Asymptotic observer design for constrained robot systems

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Indexing term: Robotics

Abstract: Two types of asymptotic observer are constructed for constrained robot systems in this paper. Since a constrained robot, in general, involves a set of differential equations and a set of algebraic equations, both differential and algebraic variables should be estimated. This gives arise to difficulty in estimating the algebraic variables which are the contact forces. The difficulty is eased by introducing a nonlinear transformation. Although the transformation causes nonlinear coupling on the input, asymptotic observers can be constructed in terms of the transformed system by a special treatment. Since both the contact force and the motion of the robot can be directly estimated, the observers may be useful for the controller design of the constrained robot system.

1 Introduction

For many operations of the robot, the robot end effector is constrained by its environment. In that case, the direct control of the contact force between the robot end effector and the constraint surface can greatly expand the task capacity. The mathematical model for the constrained robot, explicitly taking into account the contact force, has been given in References 3, 6 and 8. Several control schemes have also been developed to control directly the contact force and the robot motion based on this model [2, 7, 10]. However, all these control schemes implicitly assume that all state and algebraic variables are available. Unfortunately, this is not always true. Usually, some states are very difficult to measure and some are too expensive to be assumed. Particularly, the contact force variables may be very expensive and inadequate to be measured. Thus, it is required that we design an observer to estimate the contact force and the state variables for the constrained robot systems.

Since a constrained robot system consists of differential equations and a set of algebraic constraint equations, the contact force variables, which are the algebraic variables, may be regarded as state variables without governing differential equations. The overall system is referred to as a nonlinear singular system [6, 8]. Hence, traditional design procedures for nonlinear observers, such as the Lie-algebra observer [1], the extended linearisation observer [5], Thau's observer [4] and the variable structure system (VSS) observer [9] cannot be directly applied.

To overcome the above difficulty, the McClamroch and Wang method [7] is used to transform the constrained system into reduced unconstrained subsystems. Then, the observer is designed in terms of the reduced subsystems. The selection of our observer structure is a combination of the Kuo observer [4] and the VSS observer. We use the concept of the Kuo observer to determine the convergent property of the observer and use the idea of the VSS observer to cancel the effect caused by the nonlinear coupling in the control input. Since a linear output is desired for applying the VSS observer technique, a linear output based on a transformed system will be constructed.

In this paper, we present two types of asymptotic observer design for constrained robot systems. The constrained system and its reduced form are discussed first. Then a linear output generator is constructed. Finally, the design procedures of the observers are presented and examples are discussed.

2 Constrained robots and problem formulation

For a constrained robot, the motion of the robot end effector is constrained by its environment. The Lagrangian dynamics of the constrained robot systems, explicitly incorporating the effects of contact forces, can be modelled as [6]

$$M(q)\ddot{q} + F(q, \dot{q}) = u + J^T(q)\lambda \tag{1}$$

$$\phi(q) = 0 \tag{2}$$

where $q \in R^n$ is the generalised displacement; $M(q)$ is an $n \times n$ inertial matrix function; $F(q, \dot{q})$ is an n-dimensional vector function, containing the Coriolis, the centrifugal and the gravitational terms; $u \in R^n$ is the generalised control input; $\phi(q)$ is the m-dimensional constraint vector function; $J(q) = [\partial\phi(q)/\partial q]$ is an $m \times n$ Jacobian matrix; and $\lambda \in R^m$ is the generalised contact force vector associated with the constraints.

The constraints, given in eqn. 2, are assumed to be holonomic and frictionless. Note that, if $\phi(q)$ is identically satisfied, then also $J(q)\dot{q} = 0$. Hence, the motion of the robot end effector is constrained in the constraint manifold $S \subset R^{2n}$, defined by $S = \{(q, \dot{q})\colon \phi(q) = 0, J(q)\dot{q} = 0\}$.

Suppose the output of the constrained robot system is given by

$$y = C\begin{bmatrix} \dot{q} \\ q \end{bmatrix} \tag{3}$$

where $y \in R^p$ is the output vector and C is a $p \times 2n$ constant matrix. Our objective is to construct an observer such that the displacement q and velocity $\dot{q}$ of the robot

Paper 7732D(C13), received 20th March 1990

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and the contact force λ can be estimated. These estimated values can be used for controller design.

Since the constrained system, given in eqns. 1 and 2, contains a set of algebraic equations, it is not suitable for observer design. McClamroch and Wang [7] use a nonlinear transformation to convert the constrained system into two reduced, unconstrained subsystems in which the constraints are satisfied automatically. Our observer design will be based on these reduced subsystems. The transformation method is briefly summarised in the following:

Suppose that there exists an open set $V \subset R^{n-m}$ and a function $\Omega: V \to R^n$, such that

$$\phi(\Omega(q_2), q_2) = 0 \quad \text{for all } q_2 \in V \tag{4}$$

If rank $J(q) = m$, then according to the implicit function theorem, eqn. 4 holds for some $V = R^{n-m}$. Consider the nonlinear transformation

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = X(q) = \begin{bmatrix} q_1 - \Omega(q_2) \\ q_2 \end{bmatrix} \tag{5}$$

which is differentiable and has a differentiable inverse transformation $Q: R^n \to R^n$, such that

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = Q(x) = \begin{bmatrix} x_1 + \Omega(x_2) \\ x_2 \end{bmatrix} \tag{6}$$

Let the nonsingular Jacobian matrix of the inverse transformation be

$$T(x) = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} I_m & [\partial\Omega(x_2)/\partial x_2] \\ 0 & I_{n-m} \end{bmatrix} \tag{7}$$

Then the constrained system, given in eqns. 1 and 2, can be transformed to reduced subsystems

$$E_1\bar{M}(x_2)E_2^T\ddot{x}_2 + E_1\bar{F}(x_2, \dot{x}_2) = E_1T^T(x_2)u + E_1T^T(x_2)J^T(x_2)\lambda \tag{8}$$

$$E_2\bar{M}(x_2)E_2^T\ddot{x}_2 + E_2\bar{F}(x_2, \dot{x}_2) = E_2T^T(x_2)u \tag{9}$$

$$x_1 = 0 \tag{10}$$

where

$$\bar{M}(x_2) = T^T(x_2)M[Q(x_2)]T(x_2) \tag{11}$$

$$\bar{F}(x_2, \dot{x}_2) = T^T(x_2) \times \{F[Q(x_2), T(x_2)\dot{x}_2] + M[Q(x_2)]\dot{T}(x_2)\dot{x}_2\} \tag{12}$$

Note that the partition of the identity matrix $I_n = [E_1^T, E_2^T]$, where E_1 is an $m \times n$ matrix and E_2 is an $(n-m) \times n$ matrix, is used to partition x as $x^T = [x_1^T, x_2^T] = [(E_1x)^T, (E_2x)^T]$. The relation $E_2T^T(x_2)J^T(x_2) = 0$ is used in deriving eqn. 9. Furthermore, the constraint eqn. 2 is transformed to eqn. 10. Under this transformation, the output y becomes

$$y = C\begin{bmatrix} T(x)\dot{x} \\ Q(x) \end{bmatrix} = C\begin{bmatrix} \frac{\partial\Omega(x_2)}{\partial x_2}\dot{x}_2 \\ x_2 \\ \Omega(x_2) \\ x_2 \end{bmatrix} \tag{13}$$

which is a nonlinear relation. For simplicity, the above equation is rewritten as

$$y = C\begin{bmatrix} \frac{\partial\Omega(x_2)}{\partial x_3}\dot{x}_2 \\ \Omega(x_2) \\ \dot{x}_2 \\ x_2 \end{bmatrix} \tag{14}$$

Our problem turns out to design observers for the transformed system, eqns. 8–10 and 14. Since the differential eqn. 9 completely governs the reduced state vector x_2, and the output eqn. 14 is only in terms of x_2, the subsystem, eqns. 9 and 14, can be treated as an ordinary unconstrained nonlinear system. The contact force λ can be determined from eqn. 8. The observability of the constrained system depends on the observability of the subsystem, eqns. 9 and 14. It can be easily verified that if the $n \times m$ matrix $E_1T^T(x_2)J^T(x_2)$ is nonsingular for all $x_2 \in R^{n-m}$, then the overall system in eqns. 8–10, and 14 is observable if the subsystem of eqns. 9 and 14 is observable. Throughout this paper, we assume that the system is always observable.

As shown in eqn. 14, the output y is now nonlinear vector function of x_2. To use the VSS observer technique, a linear output is required. The generation of the linear output will be discussed in the next section.

3 Linear output generator

Consider the subsystem in eqns. 9 and 14. Since there is nonlinear coupling in control, the typical nonlinear observer design, such as the Lie-algebra observer [1] and the Kou observer [4], cannot be applied. The VSS observer [9] can handle the nonlinear coupling in control for the system with linear output. Thus, to use the idea of the VSS observer a linear output will be constructed from eqn. 14.

Since C is a $p \times 2n$ matrix, if the singular value decomposition is applied, there exist two unitary matrices U and V such that

$$C = U^T\Sigma V \tag{15}$$

where $\Sigma = [D\,|\,0]$ is a $p \times 2n$ matrix and D is a $p \times p$ diagonal matrix. Let $2n > p > m$. We further partition matrices Σ and V into

$$\Sigma = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \end{bmatrix} \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \\ V_{31} & V_{32} \end{bmatrix} \tag{16}$$

where $D_{11} \in R^{m \times m}$, $D_{22} \in R^{(p-m)\times(p-m)}$, $V_{11} \in R^{m \times m}$, $V_{12} \in R^{m \times 2(n-m)}$, $V_{21} \in R^{(p-m)\times m}$, $V_{22} \in R^{(p-m)\times 2(n-m)}$, $V_{31} \in R^{(2n-p)\times m}$, $V_{32} \in R^{(2n-p)\times 2(n-m)}$. Premultiplying both sides of eqn. 14 by U gives

$$Uy = \Sigma V\begin{bmatrix} \frac{\partial\Omega(x_2)}{\partial x_2}\dot{x}_2 \\ \Omega(x_2) \\ \dot{x}_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} D_{11}V_{11}\begin{bmatrix} \frac{\partial\Omega(x_2)}{\partial x_2}\dot{x}_2 \\ \Omega(x_2) \end{bmatrix} + D_{11}V_{12}\begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \\ D_{22}V_{21}\begin{bmatrix} \frac{\partial\Omega(x_2)}{\partial x_2}\dot{x}_2 \\ \Omega(x_2) \end{bmatrix} + D_{22}V_{22}\begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \end{bmatrix} \tag{17}$$

Note that if rank $(C) > m$, then $D_{11}V_{11}$ is nonsingular. It is convenient to use the partition of the identity matrix $I_p = [E_3^T, E_4^T]$, where E_3 is an $m \times p$ matrix and E_4 is a $(p-m) \times p$ matrix, to simplify eqn. 16 as

$$E_3 U y = D_{11} V_{11} \begin{bmatrix} \dfrac{\partial \Omega(x_2)}{\partial x_2} \dot{x}_2 \\ \Omega(x_2) \end{bmatrix} + D_{11} V_{12} \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \tag{18}$$

$$E_4 U y = D_{22} V_{21} \begin{bmatrix} \dfrac{\partial \Omega(x_2)}{\partial x_2} \dot{x}_2 \\ \Omega(x_2) \end{bmatrix} + D_{22} V_{22} \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \tag{19}$$

Since $D_{11}V_{11}$ is nonsingular, $\{\dot{x}_2^T[\partial\Omega(x_2)/\partial x_2]^T \vdots \Omega^T(x_2)\}^T$ can be solved from eqn. 18 as

$$\begin{bmatrix} \dfrac{\partial \Omega(x_2)}{\partial x_2} \dot{x}_2 \\ \Omega(x_2) \end{bmatrix} = (D_{11}V_{11})^{-1}\left[E_3 U y - D_{11} V_{12} \begin{bmatrix} \dot{x} \\ x_2 \end{bmatrix}\right] \tag{20}$$

Substituting eqn. 20 into eqn. 19 gives

$$\bar{y} = \bar{C} \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \tag{21}$$

where

$$\bar{y} = [E_4 - D_{22} V_{21} V_{11}^{-1} D_{11}^{-1} E_3] U y \tag{22}$$

$$\bar{C} = [D_{22} V_{22} - D_{22} V_{21} V_{11}^{-1} V_{12}] \tag{23}$$

The new output $\bar{y}$ is linear in state variables $\dot{x}_2$ and x_2. It will be used in our observer design. However, the observability for the subsystem eqns. 9 and 21 must be rechecked.

4 Observer design

In this section, two types of asymptotic observers will be designed for the dynamic system eqns. 8, 9 and 21. The design concept of the first observer resembles the VSS observer, and the second one is a modification of the first one.

To carry out the subsequent developments of the first observer, we make the following assumption:

Assumption 1:

(*a*) $$\begin{bmatrix} [E_2 \bar{M}(x_2) E_2^T]^{-1} E_2 T^T(x_2) \\ 0 \end{bmatrix} = P^{-1} \bar{C}^T h(x_2)$$

(*b*) $\|h(x_2)\| \leqslant H$

where

$$P^{-1} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$P_1 \in R^{(n-m) \times 2(n-m)}$, $P_2 \in R^{(n-m) \times 2(n-m)}$, P is a $2(n-m) \times 2(n-m)$ positive matrix, H is a positive number, and $P_2 \bar{C}^T \bar{C} = 0$.

Define

$$X_2 = \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix}, \hat{X}_2 = \begin{bmatrix} \hat{\dot{x}}_2 \\ \hat{x}_2 \end{bmatrix} \quad \text{and} \quad e = X_2 - \hat{X}_2$$

Then we select the observer structure as follows

$$\dot{\hat{x}}_2 = \hat{\dot{x}}_2 + G_1^0 \bar{C}(X_2 - \hat{X}_2) + P_2 R(X_2, \hat{X}_2, u) \tag{24}$$

$$\begin{aligned} E_1 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{\dot{x}}}_2 = & -E_1 \bar{F}(\hat{x}_2, \hat{\dot{x}}_2) + E_1 T^T(\hat{x}_2) u \\ & + E_1 T^T(\hat{x}_2) J^T(x_2) \hat{\lambda} \\ & + E_1 \bar{M}(\hat{x}_2) E_2^T G_3^0 \bar{C}(X_2 - \hat{X}_2) \\ & - E_1 \bar{M}(\hat{x}_2) E_2^T (E_2 \bar{M}(\hat{x}_2) E_2^T)^{-1} E_2 T^T(\hat{x}_2) u \\ & + E_1 \bar{M}(\hat{x}_2) E_2^T P_1 R(X_2, \hat{X}_2, u) \end{aligned} \tag{25}$$

$$\begin{aligned} E_2 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{\dot{x}}}_2 = & -E_2 \bar{F}(\hat{x}_2, \hat{\dot{x}}_2) \\ & + E_2 \bar{M}(\hat{x}_2) E_2^T G_2^0 \bar{C}(X_2 - \hat{X}_2) \\ & + E_2 \bar{M}(\hat{x}_2) E_2^T P_1 R(X_2, \hat{X}_2, u) \end{aligned} \tag{26}$$

where

$$R(X_2, \hat{X}_2, u) = \begin{cases} \dfrac{\bar{C}^T \bar{C} e H \|u\|}{\|\bar{C} e\|}, & \text{for } \bar{C}e \neq 0 \\ 0 \quad , & \text{otherwise} \end{cases} \tag{27}$$

and G_1^0, G_2^0 and G_3^0 are constant matrices. The structure of the observer is shown in Fig. 1.

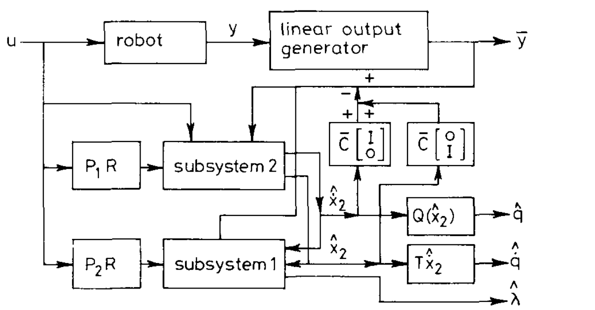

Fig. 1 *Structure of observer*

We have the following results:

Theorem 1: Consider the system in eqns 8, 9 and 21. If the following conditions are satisfied

(*a*) Assumption 1 holds

(*b*) $P\nabla\tilde{F}(x_2, \dot{x}_2) - P\begin{bmatrix} G_2^0 \bar{C} \\ G_1^0 \bar{C} \end{bmatrix} \nabla \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix}$

is uniformly negative definite for all x_2, $\dot{x}_2 \in R^{n-m}$ and some $\varepsilon > 0$, where

$$\tilde{F}(x_2, \dot{x}_2) = \begin{bmatrix} -(E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \dot{x}_2 \end{bmatrix}$$

and

$$\nabla \tilde{F} = \begin{bmatrix} \dfrac{\partial \tilde{F}}{\partial \dot{x}_2}, \dfrac{\partial \tilde{F}}{\partial x_2} \end{bmatrix} \tag{28}$$

(*c*) $E_1 T^T(x_2) J^T(x_2)$ and $E_1 T^T(\hat{x}_2) J^T(\hat{x}_2)$ are nonsingular for all x_2, $\hat{x}_2 \in R^{n-m}$, then the observer defined by eqns. 24–27 is an asymptotic observer in the sense that

$$\|x(t) - \hat{x}(t)\| \leqslant K e^{-\eta(t-t_0)} \quad \forall t \geqslant t_0 \tag{29}$$

where $\eta = (2\varepsilon e^T e / e^T P e)$ and K depends on $x(t_0)$ and $\hat{x}(t_0)$. In addition, the estimated contact force $\hat{\lambda}$ converges to λ at the same rate as $\hat{x}(t)$ to $x(t)$.

Proof: eqns. 9 and 26 can be conveniently expressed in matrix form as

$$\begin{bmatrix} \ddot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -(E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} (E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 T^T(x_2) \\ 0 \end{bmatrix} u \tag{30}$$

and

$$\begin{bmatrix} \dot{\hat{x}}_2 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -(E_2 \bar{M}(\hat{x}_2)E_2^T)^{-1} E_2 \bar{F}(\hat{x}_2, \hat{x}_2) \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} G_2^0 \bar{C} \\ G_0^1 \bar{C} \end{bmatrix} \begin{bmatrix} \dot{x}_2 - \hat{x}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} + \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} R(X_2, \hat{X}_2, u) \quad (31)$$

Choose a Lyapunov function V as

$$V(e) = \frac{1}{2} e^T P e \quad (32)$$

Then

$$\dot{V}(e) = e^T P \left\{ \tilde{F}(x_2, \dot{x}_2) - \tilde{F}(\hat{x}_2, \hat{x}_2) - \begin{bmatrix} G_2^0 \bar{C} \\ G_1^0 \bar{C} \end{bmatrix} (X_2 - \hat{X}_2) \right\} + e^T P \begin{bmatrix} (E_2 \bar{M}(x_2)E_2^T)^{-1} E_2 T^T(x_2) \\ 0 \end{bmatrix} \times u - e^T P \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} R(X_2, \hat{X}_2, u) \quad (33)$$

When $\|\bar{C}e\| \neq 0$, the last two terms in the right hand side can be further simplified as follows

$$e^T P P^{-1} \bar{C}^T h(x_2) u - e^T R(X_2, \hat{X}_2, u) \leqslant \|e^T \bar{C}^T\| \|H\| \|u\| - \frac{e^T \bar{C}^T \bar{C} e H \|u\|}{\|\bar{C}e\|} = 0$$

Thus, eqn. 33 becomes

$$\dot{V}(e) \leqslant e^T \int_0^1 P \left\{ \nabla \tilde{F}(x_2, \dot{x}_2) - \begin{bmatrix} G_2^0 \bar{C} \\ G_1^0 \bar{C} \end{bmatrix} \nabla \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} \right\} (w_s) \, ds \, e$$

where

$$w_s = s \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix} + (1 - s) \begin{bmatrix} \hat{x}_2 \\ \hat{x}_2 \end{bmatrix} \quad \text{and} \quad 0 < s < 1$$

Since

$$\nabla \tilde{F}(x_2, \dot{x}_2) - \begin{bmatrix} G_2^0 \bar{C} \\ G_1^0 \bar{C} \end{bmatrix} \nabla \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix}$$

is uniformly negative definite for some $\varepsilon > 0$, we have

$$\dot{V}(e) \leqslant -\varepsilon \|e\|^2 \quad (34)$$

Then,

$$V(e) \leqslant V[e(t_0)] e^{-\eta(t - t_0)} \quad \forall t > t_0$$

And the result is followed.

Next, we consider the estimated contact force vector $\hat{\lambda}$. If $E_1 T^T(x_2) J^T(x_2)$ and $E_1 T^T(\hat{x}_2) J^T(\hat{x}_2)$ are nonsingular for all $x_2, \hat{x}_2 \in R^{n-m}$, then λ and $\hat{\lambda}$ can be solved from eqns. 8 and 25. From eqn. 8, λ is determined as

$$\lambda = S_1(x_2, \dot{x}_2) + S_2(x_2, \dot{x}_2) u \quad (35)$$

where

$$S_1(x_2, \dot{x}_2) = [E_1 T^T(x_2) J^T(x_2)]^{-1} \{ -E_1 \bar{M}(x_2) E_2^T \times [E_2 \bar{M}(x_2) E_2^T]^{-1} E_2 + E_1 \} \bar{F}(x_2, \dot{x}_2)$$

$$S_2(x_2, \dot{x}_2) = [E_1 T^T(x_2) J^T(x_2)]^{-1} \{ E_1 \bar{M}(x_2) E_2^T \times [E_2 \bar{M}(x_2) E_2^T]^{-1} E_2 - E_1 \} T^T(x_2) \quad (36)$$

Similarly, we have

$$\hat{\lambda} = S_1(\hat{x}_2, \hat{x}_2) + S_2(\hat{x}_2, \hat{x}_2) u + [E_1 T^T(\hat{x}_2) J^T(\hat{x}_2)]^{-1} \times [E_1 \bar{M}(\hat{x}_2) E_2^T][G_2^0 - G_3^0] \bar{C}(X_2 - \hat{X}_2) \quad (37)$$

Since $\hat{x}_2$ converges to x_2, and $\hat{x}_2$ converges to $\dot{x}_2$, as time approaches to infinity, functions $S_1(\hat{x}_2, \hat{x}_2)$ and $S_2(\hat{x}_2, \hat{x}_2)$ will converge to $S_1(x_2, \dot{x}_2)$ and $S_2(x_2, \dot{x}_2)$, respectively; moreover, $\bar{C}(X_2 - \hat{X}_2)$ converges to zero. Thus, the estimated contact force vector $\hat{\lambda}$ converges to λ at the same rate as $\hat{x}_2$ to x_2. Note that the G_3^0 term is used to accelerate the convergence rate.

The above observer design, the term $R(X_2, \hat{X}_2, u)$, may be discontinuous as $\bar{C}e = 0$. Hence, there may exist chattering phenomena in the steady state. To improve the observer performance, the observer is slightly modified as follows.

Take the following assumption:

Assymption 2

(i) $$\begin{bmatrix} [E_2 \bar{M}(x_2)E_2^T]^{-1} E_2 T^T(x_2) \\ 0 \end{bmatrix} u = P^{-1} \bar{C}^T \sum_{i=1}^{n} R_i(x_2) u_i$$

(ii) $\nabla R_i(x_2) = L_i(x_2) \bar{C}, \quad \|L_i(x_2)\| \leqslant H_i, \quad i = 1, \ldots, n$

where

$$P^{-1} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$P_1 \in R^{(n-m) \times 2(n-m)}$, $P_2 \in R^{(n-m) \times 2(n-m)}$, P is a $2(n-m) \times 2(n-m)$ positive matrix, H_i is a positive number, and $P_2 \bar{C}^T \bar{C} = 0$.

Then, the observer structure is chosen as follows:

$$\dot{\hat{x}}_2 = \hat{x}_2 + G_1^0 \bar{C} \begin{bmatrix} \dot{x}_2 - \hat{x}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} \quad (38)$$

$$E_1 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{x}}_2 = -E_1 \bar{F}(\hat{x}_2, \hat{x}_2) + E_1 T^T(\hat{x}_2) J^T(\hat{x}_2) \hat{\lambda} + E_1 \bar{M}(\hat{x}_2) E_2^T G_3^0 \bar{C} \begin{bmatrix} \dot{x}_2 - \hat{x}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} + E_1 T^T(\hat{x}_2) u \quad (39)$$

$$E_2 \bar{M}(\hat{x}_2) E_2^T \dot{\hat{x}}_2 = -E_2 \bar{F}(\hat{x}_2, \hat{x}_2) + E_2 \bar{M}(\hat{x}_2) E_2^T G_2^0 \bar{C} \begin{bmatrix} \dot{x}_2 - \hat{x}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} + E_2 \bar{M}(\hat{x}_2) E_2^T P_1 \bar{C}^T \bar{C} \begin{bmatrix} \dot{x}_2 - \hat{x}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} \times \sum_{i=1}^{n} H_i \|u_i\| + E_2 T^T(\hat{x}_2) u \quad (40)$$

where G_1^0, G_2^0, and G_3^0 are constant matrices. The structure of this improved observer is similar to Fig. 1.

We have the following results.

Theorem 2: Consider the system in eqns. 8, 9 and 21. If the following conditions are satisfied

(i) Assumption 2 holds

(ii) $$P \nabla \tilde{F}(x_2, \dot{x}_2) - P \begin{bmatrix} G_2^0 \bar{C} \\ G_1^0 \bar{C} \end{bmatrix} \nabla \begin{bmatrix} \dot{x}_2 \\ x_2 \end{bmatrix}$$

is uniformly negative definite for all x_2, $\dot{x}_2 \in R^{n-m}$ and some $\varepsilon > 0$, where

$$\tilde{F}(x_2, \dot{x}_2) = \begin{bmatrix} -(E_2 \bar{M}(x_2) E_2^T)^{-1} E_2 \bar{F}(x_2, \dot{x}_2) \\ \dot{x}_2 \end{bmatrix}$$

and

$$\nabla \tilde{F} = \left[\frac{\partial \tilde{F}}{\partial \dot{x}_2}, \frac{\partial \tilde{F}}{\partial x_2} \right]$$

(iii) $E_1 T^T(x_2) J^T(x_2)$ and $E_1 T^T(\hat{x}_2) J^T(\hat{x}_2)$ are nonsingular for all x_2, $\hat{x}_2 \in R^{n-m}$, then the observer defined by eqns. 38–40 is an asymptotic observer in the sense that

$$e^T(t)Pe(t) \leqslant e^T(t_0)Pe(t_0)e^{-\eta(t-t_0)} \quad \forall t \geqslant t_0 \tag{41}$$

where

$$e(t) = \begin{bmatrix} \dot{x}_2(t) - \hat{\dot{x}}_2 \\ x_2(t) - \hat{x}_2 \end{bmatrix}, \quad \eta = \frac{2\varepsilon e^T e}{e^T P e} \tag{42}$$

In addition, the estimated contact force $\hat{\lambda}$ converges to λ.

Proof: The proof is similar to Theorem 1. Hence, it is omitted here.

Remark: In the construction of the observer, the states of the transformed system rather than the states of the original system are estimated. This indicates that it is not necessary to estimate all states for the constrained system. The complete information of the original states can be obtained by the following transformation

$$\hat{q} = \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \end{bmatrix} = Q(\hat{x}) = \begin{bmatrix} \Omega(\hat{x}_2) \\ \hat{x}_2 \end{bmatrix} \tag{43}$$

and

$$\hat{\dot{q}} = \begin{bmatrix} \hat{\dot{q}}_1 \\ \hat{\dot{q}}_2 \end{bmatrix} = T(\hat{x})\hat{\dot{x}} = \begin{bmatrix} I_m & \dfrac{\partial \Omega(x_2)}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix}_{x_2 = \hat{x}_2} \begin{bmatrix} 0 \\ \hat{\dot{x}}_2 \end{bmatrix} \tag{44}$$

Another remarkable feature of the observer is that the contact force λ can be directly estimated rather than obtained by expensive force sensors.

5 Example

In this section, an example is given to illustrate the above developments. Both asymptotic observers are constructed in terms of the same example. Their simulation results are also given for the purpose of justification. Suppose a constrained dynamic system is given by

$$\ddot{q}_1 = u_1 \tag{45}$$

$$(100 - 4\dot{q}_2^2)\ddot{q}_2 + \dot{q}_2^3 + 100q_2 - 4q_2\dot{q}_2^2 = u_2 \tag{46}$$

The constraint and output equations are

$$\phi(q_1, q_2) = q_1^2 - q_2 \tag{47}$$

$$y = \begin{bmatrix} q_1 + q_2 \\ q_2 + \dot{q}_2 \end{bmatrix} \tag{48}$$

where $q_1, q_2, u, u_2 \in R$ and $y \in R^{2\times 1}$.

By introducing the Lagrange multiplier and applying the McClamroch and Wang nonlinear transformation, eqns. 45–48 can be transformed into

$$2\dot{x}_2^2 + 2x_2\ddot{x}_2 = u_1 + \lambda \tag{49}$$

$$\ddot{x}_2 = -0.01\dot{x}_2^3 - x_2 + 0.02x_2u_1 + 0.01u_2 \tag{50}$$

$$x_1 = 0 \tag{51}$$

$$y = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_2^2 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \tag{52}$$

By singular value decomposition, a linear output can be constructed from eqn. 52 as

$$\bar{y} = [0\ 2]y = 2x_2 + 2\dot{x}_2 \tag{53}$$

From eqns. 24–27, the first observer structure is selected as follows:

$$\begin{aligned} 2\hat{\dot{x}}_2 + 2\hat{x}_2\dot{\hat{\dot{x}}}_2 &= u_1 + \hat{\lambda} + 2\hat{x}_2G_3^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) \\ &\quad - 2\hat{x}_2(0.02\hat{x}_2u_1 + 0.01u_2) \\ &\quad + [1, 0]R(x_2, \dot{x}_2, \hat{x}_2, \hat{\dot{x}}_2, u)(2\hat{x}_2) \end{aligned} \tag{54}$$

$$\begin{aligned} \dot{\hat{\dot{x}}}_2 &= -0.01\hat{\dot{x}}_2^3 - \hat{x}_2 + 2G_2^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) \\ &\quad + [1,\ 0]R(x_2, \dot{x}_2, \hat{x}_2, \hat{\dot{x}}_2, u) \end{aligned} \tag{55}$$

$$\begin{aligned} \dot{\hat{x}}_2 &= \hat{\dot{x}}_2 + 2G_1^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) \\ &\quad + [-1,\ 1]R(x_2, \dot{x}_2, \hat{x}_2, \hat{\dot{x}}_2, u) \end{aligned} \tag{56}$$

where

$$R(x_2, \dot{x}_2, \hat{x}_2, \hat{\dot{x}}_2, u) = \begin{cases} 2H\begin{bmatrix} 1 \\ 1 \end{bmatrix}\sqrt{(u_1^2 + u_2^2)} & \text{for } \begin{bmatrix} \dot{x}_2 - \hat{\dot{x}}_2 \\ x_2 - \hat{x}_2 \end{bmatrix} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{57}$$

Select $H = 1$ and the observer gains as $G_1^0 = -1.5$, $G_2^0 = 5.5$, $G_3^0 = 0.4$, then according to Theorem 1, the observer defined by eqns. 54–57 is an asymptotic observer for the system defined by eqns. 49, 50 and 53. The simulation results are shown in Figs. 2–4. It is obvious that there are

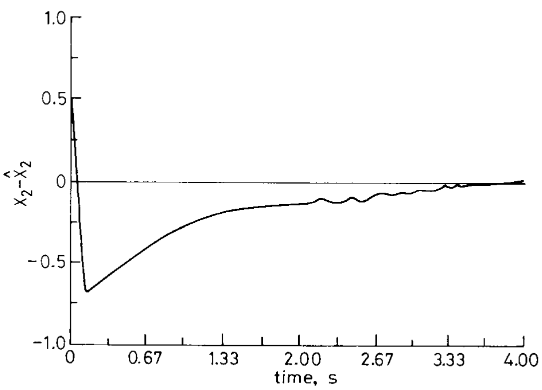

Fig. 2 *Displacement error $(x_2 - \hat{x}_2)$ of observer 1*
$\hat{x}_2(t_0) = 0.5$
$x_2(t_0) = 1.0$

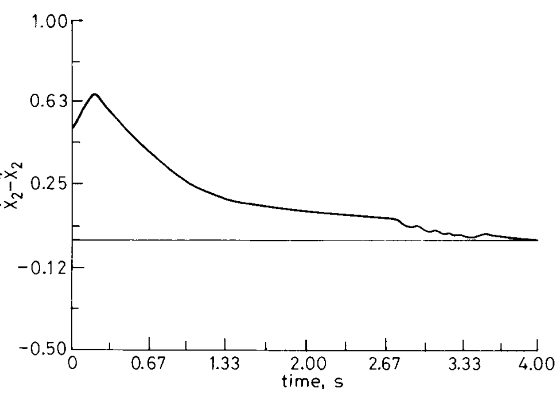

Fig. 3 *Velocity error $(\dot{x}_2 - \hat{\dot{x}}_2)$ of observer 1*
$\hat{\dot{x}}_2(t_0) = 0.5$
$\dot{x}_2(t_0) = 1.0$

some chattering phenomena as the estimated variables approach the true variables.

From eqns. 38–40, the second observer can be selected as follows:

$$\begin{aligned} 2\hat{\dot{x}}_2 + 2\hat{x}_2\dot{\hat{\dot{x}}}_2 &= u_1 + \hat{\lambda} \\ &\quad + 2\hat{x}_2G_3^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) \end{aligned} \tag{58}$$

$$\dot{\hat{\dot{x}}}_2 = -0.01\hat{\dot{x}}_2^3 - \hat{x}_2 + 0.02\hat{\dot{x}}_2 u_1 + 0.01u_2 + 2G_2^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) + 4H(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2)(\|u_1\| + \|u_2\|) \quad (59)$$

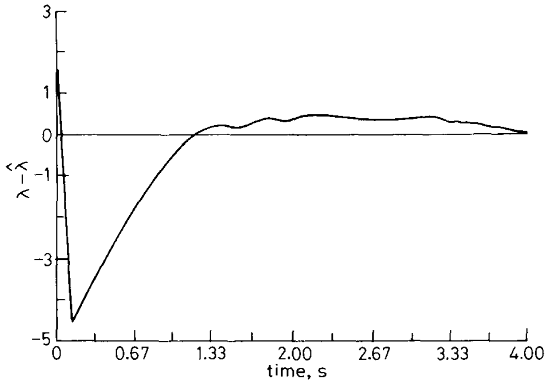

Fig. 4 *Contact force error ($\lambda - \hat{\lambda}$) of observer 1*
$\hat{\lambda}(t_0) = 2.00$
$\lambda(t_0) = 4.94$

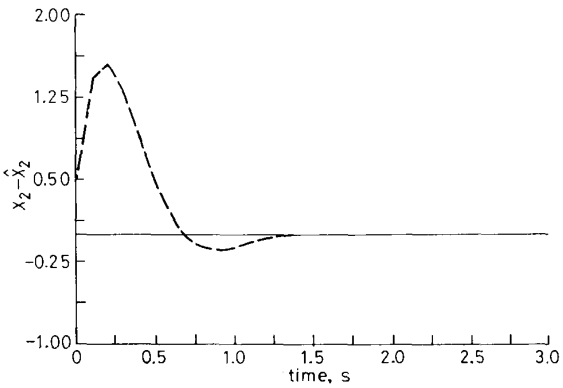

Fig. 5 *Displacement error ($x_2 - \hat{x}_2$) of observer 2*
$\hat{x}_2(t_0) = 0.5$
$x_2(t_0) = 1.0$

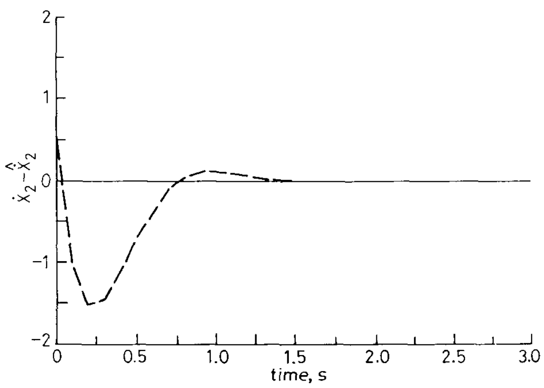

Fig. 6 *Velocity error ($\dot{x}_2 - \hat{\dot{x}}_2$) of observer 2*
$\hat{\dot{x}}_2(t_0) = 0.5$
$\dot{x}_2(t_0) = 1.0$

$$\dot{\hat{x}}_2 = \hat{\dot{x}}_2 + 2G_1^0(x_2 + \dot{x}_2 - \hat{x}_2 - \hat{\dot{x}}_2) \quad (60)$$

Select $H = 0.15$ and the observer gains as $G_1^0 = -7$, $G_2^0 = 5.5$ and $G_3^0 = 0.05$, then according to Theorem 2, the observer defined by eqns. 54–56 is an asymptotic observer for the system defined by eqns. 49, 50 and 53.

The simulation results are shown in Figs. 5–7. From those figures, we know that the estimated states $\hat{x}_2$, $\hat{\dot{x}}_2$ and $\hat{\lambda}$ will converge to the system states x_2, $\dot{x}_2$ and λ, respectively. The result is better than the first observer's.

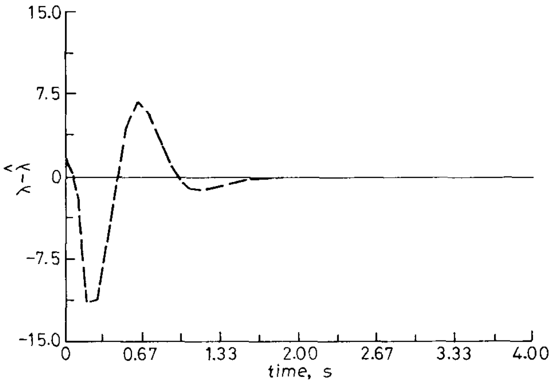

Fig. 7 *Contact force error ($\lambda - \hat{\lambda}$) of observer 2*
$\hat{\lambda}(t_0) = 2.00$ $\lambda(t_0) = 4.04$

6 Conclusion

Two asymptotic observers are constructed for the constrained robot system. It has been shown that the converging properties can be determined by the selection of the observer gain matrices G_1^0, G_2^0 an G_3^0. The difficulty caused by the nonlinear coupling in the control has been overcome by introducing the VSS observer idea at the expense of the requirement of linear output. Although the estimates of the states are based on the transformed reduced subsystems, the estimates of the original states can be recovered by applying the inverse transformation. Since the contact force, which is usually not directly available in the constrained robot system, can be estimated directly, the observer may be very useful for the controller design of a constrained robot system. The controller may design on the basis of $\hat{x}_2$, $\hat{\dot{x}}_2$ or $\hat{q}$, $\dot{\hat{q}}$; however, the stabilisation problem of the overall system should be carefully investigated. This result will be reported in a forthcoming paper.

7 References

1 BESTLE, D., and ZEITZ, M.: 'Canonical form observer design for nonlinear time-variable systems', *Int. J. of Control*, 1983, **38**, pp. 419–431
2 HUANG, H.P., and McCLAMROCH, N.H.: 'Time-optimal control for a contour following problem', *IEEE J.*, 1988, **RA-4**, pp. 140–149
3 HUANG, H.P.: 'The unified formulation of constrained robot systems', *Proc. of IEEE Intl. Conf. on Robotics and Automation*, Philadelphia, 1988, **3**, pp. 1590–1592
4 KOU, S.R.: 'Observability and observer for nonlinear dynamical systems'. Doctoral dissertation, Dept. of System Science, Washington University, Missouri, USA, 1973
5 KRENER, A.J., and RESPONDEK, W.: 'Nonlinear observers with linearizable error dynamics', *SIAM J. of Control and Optimization*, 1985, **23**, pp. 197–216
6 McCLAMROCH, N.H., and HUNG, H.P.: 'Dynamics of a closed chain manipulator'. Proc. of American Control Conference, Boston, 1985
7 McCLAMROCH, N.H., and WANG, D.: 'Feedback stabilization and tracking of constrained robots', *IEEE Trans.*, 1988, **AC-33**, pp. 419–426
8 McCLAMROCH, N.H.: 'Singular systems of differential equations as dynamical models for constrained robot systems'. Proc. of IEEE Conf. on Robotics and Automation, San Francisco, 1986, pp. 21–28
9 WALCOTT, B.L., CORLESS, M.J., and ZAK, S.H.: 'Comparative study of nonlinear state observation techniques', *Int. J. of Control*, 1987, **45**, pp. 2109–2132
10 YOSHKAWA, T.: 'Dynamical hybrid position/force control of robot manipulators-description of hand constraints and calculation of joint driving force', *IEEE J.*, 1987, **RA-3**, pp. 386–392