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VARIABLE SAMPLING RATE CONTROLLER DESIGN FOR BRUSHLESS DC MOTOR

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Abstract

The brushless DC (BLDC) motor uses a few hall sensors to determine the position of the rotor and to subsequently drive the motor. Without additional hardware, the velocity measurement depends on the consecutive sensor outputs. Therefore, the speed measurement frequency in a BLDC motor is velocity dependent. Conventional approach for the BLDC motor control uses shunt resistor to allow constant current sampling rate. However, the sampling time for the velocity measurement is still variable. To solve this problem, this paper proposes a variable sampling frequency observer (VSFO) for the velocity estimation. A constant sampling rate model-based controller can then be build around the proposed VSFO. The paper also presents a conservative controller design procedure to guarantee system stability. Experimental results showed that the proposed control is effective. The VSFO provides an access to accurate velocity measurement. It also offers additional freedom in the control design.

1. Introduction

Variable sampling rate systems arise in situations where servo signals are embedded along the moving trajectory. The systems have to move across the embedded sensors to obtain position or velocity information. Because there is no measurement available between the sensors, special treatment is necessary to ensure proper control.

This paper studied the servo control of a brushless DC motor. The BLDC motor uses three or more Hall-effect sensors to drive the commuting circuit. Because the number of sensor is few, accurate speed information is usually derived from the time required for the motor to go through consecutive sensors. It is obvious that the time between the consecutive measurements depends on the motor speed. Thus, the sampling time is not predetermined. Similar applications are frequently encountered in the industrial applications such as the computer hard disk drive and the CD-ROM servo systems. In both cases the servo information is buried along the data track and the read/write head has to move over certain sector headings to obtain the tracking information. In any case, high performance servo requirements are been imposed upon these systems, and controller design based upon variable rate sampling is becoming necessary.

The servo design for variable-rate sampling systems has recently gained some attention from the servo control community. Previous results have focused on multiple but fixed sampling rate problems [1-6]. Looking at the systems with slow sampling rate, the multi-rate control offers improvement to the system performance. Newer results are also being applied to high performance servo systems [7], [8]. They were able to achieve smoother system responses based upon the fact that control can be updated more frequent than the measurement. Moore *et al.* [9] summarized these control strategies into an N-delay input/output control, and there is already successful implementation [10]. Even though the intervals in this approach do not need to be uniform, the result still requires a fixed "N". As far as variable sampling rate is concern the results are very limited. The unpredicted sampling period really imposes a barrier on the theoretical development. In 1993, Hori published an interesting result in [11]. He considered a pure integrator system, and was able to reduce the speed-position relation into a (time) invariant system. (Thus, guaranteed error convergence.) This result is worth noting; however, it is only valid in case of pure integrator. It is completely understandable that a servo engineer would not be satisfied by such limitation. It is likely that an accurate system model exists and the controller should be able to draw information from it. The studies in [12] and [13] confirm the thought that such approaches maybe beneficial. Of course, there is still need for a more rigorous inspection.

This paper investigated the sampling behavior in the variable sampling rate systems. A novel variable sampling rate observer (VSRO) was proposed. Because the control effort was known to the system, its effect was cancelled out even when no feedback information was available. The structure of the error dynamics showed that proper design of observer gain would be necessary. With the knowledge of the observer structure, it was possible to derive sufficient condition for error convergence. A variable sampling rate observer design procedure based on singular value assignment was proposed. A BLDC motor driver servo design demonstrated the control design procedure. Experimental results showed that the VSRO based control is effective. The observer parameters control the rate at which the observation error converges. The experimental results also showed the design trade off between observation error and control error.

2. System Description

The BLDC motor driver consists of three sets of stator windings placed evenly around the motor perimeter. The rotor is usually made of permanent magnets. Depending on its design, it can be arranged into one or two pairs of poles. There is a Hall effect sensor attached to each stator winding, and there are three pairs of MOSFET circuits to direct the winding current. As the rotor approaches the stator winding, the corresponding hall sensor picks up the approaching magnetic field and sends a signal to the control circuit. The control circuit then determines the appropriate MOSFET to turn on in order to maintain rotation. The power into the motor can be controlled either through varying the supply voltage across the windings, or by Pulse Width Modulating (PWM) the MOSFET driving signal.

Measuring the time it takes between two consecutive Hall sensor signals gives an access to the rotor speed. The measurement is thus dependent on the rotor speed, and the velocity sampling is thus speed dependent.

Similar situations occur in applications like computer disk drive accessing and automobile autopilot. Due to the availability of the micro controller, one can most often describe the plant with an inherent high-speed sampling system (even though the output measurement may not be available at all the sampling instances). The fast sampling rate here can be the sampling frequency of the underlying micro controller. Let the fast sampling period be Δt, one has a fixed sampling rate discrete-time system model

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$

$$y(k) = Cx(k) \qquad (2)$$

where $A = e^{F\Delta t}$, $B = \int_0^{\Delta t} e^{F\tau} G d\tau$, and C is the same as the continuous-time system C.

At this point it is assumed that the system output pair $[A, C]$ is observable, the system model is precisely known, and that the manipulated input $u(k)$ is know at every time instances.

A prediction type discrete-time observer for the fast sampling rate system can take the form

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) \qquad (3)$$

Note $u(k)$ is know even for uneven sampling systems, and (3) can run with or without modification from the measurement.

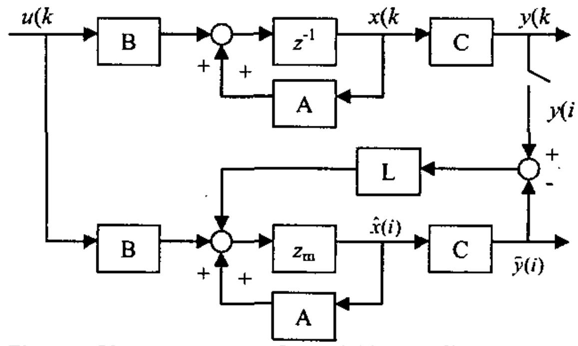

Figure 1 Observer structure for variable sampling rate system

Now consider a variable sampling rate system with "i's" stands for the measurement instances where measurements are available. Let "k_i" stands for the instance when the "i"th measurement is available. "$m_i = k_{i+1}-k_i$" stands for the number of fast samples "Δt" from the instance "i" to the instance "i+1". We can apply a feedback correction term into (3) as [13]

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) & \text{when } k \neq k_i \\ \hat{x}(k+1) = A\hat{x}(k) + Bu(k) & \\ \qquad + L(i)\left[y(i) - C\hat{x}(k)\right] & \text{when } k = k_i \end{cases} \qquad (4)$$

Then, for every instance "i", the system state is

$$x(k_{i+1}) = A^{m_i} x(k_i) + \sum_{j=0}^{m_i-1} A^{m_i-j-1} Bu(k_i + j)$$

The observer equations (4) becomes

$$\hat{x}(k) = A^{m}\hat{x}(k_i) + \sum_{j=0}^{m-1} A^{m-j-1} Bu(k_i + j)$$

for $0 < k = k_i + m < m_i$.

$$\hat{x}(k_{i+1}) = A^{m_i}\hat{x}(k_i) + \sum_{j=0}^{m_i-1} A^{m_i-j-1} Bu(k_i + j)$$

$$+ L(i)(y(k_i) - C\hat{x}(k_i))$$

for $k = k_{i+1}$.

Subtracting (8-10-4) from (8-10-1), and define $e(k) = x(k) - \hat{x}(k)$, we have

$$e(k+1) = Ae(k)$$

for $k \neq k_i$,

$$e(k_i + 1) = Ae(k_i) - L(i)Ce(k_i)$$

for $k = k_i$,

Repeating for $e(k_i+2)$ to $e(k_{(i+1)})$ one have

$$e(k_i+2) = Ae(k_i+1)$$
$$= A[Ae(k_i) - L(i)Ce(k_i)]$$
$$e(k_i+3) = Ae(k_i+2)$$
$$= A^2[A - L(i)C]e(k_i)$$
$$\vdots$$
$$e(k_{i+1}) = Ae(k_i+m_i-1)$$
$$= A^{m_i-1}[A - L(i)C]e(k_i) \tag{5}$$

Equation (5) is the observer error dynamic equation for the variable rate sampling system. Or, in a more explicit form

$$e(i+1) = A^{m_i-1}[A - L(i)C]e(i) \tag{6}$$

The error response can be calculated by

$$e(i) = A^{m_{i-1}-1}[A - L(i-1)C]A^{m_{i-2}-1}[A - L(i-2)C]\cdots$$
$$\cdots\cdots A^{m_0-1}[A - L(0)C]e(0)$$
$$e(i) = \left[\prod_{j=0}^{i-1} A^{m_j-1}[A - L(j)C]\right]e(0) \tag{7}$$

From (8-10-7) it is observed that the system is linear but is time varying. The error is only "modified" every time the system obtains a measurement. Between the measurement, the error is governed by the uncompensated system dynamics, A^{m_j-1}. The variable rate feedback can only access part of the error dynamics. Although, it may or may not be possible to design the last term in the bracket in (7) to suppress any growth in the error by the system dynamics.

Note that if the pair $[A, C]$ is observable, then it is possible to arbitrarily place the poles for $[A - L(j)C]$ with proper design of $L(j)$; however, being able to select $L(j)$ does not guarantee ability to achieve overall error convergence [14]. At this point, we would also like to note that m_i is usually dictated by the system dynamics.

Base on this observation, we propose the observable design as follow [15]:

1. Verify that $[A, C]$ is observable.
2. Take note of m_i when a measurement on the multi-rate sampling system is available.
3. Calculate A^{mi-1} and determine its maximum singular value, $\bar{\sigma}[A^{m_i-1}]$.
4. Determine $L(i)$ via pole placement method such that $\bar{\sigma}[A - L(j)C]\bar{\sigma}[A^{m_i-1}] < 1$.

The observer error dynamics is now stable, and one can make dependable state observation.

Stability Analysis:

In practice, if $m_i < M$, for all i, one possible choice for $L(i)$ is to choose $L(i) = L$, with

$$\rho = \max_i \bar{\sigma}(A^i)\bar{\sigma}(A - LC) < 1, \quad \text{for } 0 < i \le M .$$

Thus,

$$\|e(i)\| = \left\|\left[\prod_{j=0}^{i-1} A^{m_j-1}[A - LC]\right]e(0)\right\|$$
$$\le \left\|\prod_{j=0}^{i-1} A^{m_j-1}[A - LC]\right\|\|e(0)\|$$
$$\le \left[\prod_{j=0}^{i-1}\left\|A^{m_j-1}[A - LC]\right\|\right]\|e(0)\|$$
$$\le \rho^i\|e(0)\| \to 0, \text{as } i \to \infty .$$

The remaining question is how does one realize an L such that

$$\bar{\sigma}(A - LC) < \frac{1}{\max_{0<i<M} \bar{\sigma}(A^i)} .$$

Notice that the maximum singular values for the discrete-time system matrix A_d^i does not increase with i monotonically. It is important to use the maximum among the maximum singular values. Once also see the need of singular value assignment in the observer design [15],[16],[17]. At this point, it is worthwhile to note that not all the singular values in A_d-L_dC_d can be reassigned. The following design procedure helps the practice engineer to perform singular value assignment [18].

Design procedure:

1. Consider the system matrix pair $[A_d^T, C_d^T]$ is controllable. Define the singular value decomposition of C_d^T by $\tilde{P}C_d^TV = \begin{bmatrix}\Sigma_c \\ 0\end{bmatrix}$, where $\Sigma_C = \text{diag}(\rho_1, \rho_2, \ldots, \rho_m)$ is a diagonal matrix.
2. Let $P = \begin{bmatrix}0 & I_{n-m} \\ I_m & 0\end{bmatrix}$, where n is the dimension of the system and m is the dimension of the output, and partition $P\tilde{P}A_d^T = \begin{bmatrix}A_1 \\ A_2\end{bmatrix}$, compatibly with $P\tilde{P}C_d^TV = \begin{bmatrix}0 \\ \Sigma_C\end{bmatrix}$.
3. Find singular value decomposition of A_1, $WA_1Z_1 = \begin{bmatrix}\Sigma_1 & 0 \\ 0 & 0\end{bmatrix}$, where $\Sigma_1 = \text{diag}(\sigma_1, \cdots, \sigma_l)$.
4. Partition $A_2Z_1 = [A_{21} \quad \tilde{A}_{22}]$ compatibly with Z_1 A_1W and find a QR-decomposition of A_2 such that $\tilde{A}_{22}Z_2 = [A_{22} \quad 0]$ and that A_{22} is of full column rank.
5. Select $\Sigma_2 = diag(\sigma_{l+1}, \cdots, \sigma_n)$, where σ_i's are the

desired singular values that can be assigned by designing L_d.

6. Select

$$L_d = \left[\Sigma_C^{-1} A_{21} \quad -\Sigma_C^{-1} \left(\begin{bmatrix} \Sigma_2 & 0 \\ 0 & 0 \end{bmatrix} - [A_{22} \quad 0] \right) \right]^T V^T .$$

3. The PDF Controller

With the estimated speed from the observer available, it is now possible to design a conventional model based fixed rate controller. This experiment concerns the servo application; therefore the popular PDF controller is adopted.

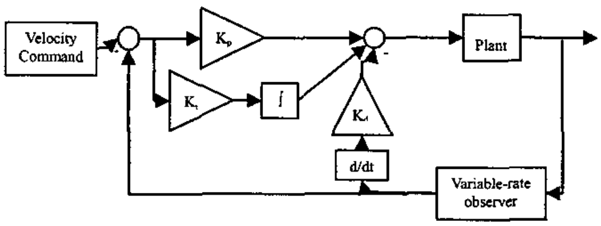

Figure 2 PDF Controller with VSRO

The controller uses the observed value for the system output. Because the observer tolerates variable sampling input, the actual system output is sampled when the signal is available. The output from the VRFO, however, is always available for the constant rate PDF controller. Instead of the conventional error derivative feedback, the PDF controller uses the output derivative directly, and it is known to be very robust compared with other control algorithms.

4. Experiment Setup

The experiment setup is based on a industrial BLDC motor.

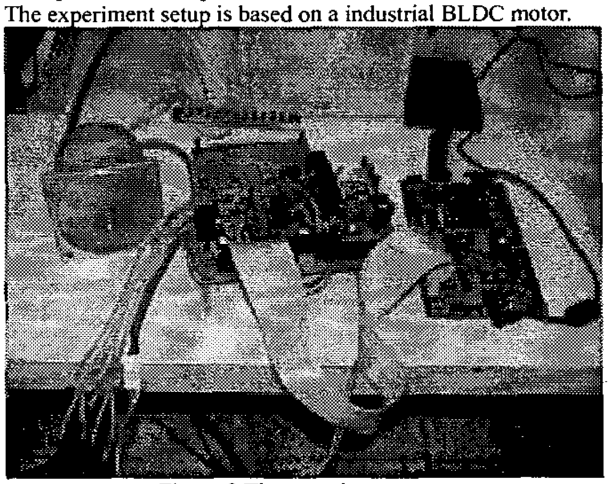

Figure 3 The experiment setup

The motor in the experiment is a KUMHO product. The rated voltage is DC 24V and rated current is 8.0 A. The torque output is 6 Kg-cm. It has three Hall sensor output which sends out a signal change for every quarter circle. Each Hall sensor produces six changes per cycle and there are a total of 24 changes per cycle. The motor takes three voltage input to supply for a three phase excitation winding. The windings must be excited in correct sequence for the motor to run normally. The control logic in the driver is implemented with a TMS320F243 DSP EVM system. The DSP is capable of generating PWM signal according to a built-in logic and it can direct the PWM signal through three output ports for the three phase driver MOSFET. The board uses a JTAG connection to communicate to the RS232 port of a PC. This port allows the downloading and the editing of the firmware.

5. Experimental Results and Discussion

The dynamic behavior of the BLDC basically exhibit first order system characteristics. Therefore, a conservative VSFO is easy to obtain, and the observer gain is a scalar. Following the conventional approach for the PDF implementation and use K_p = 180, K_i = 2, and K_d = 100 with no VSFO, Figure 4 gives the response that is generally expected. The curve in figure 4 is the measurement result using the internal clock of the DSP. The slow speed portion is not accurate because the measurement is not readily available. At 1000 rpm, the sensor signal arrives at 12000 spm or 200 Hz. This is a hardly meaningful for high performance servo control.

The following experiments implemented the VSFO with gradually changing observer gain. Figures 5 – 7 shows the responses when the observer gain is set at **L** = 1.9, 0.8, and 0.1. The feedback gain becomes high when the observer gain is set at a large value. One can see from figure 5 that high observation gain leads to faster observer response. Even though the observation error still converges, there is no guarantee for good error performance. When the observer gain is set at a slightly lower value of 0.8, figure 6 shows that the observation is more accurate. However, the feedback action seems to excite oscillation from the response. Further reduce the observer gain, one can see from figure 7 that, the observation error may suffer a little, but the overall result is a very smooth constant speed control. Notice that the smooth control is obtained with a pseudo 1KHz feedback control loop based upon a VSFO with measurement at less then 200 Hz interval.

6. Conclusions

This paper proposed a variable sampling frequency observer for the velocity estimation. The observer takes variable sampling rate measurement and made the observed output available at all time. Therefore, it is possible to implement the conventional the conventional constant sampling rate model based control algorithm. A conservative observer design procedure is also presented to allow observation error convergence. Experimental results on a commercially available BLDC motor showed that the proposed control is effective. The VSFO provides an access to accurate velocity measurement. It also offers additional freedom in the control design.

7. Acknowledgements

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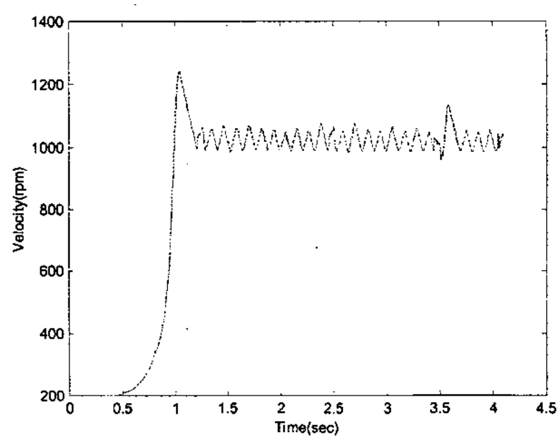

Figure 4 The BLDC control with no VSFO

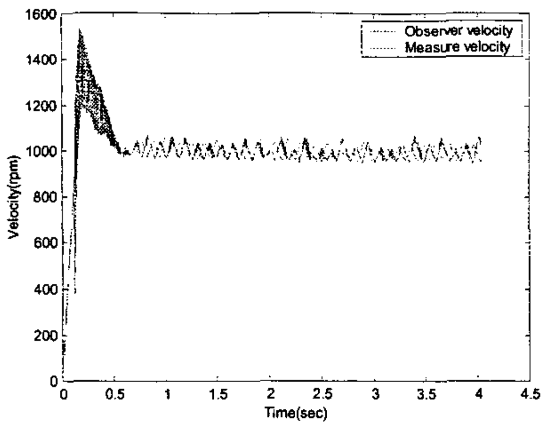

Figure 5 PDF-VSFO control with k_p = 180, k_i = 2, k_d = 100, and **L** = 1.9

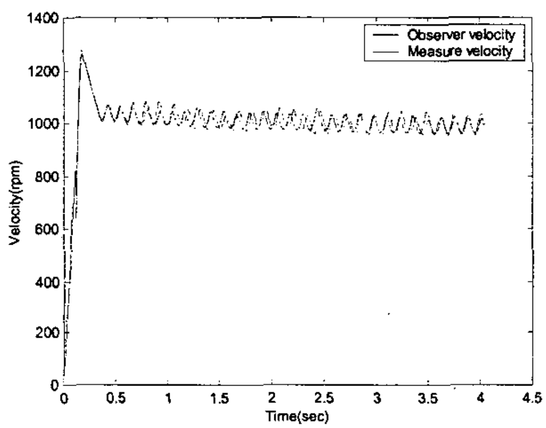

Figure 6 PDF-VSFO control with k_p = 180, k_i = 2, k_d = 100, and **L** = 0.8

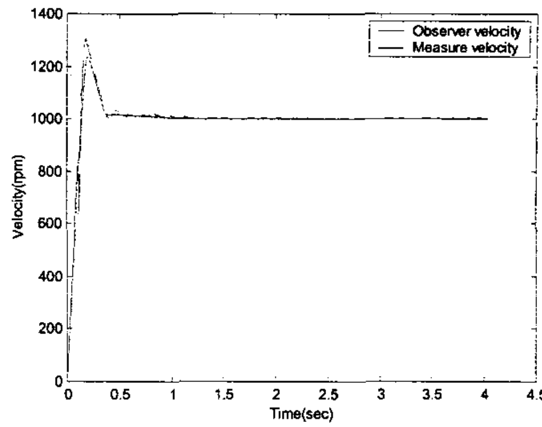

Figure 7 PDF-VSFO control with k_p = 180, k_i = 2, k_d = 100, and **L** = 0.1