Stability Analysis of a Spinning Elastic Disk Under a Stationary Concentrated Edge Load

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The natural frequencies and stability of a spinning elastic disk subjected to a stationary concentrated edge load are investigated both numerically and analytically. It is found that a stationary, conservative, compressive edge load decreases the natural frequencies of the forward and backward traveling waves, but increases the natural frequency of the so-called reflected wave. It is also found that the significance of the effect of the conservative edge load on the natural frequencies and stability of the spinning disk is solely through the transverse component of the edge load on the boundary, and not through the membrane stress field it produces inside the disk. In addition, the compressive edge load induces a stationary-type instability before the critical speed, and induces a merged-type instability after the critical speed when a reflected wave meets a forward or a backward wave. The expression for the derivative of the eigenvalues of the spinning disk with respect to the edge load is derived to verify the numerical results.

Introduction

Transverse stability is a critical constraint in the optimal design and operation of computer disk drives and circular saws. In computer disk drives, excessive transverse vibrations degrade the signal from the read/write head and can destroy the disk by a "head-crash." In the wood cutting process, saw blade vibrations result in greater kerf loss and can warp the saw permanently through overheating. Initiated by Lamb and Southwell in 1921, early research on spinning disk dynamics was concerned with verifying the critical speed theory and the effects of thermally induced membrane stress field on the natural frequencies of the spinning disk. References to these works can be found in Mote and Szymani (1978). In the past 20 years much attention has been focused on identifying the characteristic instabilities induced by the interaction between a spinning disk and a stationary load system, which represents the recording head suspension in computer disk drives or saw guides in circular saws (Iwan and Moeller, 1976; Hutton et al., 1991; Shen and Mote, 1991; Ono et al., 1991).

One unique feature in circular saw applications is the stationary concentrated in-plane load applied at the outer rim of the spinning disk during the cutting operation. The buckling analysis of a stationary circular disk subjected to general stationary in-plane loads is a classical problem that has been thoroughly studied. Srinivasan and Ramamurti (1980) considered the in-plane dynamic behavior of a stationary disk subjected to a spinning concentrated in-plane load at the outer boundary. They did not study the dynamic response associated with the bending of the disk. Regarding the effects of a concentrated edge load on the natural frequencies and stability of a spinning disk, there are two papers (Carlin et al., 1975; Radcliffe and Mote, 1977) in the literature. Carlin et al. (1975) first wrote the equation of motion for a nonrotating disk with axisymmetric in-plane stresses due to tensioning and asymmetric stresses due to a stationary concentrated normal edge load. In order to account for the effects of the disk rotation, they then superposed the axisymmetric stresses due to the centrifugal force to the membrane stress field induced by tensioning and the edge load. They found that the effects of these centrifugal stresses on blade stiffness are negligible. Radcliffe and Mote (1977) extended the work of Carlin et al. by considering a general concentrated edge load with both normal and tangential components, and were concerned primarily with the critical speed instability. Neither stationary-type nor merged-type instability is induced by their loadings. Since the equations of motion used in these two papers are nongyroscopic, they do not correspond to either a spinning disk-stationary edge load system (Iwan and Moeller, 1976), nor a stationary disk-spinning edge load system (Iwan and Stahl, 1973). On the contrary, their concentrated edge loads appear to be spinning along with the spinning disk, i.e., there is no relative motion between the disk and the edge load in the circumferential direction. Therefore these models do not represent the loading that occurs in the normal cutting process. The effects of a stationary concentrated edge load of the type that occurs in cutting on the natural frequen-

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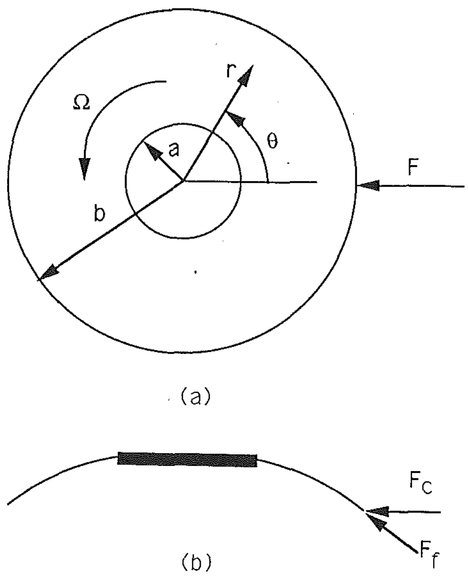

Fig. 1 A spinning disk subjected to a stationary edge load

cies and stability of a spinning disk evidently have not been studied before.

In the present paper we consider a spinning flexible annular disk clamped at the inner radius and subjected to a stationary concentrated edge load at the outer rim. Both follower and conservative edge loads are considered. A finite element procedure based on Galerkin's method is employed to calculate the eigenvalues of the system. Finally, the expression for the derivative of the eigenvalues of the spinning disk with respect to the edge load is derived to verify the numerical results.

Equation of Motion

Figure 1(a) shows a circular disk, which is clamped at the inner radius $r = a$ and subjected to a concentrated normal compressive force F at the outer radius $r = b$. The disk is rotating with constant speed Ω, while the edge load is circumferentially stationary in the space. The magnitude of the edge load remains the same when the disk vibrates laterally. The equation of motion of the system, in terms of the transverse displacement w and with respect to the stationary coordinate system (r, θ), is

$$\rho h\left(w_{,tt} + 2\Omega w_{,t\theta} + \Omega^2 w_{,\theta\theta}\right) + D\nabla^4 w + \mathbf{L}w + \hat{\mathbf{L}}w = 0 \quad (1)$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

The standard subscript notation for partial derivative is used here. The parameters ρ, h, E, and ν are the mass density, thickness, Young's modulus, and Poisson ratio of the disk, respectively. $\mathbf{L}$ is the membrane operator associated with the axisymmetrical stress field due to the centrifugal force, and $\hat{\mathbf{L}}$ is the membrane operator associated with the asymmetric stress field due to the normal edge load F at $r = b$, $\theta = 0$,

$$\mathbf{L} = -\frac{h}{r}\left[\frac{\partial}{\partial r}\left(r\sigma_r\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{1}{r}\sigma_\theta\frac{\partial}{\partial\theta}\right)\right]$$

$$\hat{\mathbf{L}} = -\frac{h}{r}\left[\frac{\partial}{\partial r}\left(r\hat{\delta}_r\frac{\partial}{\partial r} + \hat{\delta}_{r\theta}\frac{\partial}{\partial\theta}\right) + \frac{\partial}{\partial\theta}\left(\hat{\delta}_{r\theta}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\sigma}_\theta\frac{\partial}{\partial\theta}\right)\right].$$

Membrane stresses σ_r and σ_θ are due to the centrifugal force and are proportional to Ω^2. $\hat{\sigma}_r$, $\hat{\delta}_{r\theta}$, and $\hat{\delta}_\theta$ can be calculated by following a standard procedure described in Coker and Filon (1957), and the solutions are given in Carlin et al. (1975).

We consider two types of normal edge load F denoted as F_c and F_f (Celep, 1979, 1982), as shown in Fig. 1(b). F_c is a conservative force, whose direction and magnitude remains the same when the disk vibrates laterally. F_f is a follower force, which is always tangent to the radial slope of the disk at $r = b$, $\theta = 0$. In the case when $F = F_c$, the boundary conditions for Eq. (1) are

$$w = 0, \quad w_{,r} = 0 \qquad \text{at} \quad r = a \qquad (2a)$$

$$\mathbf{B}_1 w + F_c\hat{\mathbf{B}}_1 w = 0, \quad \mathbf{B}_2 w = 0 \quad \text{at} \quad r = b \qquad (2b)$$

where the boundary operators are defined as

$$\mathbf{B}_1 = \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{\partial^2}{r^2\partial\theta^2}\right) + \frac{1-\nu}{r^2}\left(\frac{\partial^3}{\partial r\partial\theta^2} - \frac{\partial^2}{r\partial\theta^2}\right)$$

$$\hat{\mathbf{B}}_1 = \frac{\delta(\theta)}{bD}\frac{\partial}{\partial r}$$

$$\mathbf{B}_2 = \frac{\partial^2}{\partial r^2} + \frac{\nu}{r}\left(\frac{\partial}{\partial r} + \frac{\partial^2}{r\partial\theta^2}\right).$$

$\delta(:)$ is the Dirac delta function. Therefore the edge load F_c affects the natural frequencies and stability of the spinning disk through two avenues: one is the membrane operator $\hat{\mathbf{L}}$ associated with the membrane stress field induced by F_c, and the other is the boundary operator $\hat{\mathbf{B}}_1$ associated with the transverse component of F_c at $r = b$, $\theta = 0$. In the case when $F = F_f$, on the other hand, the boundary conditions are the same as in Eqs. ($2a,b$), except that the term associated with boundary operator $\hat{\mathbf{B}}_1$ in ($2b$) disappears. Therefore F_f affects the natural frequencies and stability of the spinning disk only through the membrane operator $\hat{\mathbf{L}}$ associated with the membrane stress field induced by F_f.

Galerkin's Method

A particular solution of Eq. (1) is assumed to be in the form of a Fourier sine and cosine series expansion as follows:

$$w = \sum_{l=0}^{L}\left[G_l(r,t)\cos l\theta + K_l(r,t)\sin l\theta\right]. \qquad (3)$$

After substituting Eq. (3) into Eq. (1), we obtain a set of partial differential equations for the functions $G_l(r,t)$ and $K_l(r,t)$. By a finite element numerical analysis based on Galerkin's method with third-order polynomial shape functions (Ono and Maeno, 1987), we can transform these differential equations with respect to r into matrix algebraic equations for the state vector at the nodes of the elements. From the viewpoints of both computing efficiency and accuracy, the number of elements in the radial direction and the maximum number of nodal diameters are chosen to be 6 and 3, respectively. The material properties of the disk used in the calculation are: $\rho = 7.84 \times 10^3$ kg/m^3, $E = 203 \times 10^9$ N/m^2, $\nu = 0.27$, $h = 1.02$ mm (0.04 in.), $a = 101.6$ mm (4 in.), and $b = 203.2$ mm (8 in.).

Effects of Membrane Operator $\hat{\mathbf{L}}$

We first examine the case when $F = F_f$. The boundary conditions on w of this case are exactly the same as those of the freely spinning disk (without any edge load). Figures 2 and 3 show the relation between the eigenvalues $\lambda = \alpha + i\omega$ of the zero-nodal-circle modes and one-nodal-circle modes, respectively, of the spinning disk and the rotation speed Ω. ω is the natural frequency of the spinning disk, and positive α means the mode is unstable. The dashed lines are the results

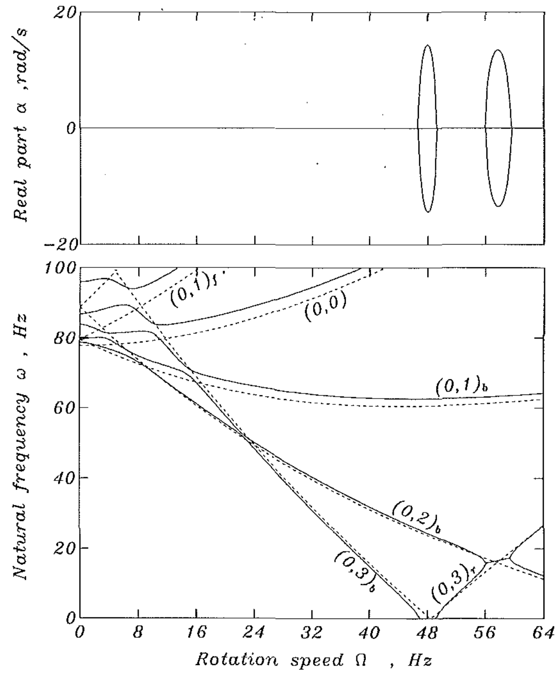

Fig. 2 Effects of follower edge load $F_f b/D = 21$ on the eigenvalues of zero-nodal-circle modes

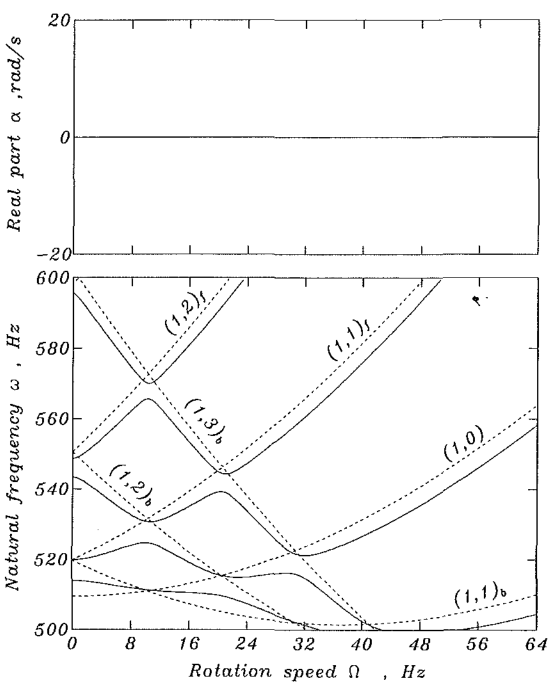

Fig. 3 Effects of follower edge load $F_f b/D = 21$ on the eigenvalues of one-nodal-circle modes

for the freely spinning disk, while the solid lines are the results for a disk under $F_f = 2000$ N (or nondimensional edge load $F_f b/D = 21$).

For a freely spinning disk the eigenvalues $\lambda_{mn} = \lambda^0_{mn}$ are all purely imaginary, i.e., $\lambda^0_{mn} = i\omega_{mn}$, where ω_{mn} are real. The corresponding eigenfunctions are complex and assume the form

$$w^0_{mn} = R_{mn}(r)e^{\pm in\theta}, \quad n = 0, 1, 2, \ldots \tag{4}$$

in which R_{mn} is a real-valued function of r. The eigenfunction corresponding to $\bar{\lambda}^0_{mn}$ is $\bar{w}^0_{mn}$, where overbar means complex conjugate. If we consider only the positive ω_{mn}, then w^0_{mn} in Eq. (4) with $+in\theta$ is a backward traveling wave with n nodal diameters and m nodal circles, which is also denoted by $(m, n)_b$. Similarly, w^0_{mn} with $-in\theta$ is a forward traveling wave $(m, n)_f$. The critical speed Ω_c for the mode (m, n) is defined as the rotation speed at which ω_{mn} of the backward traveling wave $(m, n)_b$ is zero. For Ω greater than Ω_c, this mode is a forward traveling wave, and is called a "reflected wave," denoted by $(m, n)_r$.

It is noted from Fig. 3 and other data in the higher frequency range that F_f tends to decrease the natural frequencies of the forward and backward traveling waves with nodal circle(s). However, as shown in Fig. 2, F_f tends to increase the natural frequencies of most of the zero-nodal-circle modes. Regarding the stability, Fig. 2 shows that F_f induces a stationary-type instability (an unstable mode with zero natural frequency) before the critical speed, and a merged-type instability (two unstable modes with the same natural frequencies) after the critical speed when the backward traveling wave $(0, 2)_b$ meets the reflected wave $(0, 3)_r$.

Since the effects of the follower load are through the operator $\hat{\mathbf{L}}$ in the differential equation, we can follow a procedure described in Chen and Bogy (1992a) to calculate the derivative of the eigenvalue λ_{mn} with respect to the edge load F_f,

$$\frac{\partial \lambda_{mn}}{\partial F_f} = \frac{i\langle w^0_{mn}, \hat{\mathbf{L}} w^0_{mn}\rangle}{4\pi(\omega_{mn} \pm n\Omega)F_f \int_a^b R^2_{mn}(r) r dr} \tag{5}$$

where the inner product between two complex functions w_1 and w_2 is defined as

$$\langle w_1, w_2 \rangle = \int_{-\pi}^{\pi} \int_a^b \bar{w}_1 w_2 r dr d\theta.$$

Since the inner product in the numerator of Eq. (5) is proportional to F_f, the derivative itself is independent of F_f. It is noted that the denominator in Eq. (5) is positive for a forward and a backward traveling wave, but is negative for a reflected wave (Chen and Bogy, 1992a). After integration by parts, we can calculate

$$\langle w_{mn}, \hat{\mathbf{L}} w_{mn}\rangle = \frac{1}{\rho}\int_{-\pi}^{\pi}\int_a^b \left[\left(\frac{\partial R_{mn}}{\partial r}\right)^2 r\hat{\sigma}_r + n^2 R^2_{mn}\frac{\hat{\sigma}_\theta}{r}\right] dr d\theta + \frac{F_f R_{mn}(b)}{\rho h}\frac{\partial R_{mn}(b)}{\partial r}. \tag{6}$$

While the integral term is primarily negative because the edge load is compressive, the boundary term, i.e., the second term on the right-hand side of Eq. (6), is always positive. For the numerical example in Figs. 2 and 3 we can verify by additional calculations that for the modes with nodal circle(s), the absolute value of the integral term is larger than the boundary term. On the other hand, for most of the zero-nodal-circle modes the boundary term is dominant, and the natural frequencies of the forward and the backward traveling waves tend to increase upon the application of the follower edge load.

Effects of Boundary Operator $\hat{\mathbf{B}}_1$

We next examine the case when the edge load is conservative, i.e., $F = F_c$. In this case, the edge load affects the natural frequencies and stability of the disk through both the membrane operator $\hat{\mathbf{L}}$ and the boundary operator $\hat{\mathbf{B}}_1$. When the conservative edge load with the same magnitude of the follower load in the last section, i.e., $F_c b/D = 21$, is applied, the disk will buckle (in the sense that the lowest natural frequency vanishes) even when the disk is not spinning. In order to understand how the natural frequencies of the disk change as the conservative edge load varies, we apply a

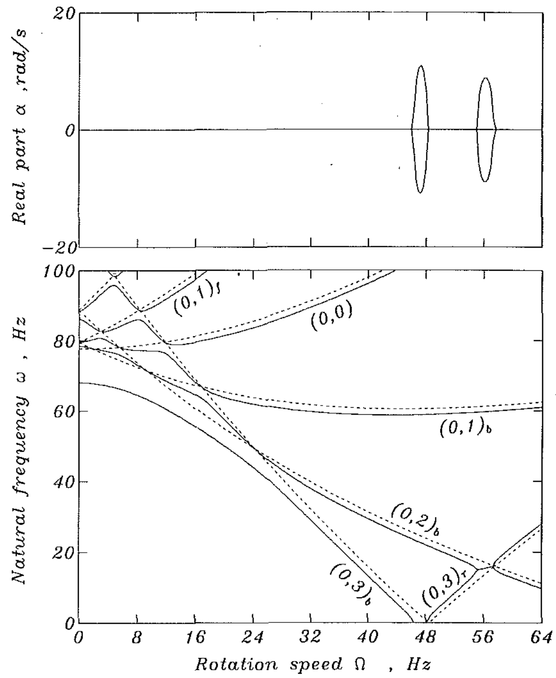

Fig. 4 Effects of conservative edge load $F_c b/D = 3.5$

smaller conservative edge load. Figure 4 shows the effects of $F_c b/D = 3.5$ on the eigenvalues of the system.

It is found that the conservative edge load decreases the natural frequencies of the forward and the backward traveling waves, and increases the natural frequency of the reflected wave. At the intersection point between two dashed curves, i.e., the degenerate eigenvalues of the freely spinning disk, one of the eigenvalues remains unchanged while the other decreases upon the application of F_c. It is also found that even though the magnitude of F_c in Fig. 4 is only one sixth the magnitude of F_f in Fig. 2, the stationary-type instability in Fig. 4 is as significant as that in Fig. 2. The conservative edge load also induces a merged-type instability when the reflected wave $(0,3)_r$ meets the backward wave $(0,2)_b$ on the left-hand side of the intersection point between these two dashed curves.

When a conservative tensile edge load with the same magnitude is applied, additional calculations show that it exhibits the opposite effects, i.e., a tensile edge load increases the natural frequencies of the forward and backward traveling waves, but decreases the natural frequency of the reflected wave. In addition, it induces a stationary-type instability just above the critical speed, and induces a merged-type instability on the right-hand side of the intersection between the dashed curves $(0,3)_r$ and $(0,2)_b$.

Additional eigenvalue calculations show that including or excluding the membrane effect $\hat{\mathbf{L}}$ alters the results by less than five percent. So, the significance of F_c on the natural frequencies and stability of the spinning disk is solely through the boundary operator $\hat{\mathbf{B}}_1$, and not through the membrane operator $\hat{\mathbf{L}}$.

If we concentrate on the effects of $\hat{\mathbf{B}}_1$ alone, we can obtain some physical insight into the stationary-type and merged-type instabilities induced by F_c by the following analysis. Assume that the spinning disk is vibrating in a backward traveling mode,

$$w = R_{mn}(r)\cos(n\theta + \omega_{mn}t + \epsilon_{mn})$$

where ϵ_{mn} is a constant phase. The velocity of the transverse direction of a particle on the spinning disk at position $r = b$, $\theta = 0$ is

$$\dot{w} = -(\omega_{mn} + n\Omega)R_{mn}(b)\sin(\omega_{mn}t + \epsilon_{mn}).$$

The transverse component of the edge load is $F_c(\partial w/\partial r)$. The work done by the edge load over one cycle from time t_1 to t_2 is

$$-\frac{1}{2}F_c(\omega_{mn} + n\Omega)R_{mn}(b)\frac{\partial R_{mn}(b)}{\partial r}\int_{t_1}^{t_2}\sin 2(\omega_{mn}t + \epsilon_{mn})dt.$$

This work is zero when the natural frequency ω_{mn} is not zero. On the other hand, this work can be positive or negative, depending on the phase ϵ_{mn} when $\omega_{mn} = 0$. This explains why instability occurs when the conservative edge load is present and the natural frequency of the mode becomes zero.

Regarding the merged-type instability, we assume that the disk is vibrating in the combination of a backward traveling wave w_{mn} and a reflected wave w_{pq},

$$w_{mn} = R_{mn}(r)\cos(n\theta + \omega_{mn}t + \epsilon_{mn})$$

and

$$w_{pq} = R_{pq}(r)\cos(q\theta - \omega_{pq}t + \epsilon_{pq})$$

where the natural frequencies ω_{mn} and ω_{pq} are assumed to be positive. It is shown in the previous paragraph that the work done on the mode shape w_{mn} by its corresponding transverse component $F_c(\partial w_{mn}/\partial r)$ is zero. However, the work done by the other force component $F_c(\partial w_{pq}/\partial r)$ over one cycle is

$$-\frac{1}{2}F_c(\omega_{mn} + n\Omega)R_{mn}(b)\frac{\partial R_{pq}(b)}{\partial r} \times \int_{t_1}^{t_2}\sin\left[(\omega_{mn} - \omega_{pq})t + \epsilon_{mn} - \epsilon_{pq}\right]dt.$$

In the case when $\omega_{mn} \neq \omega_{pq}$, this work is zero. On the other hand, when $\omega_{mn} = \omega_{pq}$, this work can be positive or negative. This explains why instability occurs when the conservative edge load is present and two frequency loci merge together.

It is emphasized here that while this simple analysis can explain why instability occurs when the natural frequency becomes zero (stationary-type) or when two natural frequencies are identical (merged-type), it does not provide explanations of why the natural frequencies tend to vanish or merge together. To provide explanations for these mechanisms, we need to study the mathematical structure of modal interactions in a manner similar to that described in Chen and Bogy (1992b).

Derivatives of Eigenvalues With Respect to F_c

We derive in this section the expression for the derivative of the eigenvalues with respect to F_c. With this analytical expression we can verify the numerical results regarding the effects of F_c on the natural frequencies of the spinning disk. Since we have shown that the boundary operator $\hat{\mathbf{B}}_1$ is much more important than the membrane operator $\hat{\mathbf{L}}$ in this regard, and the effect of $\hat{\mathbf{L}}$ alone is discussed in the previous section, we exclude $\hat{\mathbf{L}}$ here from the equation of motion for simplicity. After substituting the separable solution $w = w_{mn}e^{\lambda_{mn}t}$ into the equation of motion, we can write the equation for the eigenvalue problem in the operator form as

$$\lambda_{mn}^2\mathbf{M}w_{mn} + \lambda_{mn}\mathbf{G}w_{mn} + \mathbf{K}w_{mn} = 0 \tag{7}$$

in which

$$\mathbf{M} = 1, \quad \mathbf{G} = 2\Omega\frac{\partial}{\partial\theta}, \quad \mathbf{K} = \frac{D\nabla^4}{\rho h} + \Omega^2\frac{\partial^2}{\partial\theta^2} + \frac{\mathbf{L}}{\rho h}.$$

The boundary conditions are the same as in Eqs. ($2a$, b). For a freely spinning disk, i.e., $F_c = 0$, Eq. (7) assumes the form

$$(\lambda_{mn}^0)^2\mathbf{M}w_{mn}^0 + \lambda_{mn}^0\mathbf{G}w_{mn}^0 + \mathbf{K}w_{mn}^0 = 0 \tag{8}$$

and the boundary conditions are the same as in Eqs. $(2a, b)$, except that the operator $\hat{\mathbf{B}}_1$ disappears. The difference between eigenpair (λ_{mn}, w_{mn}) in Eq. (7) and $(\lambda^0_{mn}, w^0_{mn})$ in Eq. (8) is due to the boundary perturbation $F_c\hat{\mathbf{B}}_1 w$. This difference vanishes as F_c approaches zero.

Following a procedure described in Stakgold (1979), we first differentiate Eq. (7) and boundary conditions $(2a, b)$ with respect to F_c, and then set $F_c = 0$,

$$2\lambda^0_{mn}\lambda_{mn,F_c}\mathbf{M}w^0_{mn} + \left(\lambda^0_{mn}\right)^2\mathbf{M}w_{mn,F_c} + \lambda_{mn,F_c}\mathbf{G}w^0_{mn} + \lambda^0_{mn}\mathbf{G}w_{mn,F_c} + \mathbf{K}w_{mn,F_c} = 0 \quad (9)$$

$$w_{mn,F_c} = 0, \quad w_{mn,F_cr} = 0 \qquad \text{at} \quad r = a$$

$$\mathbf{B}_1 w_{mn,F_c} + \hat{\mathbf{B}}_1 w^0_{mn} = 0, \quad \mathbf{B}_2 w_{mn,F_c} = 0 \quad \text{at} \quad r = b.$$

We multiply the conjugate of Eq. (8) by w_{mn,F_c} and Eq. (9) by $\bar{w}^0_{mn}$, subtract and integrate over the annular region. After recognizing that $(\lambda^0_{mn})^2 = (\bar{\lambda}^0_{mn})^2$ we obtain

$$\frac{\partial\lambda_{mn}}{\partial F_c} = -\frac{iR_{mn}(b)\dfrac{\partial R_{mn}(b)}{\partial r}}{4\pi\rho h(\omega_{mn} \pm n\Omega)\int_a^b R^2_{mn}(r)rdr} \quad (10)$$

The numerator on the right-hand side of Eq. (10) is always positive. The denominator, on the other hand, is positive for a forward and a backward traveling wave, but is negative for a reflected wave. Therefore the derivative is negative for a forward and a backward wave, and is positive for a reflected wave. This verifies our numerical results showing that the conservative compressive edge load tends to decrease the natural frequencies of the forward and backward traveling waves, and tends to increase the natural frequency of the reflected wave.

Conclusions and Discussions

The effects of a stationary edge load on the natural frequencies and stability of a spinning elastic disk are examined both numerically and analytically. The results of these analyses can be summarized as follows:

(1) The stationary, conservative, compressive edge load tends to decrease the natural frequencies of the forward and backward traveling waves, but tends to increase the natural frequency of the reflected wave. In addition, it induces a stationary-type instability before the critical speed, and induces a merged-type instability after the critical speed when a reflected wave meets a forward or a backward wave.

(2) Numerical analysis shows that the significance of the conservative edge load on the natural frequencies and stability of the spinning disk is solely through the transverse component of the edge load on the boundary, and not through the membrane stress field inside the disk.

(3) The derivative of the eigenvalues of a spinning elastic disk with respect to the conservative edge load verifies the numerical results regarding the effects of the edge load on the natural frequencies of the spinning disk.

The goal in circular saw research has been to improve the stability of the spinning saw blade during cutting operation. As we have pointed out the edge load induces a stationary-type instability before the critical speed. In order to raise the operation speed of the circular saw, it becomes necessary to eliminate the instabilities induced by the edge load. In the wood cutting industry, it has been a common practice to use saw guides and a floating central collar to achieve a more stable operation. A theoretical analysis of the stabilizing effects of the saw guides and a floating collar, when the edge load is present, is important in designing new circular saws and should be undertaken.

Acknowledgments

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