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On the conceptual design of redundant-drive backlash-free geared robot manipulators

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Abstract

A systematic methodology for the conceptual design of redundant-drive backlash-free geared robot manipulators (GRMs) is presented. A GRM is considered as a combination of an equivalent open-loop chain (EOLC) and several mechanical transmission lines (MTLs), and each MTL of a series of non-fractionated units. It is shown that these units can be categorized into three family trees according to their motion types. According to the trees, MTLs can be enumerated and be classified as basic and derived MTLs. Based on the atlas of MTLs, graph and functional representations of admissible redundant-drive backlash-free GRMs can be created according to the specified form of structure matrix. It is believed that the enumerated results can expand the application of such mechanisms.

Keywords: Geared robot manipulator; Redundant-drive manipulator; Conceptual design; Backlash

1. Introduction

The robot manipulators for industrial purpose are usually designed in an open-loop configuration. The actuators are often mounted along the joints to reduce mechanical complexity. However, the dynamic performance is degraded on the other hand. The geared robot manipulators (GRMs) utilize gear trains to transmit input power to driven joints. Since the actuators can be located as close to the ground as possible, inertia load is reduced and the dynamic performance is effectively improved. By utilizing a redundant actuator to provide the control torque, Chang and Tsai [1] showed that GRMs with no backlash can be enumerated. However, the concept remained in the abstract since only the forms of admissible structure matrices were developed.

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Numerous researches have been devoted to the synthesis of GRMs [2–7]. Lin and Tsai [2] enumerated an atlas of bevel-gear-type spherical wrist mechanisms based on the results of two degree-of-freedom (dof) non-fractionated geared kinematic chains (GKCs). Chang and Tsai [3] proposed the concept of structure matrix and mechanical transmission line (MTL) to describe the power transmission among the actuators and joints. Chen and Shiue [4] showed that an MTL can be decomposed into input and transmission units which can be identified from the atlas of non-fractionated GKCs [8–13]. With the atlas of admissible units, admissible MTLs and, in turn, GRMs can be systematically enumerated.

In this paper, the 17 input and transmission units identified by Chen and Shiue [4] are simplified as 12 units by neglecting the distinction between pre- and post-connecting links. These simplified units are then classified according to their transmission characteristics. With identical transmission characteristics, a unit having the simplest form is called the basic-type unit from which the family tree can be established. With the classified units, MTLs can be enumerated and categorized into basic MTLs and derived MTLs. A basic MTL is composed of basic-type units only while a derived MTL contains non-basic-type unit(s). A systematic approach for the conceptual design of redundant-drive backlash-free GRMs is proposed based on the categorized MTLs. According to the structure matrix, the requirement of MTLs to synthesize desired GRMs can be obtained.

2. Structural characteristics

Fig. 1(a) shows the functional representation of an eight-link, three-dof redundant-drive backlash-free GRM with the g^2s^2-8 structure matrix [1]. Links 1, 4, 5 and 7 are input links and

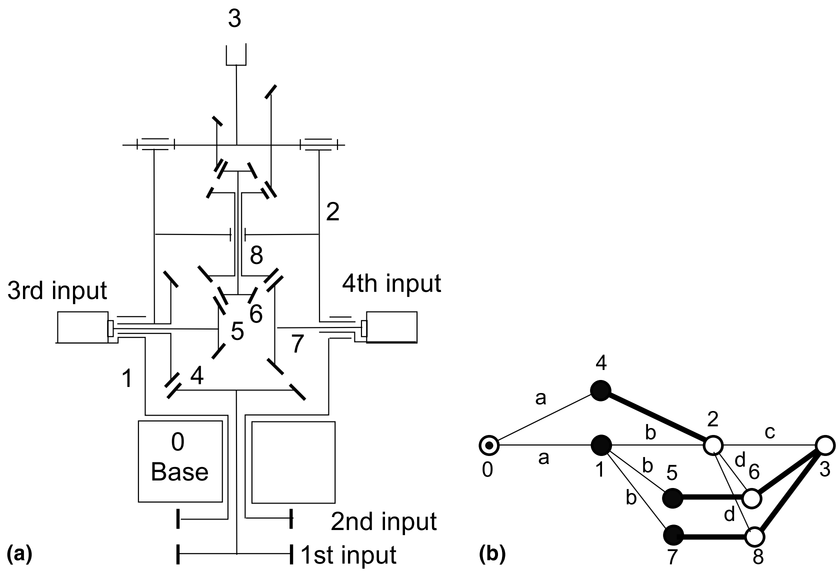


Fig. 1. A redundant-drive backlash-free GRM: (a) the functional representation; (b) the canonical graph representation.

link 3 is the end effector. Fig. 1(b) shows the corresponding canonical graph representation [14] where links are represented by vertices, revolute joints by thin edge, gear pairs by heavy edges, and thin edges are labeled according to their axes location. In the canonical graph representation, the thin-edged path starting from link 0 to link 3 forms the equivalent open-loop chain (EOLC) [14]. Links on the EOLC are called primary links while the others are called secondary links. The relative angular displacement between two adjacent primary links is referred to as a joint angle.

By denoting the displacement vectors in joint space and end-effector space as $\mathbf{\Theta}$ and $\mathbf{X}$, respectively, the velocity vectors between the two spaces can be related as

$$\dot{\mathbf{X}} = \mathbf{J}\dot{\Theta}, \tag{1}$$

where $\mathbf{J}$ is the Jacobian matrix.

By denoting the generalized forces in joint space and end-effector space as τ and $\mathbf{F}$, respectively, the torque relation between the two spaces can be expressed as

$$\tau = \mathbf{J}^{\mathrm{T}}\mathbf{F}. \tag{2}$$

Similarly, by denoting the displacement vector in the actuator space as Φ, the velocity vectors in the joint space and in the actuator space can be described as

$$\dot{\Phi} = \mathbf{A}^{\mathrm{T}}\dot{\Theta} \tag{3}$$

and the torque relation between the two spaces is expressed as

$$\tau = \mathbf{A}\xi, \tag{4}$$

where $\mathbf{A}$ is the structure matrix.

Applying Eq. (4) to the mechanism shown in Fig. 1(a), we have

$$\begin{bmatrix} \tau_{10} \\ \tau_{21} \\ \tau_{32} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ r_{2,4} & 0 & 1 & 1 \\ 0 & 0 & r_{3,6}r_{6,5} & r_{3,8}r_{8,7} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix}, \tag{5}$$

where τ_{10}, τ_{21} and τ_{32} are joint torques, ξ_1, ξ_2, ξ_3 and ξ_4 are torques applied at input links, $r_{ij} = \pm N_i/N_j$ represents the gear ratio of the gear pair composed of links i and j and the sign of the gear ratio is determined according to the positive rotation of link i which results in a positive or negative rotation of link j along their pre-defined axes of rotation, and where N_i denotes the teeth number of gear i.

Chang and Tsai [3] showed that each column in the structure matrix represents an MTL while each element in it reveals the relation between the input torque and the joint torque. Two MTLs are considered de-coupled if only primary links are shared as common links. For a de-coupled GRM, associated MTLs can be differentiated from the EOLC. The structure matrices of redundant-drive backlash-free GRMs have the following characteristics [1]:

C1. The structure matrix is an $n \times (n+1)$ matrix. Each column in the matrix has at least two non-zero elements.

C2. The sub-matrices obtained by removing any one column of the original matrix must be non-singular.

C3. The non-zero elements in each column must be consecutive.

C4. Two mechanisms are considered isomorphic if their corresponding structure matrices can be made identical by column operations.

From the above characteristics, Chang and Tsai [1] identified four admissible structure matrices for two-dof redundant-drive backlash-free GRMs and 34 for three-dof redundant-drive backlash-free GRMs.

3. Characteristics of units

Fig. 2 shows the pseudo-isomorphic graphs [13] of the four MTLs contained in Fig. 1(b). As shown in Eq. (5), three of the four MTLs influence two joints and the other is a direct drive MTL. A direct drive MTL is simply a two-link chain and the graph representations of the MTLs influencing two joints are composed of a two-link chain and a one-dof GKC. The two-link chain and the one-dof GKC are connected by a common link, which is referred to as a connecting link. Both the two-link chain and the one-dof GKC are regarded as the composing units of the redundant-drive backlash-free GRM.

Note that a connecting link is connected to primary links in adjacent units with turning pairs of the same axis label. Hence, the articulated joints on EOLC can be formed by the rearrangement of coaxial links among the primary links and connecting links. To transmit power between different axes in the GRM, each unit except the first one in an MTL should have at least two links connecting to the primary link with different axis labels. From the above discussion, the characteristics of these units, except the two-link chain, can be summarized as follows:

1. Each unit has one primary link and at least two secondary links that can be used as connecting links.
2. Primary link and connecting links are connected by turning pairs.

According to these characteristics, Chen and Shiue [4] developed the rules to identify admissible units up to five links and 17 one-dof units were found. However, it can be found that five pairs of the units are structurally isomorphic. Each pair essentially represents an identical unit but is flipped in opposite direction as connecting with other units to form an MTL. By disregarding the distinction between pre- and post-connecting links, the 17 units [4] can be reduced to 12 ones as shown in Fig. 3 in which connecting links are represented by hollow squares while the primary

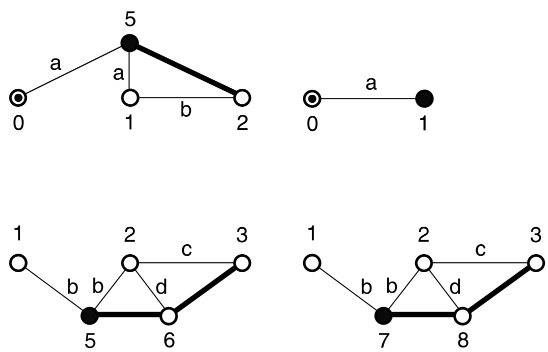


Fig. 2. Pseudo-isomorphic graphs of the MTLs in Fig. 1(b).

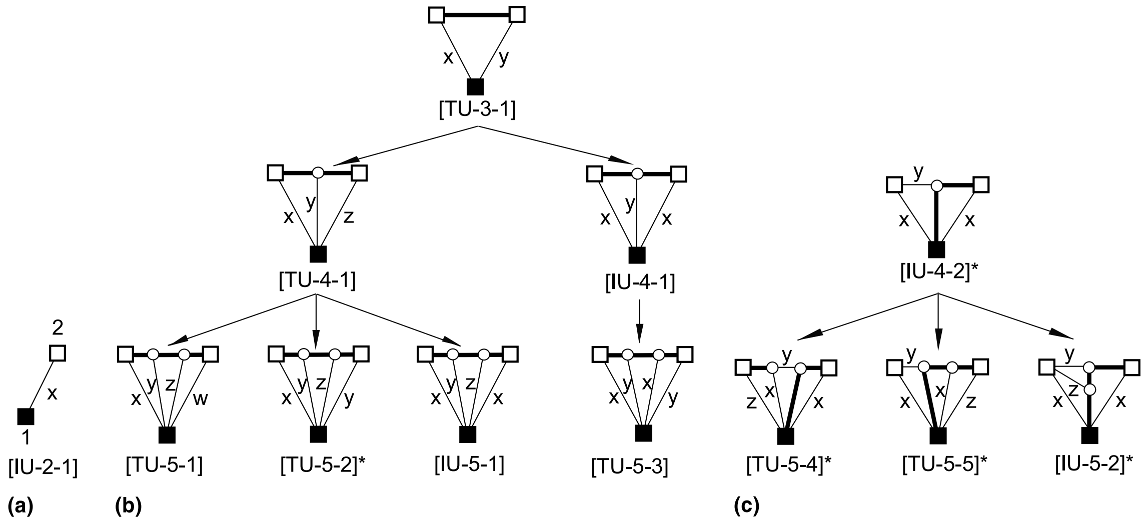


Fig. 3. Admissible units: (a) two-link chain; (b) equivalent ordinary units; (c) planetary units.

and secondary links are denoted by solid squares and circles, respectively. Note that the serial numbers of the units in Fig. 3 are renumbered and are different to those of Chen and Shiue [4]. In Fig. 3, a flippable unit is marked with an asterisk.

As the articulated joints are fixed, a unit performs one of the following types of motion:
1. all gears rotate along the fixed axis,
2. at least one gear rotates along a moving axis.

With the first kind of motion, the unit is called as the ordinary unit. For the second case, the unit is called as the planetary unit. According to motion types, units can be classified into the following three family trees:
1. direct drive unit, as the one shown in Fig. 3(a),
2. ordinary units, as the one shown in Fig. 3(b),
3. planetary units, as the one shown in Fig. 3(c).

In each of the above three trees, there is a unique basic-type unit, which has the minimum number of links and represents the simplest structural configuration with the associated motion type. It can be found that the basic type units corresponding to the three motion types are: [IU-2-1], [TU-3-1] and [IU-4-2]*. By adding idle gears or extending the geared path to other axes, derived-type units can be generated from a basic-type unit which inherits the same motion characteristics. Repeating the generating procedure to the derived-type units, next-generation derived-type units can be formed. The basic- and derived-type units constitute a family tree. The level of a unit in the hierarchy of the family tree indicates the structural complexity which provides the designer a guidance to modify the structure of the mechanism without changing the desired motion characteristics. Usually, a planetary drive has better reduction ratio and mechanical compactness than those of ordinary gear trains but it does need more design effort to arrange the moving axes in planetary drives. Based on the designer's judgment, the motion type provides a basic direction to determine what sort of gear train is to be designed for power transmission.

4. Enumeration of MTLs

It is shown that units transmit power from one to another by combining their connecting links together. The transmission between two units requires the two turning pairs connecting the primary links and the connecting link to have the same axis label so that the articulated joints can be formed. As shown in Fig. 4(a), two units are combined by using link 4 as the common link and the two primary links, link 1 and link 2, are connected with an articulated joint formed by coaxial rearrangement along *b*-axis.

A unit containing the input link must be used as the first unit in an MTL and the primary link must be the first link on the EOLC [4]. Note that the pre- and post-connecting links of the first unit must be connected to the primary link with turning pairs with the same axis label, as shown in Fig. 4(b). The first unit in an MTL is referred to as the input unit and is coded as [IU-n-#] in Fig. 3 in which n is the number of links and # represents the serial number. For other units in an MTL, pre- and post-connecting links must have different axis labels relative to the primary link in order to transmit power between different articulated joints. These units are called transmission units and are coded as [TU-n-#] in Fig. 3. The above discussion leads to the following enumeration procedure of MTLs:

Step 1. *Specify the numbers of links and joints of the desired MTL. From the family tree, n proper units are selected for an n-joint MTL.*

The number of links of an MTL can be expressed as follows:

$$m = \sum_{i=1}^{n} m_i - (n-1), \tag{6}$$

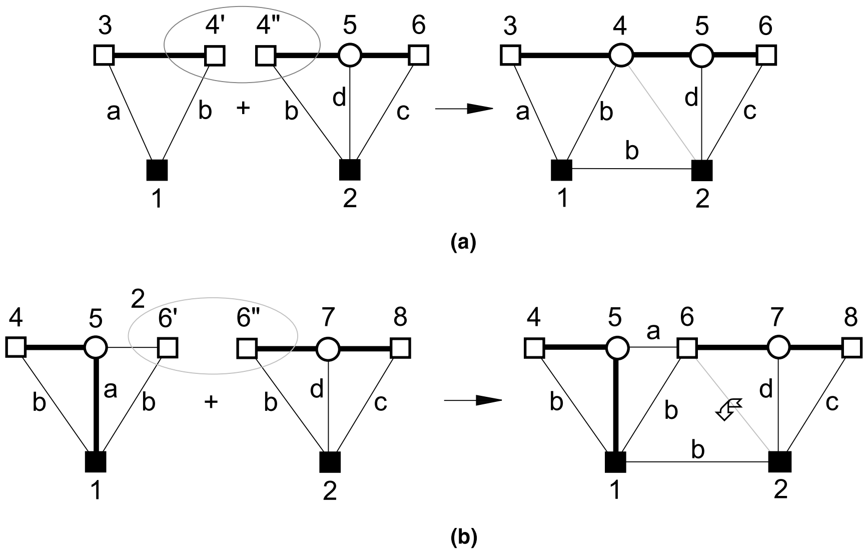


Fig. 4. Enumeration process of an MTL.

where m is the number of links of the MTL, m_i is the number of links of the ith unit and n is the number of joints of the MTL.

Eq. (6) provides the guidance to select units according to the given property of the MTL. Assume a three-joint, eight-link MTL is to be enumerated. From Eq. (6), we have

$$\sum_{i=1}^{3} m_i = 10. \quad (7)$$

As the number of links of the first unit is specified, the selection of other units can be determined. To use a basic-type unit as the first unit, only the unit [IU-2-1] or [IU-4-2]* can be chosen. Thus,

$$m_2 + m_3 = 8 \quad \text{as } m_1 = 2 \quad (8)$$

or

$$m_2 + m_3 = 6 \quad \text{as } m_1 = 4. \quad (9)$$

For the case in Eq. (8), we can choose $[m_2, m_3]$ as $[3, 5]$, $[4, 4]$ or $[5, 3]$. For the case in Eq. (9), $[m_2, m_3]$ can only be $[3, 3]$. According to the specification of number of links, proper units that have different axis labels relative to the primary link are chosen from the family trees.

Step 2. *Connect the selected units by combining the common links.*

Note that units in the family trees do not show the pre- and post-connecting links explicitly. Hence, the flipping of a unit must be taken into consideration. For the case in Eq. (9), the input unit [IU-4-2]* has a flipping pair. Hence, two configurations of MTLs can be obtained with the same unit combination.

Step 3. *Assign the input link.*

In the first unit of an MTL, the connecting link that is not used as the common link is assigned as the input link.

Step 4. *Rearrange the coaxial links to form the articulated joints.*

Repeating Steps 1–4, admissible MTLs with specified number of links and joints can be enumerated.

The enumerated results are classified as two groups. Those composed of purely basic-type units are classified as basic MTLs, and the others belong to derived MTLs, which contain one or more non-basic-type units. Figs. 5 and 6 show the graphs of admissible one- and two-joint MTLs with up to six links. In Fig. 5 and 6, basic MTLs are shown in part (a) and derived MTLs are shown in part (b). Fig. 7 shows the three-joint basic MTLs with up to eight links.

A basic MTL stands for the simplest configuration while a derived MTL is regarded as an alternative to the basic MTL. A derived MTL obtained by substituting an ordinary unit represents adding idle gears and is used for the following conditions:

1. extend the span between the corresponding joints,
2. change the gear train configuration in space,
3. obtain higher gear ratio.

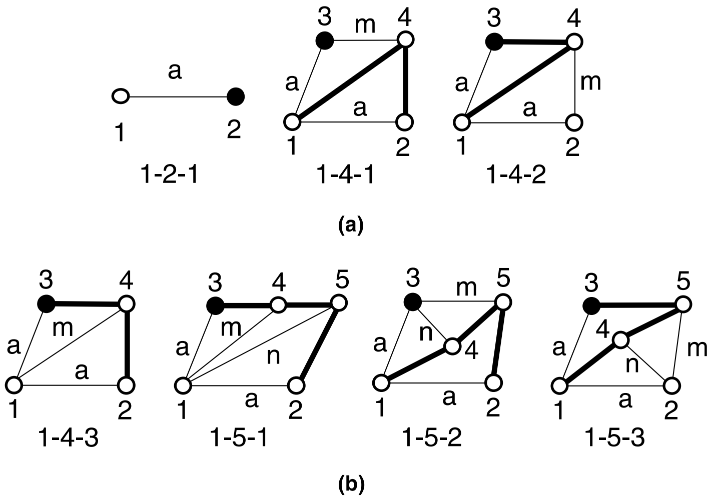


Fig. 5. One-joint MTLs with up to five links: (a) basic MTLs; (b) derived MTLs.

In contrast, a derived MTL obtained by substituting a planetary unit results in higher gear reduction between the corresponding joints.

The transmission characteristics of MTLs are determined by the composition of units. An MTL containing planetary unit(s) has larger flexibility in determining the gear ratios and results in a more compact structure than MTLs composed of ordinary units. An MTL with planetary units has the feature that more than one primary links are gears.

5. Creation of redundant-drive backlash-free GRMs

The enumeration of redundant-drive backlash-free GRMs can be completed through the following steps:

Step 1. *Specify the number of dof's of the desired mechanism.*

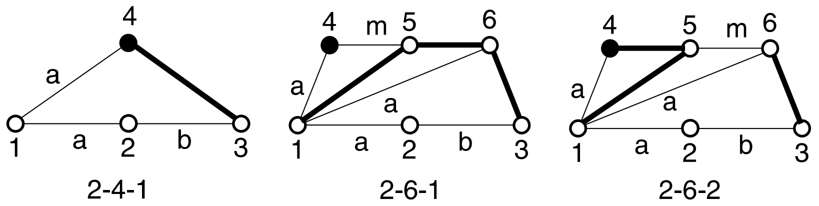
For an n-dof redundant-drive backlash-free GRM, $(n+1)$ MTLs are required. For instance, suppose a two-dof redundant-drive backlash-free GRM is desired, and thus three MTLs are required.

Step 2. *Select a proper structure matrix.*

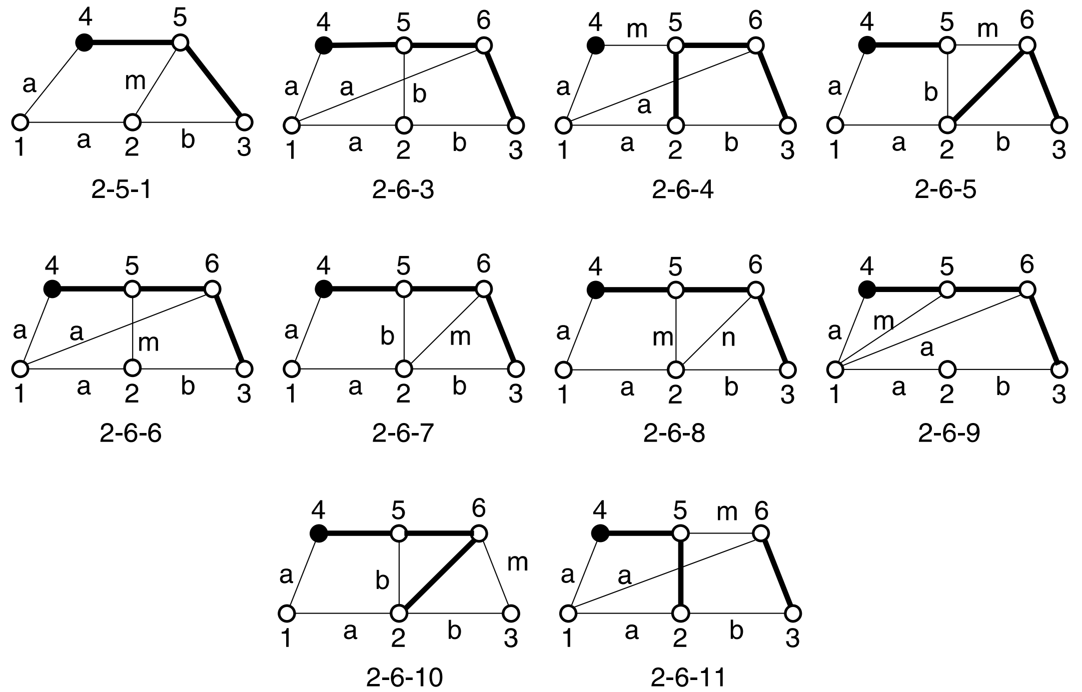
Suppose the g^3-2 structure matrix [1] is selected for the example mechanism. From the structure matrix, it is found that two two-joint MTLs and a one-joint MTL are required.

Step 3. *Select basic MTLs from the atlas.*

The selection of basic MTLs must take the transmission characteristics into account. According to Fig. 6(a), basic MTLs have two types: one contains planetary unit and the other is composed of



(a)



(b)

Fig. 6. Two-joint MTLs with up to six links: (a) basic MTLs; (b) derived MTLs.

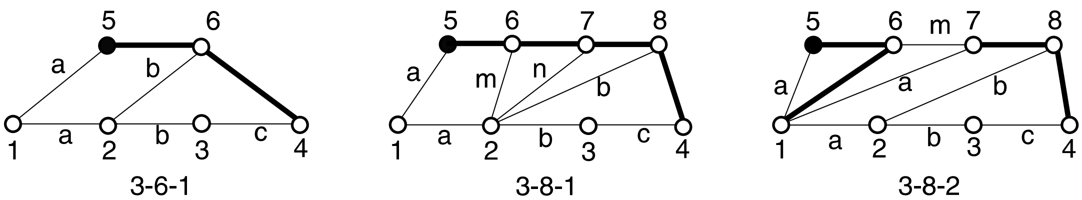


Fig. 7. Three-joint basic MTLs with up to eight links.

ordinary units only. For the example mechanism, suppose that simple configuration is required and thus MTLs without planetary units are preferred. Hence, the 2-4-1 MTL is selected for the two-joint MTL. The one-joint MTL uses the direct drive unit.

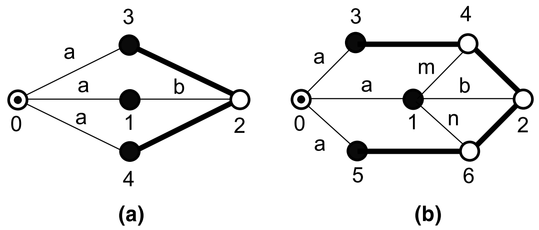


Fig. 8. Graphical representation of admissible mechanisms with g^3-2 structure matrix: (a) with basic MTLs; (b) with derived MTLs.

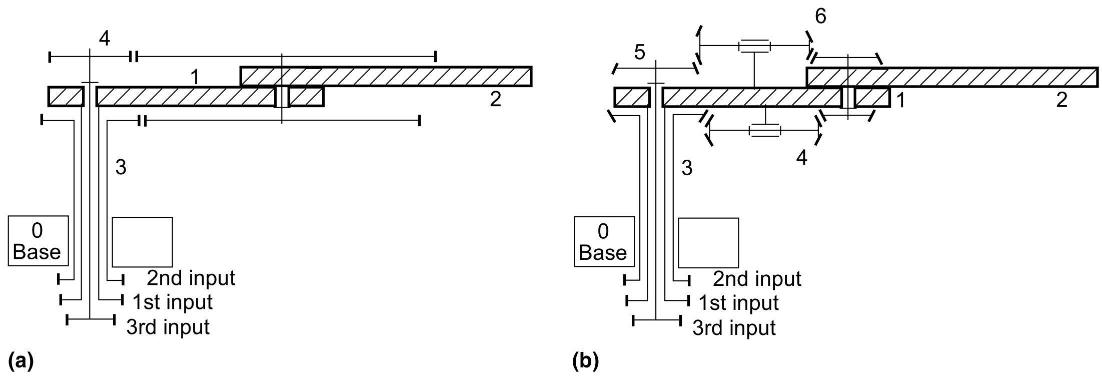


Fig. 9. Functional representation of admissible GRMs with g^3-2 structure matrix: (a) with basic MTLs; (b) with derived MTLs.

Step 4. *Combine the MTLs according to the structure matrix.*

The structure matrix reveals the joints influenced by each of the MTLs. The MTLs are combined such that the articulated joints can be driven properly. The selected MTLs are combined as shown in Fig. 8(a) which represents the simplest configuration of the desired mechanism. The associated functional representation is shown in Fig. 9(a).

Step 5. *Substitute derived MTL(s) to refine the mechanism.*

It can be observed from Fig. 9(a) that to fulfill the graph representation of Fig. 8(a), large-size gears need to be used. To reduce the gear size, adding idle gears is required. Fig. 8(b) shows an admissible mechanism with the two-joint MTLs substituted by MTL 2-5-1 and the corresponding functional representation is shown in Fig. 9(b).

With the above procedure, desired redundant-drive backlash-free GRMs can be systematically enumerated and functional requirements can also be taken into consideration to determine proper configurations of the desired mechanism.

6. Example

To obtain more physical comprehension, let us consider a three-dof redundant-drive backlash-free GRM with the g^2s^2-8 structure matrix [1] in which three two-joint MTLs and a one-joint

MTL are required. Suppose the simplest configuration is desired as a prototype design. The admissible mechanism is composed of basic MTLs and can be obtained as shown in Fig. 10(a). In the graph representation, MTL 2-4-1 is used for the three two-joint MTL and the direct drive unit is used for the one-joint MTL. The corresponding functional representation is shown in Fig. 11(a). To extend the span between the first and the second joints, MTL 2-5-1 is used to substitute the original MTL 2-4-1 which influences the first two joints as shown in Fig. 10(b). The effect of adding an idle gear is shown in Fig. 11(b). In some circumstances, the end effector is requested to perform roll motion and high torque is also required for industrial purpose such as screw fastening. The MTL 2-6-5 which has planetary unit is used to substitute one of the MTLs influencing the end effector. Also, the MTL 2-5-1 is used to substitute another MTL influencing the end effector in order to increase the space between the second and the third joints. The graph representation of the resulted mechanism is shown in Fig. 10(c), and the corresponding functional representation is shown in Fig. 11(c).

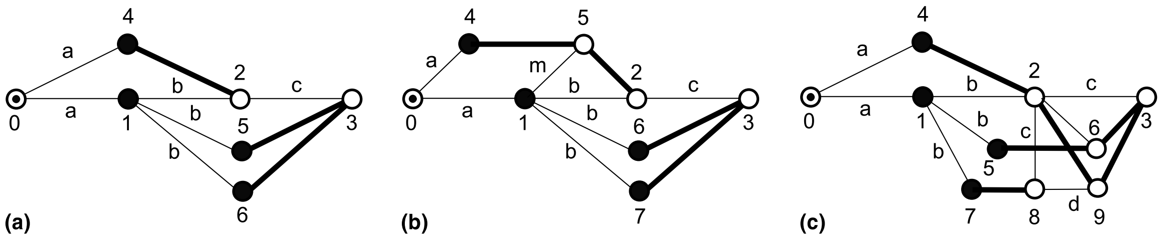


Fig. 10. Graphical representation of admissible GRMs with g^2s^2-8 structure matrix: (a) with basic MTLs; (b) an alternative with an ordinary unit; (c) an alternative with planetary units.

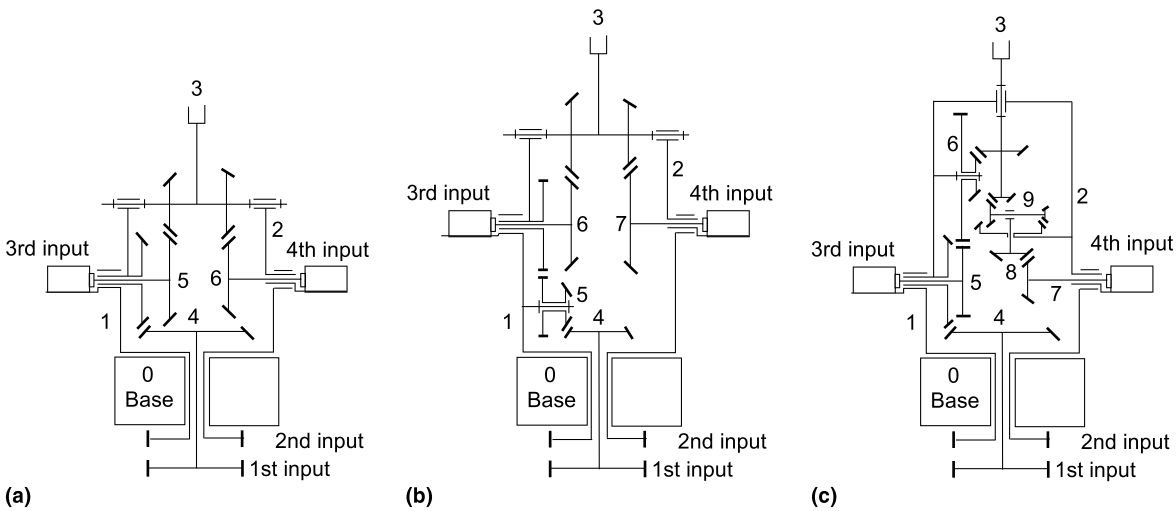


Fig. 11. Functional representation of admissible GRMs with g^2s^2-8 structure matrix: (a) with basic MTLs; (b) an alternative with an ordinary unit; (c) an alternative with planetary units.

7. Conclusion

In this paper, an efficient and systematic design methodology for the conceptual design of redundant-drive backlash-free GRMs is developed. An *n*-dof redundant-drive backlash-free GRM is considered the combination of EOLC and $(n+1)$ MTLs. Each MTL is further decomposed into several non-fractionated GKCs, which are referred to as the composing units. Twelve units are identified from the atlas of non-fractionated GKCs and are categorized into three family trees. The family trees provide designers with the kinematic insight into the mechanism and the attributes of each family tree, MTLs can be formed by combining suitable units. In turn, the desired redundant-drive backlash-free GRM is enumerated by composing proper MTLs according to the specified structure matrix. According to the graph representation, functional representation of the desired mechanism can be created to complete the design in conceptual phase. A two-dof example with the g^3-2 structure matrix and a three-dof example with the g^2s^2-8 structure matrix are presented. It is believed that the design methodology can be applied not only to all three-dof redundant-drive backlash-free GRMs but to *n*-dof cases as well.

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