RANDOM OUTCOME AND STOCHASTIC ANALYSIS OF SOME FATIGUE CRACK GROWTH DATA

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ABSTRACT

Fatigue crack propagation data of a batch of AISI 4340 steel specimens are released in the present paper. The statistical nature of the data is specially emphasized, and a probabilistic fracture mechanics model is introduced to analyze the data. The stochastic differential equation associated with the model is then solved. The solution gives us the crack exceedance probability as well as the probability distribution of the random time to reach a specified crack size. These quantities are useful in the reliability assessment of structures made of the tested material. Comparing the analytical result with the experimental result, it is found that the proposed probabilistic fracture mechanics model can reasonably explain the experimental data. For those data that cannot be fitted well by the proposed model, methods of improvement are proposed in the present paper as well.

Keywords : Fatigue crack growth, Probabilistic fracture mechanics, Random process, Stochastic analysis.

1. INTRODUCTION

The scatter of fatigue data either in the initiation phase or in the propagation phase has been observed for a long time. Along with the development of fracture mechanics, the study of fatigue scatter in the propagation phase has been emphasized for the past two decades. The need of reliability or risk assessment for some important structures such as nuclear power plant components has furthermore enhanced the development of the so-called "probabilistic fracture mechanics" [1]. One of the important issues in the probabilistic fracture mechanics analysis lies in the probabilistic modeling of fatigue crack growth phenomenon. Many probabilistic models have been proposed to capture the scatter as well as random outcome of the crack propagation data. Some of the models are purely based on direct curve fitting of the random crack growth data, including their mean value and standard deviation [2]. These models, however, have been criticized by some researchers that less crack growth mechanisms have been included in them. To overcome this difficulty, many probabilistic models adopted the crack growth equation proposed by other fatigue researchers and randomized the equation by including a random factor into the equation [3~8]. The random factor may be a random variable, a random process of time, or a random process of space. It then creates a random differential equation. The solution of the differential equation reveals the probabilistic structure of the crack growth data.

To justify the applicability of the above-mentioned probabilistic models, fatigue crack growth data are usually needed. However, it is rather time-consuming to carry out experiments to obtain a set of statistical meaningful fatigue crack growth data. To these writers' knowledge, there are only a few data sets available so far for researchers to verify their probabilistic models. Among them, the most famous data set perhaps is the one produced by Virkler, Hillberry and Goel more than twenty years ago [9]. Other data sets available are those released by the Flight Dynamics Laboratory of the US Air Force, and frequently used by Yang and Manning [10]. In fact, many probabilistic models are either lack of experimental verification or just verified by only one of the above data sets. It is suspected that a model may explain a data set well but fail to explain another data set. The universal applicability of many probabilistic models still needs to be checked carefully by other available data sets.

There are two major objectives for the present paper. The first objective is to release a fatigue crack growth data set. These data are the preliminary result of a project related to the reliability and quality assurance of structural materials. Although the number of data may not be enough for an analysis from strict statistical point

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of view, its amount is compatible to that employed by Yang and Manning [5], and is still believed to be engineering meaningful. The second objective of the present paper is to employ our experimental data to verify a probabilistic fatigue crack growth model proposed by Yang and Manning [11]. The major reason for adopting Yang and Manning's model lies in its generality. The logarithm distribution of our data as verified by a descriptive statistical analysis furthermore endorses the adoption of their model. Once this model is verified to be useful, it will be employed extensively furthermore in our project. On the other hand, if the model fails to explain our preliminary data appropriately, it will be modified or even discarded in our later study.

2. FATIGUE CRACK GROWTH EXPERIMENT

Our experimental setup consists of a dynamic testing machine, a crack closure measurement system, a crack size measurement system, a spectral analysis system, and a computer controlled systems. Compact tension (CT) specimens were made for the study. The original dimensions of the specimens were 62.5mm wide, 60.0mm long, and 12.0mm in thickness before they were cut and tested. Both constant-amplitude and random loads were applied to the specimens. In performing the tests, series of oscillating loads were generated and then converted into analog signals through an IEEE-488 GBIP card and an arbitrary waveform function generator. The controller then controlled the dynamic testing machine and transferred the analog signals to the specimen. To examine whether the input loads equal the oscillation loads we desired, the peak and trough values of each loading cycle were recorded and a spectral analyzer was used to analyze the input signals.

The crack opening load and crack size were measured by compliance method and direct current potential drop (DCPD) method, respectively. The resolution of the DCPD method was set to be 0.01mm. During the testing process, the crack size and crack-opening load were monitored continuously and discrete data points were extracted from time to time.

Several kinds of material including 2024-T351 aluminum alloy, 7075-T651 aluminum alloy, AISI 304 stainless steel and AISI 4340 high strength steel have been tested in our study. Of them, the experimental result of the AISI 4340 high strength steel will be reported and analyzed furthermore in the present paper. The chemical compositions of the material were C (0.37 ~ 0.45), Mn (0.60 ~ 0.95), Si (0.20 ~ 0.35), Ni (1.50 ~ 2.00), Cr (0.65 ~ 0.95) and Mo (0.20 ~ 0.30). Some mechanical properties were found as follows. Young's modulus: 200GPa, yield strength: 1065MPa, tensile strength: 1160MPa, ratio of elongation: 2%, ratio of area reduction: 2%, mode I fracture toughness: 52MPa, and hardness: 400HB.

Constant-amplitude fatigue tests were performed in advance to obtain some elemental material constants. After that, random loading fatigue tests were carried out. The generation of random load was based on the superposition of stationary random fluctuating components to a selected mean load. The random fluctuating components were generated from a given probability density function. Each randomly generated load representing the amplitude of the peak was followed immediately by another independently generated load representing the amplitude of the trough. The peak was added to and the trough was subtracted from the mean load to construct a cycle of the random loading. For a selected probability density function, several specimens drawn from the same batch of material were tested according to the procedure mentioned above. These specimens were subjected to different random loading histories which, however, possessing the same statistical property. In our study, Rayleigh, uniform, normal and triangular probability density functions have been used to generate the random fluctuating loads. However, only the result of Rayleigh loading will be reported herein for the sake of simplicity.

Following the procedure stated above, several different loading conditions were employed for our constant-amplitude fatigue tests. They included (1) $P_{max} = 5.5$kN, $R = 0.1$; (2)$P_{max} = 5.5$kN, $R = 0.3$; and (3) $P_{max} = 6.0$kN, $R = 0.5$; in which P_{max} is the maximum load (peak load) and R is the maximum to minimum load ratio (stress ratio). During the experiment, both crack size and crack opening load were measured and recorded. After completing the constant-amplitude fatigue tests, random loading fatigue tests were then performed. The mean load was set to be 4kN and the fluctuating random components were chosen to have a mean value of 1.43kN and a standard deviation of 0.75kN. Ten specimens were tested for each loading condition. All fatigue crack tests were continued until fracture occurred.

3. STOCHASTIC MODELING

As mentioned previously, many probabilistic models of fatigue crack growth are based on the deterministic crack growth equations. The most well-known fatigue crack growth equation is the Paris law represented by

$$\frac{da}{dN} = c\,(\Delta K)^m \tag{1}$$

in which a is the crack length, N is the number of stress cycle, c and m are material constants, and ΔK is the stress intensity factor range that is related to the applied load and material geometry. For CT specimens, special formulas can be found in a stress intensity factor handbook for the calculation of ΔK. Another famous fatigue crack growth equation is the one proposed by Elber and is sometimes called Elber's law [12],

$$\frac{da}{dN} = c'\,(\Delta K_{\mathrm{eff}})^{m'} \tag{2}$$

in which c' and m' are material constants, and ΔK_{eff} is the effective stress intensity factor range that is useful to open the crack and cause the crack tip to grow.

In Yang and Manning's probabilistic fatigue crack growth model, in order to be general enough, the above equations have been modified to be

$$\frac{da(t)}{dt} = X(t)\, g(\Delta K, R, K_{\max}, a) = X(t)\, g(\Delta K_{\text{eff}}) \quad (3)$$

In the above equation, g indicates a general and deterministic crack growth law that can be Paris law, Elber's law or any other law, R is the stress ratio, $K_{\max}$ is the maximum stress intensity factor in a stress cycle. It is noted that the discrete stress cycle N in Eqs. (1) and (2) has been replaced by a continuous time variable t in Eq. (3). The reason for doing so is to make the equation an ordinary differential equation. Physically, it is justified for a high cycle fatigue crack growth process in which the number of stress cycles can be 10^6 or even higher, to which the discrete cycle interval is almost negligible. Another thing to be noted in Eq. (3) is that a random factor $X(t)$ is added into the crack growth equation, which makes the equation to be a stochastic differential equation as stated before.

To make the calculation easier, Yang and Manning have further suggested the following simpler form of the above equation to be used for a first trial [11],

$$\frac{da(t)}{dt} = X(t)\, Q\, [a(t)]^b \quad (4)$$

in which Q and b are constants to be evaluated from the crack growth observation. The independent variable t can be interpreted as either stress cycles, flight hours, or depending on the applications. It is noted that the power-law form of $Qa(t)^b$ at the right hand side of Eq. (4) can be used to fit most fatigue crack growth data appropriately and is also compatible with the concept of Paris law. If it indicates the crack growth behavior in an average sense, then the randomness of the crack growth comes directly from the other factor $X(t)$ that should have a mean value of one. After extensive study, Yang and Manning suggested that $X(t)$ could better be modeled as a stationary lognormal random process having a mean value of 1 and a standard deviation of σ_X. If this is the case, the following normal random process can be introduced,

$$Z(t) = \ln X(t) \quad (5)$$

which should have a mean value of 0 and standard deviation of

$$\sigma_Z = \sqrt{\ln(1+\sigma_X^2)} \quad (6)$$

A general auto-covariance function of the following form is assumed for the random process $X(t)$,

$$\text{Cov}\,[X(t_1), X(t_2)] = \sigma_X^2 \exp(-\varsigma\,|\,t_2 - t_1\,|) \quad (7)$$

in which ς^{-1} indicates a measure of the correlation time for $X(t)$ and will be called "correlation time" hereafter for simplicity. The reason for using the above exponentially decaying auto-covariance function lies in its generality. By selecting appropriate values of ς, different degrees of fatigue scatter can all be fitted by the proposed probabilistic model.

Based on the above modeling and assumptions, if the deterministic part at the right hand side of Eq. (4) indicates the median crack growth rate, then the median service time for a crack to grow from size a_0 to a can be obtained by performing the necessary integration to obtain

$$\bar{t}(a) = \int_{a_0}^{a} \frac{da}{Q\, a^b} = \frac{[a_0^{-(b-1)} - a^{-(b-1)}]}{Q(b-1)} \quad (8)$$

To take the random part into consideration, the following integration of Eq. (4) can also be performed

$$\int_{a_0}^{a(t)} \frac{dv}{Q\, v^b} = \int_0^t X(\tau)\, d\tau \quad (9)$$

It indicates the crack grow from size a_0 at time 0 to size a at time t, but at a random manner. If now, a new random process $W(t)$ is defined as the integration of $X(t)$, that is

$$W(t) = \int_0^t X(\tau)\, d\tau \quad (10)$$

then $W(t)$ can also be assumed appropriately as a lognormal random process. Under this circumstance, the following associated normal random variable $Y(t)$ can be defined,

$$Y(t) = \ln W(t) \quad (11)$$

which is assumed to have a mean value $\mu_Y(t)$ and standard deviation $\sigma_Y(t)$. Their values are related to the mean value and standard deviation of $W(t)$ and will be discussed more latter.

The distribution function of crack size $a(t)$ at the service time t can be related to that of $W(t)$ through Eqs. (8) to (11) as follows

$$F_{a(t)}(a) = P[a(t) \le a] = P[W(t) \le \bar{t}(a)] = F_{W(t)}[\bar{t}(a)] \quad (12)$$

Instead, the probability that crack size $a(t)$ will exceed any given crack size a in the service interval (0, t) can be derived and expressed as

$$P^e_{a(t)}(a) = P[a(t) > a] = 1 - F_{a(t)}(a) = 1 - F_{W(t)}[\bar{t}(a)] = \Phi\left\{\frac{\mu_Y(t) - \ln[\bar{t}(a)]}{\sigma_Y(t)}\right\} \quad (13)$$

The above probability is frequently called crack exceedance probability [11].

In addition to the probability distribution of crack size, the probability distribution of time for a crack to grow from size a_0 to a can also be found based on the above model. In fact, the probability that service time $T(a)$ will be within the interval (0, t) for crack size to reach a is identical to $P^e_{a(t)}(a)$. That is,

$$F_{T(a)}(t) = P[T(a) \le t] = P^e_{a(t)}(a) = 1 - F_{W(t)}[\bar{t}(a)]$$
$$= \Phi\left\{\frac{\mu_Y(t) - \ln[\bar{t}(a)]}{\sigma_Y(t)}\right\} \qquad (14)$$

The above probability is, in fact, the probability distribution of random time to reach a given crack size a and, therefore, the notation $F_{T(a)}(t)$ was used in the equation. To summarize the concept of the above derivation, the readers can refer to Fig. 1.

To perform the calculation of Eqs. (13) and (14), the mean value and standard deviation of $W(t)$ have to be found in advance. As stated before, they are related to the statistics of $X(t)$ which, in turn, can be estimated from the real observation or experimental result. To this end, it is noted, from Eq. (10), that

$$\mu_{W(t)} = E[W(t)] = \int_0^t E[X(\tau)]\, d\tau = \mu_X t \qquad (15)$$

and

$$\sigma^2_{W(t)} = E\{[W(t) - \mu_{W(t)}]^2\}$$
$$= \int_0^t \int_0^t \mathrm{Cov}[X(\tau_1), X(\tau_2)]\, d\tau_1\, d\tau_2 \qquad (16)$$

where Cov $[X(t_1), X(t_2)]$ indicates the covariance function of $X(t)$ as introduced previously in Eq. (7). Substituting Eq. (7) into Eq. (16), carrying out the integration and then taking the square root one obtains

$$\sigma_{W(t)} = \frac{\sigma_X \sqrt{2(e^{-\varsigma t} + \varsigma t - 1)}}{\varsigma} \qquad (17)$$

where ςt can be interpreted as the service time t normalized by the correlation time ς^{-1} and therefore can be named as the "normalized service time." Moreover, the coefficient of variation of $W(t)$ can be determined from Eqs. (15) and (17) as follows

$$V_{W(t)} = \frac{\sigma_{W(t)}}{\mu_{W(t)}} = \frac{V_X \sqrt{e^{-\varsigma t} + \varsigma t - 1}}{\varsigma t} \qquad (18)$$

where V_x is the coefficient of variation of $X(t)$.

Based on the assumption of Eq. (11) and lognormal properties, the mean value and standard deviation of $Y(t)$ can be obtained as follows

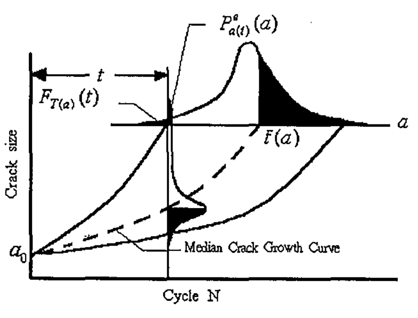

Fig. 1 Schematic diagram of crack size distribution and random time distribution

$$\mu_Y(t) = \ln\left[\frac{\mu_{W(t)}}{\sqrt{1 + V^2_{W(t)}}}\right] \qquad (19)$$

$$\sigma_Y(t) = \sqrt{\ln[1 + V^2_{W(t)}]} \qquad (20)$$

The use of $\mu_Y(t)$ and $\sigma_Y(t)$ instead of $\mu_{Y(t)}$ and $\sigma_{Y(t)}$ in the above two equations should not create any ambiguity since the random process considered is a stationary one. Based on the assumption of unit value of $X(t)$ and lognormal properties, the mean value and the coefficient of variation of $X(t)$ are determined from Eq. (5) as follows

$$\mu_X = \exp\left(\frac{\sigma_Z^2}{2}\right) \qquad (21)$$

$$V_X = \sqrt{\exp(\sigma_Z^2) - 1} \qquad (22)$$

where the time variable t is neglected for simplicity. Substituting Eqs. (21) and (22) into Eqs. (15) and (18), and then into Eqs. (19) and (20), one can obtain the following mean value and standard deviation of $Y(t)$,

$$\mu_Y(t) = \ln t + \ln \lambda \qquad (23)$$

$$\sigma_Y(t) = \sqrt{\ln[1 + \phi^2 \exp(\sigma_Z^2) - \phi^2]} \qquad (24)$$

where

$$\phi = \frac{\sqrt{2(e^{-\varsigma t} + \varsigma t - 1)}}{\varsigma t} \qquad (25)$$

and

$$\lambda = \exp\left(\frac{\sigma_Z^2}{2}\right)\sqrt{\frac{1}{1 + \phi^2 \exp(\sigma_Z^2) - \phi^2}} \qquad (26)$$

Substitution of Eq. (23) into Eqs. (13) and (14) results in the following equation for the probability of crack exceedance as well as probability distribution of random time to reach a given crack size,

$$P^e_{a(t)}(a) = F_{T(a)}(t) = \Phi\left\{\frac{\ln t - \ln \bar{t}(a)/\lambda}{\sigma_Y(t)}\right\} \qquad (27)$$

where $\bar{t}(a)$, $\sigma_Y(t)$ and λ are given by Eqs. (8), (24) and (26), respectively.

It is of great interest to notice the two extreme cases for the above stochastic crack growth model in view of Eq. (7). For the first case, the correlation time ς^{-1} approaches to zero, which indicates that $X(t)$ is a lognormal white noise random process. Under this circumstance, substituting $\varsigma^{-1} = 0$ into Eqs. (24), (25) and (26) yields $\sigma_Y(t) = 0$, $\phi = 0$, and $\lambda = \exp(\sigma_z^2/2)$. Equation (27) then becomes

$$P^e_{a(t)}(a) = F_{T(a)}(t) = U\left\{t - \bar{t}(a)\exp\left[-\left(\frac{\sigma_Z^2}{2}\right)\right]\right\} \qquad (28)$$

in which U is a unit step function. It indicates that there is a sudden jump for the associated probability at $t = \bar{t}(a)\exp(-\sigma_Z^2/2)$. In this case, there is no statistical dispersion for the crack growth assumption.

Hence, it can be considered the most un-conservative model.

For the second extreme case, the correlation time ς^{-1} approaches to infinite, which makes $X(t)$ degenerate to a lognormal random variable X. Through Taylor's expansion, one has

$$e^{-\varsigma t} = 1 - \varsigma t + \frac{(\varsigma t)^2}{2} + O(\varsigma^3 t^3) \qquad (29)$$

in which $O(\varsigma^3 t^3)$ are the higher order terms and will be neglected. By letting $\varsigma^{-1} \to \infty$ and substituting Eq. (29) into Eqs. (24), (25) and (26), one can obtain $\sigma_Y(t) = \sigma_Z$, $\phi = 1$, and $\lambda = 1$. Equation (27) then becomes

$$P^e_{a(t)}(a) = F_{T(a)}(t) = \Phi\left\{\frac{\ln t - \ln \bar{t}(a)}{\sigma_Z}\right\} \qquad (30)$$

which is identical to the result derived previously by Yang and his associates by directly assuming the random factor $X(t)$ in Eq. (3) is a lognormal random variable [10].

It indicates, from the above two extreme cases, that the probabilistic fatigue crack growth model is very versatile. Therefore, it will be used to analyze the fatigue crack growth data we have obtained in the previous section.

4. NUMERICAL RESULT

The experimental crack propagation curves of AISI 4340 steel specimens under random loading condition are shown in Fig. 2. Selected discrete data are also given in Tables 1 and 2. These data include (1) initial crack lengths and crack lengths at specified loading cycles, (2) numbers of loading cycle to reach specified crack lengths, and (3) numbers of loading cycle at which fracture occurs. The third item is considered to be the fatigue lives of the tested specimens. In the last three rows of both tables, μ indicates the mean value, σ indicates the standard deviation, and Cov indicates the coefficient of variation of the quantity shown in the respective column. They were obtained by descriptive statistics and are needed for our probabilistic analysis.

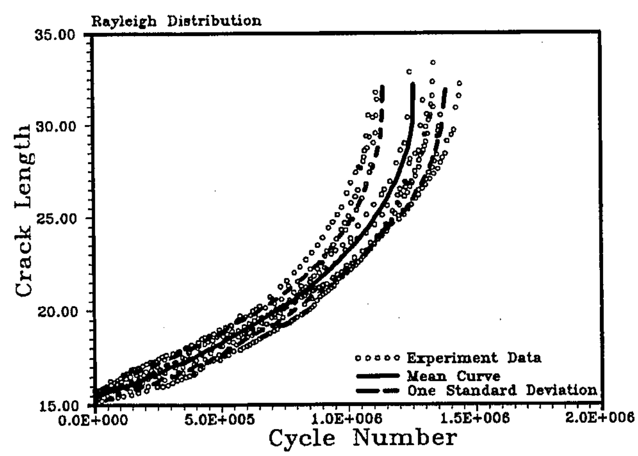

Fig. 2 Experimental result

Table 1 Distribution of crack lengths in millimeter

Test # \ N	0	300,000	600,000	800,000	1,000,000	1,200,000
1	15.16	16.16	18.10	19.61	22.06	23.52
2	15.61	17.17	19.22	20.67	22.40	23.46
3	15.62	17.21	18.98	20.44	22.31	23.74
4	15.46	16.89	18.87	20.48	22.01	23.60
5	15.31	16.48	18.53	20.79	24.81	30.83
6	15.73	17.71	19.66	21.09	23.29	24.55
7	15.56	17.15	19.70	22.04	26.29	30.03
8	15.46	16.80	18.84	20.49	24.32	28.33
9	15.65	17.61	19.82	21.52	23.69	25.60
10	15.86	16.82	18.59	20.05	22.27	23.49
μ	15.54	17.00	19.03	20.72	23.34	25.72
σ	0.193	0.451	0.536	0.663	1.358	2.761
Cov	0.012	0.026	0.028	0.032	0.058	0.107

Table 2 Distribution of cycles

Test # \ a	17 mm	19 mm	22 mm	25 mm	30 mm	Fracture
1	425,801	721,802	996,238	1,178,354	1,316,852	1,338,530
2	245,301	573,719	972,019	1,198,311	1,421,739	1,442,358
3	273,055	604,997	974,032	1,176,715	1,308,969	1,312,509
4	315,701	618,908	998,757	1,215,876	1,360,871	1,364,642
5	403,878	650,827	876,830	1,000,001	1,081,949	1,184,969
6	183,093	499,723	890,845	1,130,820	1,315,023	1,336,820
7	278,308	521,299	789,370	957,505	1,099,737	1,124,170
8	344,226	631,037	897,253	1,028,535	1,113,305	1,118,760
9	186,152	493,571	845,445	1,068,739	1,230,021	1,246,080
10	376,015	664,338	980,734	1,163,791	1,289,217	1,297,130
μ	303,153	598,022	922,152	1,111,865	1,253,768	1,276,597
σ	80,818	71,507	68,656	86,670	111,920	100,934
Cov	0.266	0.120	0.074	0.078	0.089	0.079

Our preliminary analysis of the constant-amplitude fatigue crack growth data indicates both Paris law and Elber's law can be used to model the crack growth rate of the tested AISI 4340 steel specimens. Further examination of the curves shown in Fig. 1 for the random loading test result also suggests the applicability of Eq. (3). Moreover, statistical analysis of those data shown in Tables 1 and 2 also justify the lognormal assumption needed in our probabilistic analysis. Therefore, the stochastic fatigue crack growth model introduced in the previous section is applied to analyze the fatigue crack growth data shown in Fig. 1. The independent variable in Eq. (3) is considered approximately the real stress cycle.

To begin with the stochastic analysis, the median crack growth curve has to be known in advance. For our case, it is found from the experimental data through a certain numerical algorithm that $Q = 5.8775 \times 10^{-11}$ and $b = 3.999$, which is needed for the modeling of Eq. (4). If we assume the random factor $X(t)$ is a stationary lognormal random process, then it is found that $\sigma_{X(t)} = \sigma_X = 0.24710$ and hence, $\sigma_{Z(t)} = \sigma_Z = 0.24345$. Using a repeated trial-and-error process, the correlation time of $X(t)$ is selected to be $\varsigma^{-1} = 10^5$. Having obtained these values, the crack exceedance curve for any given crack size can be found through the application of Eq. (8) and Eqs. (23) to (27). Some of the results thus calculated are shown in Fig. 3. The time distribution for the crack to grow to a specified value can be obtained from Eqs. (23) to (27) as well. Some of the results are shown in Fig. 4. In both figures, the curve indicates the analytical result and the solid circles indicate our experimental data. Comparing the analytical results with the experimental results, it is found that the stochastic fatigue crack growth model can be used to predict the random crack growth behavior rather well.

To check the influence of the correlation time on the analytical result, different values of ς^{-1} have been employed in the numerical calculation. A typical result for the random time to reach the crack value of 19mm is shown in Fig. 5. It can be seen that the dispersion of the cumulative distribution function decreases as the correlation time decreases. The continuous distribution curve will eventually become a discontinuous unit step function when the correlation time approaches to 0. On the other end, if the correlation time becomes larger and larger, the curve tends to approaches to a limit curve. The limit curve indicates the result calculated based on a lognormal random variable assumption rather than the lognormal random process assumption. In the present case, it is found when $\varsigma^{-1} = 5 \times 10^7$, the random process can almost be considered as a random variable. Under this circumstance, the simpler form of Eq. (30) rather than the more complicated Eq. (27) can be used directly to benefit.

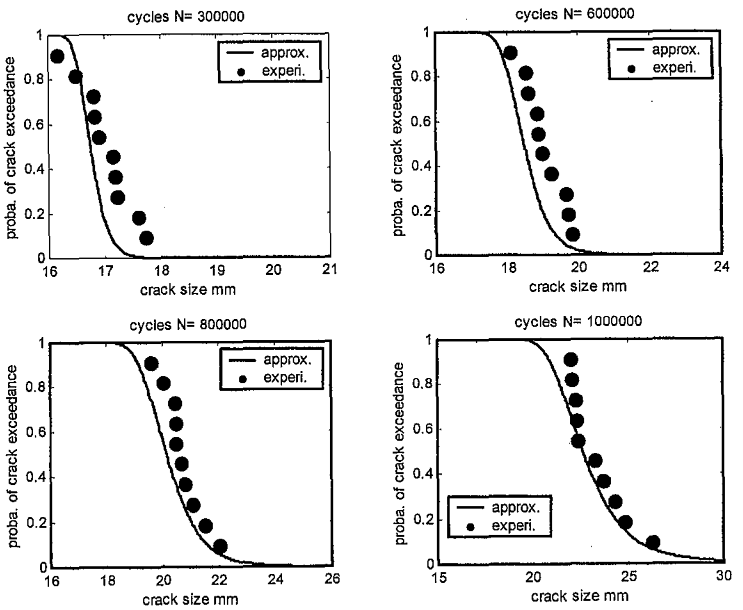

Fig. 3 Crack exceedance probability

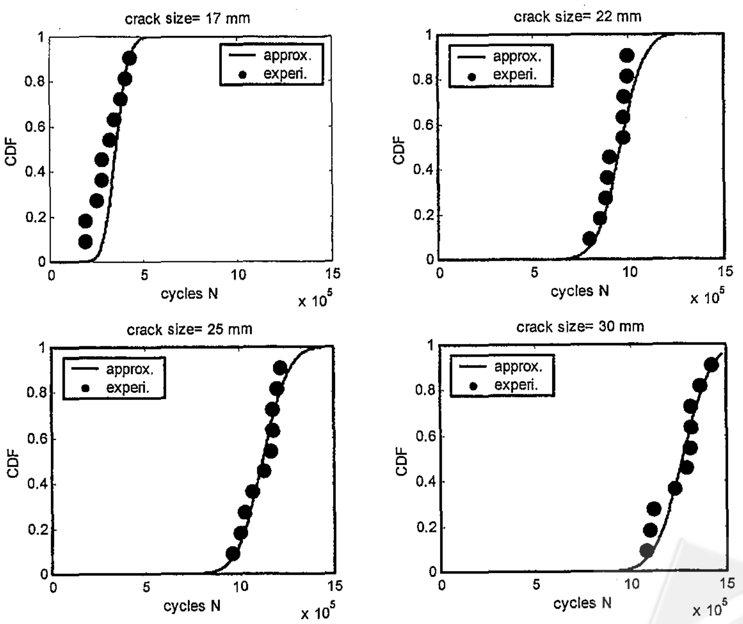

Fig. 4 Random time distribution to reach specified cracks sizes

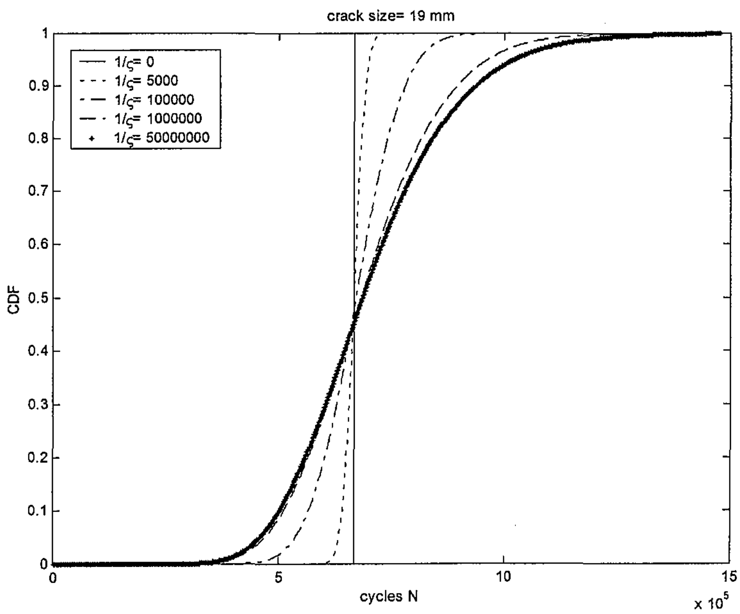

Fig. 5 The effect of correlation time

5. CONCLUDING REMARKS

A batch of fatigue crack growth data of AISI 4340 steel is released in the present paper. With a view to explain the fatigue data well, a simple stochastic differential crack growth model is introduced. The model can not only capture the average behavior of the fatigue crack growth but also reveal the statistical dispersion of the growth data. If it is proved to be accurate enough, the model can further be used for the reliability assessment of structures made of the studied material.

After the study, it is found that Yang and Manning's stochastic fatigue crack growth model can be used to fit our fatigue data rather well. Based on the established model together with the already found parametric values, we can predict the crack exceedance probability at any given service time. We can also predict the random time distribution for the crack to grow to a certain size. Both quantities are helpful for the reliability assessment of structures made of AISI 4340 steel.

In applying Yang and Manning's model, if one is not satisfied with the analysis as we have performed, he can improve the analysis very easily. There are at least two aspects for the improvement. The first one is to fit the median crack growth rate more accurate using a more complicated mathematical model. By doing so, the first order statistic can be predicted to a rather accurate degree but, of course, at the cost of increasing the amount of computation. The second aspect for improvement is to adjust the correlation length as we have shown in the present paper. The adjustment can increase or decrease the degree of scatter and, hence, improve the prediction ability of the second order statistic. If both methods fail to improve the prediction, a non-stationary random process model may then be needed to replace the stationary lognormal process assumption used in the present paper.

In our numerical calculation, the initial crack size was assumed to be a deterministic value that was obtained from the mean value of the tested ten specimens. If the randomness of the initial crack size has to be taken into account, the derived formulas can still be applied except that conditional probability concept has to be considered. And an integration process is usually needed to obtain the answers.

ACKNOWLEDGEMENT

This work is supported by the National Science Council of the Republic of China under Grant No. NSC 89-2212-E-002-152. The writers are grateful for this support.

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(Manuscript received Nov. 29, 2000,
Accepted for publication Mar. 15, 2001.)