Dynamic Loop Interactions and Multi-loop PID Controller Design

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Abstract— **In a multi-loop PID control system, a nominal model for the effective open-loop dynamics that include interactions through other loops is formulated. A standard representation for complementary sensitivity function of single loops with a global parameter is used to exempt the needs for controllers of the other loops. By making uses of such nominal models, design of controllers can be performed straightforward as doing for single loop systems. An inverse-based method for designing SISO and PID controllers is thus presented.**

I. INTRODUCTION

In many chemical process units, several controlled variables that closely related to the qualities of products are to be maintained simultaneously at independent set-points. Consequently, the control problems encountered in chemical plants are mostly of multivariables, and most of them are controlled with multiple SISO loops. Because of the coupled nature among the loops in design, the methods for designing PID controllers used in the past neither do not provide transparent access to the performance of the system, nor do they provide good efficiency in design. Until late eighties, the methods that use similar approach to design SISO systems appeared in literature [1], [2], and the multi-loop design attracts again more attentions from researchers. Lately, the development of design methods has been linked to the use of auto-tuning procedures [3]-[9]. Other developments on the design of multi-loop PID controllers include: use of describing function and Z-N rules for PI controllers [10], shaping the Gershgorin circles based on the user's specified gain margin and phase margin [11], use of detune factor to take into account loop interactions [12], [13]. In those methods above mentioned, the design of each controller in a multi-loop system has to be interacted with those of the other loops. It is our purpose to devise nominal models for the effective open-loop dynamics in a multi-loop system to include their dynamic effects in the controller design. Meanwhile, in order to design the controllers similar to the SISO approach, these nominal models should have no access to the final controllers in all other loops. As a result, the design of controllers can be performed separately as for single loop systems. An inverse-based method for designing single loop PID controllers is thus presented together with these nominal models to design the multi-loop PID controllers. Numerical examples from two processes in literature are used to illustrate the design.

II. MODELING EFFECTIVE OPEN-LOOP DYNAMICS IN MULTI-LOOP SYSTEMS

Consider a general $N \times N$ multivariable system of the following:

$$Y(s) = G(s)U(s) + D(s) \tag{1}$$

where $Y(s)$, $U(s)$ and $D(s)$ designates the output, input, and disturbance vectors, respectively. $G(s)$ is a transfer function matrix (TFM) of the following:

$$G(s) = [g_{k,j}]_{k,j=1,\cdots,N} = \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,N} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N,1} & g_{N,2} & \cdots & g_{N,N} \end{bmatrix} \tag{2}$$

An effective open-loop transmission of ith loop is referred to the total dynamic transfer from u_i to y_i when the ith loop is open and all other loops are closed. This open-loop transmission will include two parts. The first part is the direct transmission via transfer function $g_{i,i}$. The other part is via the transmission to the other loops and back to y_i. Let $G^{[i]}(s)$ designates the matrix resulting from changing $G(s)$ by moving the ith element on the diagonal (i.e. g_{ii}) to the first entry. That is:

$$G^{[i]}(s) = [g^{[i]}_{k,j}]_{k,j=1,\cdots,N} = \begin{bmatrix} g^{[i]}_{1,1} & g^{[i]}_{1,2} & \cdots & g^{[i]}_{1,N} \\ g^{[i]}_{2,1} & g^{[i]}_{2,2} & \cdots & g^{[i]}_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ g^{[i]}_{N,1} & g^{[i]}_{N,2} & \cdots & g^{[i]}_{N,N} \end{bmatrix} \tag{3}$$

In other words, each entry in $G^{[i]}(s)$ is obtained from changing the subscript indices of $G(s)$ at the same entry by the following rules for a given i :

- Replace each k or j which equals i with 1.
- Replace each k or j which equals 1 with i.

For example, to write $G^{[i]}(s)$ in terms of elements of $G(s)$ for a 3×3 system, we have:

$$G^{[1]}(s) = \begin{bmatrix} g_{1,1}(s) & g_{1,2}(s) & g_{1,3}(s) \\ g_{2,1}(s) & g_{2,2}(s) & g_{2,3}(s) \\ g_{3,1}(s) & g_{3,2}(s) & g_{3,3}(s) \end{bmatrix}$$

$$G^{[2]}(s) = \begin{bmatrix} g_{2,2}(s) & g_{2,1}(s) & g_{2,3}(s) \\ g_{1,2}(s) & g_{1,1}(s) & g_{1,3}(s) \\ g_{3,2}(s) & g_{3,1}(s) & g_{3,3}(s) \end{bmatrix}$$

$$G^{[3]}(s) = \begin{bmatrix} g_{3,3}(s) & g_{3,1}(s) & g_{3,2}(s) \\ g_{2,3}(s) & g_{2,2}(s) & g_{2,1}(s) \\ g_{1,3}(s) & g_{1,2}(s) & g_{1,1}(s) \end{bmatrix} \quad (4)$$

Let the resulting $G^{[i]}(s)$ defined above be partitioned into four blocks of the following:

$$G^{[i]}(s) = \begin{bmatrix} g_\alpha^{[i]} & G_\beta^{[i]} \\ G_\gamma^{[i]} & G_\delta^{[i]} \end{bmatrix} = \begin{bmatrix} g_{i,i} & G_\beta^{[i]} \\ G_\gamma^{[i]} & G_\delta^{[i]} \end{bmatrix} \quad (5)$$

A nominal model for the effective dynamics of loop i can be represented as following:

$$\mathbf{g}_i(s) = g_\alpha^{[i]}(s) - G_\beta^{[i]}(s)(G_\delta^{[i]})^{-1}(s)\left[G_\gamma^{[i]} \otimes H^{[i]}(s)\right]; \quad \text{for} \quad i = 1, 2, \cdots, N \quad (6)$$

where

$$H^{[i]} = [h_j]_{j=1,2,\cdots,N;\ j\neq i} \quad (7)$$

and h_j designates the single loop complementary sensitivity function as:

$$h_j = \frac{g_{c,j}g_{j,j}}{1 + g_{c,j}g_{j,j}}; \quad j = 1, 2, \cdots, N \quad (8)$$

Fig. 1 shows the block diagram of $\mathbf{g}_1(s)$ for a two-loop system. It is our purpose to model the effective open-loop dynamics (i.e. $\mathbf{g}_i(s)$) that includes the dynamic interactions from other loops for controller design. That is:

$$y_i = \mathbf{g}_i\, u_i \quad (9)$$

For a system with two loops, (6) can simply be written in terms of the dynamic relative gain (i.e. $\lambda(s)$) as:

$$\begin{aligned} \mathbf{g}_1 &= g_{1,1}\{1 - [1 - \frac{1}{\lambda(s)}]\, h_2(s)\} \\ \mathbf{g}_2 &= g_{2,2}\{1 - [1 - \frac{1}{\lambda(s)}]\, h_1(s)\} \end{aligned} \quad (10)$$

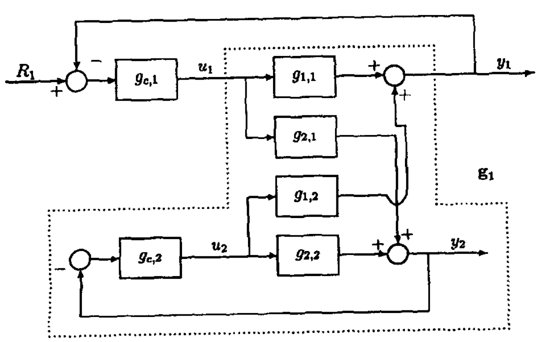

Fig. 1. A two-loop control system

where

$$\lambda(s) = \frac{1}{1 - \dfrac{g_{1,2}g_{2,1}}{g_{1,1}g_{2,2}}} \quad (11)$$

Thus, to represent the effective open loop dynamics, the complementary sensitivity functions (CSF) of each individual loops are required. From the IMC point of view, the controller in each loop is aimed at inverting the open-loop dynamics (i.e. $g_{i,i}$ in a multi-loop system). Huang *et al.* [14] had shown that the CSF of a single closed-loop system that has inverse-based controller can be represented with the the following standard form quite well, that is:

$$h_j^*(s) = \frac{\dfrac{k_{o,j}(1 + 0.4\theta_j s)e^{-\theta_j s}}{s}}{1 + \dfrac{k_{o,j}(1 + 0.4\theta_j s)e^{-\theta_j s}}{s}} \quad (12)$$

Usually, for reasonable performance, the value of $k_{o,j}\theta_j$ is given ranging from 0.5 to 0.8. In a multi-loop system, each single loop $k_{o,j}$ is tuned to be more conservative than it stands alone. Using this approximation, (6) and (10) become:

$$\mathbf{g}_i^*(s) = g_\alpha^{[i]}(s) - G_\beta^{[i]}(s)(G_\delta^{[i]})^{-1}(s)\left[G_\gamma^{[i]} \otimes (H^{[i]})^*(s)\right] \quad (13)$$

$$\begin{aligned} \mathbf{g}_1^* &= g_{1,1}\{1 - [1 - \frac{1}{\lambda(s)}]\, h_2^*(s)\} \\ \mathbf{g}_2^* &= g_{2,2}\{1 - [1 - \frac{1}{\lambda(s)}]\, h_1^*(s)\} \end{aligned} \quad (14)$$

This approximation has the advantage of that we can obtain the effective open-loop models, $\mathbf{g}_i^*(s)$, without knowing the controllers of other loops in advance. Therefore, the major difficulty in multi-loop controller design would thus be overcome.

III. MODELING ERROR DUE TO APPROXIMATION

According to the formulation given above, the modeling error due to the approximation made in derivations are given as follows. Since a standard form of $h(s)$ is used, the actual system is considered the one which is stable with multi-loop PID controllers.

A. For systems with two loops

The errors in modeling nominal $\mathbf{g}_i(s)$ for a 2×2 system can be found as:

$$\begin{aligned} \Delta\mathbf{g}_1(s) &= g_{1,1}(1 - \frac{1}{\lambda(s)})(h_2(s) - h_2^*(s)) \\ \Delta\mathbf{g}_2(s) &= g_{2,2}(1 - \frac{1}{\lambda(s)})(h_1(s) - h_1^*(s)) \end{aligned} \quad (15)$$

or, in terms of multiplicative modeling error:

$$\delta\mathbf{g}_1 = \frac{(1 - \frac{1}{\lambda(s)})(h_2(s) - h_2^*(s))}{1 - (1 - \frac{1}{\lambda(s)})\, h_2^*(s)}$$

$$\delta \mathbf{g}_2 = \frac{(1 - \frac{1}{\lambda(s)})(h_1(s) - h_1^*(s))}{1 - (1 - \frac{1}{\lambda(s)})\, h_1^*(s)} \tag{16}$$

Thus, by specifying a possible range of changes in the complementary sensitivity function regarding the diagonal elements $g_{i,i}$, the value of $\max_{\omega}|\delta \mathbf{g}_i(j\omega)|$ can be computed.

B. For systems with more than two loops

The formulation of the modeling errors to stabilize the nominal multi-loop systems that have more than two loops is quite similar to that of two loop systems. The modeling errors will be:

$$\begin{aligned} \Delta \mathbf{g}_i(s) &= g_\alpha^{[i]}(s) - G_\beta^{[i]}(s)(G_\delta^{[i]})^{-1}(s) \\ &\quad \left\{[I + G_\delta^{[i]}]^{-1}G_\gamma^{[i]} - G_\gamma^{[i]} \otimes (H^{[i]})^*(s)\right\} ; \\ &\quad \text{for} \quad i = 1, 2, \cdots, N \end{aligned} \tag{17}$$

or,

$$\delta \mathbf{g}_i(s) = \frac{\Delta \mathbf{g}_i}{\mathbf{g}_i^*} \tag{18}$$

IV. TUNING OF PID CONTROLLERS

The design procedures for PID controllers using the nominal effective open-loop models are as follows:

A. Find simple reduced order models for the effective open-loop dynamics

The nominal models of effective open-loops dynamics been given in (13). Thus, by substituting s with $j\omega$ in $\mathbf{g}_i^*$, $\omega \in [0, \omega_F]$, where ω_F is large enough, Bode' diagrams of those open-loops dynamics can be prepared. As a result, simple transfer function models can be found to fit these frequency response data by performing the following optimization problem:

$$Arg\{\mathbf{p}\} = \min_{[\mathbf{p}]} \int_0^{\omega_F} |\bar{\mathbf{g}}_i^*(j\omega) - \mathbf{g}_i^*(j\omega)|^2 d\omega \tag{19}$$

where $\mathbf{P}$ is the parameters in $\bar{\mathbf{g}}_i^*(j\omega)$.

B. Obtain the PID controllers for each of loops

The tuning rules employed for each loop are derived from the inverse-based PID controllers presented by Huang *et al.* [14]. The PID controller is given in parallel form, that is:

$$g_c(s) = k_c(1 + \frac{1}{\tau_R s} + \tau_D s) \tag{20}$$

and, the parameters are given as follows:
1. When the effective open-loop dynamics is represented in the form of FOPDT of the following:

$$g(s) = \frac{k_p\, e^{-\theta s}}{\tau s + 1} \tag{21}$$

The parameters of the a PID controller are given as:

$$\begin{aligned} \tau_R &= \tau + 0.4\theta \\ \tau_D &= \frac{0.4\tau\theta}{\tau + 0.4\theta} \end{aligned} \tag{22}$$

2. If the effective open-loop dynamics can be represented in a form of SOPDT, that is:

$$g(s) = \frac{k_p\, e^{-\theta s}}{\tau^2 s^2 + 2\tau\zeta s + 1} \tag{23}$$

The PID parameters are:

$$\begin{aligned} \tau_R &= 2\tau\zeta + 0.4\theta \\ \tau_D &= \frac{\tau^2 + 0.8\tau\zeta\theta}{2\tau\zeta + 0.4\theta} \end{aligned} \tag{24}$$

Notice that k_c is not specified in the above setting. It becomes a tuning factor and is then tuned to make the complementary sensitivity to achieve a desired maximum log modulus. This desired maximum log modulus is specified to meet the requirement for the overshoot in responding to step set-point inputs.

C. Check the feasibility of the selected tuning factors

Because of the approximation made in formulating the effective open-loop models, the stability of the nominal system has to be considered in the first priority. The controller of each main loop will be required to meet the constraint of the following:

$$\left|\frac{g_{c,i}\, \mathbf{g}_i(j\omega)}{1 + g_{c,i}\, \mathbf{g}_i(j\omega)}\right| \le \frac{1}{\max_{\omega}|\delta \mathbf{g}_i(j\omega)|} \tag{25}$$

D. Sufficiency for stability

In the previous section, the controllers $g_{c,i}$ is determined for each loop i individually. The following lemma and theorem provide the sufficient condition for designing $g_{c,i}$ in this way to stabilize the multi-loop system.

[Lemma 1] Let $G_c(s)$ be a diagonal matrix with PID controllers on diagonal entries. Then,

$$Det[I + G(s)G_c(s)] = (1 + g_{c,i}\mathbf{g}_i)\ Det[I + G_\delta^{[i]}\ G_c^{[i]}] \tag{26}$$

[Theorem 1] Assume 1): $G(s)$ is open-loop stable, 2): $\mathbf{g}_i$ of Eqn.(6) is open-loop stable for each i, and 3): Eqn.(25) is satisfied for each i. Then, stabilizing $\mathbf{g}_i$ with $g_{c,i}$ for each i will stabilize the multi-loop system.

Because of Theorem 1, the multi-loop system can be decomposed into and designed as individual loops.

E. Check the stability of the given multi-loop system

The stability of the system can be checked by using the new Nyquist stability test of Huang *et al.* [15]. If the system is found not stable, some of the loops (one or more) must have more conservative k_c values.

Notice that there is no need to check all the subsystems resulting from opening one or more loops in the multi-loop system. The following theorem will guarantee the integrity of the system, once a stable system is resulted.

[Theorem 2] If there is a diagonal matrix $G_c(s)$ such that $Det[I + G(s)\ G_c(s)]$ has no RHP zero, then, each principle minor of $[I + G(s)\ G_c(s)]$ of all orders has no RHP zero.

The proofs of these lemma and theorems will be given in the Appendix.

V. SIMULATION RESULTS

Let us use the Wood and Berry (WB) process of the following for illustration.

$$G_p(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$
$$G_L(s) = \begin{bmatrix} \frac{3.8e^{-8s}}{14.9s+1} \\ \frac{4.9e^{-3s}}{13.2s+1} \end{bmatrix} \quad (27)$$

First, the Bode' diagrams of $\mathbf{g}_1^*(s)$, and $\mathbf{g}_2^*(s)$ are prepared. Then, from these Bode' diagrams, two simple models are found by fitting the frequency responses to those of $\mathbf{g}_i^*$, $i = 1, 2$. The models are found as:

$$\bar{\mathbf{g}}_1^* = \frac{6.37\ e^{-s}}{6.1s+1}$$
$$\bar{\mathbf{g}}_2^* = \frac{-9.01\ e^{-3s}}{5.7s+1}$$

Notice that a value of 0.65 for $k_{o,i}\theta_i$ has been assigned for modeling the effective open-loop transfer function. With these two models, the PID controller settings according to the inverse-based method can be computed as:

$$\tau_{R,1} = 6.5\ ; \quad \tau_{D,1} = 0.375$$
$$\tau_{R,2} = 6.9\ ; \quad \tau_{D,2} = 0.991$$

Then, the complementary sensitivity functions for the effective open-loop are computed according to the following:

$$h_{1,eff}^*(j\omega) = \frac{g_{c,1}\ \bar{\mathbf{g}}_1^*(j\omega)}{1 + g_{c,1}\bar{\mathbf{g}}_1^*(j\omega)}$$
$$h_{2,eff}^*(j\omega) = \frac{g_{c,2}\ \bar{\mathbf{g}}_2^*(j\omega)}{1 + g_{c,2}\bar{\mathbf{g}}_2^*(j\omega)} \quad (28)$$

The values of $k_{c,1}$ and $k_{c,2}$ are thus adjusted to make the maximum log modulus ($L_{c,max}$) of $h_{i,eff}^*(j\omega)$ equal the required value (for example, 0.413 db for 10% overshoot). Notice that

$$L_{c,max} = \max_{\omega}\{\ 20 \log |h_{i,eff}^*(j\omega)|\ \} \quad (29)$$

The resulting final parameters of PID controllers are shown in Table I.

The responses to the set-point changes are shown in Fig. 2 and those from the controllers tuned with BLT-4 have also been given in the same figure for comparison.

TABLE I

PARAMETERS OF PID CONTROLLERS FOR WB PROCESS

loop no.	$L_{c,max}$	k_c	τ_R	τ_D
loop-1	0.413 db	0.379	6.5	0.375
loop-2	0.413 db	-0.094	6.9	0.991

As another example, the Ogunnaike and Ray (OR) process is considered:

$$G_p(s) =$$
$$\begin{bmatrix} \frac{0.66\ e^{-2.6s}}{6.7s+1} & \frac{-0.61\ e^{-3.5s}}{8.64s+1} & \frac{-0.0049\ e^{-s}}{9.06s+1} \\ \frac{1.11\ e^{-6.5s}}{3.25s+1} & \frac{-2.36\ e^{-3s}}{5s+1} & \frac{-0.01\ e^{-1.2s}}{7.09s+1} \\ \frac{-34.68\ e^{-9.2s}}{8.15s+1} & \frac{-46.2\ e^{-9.4s}}{10.9s+1} & \frac{-0.87(11.61s+1)\ e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}$$

$$G_L(s) = \begin{bmatrix} \frac{0.14\ e^{-12s}}{(19.2s+1)^2} \\ \frac{0.53\ e^{-10.5s}}{6.9s+1} \\ \frac{-11.54\ e^{-0.6s}}{7.01s+1} \end{bmatrix} \quad (30)$$

Three nominal models for the effective open-loop dynamics are found by fitting their frequency responses as:

$$\bar{\mathbf{g}}_1^* = \frac{0.3286\ e^{-1.194s}}{8.5615s^2 + 2.452s + 1}$$
$$\bar{\mathbf{g}}_2^* = \frac{-1.2935\ e^{-1.797s}}{6.497s^2 + 2.223s + 1}$$
$$\bar{\mathbf{g}}_3^* = \frac{0.593\ e^{-0.882s}}{0.558s^2 + 3.0712s + 1}$$

The controllers are tuned to give $L_{c,max} = 0.413$ db for each loop. The resulting PID parameters are obtained as listed in Table II. The responses to the set-point changes are given in Fig. 3.

Table III shows also the estimated values, from the nominal models, of ultimate frequency ($\omega_{u,i}$), ultimate gain ($k_{cu,i}$), and steady state gain ($k_{ss,i}$) of each loop. The results show that the estimations are very close to their actual values. If we consider the single loop transfer function ($g_{i,i}$) only, the error of estimation will be significant.

TABLE II

PARAMETERS OF PID CONTROLLERS FOR OR PROCESS

loop no.	$L_{c,max}$	k_c	τ_R	τ_D
loop-1	0.413 db	1.09	2.930	3.322
loop-2	0.413 db	-0.24	2.942	2.752
loop-3	0.413 db	4.30	4.464	0.450

TABLE III

ULTIMATE FREQUENCY, ULTIMATE GAIN, AND STEADY STATE GAIN OF EXAMPLE MULTI-LOOP SYSTEMS

	nominal	actual	g_{ii} only
WB (2×2)			
$\omega_{u,1}$	1.6376	1.6376	1.6077
$\omega_{u,2}$	0.4985	0.4940	0.5620
$k_{cu,1}$	1.9051	1.9432	2.0991
$k_{cu,2}$	-0.2591	-0.2233	-0.4203
$k_{ss,1}$	6.3701	6.3701	12.8000
$k_{ss,2}$	-9.6547	-9.6547	-19.4000
OR (3×3)			
$\omega_{u,1}$	0.5417	0.5672	0.6821
$\omega_{u,2}$	0.5467	0.5569	0.6220
$\omega_{u,3}$	1.6881	1.6881	1.6991
$k_{cu,1}$	4.5851	5.1540	7.0872
$k_{cu,2}$	-0.8917	-0.9537	-1.3843
$k_{cu,3}$	12.2699	11.9332	12.4378
$k_{ss,1}$	0.3286	0.3286	0.6600
$k_{ss,2}$	-1.2935	-1.2935	-2.3600
$k_{ss,3}$	0.5939	0.5939	0.8700

VI. CONCLUSION

In this paper, a nominal model for the effective open-loop dynamics of each loop is presented. With these effective open-loop models, the Bode' diagrams can be prepared. The results show that these nominal models can closely represent the effective open-loop dynamics. As a result, the controller in each loop can be designed like designing controller for single loop systems. The performance of each individual loop can be accessible as if they were independent of each other.

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APPENDIX

A. Proof of Lemma 1

According to the Schur's formula for partitioned determinants, i.e.:

$$Det \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} = Det[G_1] \cdot Det[G_4 - G_3\ G_1^{-1}\ G_2]$$
$$= \ Det[G_4] \cdot Det[G_1 - G_2\ G_4^{-1}\ G_2]$$

Thus, if $G^{[i]}(s)$ is partitioned as in (5), $Det[I + G(s)G_c(s)]$ can be written as:

$$Det[I + G(s)G_c(s)] = (1 + g_{c,i}\ \mathbf{g}_i)\ Det[I + G_\delta^{[i]}\ G_c^{[i]}]$$

where, $G_c^{[i]}$ designates a diagonal matrix resulting from deleting ith column and ith row from original $G_c(s)$.

B. Proof of Theorem 1

From assumption 1), each component of $G(s)$, (i.e. $g_{i,j}$), will not have unstable pole. From assumptions 2) and 3), The controller $g_{c,i}$ will stabilize $g_{i,i}(s)$ for each i and for each sub-system of order $n-1$. Then, according to Lemma 1, the multi-loop system will be stable if each $\mathbf{g}_i$ is stabilized with $g_{c,i}$ for each i. **Q.E.D.**

C. Proof of Theorem 2

By applying Schur's formula, $Det[I + G(s)\ G_c(s)]$ becomes:

$$Det[I + G(s)G_c(s)] = (1 + g_{c,i}\ \mathbf{g}_1)\ Det[I + \mathbf{g}_1\ G_c^{[1]}]$$
$$= \ (1 + g_{c,2}\ \mathbf{g}_2)\ Det[I + \mathbf{g}_i\ G_c^{[2]}]$$
$$= \ \cdots$$
$$= \ (1 + g_{c,N}\ \mathbf{g}_N)\ Det[I + G_N^N\ G_c^{[N]}]$$

As a result, if $Det[I + G(s)\ G_c(s)]$ has no RHP zero, then, each of $Det[I + G_\delta^{[i]}\ G_c^{[i]}]$, which is of $N-1$ order, has no RHP zero. In order words, the each $N-1$ order of principle minor of $[I + G(s)\ G_c(s)]$ has no RHP zero.

Then, for each principle minor of order $N-1$ that has no RHP zero, we can conclude that each principle minor of order $N-2$ will have no RHP zero, and so on. **Q.E.D.**

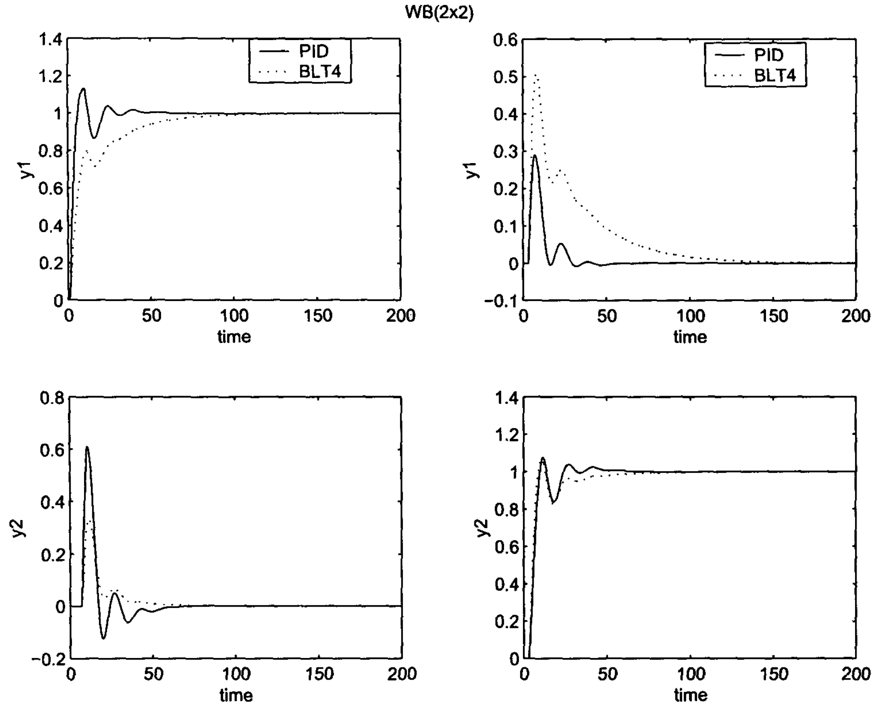

Fig. 2. Responses to set-point changes of multi-loop control for WB process with inverse-based PID controllers and BLT-4 tuned controllers

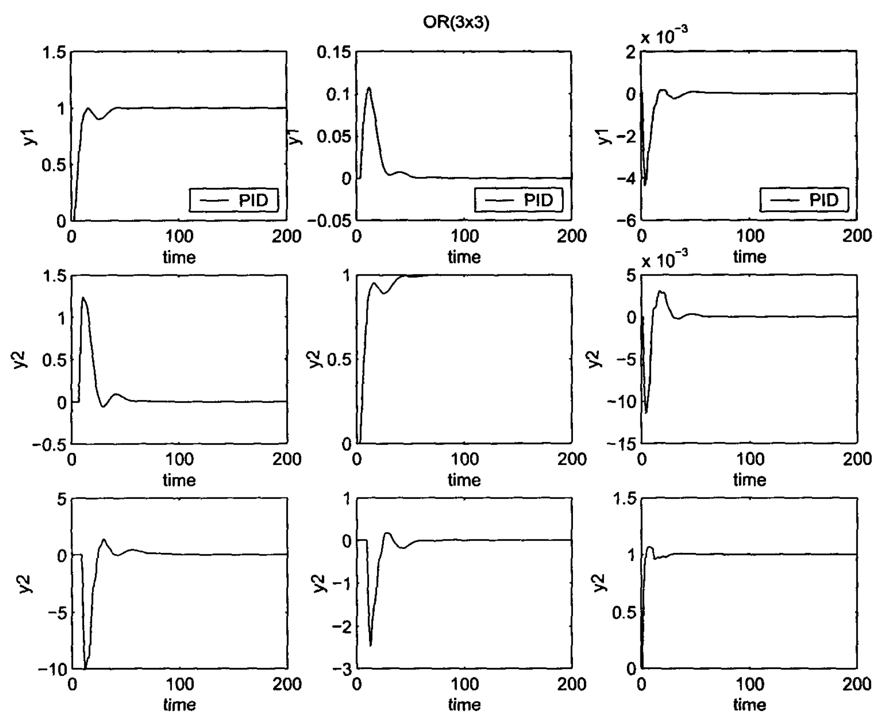

Fig. 3. Responses to set-point changes of multi-loop control for OR process with inverse-based PID controllers