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Multi-criteria fuzzy optimization for locating warehouses and distribution centers in a supply chain network

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Abstract

This study considers the planning of a multi-product, multi-period, and multi-echelon supply chain network that consists of several existing plants at fixed places, some warehouses and distribution centers at undetermined locations, and a number of given customer zones. Unsure market demands are taken into account and modeled as a number of discrete scenarios with known probabilities. The supply chain planning model is constructed as a multi-objective mixed-integer linear program (MILP) to satisfy several conflict objectives, such as minimizing the total cost, raising the decision robustness in various product demand scenarios, lifting the local incentives, and reducing the total transport time. For the purpose of creating a compensatory solution among all participants of the supply chain, a two-phase fuzzy decision-making method is presented and, by means of application of it to a numerical example, is proven effective in providing a compromised solution in an uncertain multi-echelon supply chain network.
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Keywords: Supply chain management; Uncertainty; Multiple objectives; Mixed-integer linear program (MILP); Fuzzy decision making

1. Introduction

The supply chain is an integrated process wherein a number of business entities (suppliers, manufacturers, distributors and retailers) work together in an effort to acquire raw materials, convert them into specified final products and deliver these final products to retailers (Beamon, 1998). The supply chain further fosters a new concept in management: the concept of supply chain management.

Over the past decade the world has changed from a marketplace with some large independent markets to an extremely integrated global market. The increase in competitive pressures in the global marketplace coupled with the rapid advances in information technology have brought supply chain planning into the forefront of the business practices of most manufacturing and service organizations (Gupta and Maranas, 2003). A great variety of companies, those in chemical industry included, can also benefit from this novel management scheme. Therefore, many researchers in the process systems engineering (PSE) society devote themselves to this interesting field (Applequist *et al*., 2000; Bose and Pekny, 2000; Chen *et al*., 2003; Cheng *et al*., 2003; Garcia-Flores *et al*., 2000; Gupta and Maranas, 2000; Gupta *et al*., 2000; Perea-Lopez *et al*., 2000; Pinto *et al*., 2000; Zhou *et al*., 2000, etc.).

Traditionally, the integration of supply chain networks is usually based on deterministic parameters. In practice, however, this is rarely the case as it is usually difficult to foretell prices of chemicals, market demands, availabilities of raw materials, etc., in a precise fashion (Liu and Sahinidis, 1997). A number of works are devoted to studying supply chain management under uncertain environments. For example, Gupta and Maranas (2000) incorporate uncertain demand via a normal probability function and propose a two-stage solution framework. A generalization to handle multi-period and multi-customer problems was recently proposed by Gupta and Maranas (2003). Tsiakis *et al*. (2001) use a scenario planning approach to describe demand uncertainties. Therein a number of demand scenarios with assigned non-zero probabilities is used as discrete stochastic demand quantities. All scenarios are simultaneously taken into account in the supply chain network design. However, the robustness of decision for uncertain product demands is not considered in these studies. In this paper, one of the major concerns is market demand uncertainty. The scenario-based approach will be adopted for modeling the uncertain market demands.

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The location of manufacturing and warehousing facilities has received considerable attention from academics and practitioners alike over the past four decades. Location models have been developed to answer questions such as how many facilities to establish, where to locate them, and how to distribute the products to the customers in order to satisfy demand and minimize total cost (Melachrinoudis *et al.*, 2000). However, when making location decisions, in addition to the total cost, one should also consider some other conditions such as the influence of local incentives and transport time.

In this article, the mid-term planning problem of locating warehouses and distribution centers in a supply chain network will be addressed, where multiple conflict objectives will be considered simultaneously including minimizing the total cost, raising the decision robustness to various product demand scenarios, lifting the local incentives, and reducing the total transport time. This problem can be further formulated as a multi-objective mixed-integer linear program. So the two-phase fuzzy optimization solution strategy proposed by Chen and Lee (2004a,b) can be adopted directly.

In the rest of this article, the problem statement and assumptions are outlined in Section 2. The considered uncertain issues in supply chain planning are also described. The formulation of a production and distribution–planning model is set out in Section 3. The procedure for grouping the scenario-dependent multiple conflict objectives into a scalar objective using the fuzzy sets concept is presented in Section 4. The contents of a numerical example, used to demonstrate the usefulness of the proposed method, are given in Section 5. Finally, some concluding remarks are given in Section 6.

Index/set	Dimension	Physical meaning
$c \in \mathcal{C}$	$[\mathcal{C}] = C$	Customer zones
$d \in \mathcal{D}$	$[\mathcal{D}] = D$	Distribution centers
$i \in \mathcal{I}$	$[\mathcal{I}] = I$	Products
$k \in \mathcal{K}$	$[\mathcal{K}] = K$	Transport capacity level
$m \in \mathcal{M}$	$[\mathcal{M}] = M$	All objectives
$n \in \mathcal{N}$	$[\mathcal{N}] = N$	Resources
$p \in \mathcal{P}$	$[\mathcal{P}] = P$	Plants
$s \in \mathcal{S}$	$[\mathcal{S}] = S$	Scenarios
$t \in \mathcal{T}$	$[\mathcal{T}] = T$	Periods
$w \in \mathcal{W}$	$[\mathcal{W}] = W$	Warehouses

Parameters	$*\in$	Physical meaning
FCD^{i}_{*ts}	$\{c\}$	Forecasting customer demand of i for customer c
FTC^{k}_{*}	$\{pw, wd, dc\}$	kth level fix transport cost, p to w, w to d, d to c
LI_{*}	$\{w, d\}$	Local incentive of w, d
PPD_{s}		Probability of product scenario s
PQ^{*}_{ips}	{max, min}	Maximum, minimum manufacturing quantity of product i
$Q^{\max}_{*\mathrm{s}}$	$\{pw, wd, dc\}$	Maximum transport quantity of p to w, w to d, d to c
$Q^{\min}_{*\mathrm{s}}$	$\{pw, wd, dc\}$	Minimum transport quantity of p to w, w to d, d to c
$R_{*n\mathrm{ts}}$	$\{p\}$	Total resource n at p
SQ^{+}_{*}	$\{w, d\}$	Maximum, minimum capacity of w, d, $+ \in$ {max, min}
TCL^{k}_{*}	$\{pw, wd, dc\}$	kth transport capacity level, p to w, w to d, d to c
TT_{*}	$\{pw, wd, dc\}$	Transport time of p to w, w to d, d to c
UEC_{*}	$\{w, d\}$	Unit establishing cost of w, d
UHC^{i}_{*}	$\{w, d\}$	Unit handling cost of product i for w, d
UPC^{i}_{*}	$\{p\}$	Unit production cost of product i for plant p
UTC^{k}_{*}	$\{pw, wd, dc\}$	kth level unit transport cost, p to w, w to d, d to c
α^{i}_{*}	$\{w\}$	Coefficient relating the capacity of d to flow of product i handled
β^{i}_{*}	$\{d\}$	Coefficient relating the capacity of w to flow of product i handled
ρ^{i}_{*nts}	$\{p\}$	Coefficient for resource m used in plant p for product i

Real var.	$*\in$	Physical meaning
J_{*}	$\{m\}$	Objectives
OTT		Total transportation time
PQ^{i}_{*ts}	$\{p\}$	Manufacture quantity of i
Q^{i}_{*ts}	$\{pw, wd, dc\}$	Total transport quantity, p to w, w to d, d to c
SQ_{*}	$\{w, d\}$	Capacity of w, d
TCO		Total cost
TEC		Total establishment cost of all warehouses and distribution centers
THC_{ts}		Total handling cost of scenario s in period t
TLI, LD		Total local incentive and an overall index on distribution centers
TPC_{ts}		Total production cost of scenario s in period t
TTC_{ts}		Total transportation cost for scenario s in period t

TTC_{*ts}	$\{pw, wd, dc\}$	Total transportation cost of * for scenario s in period t
TTQ^k_{*ts}	$\{pw, wd, dc\}$	kth level transport quantity, p to w, w to d, d to c
$\mu_{\mathcal{J}_m}(J_m(x))$		The membership function of fuzzy objective $\mathcal{J}_m$
Binary var.	$*\in$	Meaning when having value of 1
X_*	$\{pw, wd, dc\}$	A link between p and w, w and d, d and c exists
Y_*	$\{w, d\}$	Warehouse w or distribution center d is to be established
Z^k_{*ts}	$\{pw, wd, dc\}$	kth transport capacity level, p to w, w to d, d to c
Fuzzy var.		Physical meaning
$\mathcal{FD}$		Fuzzy set for final decision
$\mathcal{J}_m$		Fuzzy set for objective m

2. Problem description for locating warehouses and distribution centers in a supply chain network

The researchers consider a typical multi-product, multi-echelon and multi-period supply chain network originally studied by Tsiakis *et al.* (2001). The revised supply chain network consists of several existing multi-product plants at fixed places, some candidate warehouses and distribution centers at specific but undermined locations, and a number of known customer zones, as showed in Fig. 1. In this mid-term supply chain-planning problem, each customer zone places demands for one or more products. The candidate warehouses and distribution centers are described by the upper and lower bounds on their handling capacity. The establishment of warehouses and distribution centers will result in a fixed infrastructure cost. Operational costs include those associated with production, handling of material at warehouses and distribution centers, and transportation. The numbers and the locations of selected warehouses and distribution centers are left to be determined for establishment of a cost–effective supply chain network. The following assumptions are made for subsequent modeling and optimization: the whole system is operated steadily; therefore, there is no stock accumulation or depletion, and inventory can be ignored; the production capacity of each plant is related linearly to resources; the capacities of warehouses and distribution centers are related linearly to the materials that they handle; the transportation costs are piecewise linear functions of the actual flow of the product from the source stage to the destination podium; several scenarios of product demands with known probabilities are forecast over the entire planning periods. The overall problem can thus be stated as follows. Given are the manufacturing data, such as product capacity and resource constraints; the basic data for candidate warehouses and distribution centers, such as capacities and local incentives; the transportation data, such as transport time and transport capacity; all cost parameters, such as manufacturing and handling costs; and several scenarios of forecasted product demands with known probabilities. The authors are going to determine the production plan of each plant; the number, location, and the capacity of warehouses and distribution centers to be set up; the transportation plan of each warehouse and distribution center; and all types of costs. The target is to integrate the multi-echelon decisions simultaneously to minimize the total cost and the transport time, and to elevate local incentives and the robustness of all considered design objectives to product demand uncertainties as much as possible.

In the market, the participants of a supply chain not only faces the uncertainties of product demands and raw material supplies but also faces the uncertainties of commodity prices and costs (Liu and Sahinidis, 1997). The authors will also

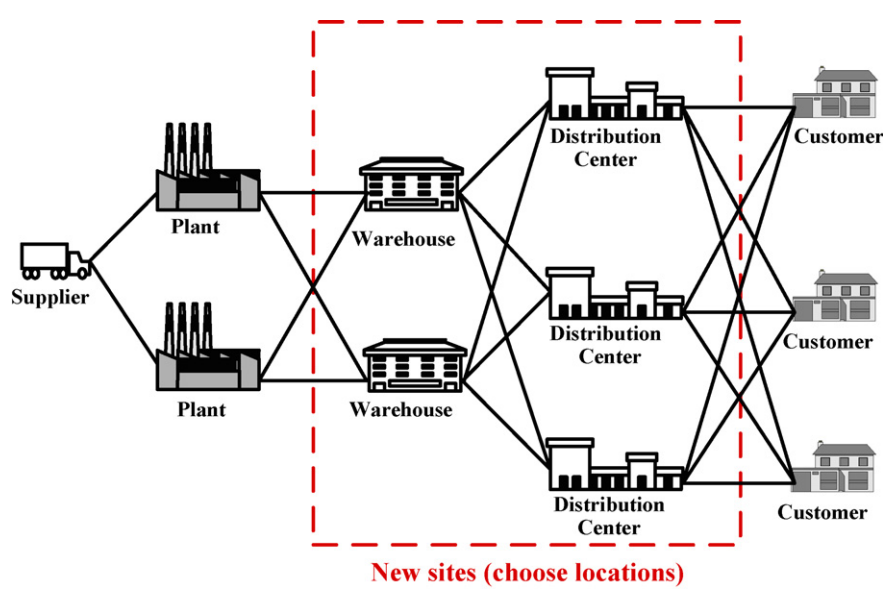

Fig. 1. The studied supply chain network.

address issues of demand uncertainty in the mid-term planning problem. The first concern in incorporating uncertainties into supply chain modeling and optimization is the determination of suitable representation of the uncertain parameters (Gupta and Maranas, 2003). Several distinct methods are frequently mentioned for representing uncertainty. For example, the fuzzy-based approach (Giannoccaro *et al.*, 2003; Liu and Sahinidis, 1997; Petrovic *et al.*, 1998, 1999), wherein the forecast parameters are considered as fuzzy numbers with accompanied membership functions; the scenario-based approach (Gupta and Maranas, 2003), in which several discrete scenarios with associated probability levels are used to describe expected occurrence of particular outcomes. To simplify the subsequent mathematical calculations, the discrete scenario-based approach for modeling uncertain mid-term product demands is adopted. Previous experience concerning uncertain product demands in short-term supply chain management problems (Chen and Lee, 2004a,b) gives strong support for applying the scenario-based approach. The mid-term planning problem considering multiple conflict objectives, including uncertain product demands, will be addressed in this article.

For applying the discrete cases representation for modeling uncertain demands, several possible outcomes for demand forecasting, $\mathrm{FCD}_s, s \in \mathcal{S}$, with known probabilities, PPD_s, should be determined at first with the restriction of $\sum_{\forall s \in \mathcal{S}} \mathrm{PPD}_s = 1$. Then, all variables will become scenario-dependent, and the expected value of any variable will be the weighted average of those scenario-dependent values. That is, for any variable v, one has to solve for several scenario-dependent values, $v_s, s \in \mathcal{S}$, and the expected value of v can be taken as $\sum_{\forall s \in \mathcal{S}} \mathrm{PPD}_s v_s$. In such a case the deterministic supply chain model can be easily extended to cope with the uncertain demand conditions (Chen and Lee, 2004a,b).

3. Supply chain modeling with demand uncertainty

A general supply chain that consists of three different levels of enterprises is considered. The first level enterprise is the retailer from which the products are sold to customers. The second level enterprise is the distribution center (DC) and/or warehouse using different types of transport capacity to deliver products from the plant side to the retailer side. The third level enterprise is the plant or the manufacturer that batch-manufactures one product over one period. In the following, the integrated multi-echelon supply chain model of Tsiakis *et al.* (2001) is extended for optimal decisions. The scenario-based representation for uncertain product demands is considered in the modeling. The indices, sets and parameters designed for modeling the supply chain network with product demand uncertainty are given in the nomenclature. Therein, parameters are divided into two categories: the cost parameters, including product cost, handling cost and transport cost; and other parameters describing the system information, such as handling and transport capacity or forecasting customer demand. Two kinds of variables are used: the binary variables that act as policy decisions to establish warehouses and distribution centers, along with using economies of scale for manufacturing or transportation, and the continuous variables that include manufacturing quantities, handling capacity and transport quantities.

3.1. Network structure constraints

All relevant network structural constraints between all plants, warehouses, DCs and customer zones can be summarized as follows.

$$X_{pw} \le Y_w, \qquad X_{wd} \le Y_w \tag{1}$$

$$X_{wd} \le Y_d, \qquad X_{dc} \le Y_d \tag{2}$$

$$\sum_{\forall w \in \mathcal{W}} X_{wd} = Y_d, \qquad \sum_{\forall d \in \mathcal{D}} X_{dc} = 1 \tag{3}$$

$$\forall\, p \in \mathcal{P}, w \in \mathcal{W}, d \in \mathcal{D}, c \in \mathcal{C}$$

Eq. (1) denotes that a link between a plant p and a warehouse w or between a warehouse w and a DC d can exist only if warehouse w exists. Similar constraints can apply to the link between DCs and customer zones, as shown in Eq. (2). To simplify the problem, it is assumed that a DC can only be served by a single warehouse, and a customer zone can only be served by a single DC, as shown in Eq. (3).

3.2. Transport constraints

The transportation constraints at considered periods, $t \in \mathcal{T}$, in different economic scales are given below.

$$\mathrm{TCL}_*^{k-1} Z_{*ts}^k < \mathrm{TTO}_{*ts}^k \le \mathrm{TCL}_*^k Z_{*ts}^k \tag{4}$$

$$\sum_{\forall k \in \mathcal{K}} Z_{*ts}^k \le 1 \tag{5}$$

$$\sum_{\forall i \in \mathcal{I}} Q_{*ts}^i = \sum_{\forall k \in \mathcal{K}} Q_{*ts}^k \tag{6}$$

$$Q_{*s}^{\min} X_* \le \sum_{\forall i \in \mathcal{I}} Q_{*ts}^i \le Q_{*s}^{\max} X_* \tag{7}$$

$$Q_{*ts}^i \ge 0 \tag{8}$$

where $* \in \{pw, wd, dc\}$,
$\forall\, p \in \mathcal{P}, w \in \mathcal{W}, d \in \mathcal{D}, c \in \mathcal{C}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}$

Eqs. (4) and (5) imply that several transport capacity levels with various unit transport costs can be used, as depicted in Fig. 2 for a three-level case, and at most one transport capacity can be chosen at each period. In Eq. (6), the transport quantities from plants to warehouses, from warehouses to DCs, or from DCs to customer zones at each period are respectively translated into total transport quantities. Eq. (7) says that the total transport quantities have lower/upper bonds, for all the existing links.

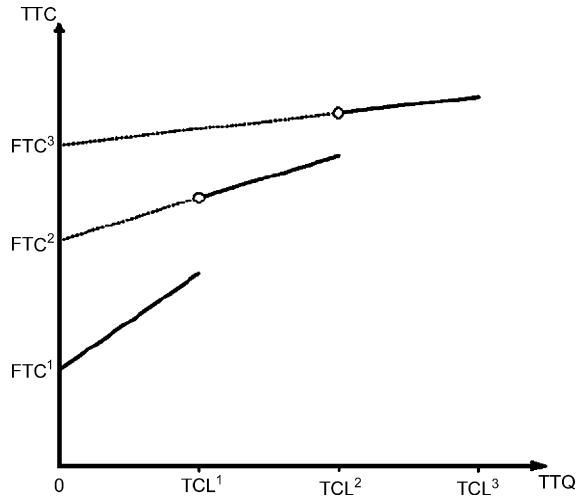

Fig. 2. Piecewise linear relation (solid lines) between transport cost, TTC, and shipment quantity, TTQ.

3.3. Material balances constraints

For mid-term design, it is assumed that the operation is in a constant state. Therefore, there is no stock accumulation or depletion. All material balance constraints can thus be summarized as follows.

$$\mathrm{PQ}^i_{pts} = \sum_{\forall w \in \mathcal{W}} Q^i_{pwts} \tag{9}$$

$$\sum_{\forall p \in \mathcal{P}} Q^i_{pwts} = \sum_{\forall d \in \mathcal{D}} Q^i_{wdts} \tag{10}$$

$$\sum_{\forall w \in \mathcal{W}} Q^i_{wdts} = \sum_{\forall c \in \mathcal{C}} Q^i_{dcts} \tag{11}$$

$$\sum_{\forall d \in \mathcal{D}} Q^i_{dcts} = \mathrm{FCD}^i_{cts} \tag{12}$$

$$\mathrm{PQ}^t_{ips} \geq 0, \quad \forall\, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S} \tag{13}$$

Eq. (9) says that the production of a product i by a plant p must be equal to the total flow of the product i to all warehouses. Eq. (10) states that the total flow of a product i from all plants to a warehouse w must be equal to the total flow of the product i from the warehouse w to all DCs.

Similarly, Eq. (11) means that the total flow of a product i from all warehouses to a DC d must equal to the total flow of a product i from the DC d to all customer zones. Eq. (10) denotes that the total flow of a product i from all DCs to a customer zone c must equal to the forecast customer demands.

3.4. Production resource constraints

An important issue in designing the network is the ability of the plants to satisfy the demands of customers. The production of each product at any plant thus has some limitations as follows.

$$\mathrm{PQ}^{\min}_{ips} \leq \mathrm{PQ}^i_{pts} \leq \mathrm{PQ}^{\max}_{ips} \tag{14}$$

$$\sum_{\forall i \in \mathcal{I}} \rho^i_{pnts} \mathrm{PQ}^i_{pts} \leq R_{pnts}, \quad \forall\, p \in \mathcal{P}, n \in \mathcal{N}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S} \tag{15}$$

Here, Eq. (14) states that each plant has its own maximum and minimum product capacities. Many plants may apply the same resources (equipment, utilities, manpower, etc.) to produce different products at different production stages. The limitations of resource utilization can be seen in Eq. (15).

3.5. Capacity constraints

All capacity constraints for warehouses and DCs are listed in the following.

$$\mathrm{SQ}^{\min}_w Y_w \leq \mathrm{SQ}_w \leq \mathrm{SQ}^{\max}_w Y_w \tag{16}$$

$$\mathrm{SQ}^{\min}_d Y_d \leq \mathrm{SQ}_d \leq \mathrm{SQ}^{\max}_d Y_d \tag{17}$$

$$\mathrm{SQ}_w \geq \sum_{\forall i \in \mathcal{I}} \sum_{\forall d \in \mathcal{D}} \alpha^i_w Q^i_{wdts} \tag{18}$$

$$\mathrm{SQ}_d \geq \sum_{\forall i \in \mathcal{I}} \sum_{\forall c \in \mathcal{C}} \beta^i_d Q^i_{dcts}, \quad \forall\, w \in \mathcal{W}, d \in \mathcal{D}, c \in \mathcal{C}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S} \tag{19}$$

Eq. (16) means that the capacity of a warehouse w has its lower bounds $\mathrm{SQ}^{\min}_w$ and upper bounds $\mathrm{SQ}^{\max}_w$, if the warehouse is established. Similar constraints apply to the capacities of the distribution centers, as shown in Eq. (17). It is assumed that the capacities of the warehouses and the distribution centers are related linearly to the materials that they handle, as expressed in Eqs. (18) and (19).

3.6. Costs

The establishment costs of warehouses and distribution centers, production costs, all material handling costs, and transportation costs are given below.

$$\mathrm{TEC} = \sum_{\forall w \in \mathcal{W}} \mathrm{UEC}_w Y_w + \sum_{\forall d \in \mathcal{D}} \mathrm{UEC}_d Y_d \tag{20}$$

$$\mathrm{TPC}_{ts} = \sum_{\forall i \in \mathcal{I}} \sum_{\forall p \in \mathcal{P}} \mathrm{UPC}^i_p \mathrm{PQ}^i_{pts} \tag{21}$$

$$\mathrm{THC}_{ts} = \sum_{\forall i \in \mathcal{I}} \sum_{\forall w \in \mathcal{W}} \mathrm{UHC}^i_w \left(\sum_{\forall p \in \mathcal{P}} Q^i_{pwts} \right) + \sum_{\forall i \in \mathcal{I}} \sum_{\forall d \in \mathcal{D}} \mathrm{UHC}^i_d \left(\sum_{\forall w \in \mathcal{W}} \mathrm{Q}^i_{wdts} \right) \tag{22}$$

$$\mathrm{TTC}_{*ts} = \sum_{\forall k \in \mathcal{K}} (\mathrm{FTC}^k_* Z^k_{*ts} + \mathrm{UTC}^k_* \mathrm{TTQ}^k_{*ts}) \tag{23}$$

$$\mathrm{TTC}_{ts} = \sum_{\forall p \in \mathcal{P}} \sum_{\forall w \in \mathcal{W}} \mathrm{TTC}^{t}_{pws} + \sum_{\forall w \in \mathcal{W}} \sum_{\forall d \in \mathcal{D}} \mathrm{TTC}^{t}_{wds} + \sum_{\forall d \in \mathcal{D}} \sum_{\forall c \in \mathcal{D}} \mathrm{TTC}^{t}_{dcs}, \tag{24}$$

where $* \in \{pw, wd, dc\}, \forall\, p \in \mathcal{P}, w \in \mathcal{W}, \in \mathcal{D}, c \in \mathcal{C}, i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}$

Eq. (20) gives the establishment costs of warehouses and distribution centers at candidate locations. In Eq. (21), the total production costs are the summations of the production quantity of product i multiply the unit production cost UPC_{ip}. Eq. (22) states that the total material handling costs can be expressed as a linear function of each product being handled at warehouses and distribution centers. Eq. (23) denotes transport costs for the plant and DC, respectively. Here, the transport cost is a composite of transport level-dependent fixed cost and a transport quantity-dependent carrying cost. This would cause a discontinuous piecewise linear transport cost, as illustrated in Fig. 2 with skipped subscripts. Finally, Eq. (24) is the total transportation cost (Gjerdrum *et al.*, 2001).

3.7. Multiple objectives for optimal planning

Several conflicting objectives such as minimizing the total cost, maximizing the robustness of selected objectives to demand uncertainties, maximizing the local incentives, and minimizing the total transport time can be considered simultaneously for the supply chain network design, as stated in the following.

3.7.1. Objective 1: minimizing the total cost

The total cost is a summation of the total establishment costs, the total production costs, the total handling costs, and the total transportation costs, such as

$$\min_{x \in \Omega'} \mathrm{TCO} = \mathrm{TEC} + \sum_{\forall t \in \mathcal{T}} \sum_{\forall s \in \mathcal{S}} \mathrm{PPD}_s(\mathrm{TPC}_{ts} + \mathrm{THC}_{ts} + \mathrm{TTC}_{ts})$$

Where Ω' is the feasible searching space which is a composite of all constraints, Eqs. (1)–(24), and x denotes the decision vector

$$x = \{Y_w, Y_d, X_*; Q^{i}_{*ts}\mathrm{TTQ}^{k}_{*ts}, Z^{k}_{*ts}; \mathrm{PQ}^{i}_{ips}, \mathrm{SQ}_w, \mathrm{SQ}_d; * \in \{pw, wd, dc\}; \forall\, i \in \mathcal{I}, p \in \mathcal{P}, w \in \mathcal{W}, d \in \mathcal{D}, c \in \mathcal{C}, k \in \mathcal{K}, t \in \mathcal{T}, s \in \mathcal{S}\}$$

3.7.2. Objective 2: maximizing the robustness to various scenarios

It has been mentioned that all operating variables are scenario-dependent when the explicit scenario-based approach is applied to model the uncertain product demands. However, the total cost realization might be unacceptably high for certain scenarios with especially low probabilities (Suh and Lee, 2001). It is thus significant to reduce the variability of objective values J_s for any realization of scenarios. An important issue in enforcing the robustness to uncertainties is the choice of a variability metric (Ahmed and Sahinidis, 1998). For the total cost to be minimized, the decision maker usually does not care if the objective value J_s is lower than the expected mean value J. Thus, the upper partial mean (UPM) is used as the measure of robustness where only costs above the expectation are penalized and are weighted by probabilities of related scenarios, PPD_s.

$$\mathrm{UPM} = \sum_{\forall s \in \mathcal{S}} \mathrm{PPD}_s \max\{0, J_s - J\}$$

As the upper partial mean decreases, the robustness of total cost will increase, thus one can define the robustness index (RI) as below, where the nonlinear objective function is further reformulated to equivalent linear form with additional constraint, Eq. (25).

$$\max_{x \in \Omega''} \mathrm{RI} = -\mathrm{UPM} = -\sum_{\forall s \in \mathcal{S}} \mathrm{PPD}_s \mathrm{UPM}_s$$

where

$$\mathrm{UPM}_s \geq 0, \qquad \mathrm{UPM}_s \geq J_s - J, \quad \forall\, s \in \mathcal{S} \tag{25}$$

Notably, there is an additional constraint in the new feasible searching space, $\Omega'' = \Omega' \cap \{\text{Eq. (25)}\}$.

3.7.3. Objective 3: maximizing the local incentives

When dealing with the location–allocation problem, there are many factors worth being considered, such as traffic facilities, labor quality, tax breaks, laws, etc. More traffic facilities such as highways, railroads, harbors, airports, etc., will decrease transport risks. Higher labor quality will have higher work efficiency. A meaningful local incentive can be defined by first identifying all important factors which cause great impact on the location–allocation problem, and second, giving weight to each factor according to its importance, and subjectively scoring the factors of each candidate location. The weighted average of these scores can be defined as the local incentive of each candidate location. Although the target is to maximize the average local incentive of all chosen locations, it will cause the nonlinear term as shown below.

$$\max_{x \in \Omega'} \mathrm{TLI} = \frac{\sum_{\forall w \in \mathcal{W}} \mathrm{LI}_w Y_w}{\sum_{\forall w \in \mathcal{W}} Y_w} + \frac{\sum_{\forall d \in \mathcal{D}} \mathrm{LI}_d Y_d}{\sum_{\forall d \in \mathcal{D}} Y_d}$$

Thus the following model will be applied to simplify the solution procedure.

$$\max_{x \in \Omega'} \mathrm{TLI} = \min\{\mathrm{LI}_w + U(1 - Y_w) \mid \forall\, w \in \mathcal{W}\} + \min\{\mathrm{LI}_d + U(1 - Y_d) \mid \forall\, d \in \mathcal{D}\}$$

The model raises the minimum local incentives of the warehouses and the distribution centers as high as possible, where U is a large positive value. For those candidate locations of warehouses or distribution centers that are not chosen, their local incentives will be ignored. The above nonlinear objective

can be re-formulated as the following linear form.

$$\max_{x \in \Omega'''} \text{TLI}$$

where $\Omega''' = \Omega' \cap \{\text{Eq. (26)}\}$.

$$\begin{aligned} &\text{TLI} \le \text{LI}_w + U(1 - Y_w) + \text{LD} && \forall w \in \mathcal{W} \\ &\text{LD} \le \text{LI}_d + U(1 - Y_d) && \forall d \in \mathcal{D} \end{aligned} \tag{26}$$

3.7.4. Objective 4: minimizing the total transport time

Decreasing the transport time cannot only reduce the inventory levels, but can also increase the customer service levels. So, reduction of transport time is an important topic when coping with allocation–location problems. One can set the total transport time as the objective to be minimized, as shown below.

$$\min_{x \in \Omega'} \text{OTT} = \sum_{\forall p \in \mathcal{P}} \sum_{\forall w \in \mathcal{W}} \text{TT}_{pw} X_{pw} + \sum_{\forall w \in \mathcal{W}} \sum_{\forall d \in \mathcal{D}} \text{TT}_{wd} X_{wd} + \sum_{\forall d \in \mathcal{D}} \sum_{\forall c \in \mathcal{C}} \text{TT}_{dc} X_{dc}$$

In summary, the mid-term supply chain-planning model can be constructed as a multi-objective mixed-integer linear program (MILP). Notably, all objectives expressed below are set to a maximum for simplifying the discussion. The feasible searching space Ω is a composite of all constraints, Eqs. (1)–(26)

$$\max_{x \in \Omega} (J_1(x), \ldots, J_{\text{M}}(x)) = (-\text{TCO}, \text{RI}, \text{TLI}, -\text{OTT}) \tag{27}$$

4. Supply chain optimization with uncertain demands

The conventional approaches for solving the multi-objective optimization problems are usually searching for efficient (Pareto-optimal) solutions that can best attain the prioritized objectives. Users, on the whole, have to provide a subjective account of each objective. The fuzzy optimization approach, on the other hand, can supply a single, yet unprejudiced final decision as stated in the following.

By considering the uncertain property of human thinking, it is quite intuitive to assume that the decision maker has a fuzzy goal, $\mathcal{J}_{\text{m}}$, to describe a maximizing objective J_{m} with an acceptable interval $[J_{\text{m}}^0, J_{\text{m}}^1]$. It would be quite satisfactory as the objective value is greater than J_{m}^1, and unacceptable as the profit is less than J_{m}^0, the minimum acceptable objective value such that the company would like to enter to negotiation for a fair deal in the multi-enterprise network. A strictly monotonic increasing membership function, $\mu_{\mathcal{J}_{\text{m}}}(J_{\text{m}}(x)) \in [0, 1]$, can be used to characterize such a transition from maximizing a numerical

Table 1
Scenarios of forecasting product demands and probabilities of an illustrative example

FCD_{cts}^{i}																							
i	c	t	s			i	c	t	s			i	c	t	s			i	c	t	s		
			1	2	3				1	2	3				1	2	3				1	2	3
1	1	1	10	18	28	2	3	1	105	145	105	3	5	1	32	72	82	4	7	1	40	80	88
1	1	2	8	18	30	2	3	2	82	143	125	3	5	2	22	73	92	4	7	2	35	82	92
1	1	3	5	19	46	2	3	3	55	143	145	3	5	3	12	72	98	4	7	3	24	82	98
1	2	1	0	0	0	2	4	1	0	0	0	3	6	1	0	0	0	4	8	1	35	49	55
1	2	2	0	0	0	2	4	2	0	0	0	3	6	2	0	0	0	4	8	2	31	50	75
1	2	3	0	0	0	2	4	3	0	0	0	3	6	3	0	0	0	4	8	3	25	49	81
1	3	1	0	0	0	2	5	1	0	0	0	3	7	1	0	0	0	5	1	1	0	0	0
1	3	2	0	0	0	2	5	2	0	0	0	3	7	2	0	0	0	5	1	2	0	0	0
1	3	3	0	0	0	2	5	3	0	0	0	3	7	3	0	0	0	5	1	3	0	0	0
1	4	1	9	13	15	2	6	1	0	0	0	3	8	1	0	0	0	5	2	1	51	70	76
1	4	2	5	15	17	2	6	2	0	0	0	3	8	2	0	0	0	5	2	2	41	71	86
1	4	3	4	17	20	2	6	3	0	0	0	3	8	3	0	0	0	5	2	3	23	72	96
1	5	1	0	0	0	2	7	1	9	14	17	4	1	1	226	276	283	5	3	1	0	0	0
1	5	2	0	0	0	2	7	2	8	14	27	4	1	2	180	277	293	5	3	2	0	0	0
1	5	3	0	0	0	2	7	3	8	16	37	4	1	3	150	277	316	5	3	3	0	0	0
1	6	1	45	50	60	2	8	1	0	0	0	4	2	1	103	173	203	5	4	1	0	0	0
1	6	2	40	51	70	2	8	2	0	0	0	4	2	2	80	174	223	5	4	2	0	0	0
1	6	3	32	52	80	2	8	3	0	0	0	4	2	3	50	174	233	5	4	3	0	0	0
1	7	1	0	0	0	3	1	1	0	0	0	4	3	1	80	236	266	5	5	1	0	0	0
1	7	2	0	0	0	3	1	2	0	0	0	4	3	2	54	231	282	5	5	2	0	0	0
1	7	3	0	0	0	3	1	3	0	0	0	4	3	3	38	231	298	5	5	3	0	0	0
1	8	1	0	0	0	3	2	1	55	105	155	4	4	1	0	0	0	5	6	1	0	0	0
1	8	2	0	0	0	3	2	2	40	101	125	4	4	2	0	0	0	5	6	2	0	0	0
1	8	3	0	0	0	3	2	3	20	100	215	4	4	3	0	0	0	5	6	3	0	0	0
2	1	1	0	0	0	3	3	1	0	0	50	4	5	1	0	0	0	5	7	1	0	0	0
2	1	2	0	0	0	3	3	2	0	0	71	4	5	2	0	0	0	5	7	2	0	0	0
2	1	3	0	0	0	3	3	3	0	0	83	4	5	3	0	0	0	5	7	3	0	0	0
2	2	1	199	399	499	3	4	1	126	141	150	4	6	1	0	0	0	5	8	1	0	0	0
2	2	2	150	398	519	3	4	2	100	142	161	4	6	2	0	0	0	5	8	2	0	0	0
2	2	3	120	397	549	3	4	3	76	144	182	4	6	3	0	0	0	5	8	3	0	0	0

objective value $J_{\mathrm{m}}(x)$ to degree-of-satisfaction for $\mathcal{J}_{\mathrm{m}}$ (Zadeh, 1965). It is noted that the design performance of the fuzzy method completely depends on the membership function. Different membership functions will have different outcomes. For practical consideration, a reasonable unprejudiced procedure is expected for providing reasonable limiting values for the objective. Without loss of generality, the authors first adopt the linear membership function since it has been proven in providing qualified solutions for many applications (Liu and Sahinidis, 1997).

$$\mu_{\mathcal{J}_{\mathrm{m}}}(x) = \begin{cases} 1; & \text{for} \quad J_{\mathrm{m}}(x) \geq J_{\mathrm{m}}^{1} \\ \dfrac{J_{\mathrm{m}}(x) - J_{\mathrm{m}}^{0}}{J_{\mathrm{m}}^{1} - J_{\mathrm{m}}^{0}} & \text{for} \quad J_{\mathrm{m}}^{0} \leq J_{\mathrm{m}}(x) \leq J_{\mathrm{m}}^{1} \\ 0; & \text{for} \quad J_{\mathrm{m}}^{0} \geq J_{\mathrm{m}}(x) \end{cases} \quad \forall m \in \mathcal{M} \tag{28}$$

Here, x denotes the argument vector. The effective range of the membership function $[J_{\mathrm{m}}^{0}, J_{\mathrm{m}}^{1}]$, can be determined dispassionately as follows. For those objectives, $J_{\mathrm{m}}, m \in \mathcal{M}$, one can use the most optimistic expectation as the upper limit, $\overline{J_{\mathrm{m}}^{1}} = J_{\mathrm{m}}(x_{m}^{*})$, where x_{m}^{*} is the optimal solution of the single objective maximizing problem, $\max_{x \in \Omega} J_{\mathrm{m}}(x)$, and choose the most pessimistic expectation, $\underline{J_{\mathrm{m}}^{0}}$, as the lower limiting value (Zimmermann, 1978; Sakawa, 1993), where

$$\underline{J_{\mathrm{m}}^{0}} = \min\{J_{\mathrm{m}}(x_{i}^{*}), i \in \mathcal{M}\}, \quad \forall m \in \mathcal{M} \tag{29}$$

One can, thus, impersonally determine the effective range of membership functions with the restriction of $\underline{J_{\mathrm{m}}^{0}} \leq J_{\mathrm{m}}^{0} < J_{\mathrm{m}}^{1} \leq \overline{J_{\mathrm{m}}^{1}}$. The original multi-objective optimization problem is now equivalent to looking for a suitable decision vector that can provide the maximal degree-of-satisfaction for the aggregated fuzzy objectives, $\mathcal{J}_{1}(x) \cap \ldots \cap \mathcal{J}_{\mathrm{m}}(x)$. When simultaneously considering all fuzzy objectives, the final fuzzy decision, $\mathcal{FD}(x)$, can be interpreted as the fuzzy intersection between all fuzzy objectives.

$$\mathcal{FD}(x) = \mathcal{J}_{1}(x) \cap \ldots \cap \mathcal{J}_{M}(x) \tag{30}$$

The final overall satisfactory level, $\mu_{\mathcal{FD}}(x)$, can be determined by aggregating the degree-of-satisfaction for all

Table 2
Fixed transport costs of an illustrative example

$\mathrm{FTC}_{pw}^{k}, \mathrm{FTC}_{wd}^{k}$																			
k	p	w	d	\$	k	p	w	d	\$	k	p	w	d	\$	k	p	w	d	\$
1	1	1		100	1		1	2	700	1		2	4	550	1		3	6	1250
2	1	1		200	2		1	2	1400	2		2	4	1100	2		3	6	2500
3	1	1		300	3		1	2	2100	3		2	4	1650	3		3	6	3750
4	1	1		400	4		1	2	2800	4		2	4	2200	4		3	6	5000
1	1	2		700	1		1	3	700	1		2	5	1300	1		3	7	800
2	1	2		1400	2		1	3	1400	2		2	5	2600	2		3	7	1600
3	1	2		2100	3		1	3	2100	3		2	5	3900	3		3	7	2400
4	1	2		2800	4		1	3	2800	4		2	5	5200	4		3	7	3200
1	1	3		700	1		1	4	200	1		2	6	800	1		4	1	250
2	1	3		1400	2		1	4	400	2		2	6	1600	2		4	1	500
3	1	3		2100	3		1	4	600	3		2	6	2400	3		4	1	750
4	1	3		2800	4		1	4	800	4		2	6	3200	4		4	1	1000
1	1	4		300	1		1	5	600	1		2	7	800	1		4	2	600
2	1	4		600	2		1	5	1200	2		2	7	1600	2		4	2	1200
3	1	4		900	3		1	5	1800	3		2	7	2400	3		4	2	1800
4	1	4		1200	4		1	5	2400	4		2	7	3200	4		4	2	2400
1	2	1		800	1		1	6	300	1		3	1	900	1		4	3	500
2	2	1		1600	2		1	6	600	2		3	1	1800	2		4	3	1000
3	2	1		2400	3		1	6	900	3		3	1	2700	3		4	3	1500
4	2	1		3200	4		1	6	1200	4		3	1	3600	4		4	3	2000
1	2	2		100	1		1	7	150	1		3	2	950	1		4	4	100
2	2	2		200	2		1	7	300	2		3	2	1900	2		4	4	200
3	2	2		300	3		1	7	450	3		3	2	2850	3		4	4	300
4	2	2		400	4		1	7	600	4		3	2	3800	4		4	4	400
1	2	3		900	1		2	1	700	1		3	3	100	1		4	5	800
2	2	3		1800	2		2	1	1400	2		3	3	200	2		4	5	1600
3	2	3		2700	3		2	1	2100	3		3	3	300	3		4	5	2400
4	2	3		3600	4		2	1	2800	4		3	3	400	4		4	5	3200
1	2	4		600	1		2	2	100	1		3	4	600	1		4	6	550
2	2	4		1200	2		2	2	200	2		3	4	1200	2		4	6	1100
3	2	4		1800	3		2	2	300	3		3	4	1800	3		4	6	1650
4	2	4		2400	4		2	2	400	4		3	4	2400	4		4	6	2200
1		1	1	100	1		2	3	750	1		3	5	1200	1		4	7	300
2		1	1	200	2		2	3	1500	2		3	5	2400	2		4	7	600
3		1	1	300	3		2	3	2250	3		3	5	3600	3		4	7	900
4		1	1	400	4		2	3	3000	4		3	5	4800	4		4	7	1200

objectives, $\mu_{\mathcal{J}_m}(x)$, via specific t-norm, T.

$$\mu_{\mathcal{FD}}(x) = T(\mu_{\mathcal{J}_1}(x), \ldots, \mu_{\mathcal{J}_m}(x)) \tag{31}$$

The best solution x^* with the maximal firing level, $\mu_{\mathcal{FD}}(x^*)$, should be selected.

$$\mu_{\mathcal{FD}}(x^*) = \max_{x \in \Omega} \mu_{\mathcal{FD}}(x) \tag{32}$$

Several t-norms can be chosen for T, wherein the three most popular selections are shown below (Klir and Yuan, 1995).

$$T(\mu_{\mathcal{J}_1}(x), \ldots, \mu_{\mathcal{J}_M}(x)) \tag{33}$$

$$= \begin{cases} \min(\mu_{\mathcal{J}_1}(x), \ldots, \mu_{J_M}(x)) & T = \text{minimum} \\ \mu_{\mathcal{J}_1}(x) \times \ldots \times \mu_{J_M}(x) & T = \text{product} \\ \dfrac{1}{M}(\mu_{\mathcal{J}_1}(x) + \ldots + \mu_{J_M}(x)) & T = \text{"average"} \end{cases} \tag{34}$$

Therein the minimum t-norm concerns the worst scenario only, but it may result in a non-compensatory solution (Li and Lee, 1993). On the other hand, both the product t-norm and the average operator can provide a compensatory result; however, they may cause an unbalanced solution between all fuzzy terms due to their inherent character. In order to avoid numerical difficulties caused by a highly nonlinear property of product t-norm, Chen and Lee (2004a,b) adopt the average operator, although it is not a t-norm. The successful application experience of combining the advantages of minimum and average operators for calculating the satisfactory level of fuzzy decisions in a short-term supply chain problem (Chen and Lee, 2004a,b) is thus applied to solving multi-objective mid-term planning problems.

Step 1. Determining the membership function for each fuzzy objective based on the expected upper/lower bounds for the objective value, as shown in Eq. (28), where $\underline{J_m^0} \le J_m^0 \le J_m^1 \le \overline{J_m^1}$, $\forall m \in \mathcal{M}$.

Step 2. (Phase I) Considering all fuzzy objectives and using the minimum operator, maximizing the degree of satisfaction for the worst situation.

$$\max_{x \in \Omega} \mu_{\mathcal{FD}} = \max_{x \in \Omega} \min(\mu_{\mathcal{J}_1}, \mu_{\mathcal{J}_2}, \ldots, \mu_{\mathcal{J}_M}) \equiv \mu_{\min} \tag{35}$$

Step 3. (Phase II) Applying the average operator, maximizing the overall satisfactory level with guaranteed minimal fulfillment for all fuzzy objectives.

$$\max_{x \in \Omega^+} \mu_{\mathcal{FD}}(x) = \max_{x \in \Omega^+} \frac{1}{M}(\mu_{\mathcal{J}_1}(x) + \ldots + \mu_{\mathcal{J}_M}(x)) \tag{36}$$

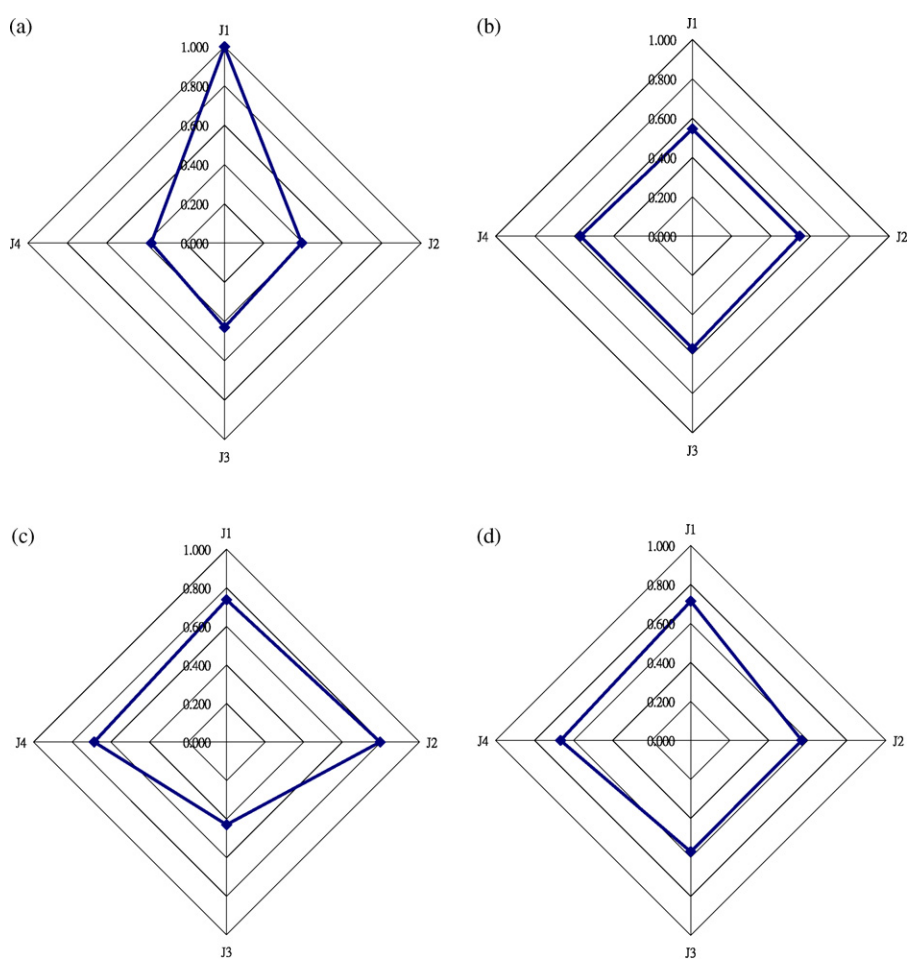

Fig. 3. Radar plots using single objective (a), or minimum (b) and average operator (c) (single-phase optimization) and proposed two-phase optimization method (d).

where

$$\Omega^{+} = \Omega \cap \{\mu_{\mathcal{J}_m} \geq \mu_{\min} | \forall m\} \tag{37}$$

It is noted that, in the proposed two-phase optimization approach, parameters in these objective functions are not usually independent and they cannot be completely separated. Though a global solution is not guaranteed, the proposed strategy can provide a definite procedure to reach a single compensatory solution among all participants of the supply chain.

5. Numerical example

Consider a typical supply chain consisting of 2 plants, 4 candidate warehouses, 7 candidate distribution centers, 8 customer zones, and 5 products. Two plants manufacture 5 different types of products and are located in two different locations. Each plant produces several products using a number of shared production resources. There are 4 candidate locations of warehouses and 7 candidate locations of distribution centers. Each candidate warehouse and distribution center has its own establishing cost, capacity, and local incentive. The whole planning horizon is 3 periods. The product demand scenarios are shown in Table 1 and the assigned probabilities are $PPD_{s=1} = 0.4$, $PPD_{s=2} = 0.3$ and $PPD_{s=3} = 0.3$, for the case study. Also, in order to simplify the problem, the fluctuating rate for cost parameters is neglected. Other indices and sets are $[\mathcal{K}] = 4$ and $[\mathcal{N}] = 6$.

Values of all fixed transport cost parameters are listed in Tables 2 and 3, unit transportation cost and transportation time are shown in Table 4, resource coefficients are listed in Table 5, and other parameters in Table 6.

The problem includes 11,478 equations, 7,622 continuous variables, and 3,415 binary variables. To solve this mixed-integer linear programming problem for the supply chain model, the Generalized Algebraic Modeling System (GAMS, Brooke *et al.*, 2003), a well-known high-level modeling system for mathematical programming problems, is used as the solution environment. The MILP solver used is CPLEX 7.5.

One can first apply the single objective programming method to minimize the total cost, the most common method in the traditional supply chain planning. Then, the result is projected, caused by single objective programming, to the membership functions, such as shown in Fig. 3 and Table 8. Obviously, the satisfaction levels are extremely unbalanced, since the objective function is only taking the total cost into consideration. So, one should consider all objectives simultaneously, and use

Table 3
Fixed transport costs of an illustrative example

FTC^{k}_{dc}																											
k	*d*	*c*	$	*k*	*d*	*c*	$	*k*	*d*	*c*	$	*k*	*d*	*c*	$	*k*	*d*	*c*	$	*k*	*d*	*c*	$	*k*	*d*	*c*	$
1	1	1	100	1	2	1	700	1	3	1	700	1	4	1	300	1	5	1	600	1	6	1	300	1	7	1	200
2	1	1	200	2	2	1	1400	2	3	1	1400	2	4	1	600	2	5	1	1200	2	6	1	600	2	7	1	400
3	1	1	300	3	2	1	2100	3	3	1	2100	3	4	1	900	3	5	1	1800	3	6	1	900	3	7	1	600
4	1	1	400	4	2	1	2800	4	3	1	2800	4	4	1	1200	4	5	1	2400	4	6	1	1200	4	7	1	800
1	1	2	700	1	2	2	100	1	3	2	800	1	4	2	600	1	5	2	1300	1	6	2	1000	1	7	2	900
2	1	2	1400	2	2	2	200	2	3	2	1600	2	4	2	1200	2	5	2	2600	2	6	2	2000	2	7	2	1800
3	1	2	2100	3	2	2	300	3	3	2	2400	3	4	2	1800	3	5	2	3900	3	6	2	3000	3	7	2	2700
4	1	2	2800	4	2	2	400	4	3	2	3200	4	4	2	2400	4	5	2	5200	4	6	2	4000	4	7	2	3600
1	1	3	500	1	2	3	700	1	3	3	200	1	4	3	600	1	5	3	1000	1	6	3	900	1	7	3	700
2	1	3	1000	2	2	3	1400	2	3	3	400	2	4	3	1200	2	5	3	2000	2	6	3	1800	2	7	3	1400
3	1	3	1500	3	2	3	2100	3	3	3	600	3	4	3	1800	3	5	3	3000	3	6	3	2700	3	7	3	2100
4	1	3	2000	4	2	3	2800	4	3	3	800	4	4	3	2400	4	5	3	4000	4	6	3	3600	4	7	3	2800
1	1	4	100	1	2	4	700	1	3	4	500	1	4	4	200	1	5	4	700	1	6	4	400	1	7	4	300
2	1	4	200	2	2	4	1400	2	3	4	1000	2	4	4	400	2	5	4	1400	2	6	4	800	2	7	4	600
3	1	4	300	3	2	4	2100	3	3	4	1500	3	4	4	600	3	5	4	2100	3	6	4	1200	3	7	4	900
4	1	4	400	4	2	4	2800	4	3	4	2000	4	4	4	800	4	5	4	2800	4	6	4	1600	4	7	4	1200
1	1	5	700	1	2	5	1200	1	3	5	1000	1	4	5	800	1	5	5	100	1	6	5	100	1	7	5	600
2	1	5	1400	2	2	5	2400	2	3	5	2000	2	4	5	1600	2	5	5	200	2	6	5	200	2	7	5	1200
3	1	5	2100	3	2	5	3600	3	3	5	3000	3	4	5	2400	3	5	5	300	3	6	5	300	3	7	5	1800
4	1	5	2800	4	2	5	4800	4	3	5	4000	4	4	5	3200	4	5	5	400	4	6	5	400	4	7	5	2400
1	1	6	300	1	2	6	800	1	3	6	1000	1	4	6	600	1	5	6	700	1	6	6	100	1	7	6	500
2	1	6	600	2	2	6	1600	2	3	6	2000	2	4	6	1200	2	5	6	1400	2	6	6	200	2	7	6	1000
3	1	6	900	3	2	6	2400	3	3	6	3000	3	4	6	1800	3	5	6	2100	3	6	6	300	3	7	6	1500
4	1	6	1200	4	2	6	3200	4	3	6	4000	4	4	6	2400	4	5	6	2800	4	6	6	400	4	7	6	2000
1	1	7	200	1	2	7	800	1	3	7	600	1	4	7	300	1	5	7	500	1	6	7	500	1	7	7	100
2	1	7	400	2	2	7	1600	2	3	7	1200	2	4	7	600	2	5	7	1000	2	6	7	1000	2	7	7	200
3	1	7	600	3	2	7	2400	3	3	7	1800	3	4	7	900	3	5	7	1500	3	6	7	1500	3	7	7	300
4	1	7	800	4	2	7	3200	4	3	7	2400	4	4	7	1200	4	5	7	2000	4	6	7	2000	4	7	7	400
1	1	8	1100	1	2	8	1100	1	3	8	400	1	4	8	1000	1	5	8	1200	1	6	8	1600	1	7	8	1200
2	1	8	2200	2	2	8	2200	2	3	8	800	2	4	8	2000	2	5	8	2400	2	6	8	3200	2	7	8	2400
3	1	8	3300	3	2	8	3300	3	3	8	1200	3	4	8	3000	3	5	8	3600	3	6	8	4800	3	7	8	3600
4	1	8	4400	4	2	8	4400	4	3	8	1600	4	4	8	4000	4	5	8	4800	4	6	8	6400	4	7	8	4800

Table 4
Unit transportation costs and transportation times of an illustrative example

$UTC^k_{pw}, UTC^k_{wd}, UTC^k_{dc}$(\$) and $TT_{pw}, TT_{wd}, TT_{dc}$(h)																	
p	*w*	*d*	\$	h	*w*	*d*	*c*	\$	h	*d*	*c*	\$	h	*d*	*c*	\$	h
1	1		5	40	3	1		45	30	2	1	32.5	50	5	1	30	40
1	2		35	70	3	2		47.5	20	2	2	42.5	60	5	2	65	80
1	3		35	30	3	3		5	60	2	3	30	70	5	3	50	70
1	4		15	40	3	4		30	30	2	4	35	80	5	4	35	60
2	1		40	10	3	5		60	10	2	5	60	40	5	5	42.5	10
2	2		5	60	3	6		62.5	30	2	6	40	50	5	6	35	30
2	3		45	20	3	7		40	30	2	7	40	50	5	7	25	40
2	4		30	30	4	1		12.5	40	2	8	55	10	5	8	60	20
	1	1	5	20	4	2		30	50	3	1	35	20	6	1	35	30
	1	2	35	10	4	3		25	50	3	2	40	40	6	2	50	30
	1	3	35	30	4	4		5	70	3	3	30	60	6	3	45	70
	1	4	10	20	4	5		40	20	3	4	25	40	6	4	20	30
	1	5	30	40	4	6		27.5	10	3	5	50	60	6	5	30	10
	1	6	15	50	4	7		15	30	3	6	50	60	6	6	30	60
	1	7	7.5	80		1	1	50	40	3	7	30	20	6	7	25	20
	2	1	35	40		1	2	35	30	3	8	52.5	60	6	8	52.5	20
	2	2	5	70		1	3	25	10	4	1	37.5	30	7	1	35	20
	2	3	37.5	40		1	4	37.5	30	4	2	30	20	7	2	45	50
	2	4	27.5	50		1	5	35	70	4	3	30	10	7	3	35	20
	2	5	65	30		1	6	30	50	4	4	37.5	50	7	4	25	30
	2	6	40	80		1	7	25	70	4	5	40	70	7	5	30	70
	2	7	40	20		1	8	50	20	4	6	30	40	7	6	25	10
										4	7	25	50	7	7	40	60
										4	8	50	60	7	8	60	40

multi-objective programming methods to elevate satisfaction level of individual objectives.

According to the problem description, mathematical formulation, and parameter design mentioned previously, one can solve the multi-objective mixed-integer linear program by using the fuzzy procedure discussed in Section 4.

Step 1. Select suitable ranges for defining membership functions. Relevant lower/upper limits, $\underline{J}^0_m$ and $\overline{J}^1_m$, and selected effective ranges, $[\underline{J}^0_m, \overline{J}^1_m]$, for membership functions are shown in Table 7. As mentioned previously, one can subjectively select values for $\underline{J}^0_m$ and $\overline{J}^1_m$ for each objective if meaningful lower/upper bounds can be expected. One can, thus, directly use $[\underline{J}^0_m, \overline{J}^1_m]$ as the effective range for defining fuzzy objectives such as local incentives. Using $\overline{J}^1_m$ as the upper bound is suggested, and the second lower value as the lower bound such as the total cost and transport time. Apply the second largest value as the upper bound and the second lower value as the lower bound for robustness measurement.

Step 2. (Phase I) To maximize the degree of satisfaction for the worst objective by using the minimum operator. The result is $\mu_{min} = 0.55$.

Table 5
Resource coefficients of an illustrative example

Resource coefficient ρ^i_{pnts}																			
i	*p*	*n*		*i*	*p*	*n*		*i*	*p*	*n*		*i*	*p*	*n*		*i*	*p*	*n*	
1	1	1	0.0	2	1	1	0.4	3	1	1	0.3	4	1	1	0.2	5	1	1	0.3
1	1	2	0.0	2	1	2	0.4	3	1	2	0.4	4	1	2	0.5	5	1	2	0.2
1	1	3	0.2	2	1	3	0.0	3	1	3	0.4	4	1	3	0.1	5	1	3	0.0
1	1	4	0.3	2	1	4	0.0	3	1	4	0.0	4	1	4	0.1	5	1	4	0.4
1	1	5	0.4	2	1	5	0.2	3	1	5	0.0	4	1	5	0.0	5	1	5	0.3
1	1	6	0.5	2	1	6	0.3	3	1	6	0.2	4	1	6	0.0	5	1	6	0.0
1	2	1	0.6	2	2	1	0.4	3	2	1	0.3	4	2	1	0.2	5	2	1	0.0
1	2	2	0.0	2	2	2	0.1	3	2	2	0.1	4	2	2	0.7	5	2	2	0.2
1	2	3	0.0	2	2	3	0.7	3	2	3	0.1	4	2	3	0.0	5	2	3	0.1
1	2	4	0.2	2	2	4	0.0	3	2	4	0.3	4	2	4	0.0	5	2	4	0.3
1	2	5	0.1	2	2	5	0.0	3	2	5	0.0	4	2	5	0.0	5	2	5	0.1
1	2	6	0.4	2	2	6	0.2	3	2	6	0.0	4	2	6	0.2	5	2	6	0.4

Table 6
Other parameters of an illustrative example

Unit handling cost, UHC_*^i								Max prod. PQ_{ips}^{max}			Unit estab. cost, UEC_*			Loc. Inc. LI_*		
I	w	d	\$	i	w	d	\$	i	p		w	d	\$	w	d	
1	1		15	2		1	3	1	1	150	1		190000	1	1	30
1	2		15	2		2	3	2	1	600	2		80000	2		60
1	3		2	2		3	15	3	1	400	3		100000	3		70
1	4		2	2		4	15	4	1	1000	4		120000	4		40
2	1		15	2		5	8	5	1	100		1	80000		1	70
2	2		15	2		6	8	1	2	200		2	70000		2	30
2	3		2	2		7	8	2	2	700		3	80000		3	90
2	4		2	3		1	3	3	2	400		4	110000		4	50
3	1		15	3		2	3	4	2	700		5	110000		5	80
3	2		15	3		3	15	5	2	150		6	70000		6	40
3	3		2	3		4	15					7	110000		7	60
3	4		2	3		5	8	Max cap. SQ_*^{max}			Unit prod. cost, UPC_p^i			Resource R_{pnts}		
4	1		15	3		6	8	w	d							
4	2		15	3		7	8	1		2700	i	p	\$	p	n	
4	3		2	4		1	3	2		1500	1	1	140	1	1	600
4	4		2	4		2	3	3		1800	2	1	140	11	2	800
5	1		15	4		3	15	4		2000	3	1	100	11	3	300
5	2		15	4		4	15		1	1200	4	1	60	11	4	150
5	3		5	4		5	8		2	1000	5	1	40	11	5	200
5	4		2	4		6	8		3	1000	1	2	130	11	6	340
1		1	3	4		7	8		4	1500	2	2	130	22	1	310
1		2	3	5		1	3		5	1400	3	2	100	22	2	300
1		3	15	5		2	3		6	1000	4	2	60	22	3	200
1		4	15	5		3	15		7	1400	5	2	40	22	4	150
1		5	8	5		4	15	k		TCL_*^k	Q_{*s}^{max}			22	5	350
1		6	8	5		5	8	1		500	2000			22	6	200
1		7	8	5		6	8	2		1000	Q_{*s}^{min}			α_w^i		
SQ_w^{min}				SQ_d^{min}				3		1500	1			1		
1				1				4		2000	Note : $* \in \{pw, wd, dc\}$			β_d^1		
														1		

Step 3. (Phase II) Re-optimize the problem with new constraints of guaranteed minimum satisfaction for all fuzzy objectives. The results will be shown and discussed in the following.

The radar plots for the total cost, robustness measure, local incentives, and transport time are shown in Fig. 3, where the numerical value of each objective indicates its satisfaction level, and the resulting objective and membership function values are listed in Table 8.

As shown in Fig. 3(a) and Table 8, when one make a decision by considering a single objective such as minimizing the total cost (J_1), the satisfaction levels for other conflict objectives would be quite low (0.39, 0.43, 0.39) though the satisfaction level concerning total cost can be as high as 1. From the results obtained by directly selecting ‘‘minimum’’ as the t-norm (see Fig. 3(b)), one can get a more balanced level of satisfaction among all objectives where the degrees of satisfaction are all around 0.55. It means that the overall satisfaction levels are not concerning, though one can maximize the minimum satisfaction level of the worst objective. By using ‘‘average operator’’ to guarantee a unique solution, however, the results are unbalanced with a lower degree of satisfaction for local incentives (0.43, as shown in Fig. 3(c). On the other hand, the high robustness measure is given very high emphasis. Obviously this is not desirable for obtaining a compromise solution. Overcoming the drawbacks of the single-phase method, the proposed two-phase method can incorporate advantages of these two t-norms. The minimum operator is used in phase I to find the maximal satisfaction for the worst situation (0.55), and the average operator is applied in phase II to maximize the overall satisfaction with guaranteed minimal

Table 7
Parameters for defining membership functions for objectives

m	J_m	J_m^0	J_m^0	J_m^1	$\overline{J_m^1}$
1	–TCO	−1,913, 513	−1,584, 529	−1,219,554	−1,219,554
2	RI	−189, 103	−163, 203	−124, 319	0
3	TLI	60	60	130	130
4	–OTT	−1270	−800	−290	−290

Table 8
Resulting objective and membership function values

Operator		TCO	RI	TLI	OTT
Single objective (J_1)	J_m	1, 219, 554	−151, 557	90	610
	$\mu_{\mathcal{J}m}$	1.00	0.39	0.43	0.37
Minimum	J_m	1, 330, 728	−144, 736	100	510
	$\mu_{\mathcal{J}m}$	0.55	0.55	0.57	0.57
Average	J_m	1, 315, 214	−133, 465	90	450
	$\mu_{\mathcal{J}m}$	0.74	0.80	0.43	0.69
Two-phase	J_m	1, 323, 772	−143,685	100	460
	$\mu_{\mathcal{J}m}$	0.71	0.57	0.57	0.67

fulfillment for all fuzzy objectives. The average satisfaction level is increased from 0.56 (by applying the minimum operator) to 0.63, as shown in Fig. 3(d).

The optimal network structures are shown in Figs. 4 and 5, where triangle means plant, square means warehouse, hexagon means distribution center, and circle means customer zone. The values in the network structures are the total transport quantities through the whole planning horizon. Fig. 4(a) is the result of considering total cost, where other objectives are not taken into account. The transport time is quite large in this case, as shown

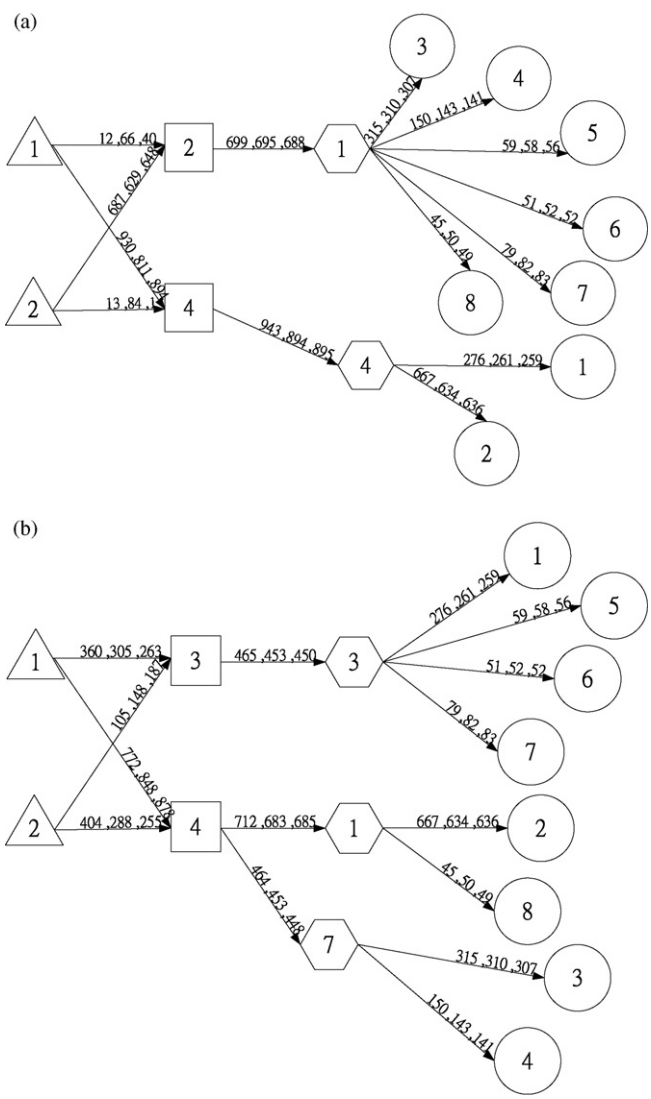

Fig. 4. Optimal network structure using single objective (a) and minimum operator (b).

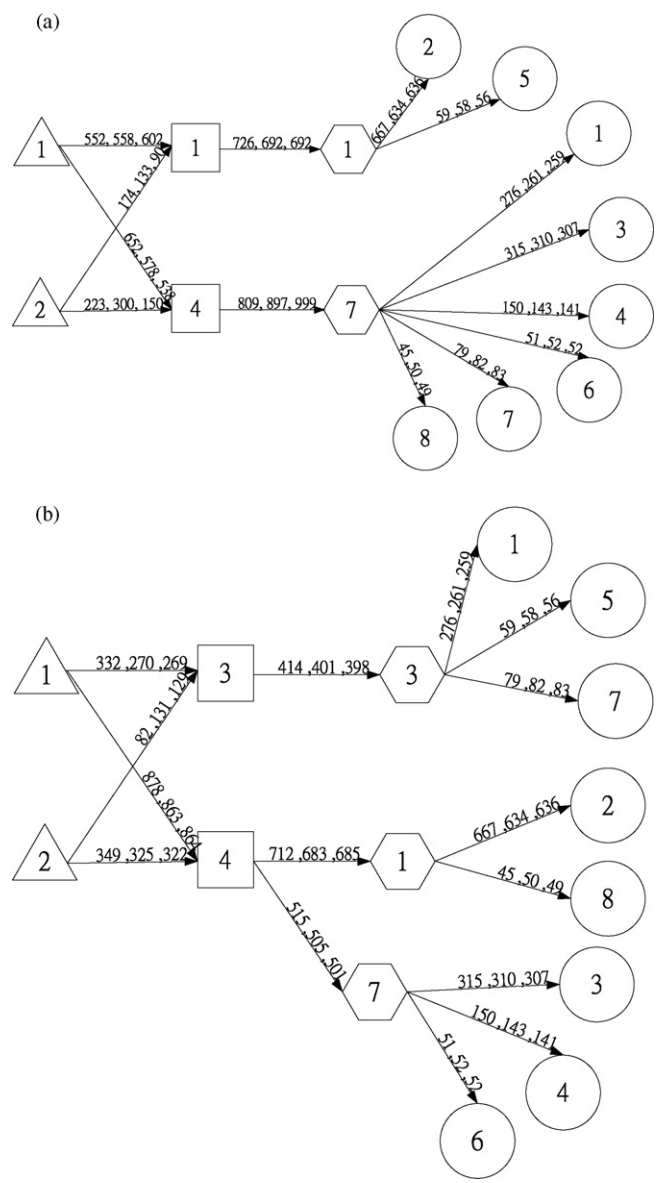

Fig. 5. Optimal network structure using average operator (a) and two-phase optimization method (b).

in Table 8. The result of ''average operator'', Fig. 5(a), also does not care about the worst objective, where the local incentive is quite low. The results of ''minimum operator'' and the two-phase method are quite similar, where customer zone 6 is serviced by distribution center 3 and 7, respectively. The supply chain network designed by the two-phase method can provide superior overall performance.

6. Conclusion

This paper investigates the simultaneous optimization of multiple conflict objectives problem in a typical supply chain network with market demand uncertainties. The demand uncertainty is modeled as discrete scenarios with given probabilities for different expected outcomes. In addition to the total cost, the project considers the influence of local incentives and transport time to location decision. The problem is formulated as a mixed-integer linear programming (MILP) model to achieve minimum total cost, maximum robustness to demand uncertainties, maximum local incentives, and minimum total transport time. To find the degree of satisfaction of the multiple objectives, the linear increasing membership function is used; the final decision is acquired by fuzzy aggregation of the fuzzy goals, and the best compromised solution can be derived by maximizing the overall degree of satisfaction for the decision. The implementation of the proposed fuzzy decision-making method, as one can see in the case study, demonstrates that the method can provide a compensatory solution for the multiple conflict objectives problem in a supply chain network with demand uncertainties.

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供應鏈網路中最佳倉儲與配銷中心選址多目標模糊決策

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摘　要

本研究旨在探討供應鏈網路中的最佳倉儲與配銷中心選址策略問題。考慮一個多產品、多階層、多規劃週期的生產配送網路，其包含了固定地點的數個生產多產品之工廠與數個顧客區，和數個未決定建立地點之候選倉儲與候選配銷中心。需決定之決策變數有各工廠的生產計畫及所需原物料，倉儲和配銷中心的數量、處理量與建立地點，供應鏈網路架構之配送計畫。此外，在建構本模式時，使用現實工業界中常見的經濟批量運送方式。並且使用數個不同的需求量方案來處理顧客對產品需求的不確定性。在傳統供應鏈規劃中處理這種類型的問題時，常以系統總成本或收益作為目標函數來進行單目標規劃，而本研究同時考慮整體成本最小化、韌性指標最大化、地點選擇分數最大化與運輸時間最小化等四個績效指標。為了能同時滿足四個目標，本模式將形成一個多目標混合整數非線性規劃問題。再利用模糊多目標規劃法簡單易懂與求解方便的優點來對所建構模式求解，並使用兩階段求解方法，在階段一找出各目標可接受的最低滿意度，階段二中則基於這個各目標可接受的最低滿意度上，尋求各目標隸屬度函數平均的最大。藉由兩階段法，使得多目標規劃中的各目標函數能在彼此互斥的關係下，相互妥協。最後，我們提出一個模擬範例，應用多目標規劃法來求解，並發現使用兩階段法，可使多目標問題有相當滿意的解。