Proceedings of the 38th
Conference on Decision & Control
Phoenix, Arizona USA • December 1999

Dual Stroke and Phase Control and System Identification of Linear Compressor of a Split-Stirling Cryocooler

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Abstract

A new dual control scheme is proposed to the stroke and phase compensation on the displacement of linear compressor of a split-Stirling cryocooler. This dual controller has two connected stroke and phase control loops, whose dynamic models are identified experimentally. These loops individually provide corrections on the stroke and phase of the compressor displacement to follow a sinusoidal command. The experimental results show that the dual stroke and phase control system is robust to parameter changes and external disturbances.

Keywords: split-Stirling cryocooler, linear compressor, dual controller, phase control

Introduction

Miniature split-type Stirling cryocoolers are characterized by complete separation of the expansion cylinder from the compressor cylinder, drive motor and crankcase. The working fluid provides gas force by an actuating piston oscillating in the compressor cylinder with a phase-shift relative to the displacer motion in the expansion cylinder. The stroke variation of the displacer indicates the volume change in the expansion space, and its pressure-volume diagram illustrates the work done by the gas on the displacer. The net area of the diagram will represent the heat transferred to the expansion space, a positive work, so there is some refrigeration effect, and the tip of the cold finger will become cold. The optimal P-V diagram condition of cryocooler depends not only on the stroke but also on the phase angle of pressure phasors[1,2].

In addition to the stroke of the compressor that determines the available refrigeration, the phase control of the compressor is also important for the improvement of the cooling performance [3-5]. Stroke controls were present in most of the documentation of the motion control of linear motors, but few talked about both phase and stroke controls [6,7]. Stolfi and Daniels [8] employed local feedback loop to control the phase and frequency of the displacer and piston, to which the reference lagged the signal to the displacer, producing the desired piston/displacer phase angle. The repetitive control on a periodically moving mechanism may be an alternative solution. Point-to-point command tracking usually results in a precise positioning control on the system. However, direct tracking the displacement command must yield heavy control efforts that may cause saturation of actuators[9].

The refrigeration performance of the cryocooler studied in this paper depends on the stroke and phase of the working fluid. A precise displacement tracking of a sinusoidal command is not necessary for the compressor piston, thereby saving a lot of driving energy. In the linear range of operation, the frequency response of the linear compressor has fixed magnitude and phase angle at a specific operation frequency. It is required, however, that the stroke and phase are to be controlled separately. Therefore, a new dual controller is proposed to provide new dynamics to the system. This controller consists of a stroke control loop and a phase control loop, connected internally with the linear compressor. First, a dominant linear model with time delay is identified for the linear compressor, whose bandwidth of operation is then specified. Second, the loop transfer functions are determined by a nonparametric identification technique, and the controller parameters are designed by the root-locus method. The robustness tests on the change of load and rejection of disturbances are also performed.

Dynamics of Linear Compressor

The mechanical structure of the linear compressor in this research is shown in Fig. 1. The armature is a cylinder wound with coils through which the input current is applied, and the stator is composed by two separate rings of permanent magnets. A linear variable-differential transformer (LVDT) is placed at the tail of the piston for measuring its displacement [10,11]. Assumptions are made for the following derivation of linear system models:

(1) The air-gap flux density and coil inductance are invariant as the armature moves.

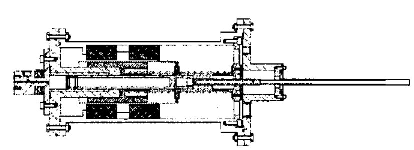

Fig. 1. Mechanical structure of linear compressor

(2) The friction between the cylinder and its wall is described by an equivalent viscous damping coefficient.
(3) The springs are operated in the linear range.

The electrical equation of the linear compressor[12] can be expressed as

$$V = L\frac{dI}{dt} + RI + B\lambda\frac{dX}{dt} \quad (1)$$

and the mechanical equation of motion can be described by

$$m\frac{d^2X}{dt^2} + c\frac{dX}{dt} + kX = \lambda IB - A\Delta P \quad (2)$$

where L and R are coil inductance and resistance, I and V are driving current and voltage, B is the magnetic flux density in the air gap between the stator and moving coil, the piston displacement is denoted by X, the effective length of the coil is λ, the pressure difference between the left and right chambers is ΔP, the cross-section area of the piston is A, and m, k, c denote the equivalent mass, spring constant and viscous damping coefficient, respectively. Taking the Laplace transformation of the above equations yields

$$X(s) = G(s)V(s) + W(s)\Delta P(s) \quad (3)$$

where

$$G(s) = \frac{B\lambda}{mLs^3 + (mR + cL)s^2 + [cR + B^2\lambda^2 + Lk]s + Rk} \quad (4)$$

and

$$W(s) = \frac{(Ls + R)A}{mLs^3 + (mR + cL)s^2 + [cR + B^2\lambda^2 + Lk]s + Rk} \quad (5)$$

are, respectively, the transfer functions of the driving voltage and pressure difference with respect to the piston displacement of the linear compressor. It is clearly understood that the displacement of the compressor is mainly determined by the driving voltage and pressure difference, and the latter changes with the cooling or ambient temperature, working gas pressure, operating frequency, etc. Therefore, from the control system point of view, the term $W(s)\Delta P(s)$ can be deemed as a bounded uncertainty, to which the controller has to be designed robust so that the displacement follows the reference command.

To investigate the importance of system parameters on the system dynamics, the characteristic equation can be expressed in an alternative form:

$$[ms^2 + cs + k][Ls + R] + B^2l^2s = 0 \quad (6)$$

in which the term B^2l^2 accounts for the back emf induced in the armature. If the root-locus is drawn with respect to B^2l^2, one of the open-loop poles is real and the other two are complex conjugates. The latter are dominant poles, which are determined by the mass, spring constant and damping coefficient of the armature. The specifications of the linear compressor are listed in Table 1, and the frequency response is identified experimentally as shown in Fig. 2.

Cylinder mass	0.717 kg
Damping coefficient	0.116 Nt.sec/m
Spring constant	20000 Nt/m
Bandwidth	190.5 Hz
Max. output force	35.2 Nt
Length of coil	32.8 m
Coil resistance	3 Ohm
Coil inductance	0.00142 H
Rated current	6.24 A (rms)
Rated voltage	30 V
Air-gap flux density	0.25 Tesla
Length of coil	32.8 m

Table 1. Specifications of the linear compressor

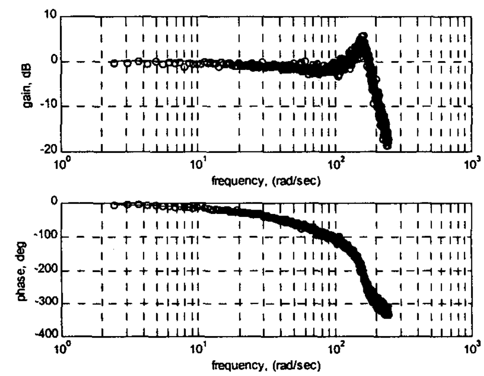

Fig. 2. Frequency response of the linear compressor (○: experiments, —:curve fitting)

System Identification and Dual Controller Design

The basic structure of the proposed dual controller consists of a stroke and a phase control loop, as shown in Fig. 3. A full-wave rectifier, subtractor, and integrator compose the stroke control loop. The input of the integrator is the dc signal, representing the stroke difference between the reference command and the displacement of the compressor measured by the linear variable differential transformer (LVDT). Then, the output of the integrator corrects the stroke and flows into a four-quadrant multiplier to join with the phase correction from the phase control loop.

The purpose of the phase controller is to adjust the piston displacement in-phase to the reference command. Leading a phase angle θ is equivalent to lagging a phase $2\pi - \theta$, and the latter is easier to implement with electronic circuits. For phase adjustment, the phase comparator takes the phase difference of a square wave reference command and the piston displacement, and the

phase controller makes corrections to compensate the phase delay through an integration.
Both the stroke and phase controllers are designed as integrators. Their inputs are step signals rectified through feedback loops. The stead-state error for a step input in each loop with one integrator or more becomes zero. The integral gains are then determined by root loci according to additional performance requirements.

A dc voltage shift may exist in the displacement output of the linear compressor due to constant loads. In addition to the stroke and phase of the displacement, a dc reference level must be specified and adjusted in order to track the sinusoidal command. A dc compensator, whose input is the rectified dc content of the piston displacement measured by a LVDT, accomplishes this. A corresponding dc voltage is then added to the output of the four-quadrant multiplier, through which the corrections from stroke and phase control loops composes a sinusoidal driving command to the PWM driver. Finally, the PWM driver amplifies the corrected signal to the driving force into the linear compressor.

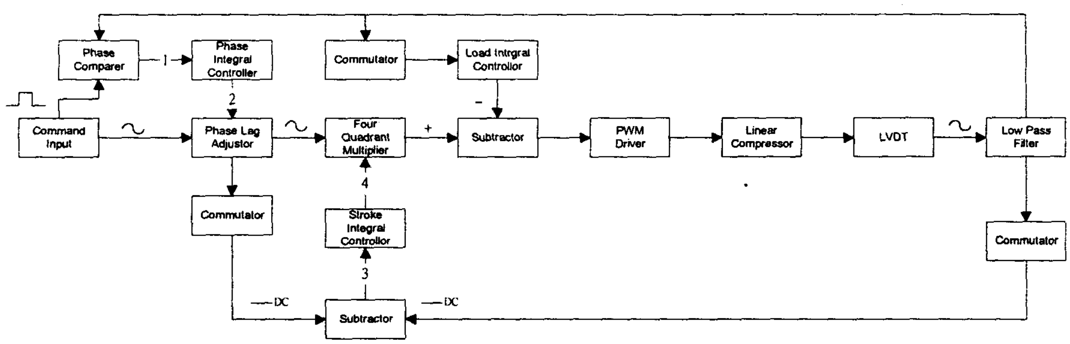

Fig. 3. Dual control system configuration

To tune the proper gains for the integrators in the stroke and phase control loops, their loop dynamics must be identified and expressed approximately in a linear model.

Dynamic Model of Stroke Control Loop

The stroke signal S_O at node 3 in Fig. 3 is a dc voltage rectified from the measured compressor displacement, while the input of the stroke control loop is denoted by S_I at node 4. The step response method [13] is used to identify the open-loop dynamics between S_I and S_O, where the input is taken as a step and the recorded output constitutes the model. During the identification of the stroke model, the signal at node 2 in Fig. 3 is supplied with a constant voltage so as to isolate the interference from the phase control loop. The sampling period is chosen at 0.005 second, and the step input voltage from node 4 is 10 V. For a better design of a robust controller near the bandwidth of the system, the compressor is operated at 25 Hz. The resulting dynamics model of the stroke control loop becomes

$$G_S(s) = \frac{S_O}{S_I} = \frac{K_S}{1 + s/P_S} e^{-T_{ds}s} \tag{7}$$

where $K_S = 3.9$, $P_S = 0.86863$, and the time delay parameter $T_{ds} = 005$. The frequency response of the stroke control loop is shown in Fig. 4. Figure 5 illustrates the block diagram of the stroke control loop, and its root-locus with respect to the integral gain K_1 is shown in Fig. 6. To eliminate undesirable residual oscillations, both the closed-loop poles are assigned at 0.43 where the corresponding integration gain K_1 is 0.05.

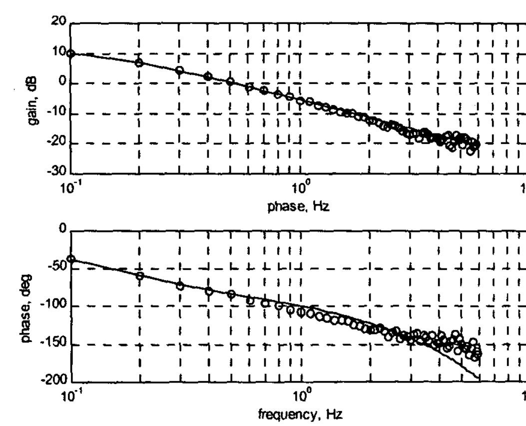

Fig. 4. Frequency response of the stroke control loop
(○: experiments, —:curve fitting)

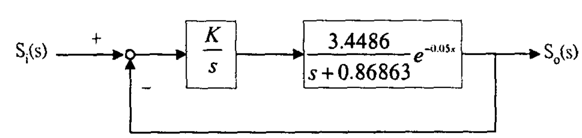

Fig. 5. Block diagram of stroke control loop

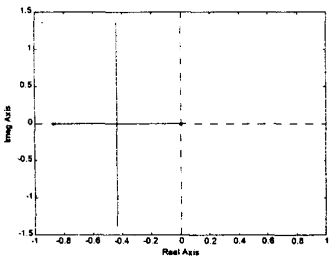

Fig. 6. Root locus of the stroke control loop for values of K_1

Dynamic Model of the Phase Control Loop

In the phase control loop, the phase comparator takes the phase difference of the reference square TTL command and the piston displacement to provide a corresponding voltage. This calibrated output voltage is linearly related to the phase lead or lag angles. The signals for the identification of the dynamic model of the phase control loop are collected at nodes 1 and 2, while the signal at node 4 is supplied with a constant voltage so as to isolate the interfering dynamics from the stroke control loop. At the same operating point as in the identification of the stroke control model, the resulting dynamics model of the phase control loop becomes

$$G_P(s) = \frac{P_O}{P_I} = \frac{K_P}{1 + s/P_P} e^{-T_{dp}s} \qquad (8)$$

Where $K_P = 17.987$, $P_P = 2.0872$, and the time delay constant T_{dp} is 0.06. Figure 7 shows the frequency response of the phase control loop. The block diagram of the phase control loop is shown in Fig. 8, in which the integral gain K_2 is also determined to be 0.06 by the root-locus design.

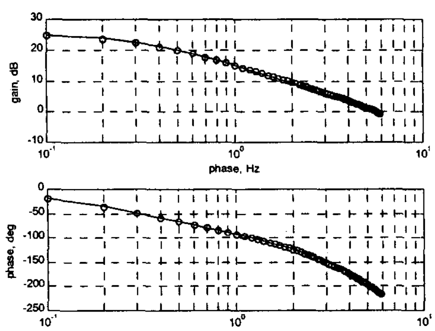

Fig. 7. Frequency response of the phase control loop (○: experiments, —:curve fitting)

Experiments

The bandwidth of the linear compressor is about 30 Hz according to the frequency response in Fig. 2, and the corresponding phase angle is around -260 degree. The open-loop time response to a 30 Hz sinusoidal command is shown in Fig. 9. It is desirable that the compressor is driven up to 30 Hz without phase delay to the reference signal. Either stroke controller or phase controller alone does not work for the compressor to follow a reference command, as shown in Figs. 10 and 11. Concurrent controls on the stroke and phase with separate loops apparently improves the performance as shown in Fig. 12.

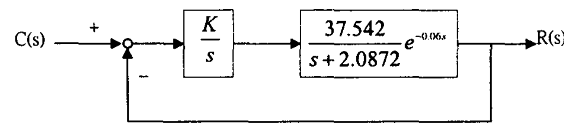

Fig. 8. Block diagram of phase control loop

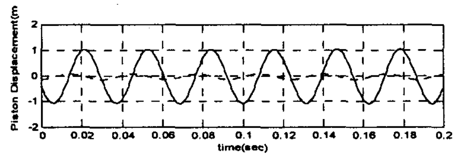

Fig. 9. Open-loop response to a 30 Hz sinusoidal command (solid curve: command, dotted curve: output)

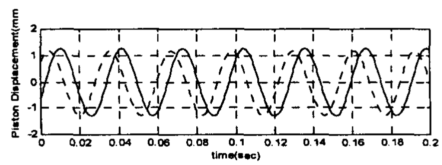

Fig. 10. Response of the compressor with only stroke control (solid curve: 30Hz command, dotted curve: output)

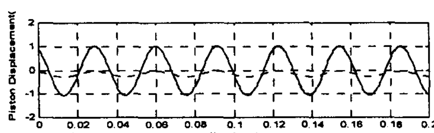

Fig. 11. Response of the compressor with only phase control (solid curve: 30Hz command, dotted curve: output)

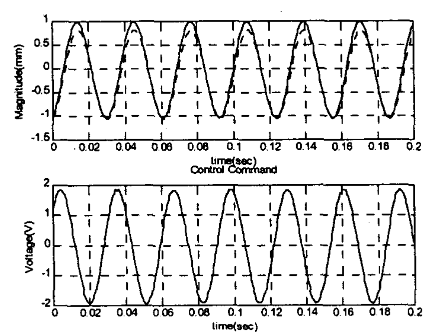

Fig. 12. Response of the compressor with dual stroke and phase control (solid curve: 30Hz command, dotted curve: output)

The proposed dual controller has to be robust to the change of system parameters and external disturbances. In the above experiments, the piston of the linear compressor moves in the horizontal direction, hence no gravity effect on the system operation. However, the gravity force produces a constant load on the piston when it moves in the vertical direction. This loading effect can be regarded as an external disturbance or a change of system dynamics. The robustness test is then performed by suddenly shift the compressor from the horizontal position to the vertical position during the operation. Figure 13 shows that the linear compressor is not only robust to the change of system dynamics, but also able to reject disturbances.

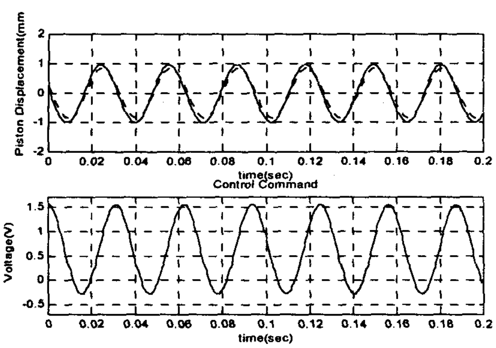

Fig. 13 Robustness test

Summary and Conclusions

This paper proposes a new concept of dual stroke and phase control on the linear compressor of a split-Stirling cryocooler. This dual control configuration consists of a stroke and a phase control loop. The transfer function of each loop is obtained by a nonparametric identification -- step response method. Root-locus diagrams then determine integral gains to guarantee a zero stead-state error and less residual output oscillations. Each control loop provides stroke or phase correction to composed a displacement control signal through a four-quadrant multiplier, while a dc compensator compensates the control reference level. Although the open-loop frequency response has fixed phase angle and magnitude at a certain operation frequency, the proposed dual controller is able to compensate the output phase and stroke separately. Therefore, the piston of the linear compressor successfully follows the sinusoidal displacement command both in stroke and phase, with moderate control effort. The robustness of the system is also verified by the change of loading. The new dual controller can be implemented to any split-type cryocoolers when the phase and stroke of the working fluid need to be controlled separately.

Acknowledgments

The authors greatly acknowledge the support of the National Science Council under Contract No. NSC84-0210-D-002-027 in Taiwan, Republic of China.

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