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AN OBSERVER DESIGN FOR CONSTRAINED ROBOT SYSTEMS

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ABSTRACT

An asymptotic observer is constructed for constrained robot systems in this paper. Since a constrained robot, in general, involves in a set of differential equations and a set of algebraic equations, both differential and algebraic variables should be estimated. This gives arise to difficulty in estimating the algebraic variables which are the contact forces. The difficulty is eased by introducing the reduced model of constrained systems developed by McClamroch and Wang. By using this model, the observer design is similar to designing an observer for an unconstrained nonlinear system. Since both the contact force and the motion of the robot can be directly estimated, our observer may be useful for the controller design of the constrained robot system.

I. INTRODUCTION

For many operations of the robot, the robot end effector is constrained by its environment. In that case, the direct control of the contact force between the robot end effector and the constraint surface can greatly expand the task capacity. The mathematical model for the constrained robot, explicitly taking into account the contact force, has been given in [3,6,8]. Several control schemes have also been developed to directly control the contact force and the robot motion based on this model [2,7,10]. However, all these control schemes implicitly assume that all state and algebraic variables are available. Unfortunately, this is not always true. Usually, some states are very difficult to measured and some are too expensive to be sensed. Particularly, the contact force variables may be very expensive and inadequate to be measured. Thus, it is required that we design an observer to estimate the contact force and the state variables for the constrained robot systems.

Since a constrained robot system consists of differential equations and a set of algebraic constraint equations, the contact force variables, which are the algebraic variables, may be regarded as state variables without governing differential equations. The overall system is referred to as an nonlinear singular systems [6,8]. Hence, traditional design procedures for nonlinear observers such as the Lie-algebra observer [1], the extended linearization observer [5], Thau's observer [4], and the VSS (Variable Structure System) observer [9] can not be directly applied.

In order to overcome the above difficulty, the McClamroch and Wang's method [7] is used to transform the constrained system into reduced unconstrained subsystems. Then the observer is designed in terms of the reduced subsystems. The selection of our observer structure is sort of combination of Kuo's observer [4] and the VSS observer. We use the concept of Kuo's observer to determine the convergent property of the observer and use the idea of the VSS observer to cancel the effect caused by the nonlinear coupling in the control input. Since a linear output is desired for applying the VSS observer technique, a linear output based on transformed system will be constructed.

In this paper, we present an observer design for constrained robot sytems. The constrained system and its reduced form are discussed first. Then a linear output generator is constructed. Finally, the design procedure of the observer is presented.

II. PROBLEM FORMULATION

For a constrained robot, the motion of the robot end effector is constrained by its environment. The Lagrangian dynamics of the constrained robot systems, explicitly incorporating the effects of contact forces, can be modeled as [6]

$$M(q)\ddot{q} + F(q,\dot{q}) = u + J^T(q) \tag{1}$$

$$\phi(q) = 0 \tag{2}$$

where $q \in R^n$ is the generalized displacement; $M(q)$ is an $n \times n$ inertial matrix function; $F(q,\dot{q})$ is an n dimensional vector function, containing the Coriolis, the centrifugal and the gravitational terms; $u \in R^n$ is the generalized control input; $\phi(q)$ is the m dimensional constraint vector function; $J(q) = \dfrac{\partial \phi(q)}{\partial q}$ is an $m \times n$ Jacobian matrix; $\lambda \in R^m$ is the generalized contact force vector associated with the constraints.

The constraints, given in Eq.(2), are assumed to be holonomic and frictionless. Note that if $\phi(q)$ is identically satisified then also $J(q)\dot{q} = 0$. Hence the motion of the robot end effector is constrained in the constraint manifold $S \subset R^{2n}$ defined by $S = \{(q,\dot{q}) : \phi(q) = 0,\ J(q)\dot{q} = 0\}$.

Suppose the output of the constrained robot system is given by

$$y = Cq \tag{3}$$

where $y \in R^p$ is the output vector and C is a $p \times n$ constant matrix. Our objective is to construct an observer such that the displacement q and velocity $\dot{q}$ of the robot and the contact force λ can be estimated. These estimated values can be used for controller design.

Since the constrained system, given in Eqns. (1) and (2), contains a set of algebraic equations, it is not suitable for observer design. McClamroch and Wang [7] use an nonlinear transformation to convert the constrained system into two reduced unconstrained subsystems at which the constraints are satisified automatically. Our observer design will be based on these reduced subsystems. The transformation method is briefly summarized in the following.

Suppose that there exists an open set $V \subset R^{n-m}$ and a function $\Omega : V \to R^n$ such that

$$\phi(\Omega(q_2), q_2) = 0 \quad \text{for} \quad \text{all} \quad q_2 \in V. \tag{4}$$

If rank J(q)=m, then according to the implicit function theorem Eq.(4) holds for some $V = R^{n-m}$. Consider the nonlinear transformation

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = X(q) = \begin{bmatrix} q_1 - \Omega(q_2) \\ q_2 \end{bmatrix} \tag{5}$$

which is differentiable and has a differentiable inverse transformation $Q : R^n \to R^n$ such that

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = Q(x) = \begin{bmatrix} x_1 + \Omega(x_2) \\ x_2 \end{bmatrix} \tag{6}$$

Let the nonsingular Jacobian matrix of the inverse transformation be

$$T(x) = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} I_m & \frac{\partial \Omega(x_2)}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix} \tag{7}$$

Then the constrained system, given in Eqns. (1) and (2), can be trandformed to reduced subsystems

$$E_1\bar{M}(x_2)E_2^T\ddot{x}_2 + E_1\bar{F}(x_2,\dot{x}_2) = E_1T^T(x_2)u + E_1T^T(x_2)J^T(x_2)\lambda \tag{8}$$

$$E_2\bar{M}(x_2)E_2^T\ddot{x}_2 + E_2\bar{F}(x_2,\dot{x}_2) = E_2T^T(x_2)u \tag{9}$$

$$x_1 = 0 \tag{10}$$

where

$$\bar{M}(x_2) = T^T(x_2)M(Q(x_2))T(x_2) \tag{11}$$

$$\bar{F}(x_2,\dot{x}_2) = T^T(x_2)\left[F(Q(x_2), T(x_2)\dot{x}_2) + M(Q(x_2))\dot{T}(x_2)\dot{x}_2\right] \tag{12}$$

Note that the partition of the identity matrix $I_n = \left[E_1^T,\ E_2^T\right]$, where E_1 is an $m \times n$ matrix and E_2 is an $(n-m) \times n$ matrix, is used to partition x as $x^T = \left[x_1^T, x_2^T\right]^T = \left[(E_1 x)^T, (E_2 x)^T\right]^T$. The relation $E_2 T^T(x_2) J^T(x_2) = 0$ is used in deriving Eq.(9). Furthermore, the constraint equation (2) is transformed to Eq.(10). Under this trasnsformation, the output y becomes

$$y = CQ(x_2) \tag{13}$$

which is an nonlinear relation.

Our problem turns out to design an observer for the transformed system (8)-(10) and (13). Since the differential equation (9) completely governs the reduced state vector x_2 and the output equation (13) is only in terms of x_2, the subsystem (9) (13) can be treated as an ordinary unconstrained nonlinear system. The contact force λ can be determined from Eq.(8). The observability of the constrained system depends on the observability of the subsystem (9) (13). It can be easily verified that if the $m \times m$ matrix $E_1 T^T(x_2) J^T(x_2)$ is nonsingular for all $x_2 \in R^{n-m}$, then the overall system (8)(9)(10)(13) is observable if the subsystem (9)(13) is observable. Throughout this paper, we assume that the system is always observable.

As shown in Eq.(13), the output y now is an nonlinear vector function of x_2. In order to use the VSS observer technique, a linear output is required. The generation of the linear output will be discussed in the next section.

III. LINEAR OUTPUT GENERATOR

Consider the subsystem (9)(13). Since there is nonlinear coupling in control, the typical nonlinear observer design, such as the Lie-algebra observer [1] and the Kou's observer [4], can not be applied. The VSS observer [9] can handle the nonlinear coupling in control for the system with linear output. Thus, in order to use the idea of the VSS observer a linear output will be constructed from Eq.(13).

Since C is a $p \times n$ matrix, by applying the singular value decomposition, there exist two unitary matrices U and V such that

$$C = U^T \Sigma V \tag{14}$$

where $\Sigma = [D\,|\,0]$ is a $p \times n$ matrix and D is a $p \times p$ diagonal matrix. Let $n > p > m$. We further partitiopn matrices Σ and V into

$$\Sigma = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \\ V_{31} & V_{32} \end{bmatrix} \tag{15}$$

where $D_{11} \in R^{m \times m}$, $D_{22} \in R^{(p-m)\times(p-m)}$, $V_{11} \in R^{m \times m}$, $V_{12} \in R^{m \times (n-m)}$, $V_{21} \in R^{(p-m) \times m}$, $V_{22} \in R^{(p-m)\times(n-m)}$, $V_{31} \in R^{(n-p)\times m}$, $V_{32} \in R^{(n-p)\times(n-m)}$. Premultiply both sides of Eq.(13) by U, we have

$$Uy = \Sigma V \begin{bmatrix} \Omega(x_2) \\ x_2 \end{bmatrix} = \begin{bmatrix} D_{11} V_{11} \Omega(x_2) + D_{11} V_{12} x_2 \\ D_{22} V_{21} \Omega)x_2) + D_{22} V_{22} x_2 \end{bmatrix} \tag{16}$$

It is convenient to use the partition of the identity matrix $I_p = \left[E_3^T, E_4^T\right]$, where E_3 is an $m \times p$ matrix and E_4 is a $(p-m) \times p$ matrix, to simplify Eq.(16) as

$$E_3 Uy = D_{11} V_{11} \Omega(x_2) + D_{11} V_{12} x_2 \tag{17}$$
$$E_4 Uy = D_{22} V_{21} \Omega(x_2) + D_{22} V_{22} x_2 \tag{18}$$

Since $D_{11} V_{11}$ is nonsingular, $\Omega(x_2)$ can be solved from Eq.(17) as

$$\Omega(x_2) = (D_{11} V_{11})^{-1} \left[E_3 Uy - D_{11} V_{12} x_2\right] \tag{19}$$

Substituting (19) into (18) gives

$$\left[E_4 - D_{22} V_{21} (D_{11} V_{11})^{-1} E_3\right] Uy = \left[D_{22} V_{22} - D_{22} V_{21} V_{11}^{-1} V_{12}\right] x_2 \tag{20}$$

For simplicity, we define

$$\bar{y} = \left[E_4 - D_{22} V_{21} V_{11}^{-1} D_{11}^{-1} E_3\right] Uy \tag{21}$$
$$\bar{C} = \left[D_{22} V_{22} - D_{22} V_{21} V_{11}^{-1} V_{12}\right] \tag{22}$$

Then, Eq.(20) is rewritten as

$$\bar{y} = \bar{C} x_2. \tag{23}$$

The new output $\bar{y}$ is linear in state variable x_2 only. It will be used in our observer design. However, the observability for the subsystem (9)(23) must be re-checked.

IV. OBSERVER DESIGN

In this section, we construct an asymptotic observer for the dynamic system (8)(9)(23). In order to carry our subsequent development, we make the following assumption

Assumption 1

(1) $\left[E_2 \bar{M}(x_2) E_2^T\right]^{-1} E_2 T^T(x_2) = \bar{C}^T h(x_2)$;
(2) $\|h(x_2)\| \le H$ where H is a scalar.

Now we select the observer structure as follows

$$\dot{\hat{x}}_2 = \hat{\dot{x}}_2 - G_1^o \bar{C}(x_2 - \hat{x}_2) \tag{24}$$

$$\begin{aligned} E_1 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{\dot{x}}}_2 &= -E_1 \bar{F}(\hat{x}_2, \hat{\dot{x}}_2) + E_1 T^T(\hat{x}_2) u \\ &+ E_1 T^T(\hat{x}_2) J^T(x_2) \hat{\lambda} + E_1 \bar{M}(\hat{x}_2) E_2^T G_3^o \bar{C}(x_2 - \hat{x}_2) \end{aligned} \tag{25}$$

$$\begin{aligned} E_2 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{\dot{x}}}_2 &= -E_2 \bar{F}(\hat{x}_2, \hat{\dot{x}}_2) + E_2 \bar{M}(\hat{x}_2) E_2^T G_2^o(x_2 - \hat{x}_2) \\ & R(x_2, u) \end{aligned} \tag{26}$$

where

$$R(x_2, u) = \begin{cases} \dfrac{-\bar{C}^T \bar{C}(x_2 - \hat{x}_2)}{\|\bar{C}(x_2 - \hat{x}_2)\|} H\|u\|, & \text{for } \bar{C}(x_2 - \hat{x}_2) \ne 0; \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

and G_1^o, G_2^o, and G_3^o are constant matrices.

We have the following results.

Theorem: Consider the system (8)(9)(23). If the following conditions are satisfied

(1) Assumption 1 holds,

(2) $\nabla \bar{p}(x_2, \dot{x}_2) + \begin{bmatrix} -G_2^o \bar{C} \\ G_1^o \bar{C} \end{bmatrix} \nabla x_2$ is uniformly negative definite for some $\epsilon > 0$, where

$$\nabla \bar{p} = \left[\frac{\partial \bar{p}}{\partial x_2}, \frac{\partial \bar{p}}{\partial \dot{x}_2}\right]$$

and

$$\bar{p}(x_2, \dot{x}_2) = \begin{bmatrix} (E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \dot{x}_2 \end{bmatrix} \tag{28}$$

(3) $E_1 T^T(x_2) J^T(x_2)$ and $E_1 T^T(\hat{x}_2) J^T(\hat{x}_2)$ are nonsingular for all $x_2, \hat{x}_2 \in R^{n-m}$,

then the observer defined by Eqns.(24)-(27) is an asymptotic observer; i.e.,

$$\|x(t) - \hat{x}(t)\| \le K e^{-\epsilon t} \quad \forall t \ge 0 \tag{29}$$

where K depends on $x(0)$ and $\hat{x}(0)$. In addition, the estimated contact force $\hat{\lambda}$ converges to λ at the same rate as $\hat{x}(t)$ to $x(t)$.

Proof: Eqns.(9) and (26) can be conveniently expressed in matrix form as

$$\begin{bmatrix} \ddot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -(E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} (E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 T^T(x_2) \\ 0 \end{bmatrix} u \tag{30}$$

and

$$\begin{bmatrix} \dot{\hat{\dot{x}}}_2 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -(E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \hat{\dot{x}}_2 \end{bmatrix} + \begin{bmatrix} G_2^o \\ -G_1^o \end{bmatrix} \bar{C}(x_2 - \hat{x}_2) + \begin{bmatrix} R(x_2, u) \\ 0 \end{bmatrix} \tag{31}$$

Let the estimate errror be e

$$e = \begin{bmatrix} e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \hat{\dot{x}}_2 - \dot{x}_2 \\ \hat{x}_2 - x_2 \end{bmatrix} \tag{32}$$

Choose a Lyapunov function V as

$$V(e) = \frac{1}{2}e^T e \tag{33}$$

Then

$$\dot{V}(e) = e^T\{\left[\bar{p}(\hat{x}_2, \hat{\dot{x}}_2) - \bar{p}(x_2, \dot{x}_2)\right] + \begin{bmatrix} -G_2^o\bar{C} \\ G_1^o\bar{C} \end{bmatrix}(\hat{x}_2 - x_2)\}$$
$$+ e^T\begin{bmatrix} \frac{-\bar{C}^T\bar{C}(x_2 - \hat{x}_2)H\|u\|}{\|\bar{C}(x_2 - \hat{x}_2)\|} \\ 0 \end{bmatrix}$$
$$- e^T\begin{bmatrix} (E_2\bar{M}(x_2)E_2^T)^{-1}E_2^T(x_2)u \\ 0 \end{bmatrix} \tag{34}$$

The last two terms in the right hand side can be further simplified as follows

$$e^T\begin{bmatrix} \frac{-\bar{C}^T\bar{C}(x_2 - \hat{x}_2)H\|u\|}{\|\bar{C}(x_2 - \hat{x}_2)\|} \\ 0 \end{bmatrix} - e^T\begin{bmatrix} \bar{C}^T h(x_2)u \\ 0 \end{bmatrix}$$
$$\leq -\|\bar{\bar{C}}e\|H\|u\| + \|e^T\bar{\bar{C}}^T h(x_2)\|\|u\|$$
$$\leq -\|\bar{\bar{C}}e\|H\|u\| + \|\bar{\bar{C}}e\|H\|u\| = 0$$

where $\bar{\bar{C}} = [\bar{C}, 0]$.
Thus, Eq.(34) becomes

$$\dot{V}(e) \leq e^T\{\left[\bar{p}(\hat{x}_2, \hat{\dot{x}}_2) - \bar{p}(x_2, \dot{x}_2)\right] + \begin{bmatrix} -G_2^o\bar{C} \\ G_1^o\bar{C} \end{bmatrix}(\hat{x}_2 - x_2)\}$$
$$\leq e^T\int_0^1\{(\nabla\bar{p} + \begin{bmatrix} -G_2^o\bar{C} \\ G_1^o\bar{C} \end{bmatrix}\nabla x_2(w_s)\}e\, ds$$

where $w_s = s\begin{bmatrix} x_2 \\ \dot{x}_2 \end{bmatrix} + (1-s)\begin{bmatrix} \hat{x}_2 \\ \hat{\dot{x}}_2 \end{bmatrix}$ and $0 \leq s \leq 1$.

Since $\nabla\bar{p}(x_2, \dot{x}_2) + \begin{bmatrix} -G_2^o\bar{C} \\ G_1^o\bar{C} \end{bmatrix}\nabla x_2$ is uniformly negative definite for some $\epsilon > 0$, we have

$$\dot{V}(e) \leq -\epsilon\|e\|^2 \tag{35}$$

Then, Eq.(29) follows from Eq.(35).

Next, we consider the estimated contact force vector $\hat{\lambda}$. If $E_1T^T(x_2)J^T(x_2)$ and $E_1T^T(\hat{x}_2)J^T(\hat{x}_2)$ are nonsingular for all $x_2, \hat{x}_2 \in R^{n-m}$, then λ and $\hat{\lambda}$ can be solved from Eqns. (8) and (25). From Eq. (8), λ is determined as

$$\lambda = \left[E_1T^T(x_2)J^T(x_2)\right]^{-1}\{E_1\bar{M}(x_2)E_2^T(E_2\bar{M}(x_2)E_2^T)^{-1}$$
$$(-E_2\bar{F}(x_2, \dot{x}_2) + E_1\bar{F}(x_2, \dot{x}_2)\} + \left[E_1T^T(x_2)J^T(x_2)\right]^{-1}$$
$$\left[E_1\bar{M}(x_2)E_2^T(E_2\bar{M}(x_2)E_2^T)^{-1}E_2T^T(x_2) - E_1T^T(x_2)\right]u$$
$$\triangleq S_1(x_2, \dot{x}_2) + S_2(x_2, \dot{x}_2)u \tag{36}$$

Similarly, we have

$$\hat{\lambda} = S_1(\hat{x}_2, \hat{\dot{x}}_2) + S_2(\hat{x}_2, \hat{\dot{x}}_2)u + \left[E_1T^T(\hat{x}_2)J^T(\hat{x}_2)\right]^{-1}$$
$$E_1\bar{M}(x_2)E_2^TG_3^o\bar{C}(x_2 - \hat{x}_2) \tag{37}$$

Since $\hat{x}_2$ converges to x_2 and $\hat{\dot{x}}_2$ converges to $\dot{x}_2$ as time approaches to infinity, functions $S_1(\hat{x}_2, \hat{\dot{x}}_2)$and $S_2(\hat{x}_2, \hat{\dot{x}}_2)$ will converge to $S_1(x_2, \dot{x}_2)$ and $S_2(x_2, \dot{x}_2)$, respectively; moreover, $\bar{C}(x_2 - \hat{x}_2)$ converges to zero. Thus, the estimated contact force vector $\hat{\lambda}$ converges to λ at the same rate as $\hat{x}_2$ to x_2. Q.E.D.

Remark: In the construction of the observer, the states of the transformed system rather than the states of the original system are estimated. This indicates that it is not necessary to estimate all states for the constrained system. The complete information of the original states can be obtained by the following transformation

$$\hat{q} = \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \end{bmatrix} = Q(\hat{x}) = \begin{bmatrix} \Omega(\hat{x}_2) \\ \hat{x}_2 \end{bmatrix} \tag{38}$$

and

$$\hat{\dot{q}} = \begin{bmatrix} \hat{\dot{q}}_1 \\ \hat{\dot{q}}_2 \end{bmatrix} = T(\hat{x})\hat{\dot{x}} = \begin{bmatrix} I_m & \frac{\partial\Omega(x_2)}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix}_{x_2 = \hat{x}_2}\begin{bmatrix} 0 \\ \hat{\dot{x}}_2 \end{bmatrix} \tag{39}$$

Another remarkable feature of the observer is that the contact force λ can be directly estimated rather than obtained by expensive force sensors.

V. CONCLUSION

An asymptotic observer is constructed for the constrained robot system. It has been shown that the converging properties can be determined by the selection of the observer gain matrices G_1^o, G_2^o and G_3^o. The difficulty caused by the nonlinear coupling in the control has been overcome by introducing the VSS observer idea at the expense of the requirement of linear output. Although the estimate are based on the transformed reduced subsystems, the estimate of the original state can be recovered by applying the inverse transformation. Since the contact force, which is usually not directly available in the constrained robot system, can be estimated directly, the observer may be very useful for the controller design of a constrained robot system. The controller may design on the basis of $\hat{x}_2, \hat{\dot{x}}_2$ or $\hat{q}, \hat{\dot{q}}$; however, the stabilization problem of the overall system should be carefully investigated. This result will be reported in the forthcoming paper.

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