」的研究計劃。該計劃係針對租金管制如何影響房地產投資決策，而 Kanak 教授則針對此政策如何影響持有房屋的金融機構的風險。因此，本人預計和 Kanak 聯合發表一篇論文。第一部份簡介租金管制的緣由及影響效果；第二部份談租金管制對投資決策的影響；第三部份談租金管制對金融機構風險管理決策的影響。最後一部份，則為結論。

在劍橋九天的時間，大部份皆在 Kanak 教授位於 Magdalene College 的研究室，進行研究計劃的密集討論。在一些空檔時間，也趁機參觀了劍橋大學校園，尤其進入了一些一般遊客不能參訪的學院地方；例如，包括國王學院及聖約翰學院的院內庭園。此外，也有幸拜訪劍橋大學土地經濟學系的一些教授，如 Ian Hodge 及 Shaun Bond 等人。

6 月 27 日至 29 日到倫敦參訪，並和曼徹斯特大學 Dean A. Paxson 教授討論房地產市場衍生性商品風險管理研究計劃的合作事宜。

Paxson 教授雖在 Manchester 大學教書，但其家位於靠近 Heathrow 機場的 Richmond 區。由於 6 月底為暑假期間，因此，有機會至其家中相聚討論。Paxson 教授目前有興題的主題為和房地產商品 Value-at-Risk 相關的議題。目前文獻大多談論金融機構 Value-at-Risk 的議題，而 Paxson 則用典型的 option pricing formula 來談論房地產商品 Value-at-Risk 的衡量。本人建議 Paxson 教授應將房地產及金融機構資產的差異釐清，再來從事房地產商品 Value-at-Risk 的衡量。我們彼此並談論發展出一篇 Working Paper 的可能性。

在倫敦待的時間並不長，大部份係在 Paxson 家中討論；其餘閒暇時間，則至其家中附近的 Richmond 社區、Cass Business School，及 Covet Garden 參觀。

6 月 30 至 7 月 4 日至法國巴黎參訪，和 Michel Baroni (ESSEC Business School 教授) 討論有關歐洲住宅市場的衍生性商品價格指數建構的研究計劃合作事宜。在 Cascais 時已聆聽 Michel Baroni 討論巴黎衍生性商品價格指數建構的研究。此次至巴黎主要是和其討論巴黎之外的衍生性商品價格指數的建構。雖然美國也有此市場，但由於 Baroni 教授對歐洲資料較熟悉，因此，雙方討論到以其他國家如英國及荷蘭的資料，以做為雙方合作 Working paper 的資料。

在法國期間也趁閒暇時間，由 Baroni 教授帶領參觀 Pantheon-Sorbonne 大學以及羅浮宮、聖母院等景點。最後，則於 7 月 5 日 11:20 由巴黎戴高樂機場搭乘 BR88 班機，而在 7 月 6 日 6:35 抵達桃園中正機場。