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A modified model for the estimation of fatigue life derived from random vibration theory[1]

H.Y. Liou, W.F. Wu*, C.S. Shin

Department of Mechanical Engineering, National Taiwan University, 10617 Taipei, Taiwan, Republic of China

Abstract

When a component is subjected to variable-amplitude loading, if the fundamental stress–life cycle relationship and an accumulation rule are given, then the fatigue damage or fatigue life of the component can be calculated and/or estimated. In the present paper, random vibration theory is incorporated into the analysis of the above problem. Several formulas are thus derived. Experimental work is then carried out to verify the derived formulas. Comparison is made among the results calculated based on different formulas, different accumulation rules and different random loading. It is concluded that the derived formulas do provide us with quick prediction of the fatigue damage or fatigue life when a component is subjected to variable-amplitude loading that has a certain random nature. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Fatigue damage; Fatigue life; Damage accumulation rule; Random vibration; Statistical extremes

1. Introduction

Structures and mechanical components are frequently subjected to oscillating loads of random nature. Random vibration theory has been introduced for more then three decades to deal with all kinds of vibration behavior when random loading is concerned. Since fatigue is one of the major causes when component failure is considered, fatigue life prediction has become a major subject in almost any random vibration book [1–4]. Aside from the fatigue crack propagation problem, when the fatigue damage accumulation problem is concerned, random vibration theory is applied in conjunction with a damage rule to derive the expected fatigue damage and/or fatigue life of a structure or component when material properties and the environment loading are known in advance. The most frequently used fatigue damage rule in published random vibration books is the famous Miner's rule, and many ready-to-use formulas have been derived in those books. These formulas have been used in many practical applications [5–8].

Miner's linear damage rule is easy to apply. However, it does not take into account the sequence or memory effect owing to overloads or overstrains occurred in a variable-amplitude loading history. The result is a non-conservative, longer life prediction than the actual fatigue life of a component. In order to improve the fatigue life prediction, Morrow has proposed a nonlinear plastic work interaction damage rule [9]. It modifies Miner's linear cycle ratio cumulative damage rule by multiplying a given stress level's cycle ratio damage term by a nonlinear factor to incorporate the interactive effect. According to Morrow, this nonlinear damage rule along with overload parameter values gave accurate fatigue life prediction in the long life region and slightly conservative life prediction in the short life region for investigative experiments performed using several selected variable-amplitude applied stress histories on smooth cylindrical laboratory specimens of 1020 steel.

Lambert has extended Morrow's plastic work interaction damage rule to the case of a continuously distributed random stress history as usually considered in a random vibration book [10]. Using an assumed maximum peak in a random stress history, he showed that the predicted fatigue life was reduced by 10–15% from that evaluated based on Miner's rule. He also concluded that most of the fatigue damage was caused by those stress peaks at values between two and four times the root-mean-square stress level. These conclusions, however, are short of experimental verification.

The major purpose of the present paper is to derive ready-to-use formulas for the prediction of fatigue damage and fatigue life when a component is subjected to statistically defined random stresses. Morrow's plastic work interaction damage rule is considered, in particular, in the derivation in which the maximum stress peak should be taken into

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* Corresponding author.

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consideration. Unlike Lambert's arbitrarily selected value, Gumbel's asympototic theory of statistical extremes [11] is used to obtain the maximum stress peak without any additional assumption. To verify the applicability of the derived formulas, a series of fatigue tests have been performed and some of the analytical as well as experimental results are presented in the present paper.

2. Prediction of fatigue damage and fatigue life

In a variable-amplitude stress condition, according to Morrow's plastic work interaction rule, the fatigue damage done by the stress of amplitude σ_i, can be written as

$$D_i = \frac{n_i}{N_{fi}}\left(\frac{\sigma_i}{\sigma_m}\right)^d \tag{1}$$

in which σ_m is the maximum stress amplitude in the stress history considered, n_i is the number of stress peak at level σ_i, N_{fi} is the number of stress peaks to cause failure if constant amplitude σ_i is considered, and d is Morrow's plastic work interaction exponent. The exponent can be interpreted as the material's sensitivity to the variable-amplitude stress history. From Eq. (1), if all different cyclic stresses are considered, the total fatigue damage becomes

$$D = \sum_i D_i = \sum_i \frac{n_i}{N_{fi}}\left(\frac{\sigma_i}{\sigma_m}\right)^d \tag{2}$$

and the fatigue failure occurs when the damage index D reaches 1. It is interesting to note that Morrow's plastic work interaction damage rule has exactly the same form as the Corten–Dolan cumulative damage theory [12] although they were derived based on different arguments and different assumptions. The same damage rule has also been derived by Martin using a third different approach [12]. This nonlinear damage rule reduces to the famous Miner's rule if the exponent d is chosen to be zero, as can be seen from Eqs. (1) and (2).

In random vibration books, the stress history applied to a structure or mechanical component is usually considered a random process which has continuously distributed stress amplitudes abiding by the unit-measure probability law. Under this circumstance, many parameters in Eq. (2) become functions of stress amplitude, and the summation sign should be replaced by the integration sign. Owing to the probability measure, the damage index should also be interpreted in an expected sense. Following a procedure employed by Crandall and Mark [1], the expected fatigue damage of a component when subjected to well-defined random loading for a duration of T can be derived as

$$E[D(T)] = \nu_o^+ T \int_0^\infty \frac{p(\sigma)}{N_f(\sigma)}\left(\frac{\sigma}{\sigma_m}\right)^d \mathrm{d}\sigma \tag{3}$$

in which $E[\cdot]$ is the expected value of the bracketed quantity, ν_0^+ is the expected number of the zero up-crossing rate which can be considered as the expected number of stress cycle per unit time for a narrow-band random process, $p(\sigma)$ is the probability density function of stress amplitudes, and $N_f(\sigma)$ is now distributed as a function of stress amplitude. The stress amplitude is usually referred to as stress peak in random vibration theory.

According to random vibration theory, if the stress history is a zero-mean, narrow-band Gaussian random process, then the probability density function of the stress amplitudes has the following Rayleigh form [1–4]

$$P(\sigma) = \frac{\sigma}{\sigma_{rms}^2}\exp\left(-\frac{\sigma^2}{2\sigma_{rms}^2}\right), \quad \sigma > 0 \tag{4}$$

in which σ_{rms} is the standard deviation of the random stress process (but not stress amplitudes). Traditional random vibration theory considers that, under constant-amplitude loading, the stress-life cycle relationship follows the following Basquin's equation

$$N_f \sigma^p = C \tag{5}$$

in which p and C are fitting constants. Substituting Eqs. (4) and (5) into Eq. (3) one obtains

$$E[D(T)] = \frac{\nu_0^+ T}{C\sigma_{rms}^2}\int_0^\infty \frac{\sigma^{p+d+l}}{\sigma_m^d}\exp\left(-\frac{\sigma^2}{2\sigma_{rms}^2}\right)\mathrm{d}\sigma \tag{6}$$

The above expression for the expected fatigue damage of a component when subjected to random loading is derived based on Morrow's nonlinear damage rule as well as random vibration theory. The integration in Eq. (6) is performed from zero to infinite, which includes all possible values of the stress amplitude. However, when performing experimental fatigue tests in the laboratory, a clipping stress is usually selected for the generated random loading history to protect the test machine. Under this circumstance, the clipping stress becomes the maximum stress amplitude in Eq. (6) and, when carrying out the numerical integration, all stress amplitudes greater than the maximum stress should be treated as the maximum stress. The result can also be found analytically as

$$\begin{aligned} E[D(T)] = {} & \frac{\nu_0^+ T}{C\sigma_m^d}(\sqrt{2}\sigma_{rms})^{p+d}\gamma\left(\frac{p+d}{2}+1, \rho\right) \\ & + \frac{\nu_0^+}{C}(\sigma_m)^{p-d}(\sqrt{2}\sigma_{rms})^d \\ & \times\left[\Gamma\left(\frac{d}{2}+1\right) - \gamma\left(\frac{d}{2}+1, \rho\right)\right] \end{aligned} \tag{7}$$

in which $\rho = \sigma_m^2/2\sigma_{rms}^2$, and the special functions Γ and γ represent a gamma and an incomplete gamma function defined, respectively, by

$$\Gamma(x) = \int_0^\infty \xi^{x-1} e^{-\xi} d\xi \tag{8}$$

and

$$\gamma(x, \alpha) = \int_0^\alpha \xi^{x-1} e^{-\xi} d\xi \tag{9}$$

It is interesting to note that when Morrow's plastic interaction exponent is selected to be zero, the expression of Eq. (7) becomes

$$E[D(T)] = \frac{\nu_0^+ T}{C}(\sqrt{2}\sigma_{\mathrm{rms}})^p \gamma\left(\frac{p}{2} + 1, \rho\right) + \frac{\nu_0^+ T}{C}\sigma_{\mathrm{m}}^p \exp(-\rho) \tag{10}$$

which is also the result when Miner's rule is considered as mentioned previously. If no clipping stress is set and the theoretical maximum stress can be extended to infinity as that considered in random vibration theory, then the above expression can further be reduced to be

$$E[D(T)] = \frac{\nu_0^+ T}{C}(\sqrt{2}\sigma_{\mathrm{rms}})^p \Gamma\left(\frac{p}{2} + 1\right) \tag{11}$$

The above equation is derived based on Miner's rule and can be found in almost any random vibration book [1–4].

In using Eq. (7), Eq. (10) or Eq. (11) in prediction the fatigue damage of a component when subjected to random loading of narrow-band Gaussian type for a duration of T, the fitting constants C and p should be obtained from the material's S–N curve, and parameters ν_0^+, σ_{rms} and ρ can be found from the given random loading. The selection of d in Eq. (7) can be referred to Morrow [9]. Very often, one may expect to predict the fatigue failure time or fatigue life rather than the fatigue damage. Under this circumstance, one can set the expected damage at the left-hand side of the above equations to be 1 and then solve the equations for the unknown T. The above prediction is the major concern in random vibration books. In real practices, however, some modification or simplification may be made. For example, cyclic counting rather than continuous time is usually adopted in the fatigue analysis. Therefore, the term $\nu_0^+ T$ in Eqs. (7), (10) and (11) may be replaced by N indicating the number of stress cycle. Further, cyclic stress amplitudes rather than the continuous analog signal of a random stress process is considered in the fatigue analysis. For the case of probability density function shown in Eq. (4), the standard deviation of the stress amplitude can be found to be

$$\sigma_\Sigma = \sigma_{\mathrm{rms}}\sqrt{2 - \frac{\pi}{2}} \tag{12}$$

If the above concept is taken into consideration, then Eqs. (7), (10) and (11) can be modified. For example, Eq. (7) is modified to be

$$\begin{aligned} E[D(N)] = {} & \frac{N}{C\sigma_{\mathrm{m}}^d}\left(\frac{\sigma_\Sigma}{1 - \pi/4}\right)^{p+d} \gamma\left(\frac{p+d}{2} + 1, \rho\right) \\ & + \frac{N}{C}(\sigma_{\mathrm{m}})^{p-d}\left(\frac{\sigma_\Sigma}{1 - \pi/4}\right)^d \\ & \times\left[\Gamma\left(\frac{d}{2} + 1\right) - \gamma\left(\frac{d}{2} + 1, \rho\right)\right] \end{aligned} \tag{13}$$

where ρ can now be calculated from $\rho = (1 - \pi/4)\sigma_{\mathrm{m}}^2/\sigma_\Sigma^2$. To use the above equation, the maximum stress amplitude σ_{m} had better be known if it is not given. The resolution of this problem will be discussed further in the next paragraph.

As mentioned previously, when fatigue tests are performed in a laboratory, the maximum stress amplitude needed in applying Morrow's damage rule can usually be selected as the clipping stress. However, in real situations, one may know in advance the statistics of the random loading, but does not know exactly what value of the maximum amplitude will be applied to a structure or component. This may prevent us from applying Morrow's nonlinear rule to the prediction of fatigue damage and fatigue life such as that shown in Eq. (13). To proceed with the prediction, an empirically assumed maximum stress amplitude has been used by Lambert [10]. In the present paper, however, a theoretically meaningful method will be proposed to overcome this shortcoming. It is based on Gumbel's asymptotic theory of statistical extremes [11]. According to Gumbel, the probability distribution of the largest value among a sequence of N identically distributed random values can be determined by the theory of standard order statistics. In our current problem, let $\sigma_1, \sigma_2, \ldots$ and σ_N be the stress amplitudes which are identically distributed such as those shown in Eq. (4), the cumulative distribution of the maximum amplitude is then given by

$$F_{\Sigma_{\mathrm{m}}}(\sigma_{\mathrm{m}}) = Pr[\max(\sigma_i,\ i = 1, 2 \ldots N) \le \sigma_{\mathrm{m}}] = [F(\sigma)]^N \tag{14}$$

in which Pr is the abbreviation for 'probability' and F

Table 1
Statistical properties of random loading (in MPa)

Random loading	*A*	*B*	*C*	*D*	*E*
Standard deviation of stress process (σ_{rms})	251.42	223.08	193.32	162.91	142.53
Expected value of stress amplitude (μ_Σ)	314.93	279.59	242.17	203.98	178.51
Standard deviation of stress amplitude (σ_Σ)	164.62	146.14	126.58	106.62	93.34

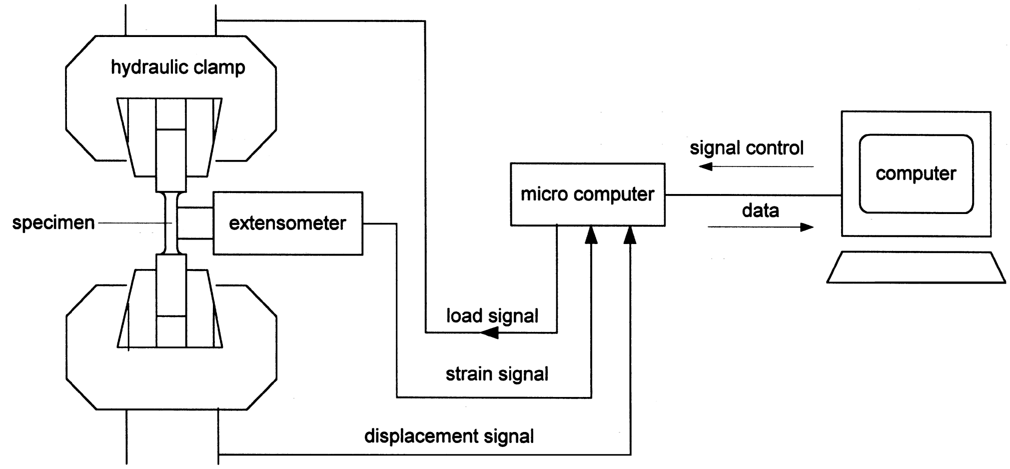

Fig. 1. Experimental setup.

indicates the cumulative distribution of a density function such as Eq. (4). According to the asympototic theory of statistical extremes, if N is large enough, then Eq. (14) approaches to the following Type I asymptotic distribution of the largest value [11]

$$F_{\Sigma_m}(\sigma_m) = \exp\{-\exp[-\alpha_N(\sigma_m - \mu_N)]\} \quad (15)$$

in which μ_N is named the characteristic parameter and α_N is an inverse measure of dispersion. The expected value of the maximum amplitude can therefore be derived based on Eq. (15) as well as the cumulative distribution function $F(\alpha)$. In particular, if Eq. (4) is considered, the expected maximum amplitude is found to be [13]

$$E[\Sigma_m] = \sigma_{rms}\left(\sqrt{2\ln N} + \frac{0.577}{\sqrt{2\ln N}}\right) \quad (16)$$

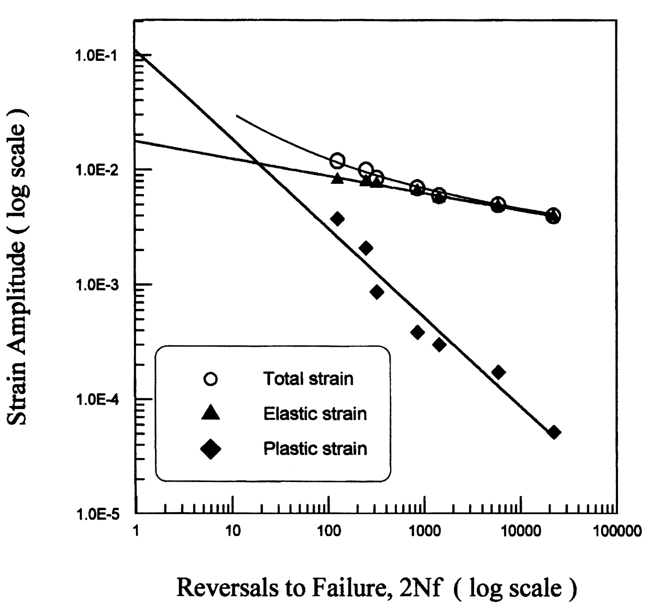

Fig. 2. Strain–life curve.

where 0.577 is a truncated Euler's constant. Eq. (16) is interpreted in a probabilistic sense as the theoretically derived 'expected' maximum stress amplitude. In real practices, it can be used to replace the arbitrarily assumed maximum stress amplitude adopted by Lambert when applying Morrow's damage rule. Under such circumstances and in consideration of Eq. (12), Eq. (16) can be re-written as

$$\sigma_m = \sigma_\Sigma\left(\frac{\sqrt{2\ln N} + 0.577\sqrt{2\ln N}}{\sqrt{2 - \pi 2}}\right) \quad (17)$$

Eq. (17) in conjunction with Eq. (13) can be used to predict the fatigue damage of a component when subjected to random loading having or resembling to narrow-band Gaussian type. They can also be used to predict the fatigue life of a component if so desired by setting the left-hand side of Eq. (13) to be 1, and then solving the equations for N. Unlike those documented in random vibration books, Morrow's nonlinear fatigue damage rule is considered in the prediction.

3. Experiment and results

In association with other fatigue-related research projects, a large amount of fatigue tests have been performed. Some of the results can be used to verify the accuracy of those formulas derived in the preceding section and will be introduced, in particular, in the present section. The experimental setup is schematically shown in Fig. 1. First of all, through standard test procedure, the static mechanical properties of the material were found to be: Young's modulus, 73.5 GPa; yield strength, 503.0 MPa; and ultimate strength, 647.2 MPa. All of them were measured as averages. Fatigue tests were then performed. The specimens were made of 7075-T651 aluminum alloy and machined according to ASTM E606 standard. They were cylindrical and polished smoothly in the gage section.

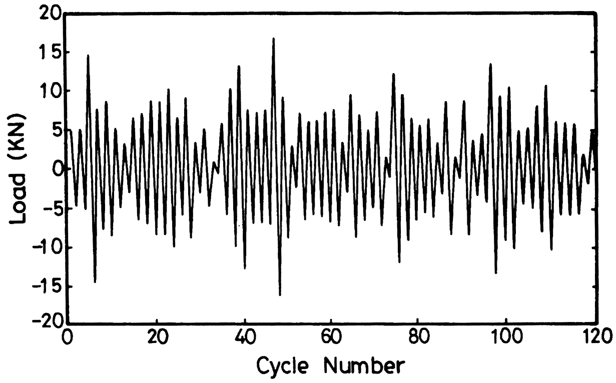

Fig. 3. A segment of random loading history E.

An MTS servo-hydraulic test machine was used which has an output wave form frequency of 0.6 Hz. Before carrying out random loading tests, fully reversed, push–pull, total strain controlled, tension–compression ($R = -1$) low-cycle fatigue tests were performed for a wide range of constant-amplitude strains. The stress–strain hysteresis loops were recorded on an X–Y recorder. The total strain range and the stress range were determined from the width and height of the monitored hystresis loop and then analyzed following a way proposed by other fatigue experimenters. To be precise, the total strain range (ϵ) was divided into an elastic part (ϵ_e) and a plastic part (ϵ_p). And for all test specimens, the empirical strain–life relationship

$$\epsilon = \epsilon_e + \epsilon_p = \frac{\sigma_f}{E}(2N_f)^b + \epsilon_f(2N_f)^c \tag{18}$$

was found and is shown in Fig. 2. As indicated in the figure, Eq. (18) consists of an elastic strain–life relationship and a plastic strain–life relationship. The latter relationship is known as the Coffin–Mansion relationship. In the present study, the fatigue ductility coefficient ϵ_f is found to be 0.116 and the fatigue ductility exponent c is found to be -0.776. The other relationship in Eq. (18) can be reformulated as

$$\epsilon_e = \frac{\sigma}{E} = \frac{\sigma_f}{E}(2N_f)^b \tag{19}$$

which, in fact, exhibits a Basquin's equation of

$$\sigma = \sigma_f(2N_f)^b \tag{20}$$

Compared to Eq. (5), it can be shown that $C = 1/2\sigma_f^{1/b}$ and $p = -1/b$. In the present study, from Fig. 2, it is found that fatigue strength coefficient $\sigma_f = 1286.7601$ MPa and the fatigue strength exponent $b = -0.1486$. The empirical constants needed in applying Eq. (5) are found to be $p = 6.7290$ and $C = 4.2097 \times 10^{20}$ accordingly.

With a view to verifying the prediction formulas derived previously, stress-controlled random loading fatigue tests were performed in particular for the present study. To imitate various kinds of narrow-band Gaussian random loading, five sets of random stress amplitudes, each having the probability density function as shown in Eq. (4), were selected for the generation of random loading. The statistical properties of each set of random loading are summarized in Table 1 in which the standard deviation of the stress amplitude should not be confused with the standard deviation of the continuous random stress process considered in random vibration books. For each set of random loading shown in Table 1, a long series of random stress amplitudes were generated to resemble the alternating peaks and troughs of a zero-mean random stress history. Fifteen random stress histories, such as the one shown in Fig. 3, were thus generated for each random loading and then applied to the specimens until the specimens fractured.

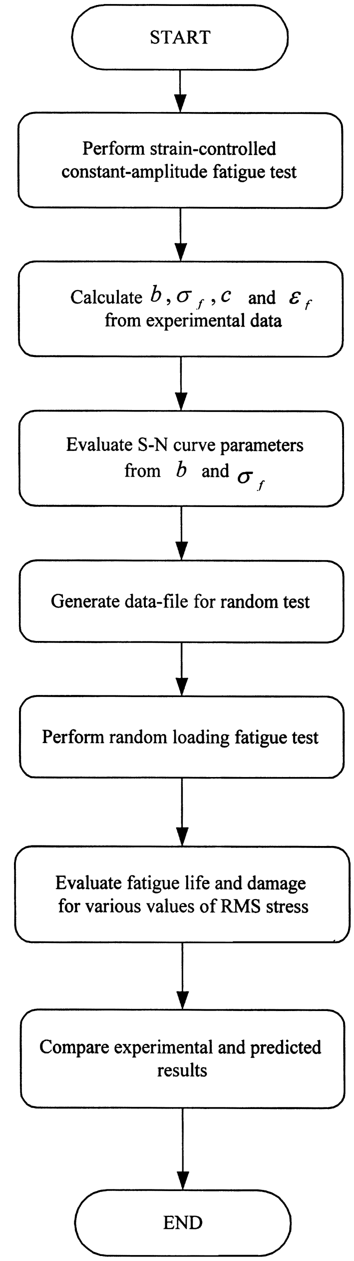

Fig. 4. Flow chart of the study.

Table 2
Failure cycles and fatigue damage by cycle-by-cycle calculation (under random loading E)

Specimen no.	Failure cycle	Damage (Miner's rule)	Plastic work interaction rule		
			$D(d = -0.25)$	$D(d = -0.35)$	$D(d = -0.45)$
ART-El	10803	0.7477	0.8492	0.8947	0.9431
ART-E2	9783	0.6908	0.7836	0.8249	0.8689
ART-E3	10419	0.7098	0.8087	0.8528	0.8998
ART-E4	10441	0.7296	0.8280	0.8718	0.9183
ART-E5	10102	0.6850	0.7785	0.8202	0.8646
ART-E6	9921	0.6979	0.7912	0.8327	0.8770
ART-E7	10381	0.7148	0.8149	0.8596	0.9073
ART-E8	9732	0.6960	0.7885	0.8299	0.8740
ART-E9	10146	0.7052	0.8008	0.8435	0.8889
ART-ElO	9978	0.6954	0.7896	0.8316	0.8764
ART-E11	9836	0.6855	0.7784	0.8198	0.8639
ART-E12	10237	0.7099	0.8063	0.8493	0.8950
ART-E13	10292	0.7120	0.8087	0.8519	0.8978
ART-E14	9344	0.6540	0.7423	0.7817	0.8237
ART-E15	10533	0.7265	0.8255	0.8696	0.9165
Mean	10131	0.7034	0.8001	0.8423	0.8877
Std.	324.51	0.0220	0.0256	0.0270	0.0286

During the test process, stress amplitudes in addition to fatigue lives were recorded for the future analysis of fatigue damage. The entire test and analytical procedure can be summarized in a simple flow chart, shown in Fig. 4, in which the damage evaluation will be discussed further in the next section.

4. Verification of prediction formulas

The most straightforward method to calculate the fatigue damage under random loading is the use of Eq. (2) which is named cycle-by-cycle calculation in the present study. Based on the test result, the fatigue damage for each specimen was calculated and added together until it fractured. The result was then tabulated in tables such as the one shown in Table 2 where the label in the first column indicates the name of the specimen, the quantity in the second column indicates the fracture cycle, and quantities in other columns indicate the fatigue damage calculated based on different rules. For a completely accurate prediction, the calculated damage should be 1 when the specimen fractures. Judging from Table 2, all the predictions are on the non-conservative side, but the plastic work interaction rule does improve the prediction over the widely used Miner's rule. Only the result of random loading condition E is shown in

Table 3
Fatigue damage by random vibration theory (under random loading E)

Specimen no.	Miner's rule	Plastic work interaction rule		
		$D(d = -0.25)$	$D(d = -0.35)$	$D(d = -0.45)$
ART-El	0.7949	0.9258	0.9947	0.9966
ART-E2	0.7198	0.8384	0.9008	0.9025
ART-E3	0.7666	0.8929	0.9593	0.9612
ART-E4	0.7683	0.8948	0.9614	0.9632
ART-E5	0.7433	0.8677	0.9302	0.9319
ART-E6	0.7300	0.8502.	0.9135	0.9152
ART-E7	0.7638	0.8896	0.9558	0.9577
ART-E8	0.7161	0.8340	0.8961	0.8980
ART-E9	0.7466	0.8695	0.9342	0.9360
ART-ElO	0.7342	0.8531	0.9187	0.9205
ART-E11	0.7237	0.8429	0.9057	0.9074
ART-E12	0.7533	0.8775	0.9426	0.9444
ART-E13	0.7573	0.8820	0.9476	0.9495
ART-E14	0.6875	0.8008	0.8604	0.8620
ART-E15	0.7750	0.9027	0.9698	0.9717
Mean	0.7455	0.8682	0.9328	0.9346
Std.	0.0276	0.0328	0.0345	0.0346

Table 4
Failure cycle predicted by different methods (under random loading E)

Method	Cycles to failure			
Experimental value	10 131			
Predicted values	Miner's rule	Plastic work interaction rule		
		$d = -0.25$	$d = -0.35$	$d = -0.45$
Random vibration (constant clipping stress)	13 591	11 669	10 861	10 840
Random vibration (associated with statistical extreme)	13 700	12 100	11 500	10 950

the present paper for the sake of simplicity. For other sets of random loading, except that of random loading A, similar results were also found, as will be summarized in a later table.

The above tedious cycle-by-cycle calculation of fatigue damage can, in fact, be greatly simplified if random vibration theory is applied. Through the use of Eqs. (7) and (10), or Eq. (13), the fatigue damage can not only be calculated afterwards, but can be predicted in advance if the statistical properties are already known, as in the present study. To be compatible with Table 2, the result thus calculated for random loading condition E is shown in Table 3. In comparison with the result of Table 2, not only the use of formulas derived from random vibration theory is justified, but a more accurate result is obtained.

Another advantage for the use of random vibration theory in the present study is the ability to predict the fatigue life. As introduced previously, through the use of Eq. (13), or Eqs. (13) and (17), the fatigue life can be estimated in advance. The prediction result for the case of random loading E is shown in Table 4. The experimental result is also shown in the same table. Since random vibration theory is considered, both the experimental result and the predicted result should be interpreted in averages. Again, it is found that the predicted average fatigue lives are very close to the experimentally obtained average fatigue life. The accuracy of the prediction can be improved if the plastic work interaction rule is considered. In Table 4, it is also interesting to note that if the maximum clipping stress is not known in advance, a very accurate result can still be obtained through the use of statistics of extremes.

As shown in Table 1 and stated before, five sets of random loading were considered in the present study. For each set of random loading, 15 sample stress histories that are different in reality, but have the same statistics, were generated. Each sample history was then applied to a fatigue specimen. Thus, there were, in total, 75 fatigue specimens tested in the present study. Owing to limited space, only the result for the case of random loading E is presented in the present paper. Similar results, however, were also found for other cases of random loading. To summarize the above random loading test result, Fig. 5 and Table 5 were constructed. From Table 5, it is found that the formula derived based on plastic work interaction rule and random vibration theory provides better fatigue damage prediction for all cases. Further, in applying the plastic work interaction rule, the best prediction is obtained when the lowest value of the interaction exponent is selected. From Fig. 5, it is interesting to note that a kind of stress–strain relationship is observed. Since it is not closely related to the topic presented in the present paper, it will not be discussed further.

5. Conclusions

Through the present study, the following conclusions can be drawn when considering the fatigue damage and fatigue life of a component when subjected to random loading.

1. Similar to that observed by other researchers, when a component is subjected to variable-amplitude loading or random loading, Morrow's plastic work interaction damage rule gives us rather accurate fatigue life prediction in the long life region and slightly conservative life

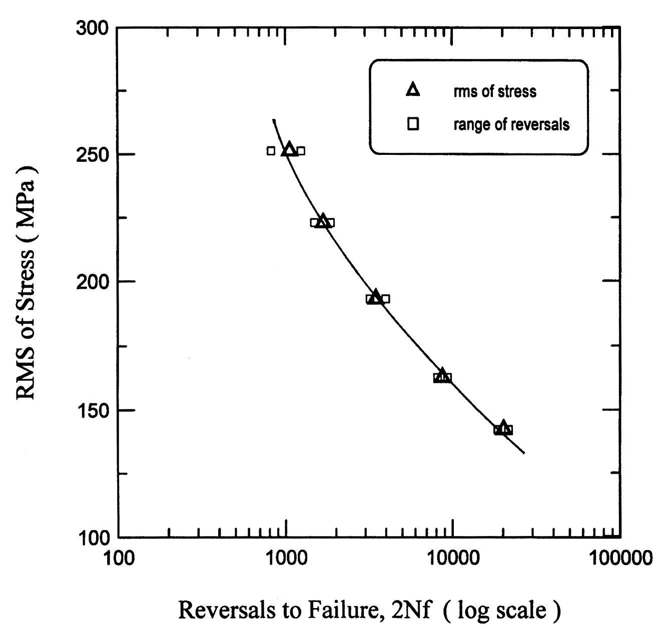

Fig. 5. Random loading vs failure cycles.

Table 5
Summary of the result under different random loading

Set	Fatigue damage								
	Average failure cycle	Cycle-by-cycle				Random vibration theory (constant clipping stress)			
		Miner's rule	Plastic work interaction rule			Miner's rule	Plastic work interaction rule		
			$d = -0.25$	$d = -0.35$	$d = -0.45$		$d = -0.25$	$d = -0.35$	$d = -0.45$
A	529	1.0247	1.0539	1.0725	1.0821	0.9333	0.9584	0.9640	0.9700
B	841	0.8907	0.9262	0.9418	0.9578	0.8774	0.9058	0.9263	0.9280
C	1743	0.8600	0.9157	0.9384	0.9621	0.8628	0.8992	0.9461	0.9619
D	4373	0.7510	0.8250	0.8577	0.8921	0.7713	0.8646	0.8935	0.9201
E	10131	0.7034	0.8001	0.8423	0.8877	0.7455	0.8682	0.9328	0.9346

prediction in the short life region. The degree of accuracy depends on the selection of an appropriate plastic work interaction exponent that, at present, can only be found on a trial basis. In the present study, three values of -0.25, -0.35 and -0.45 were selected for the exponent, and the result indicates that the use of the smallest exponent led to the best prediction for most cases which have longer fatigue lives. For the special case when the fatigue life is very short, the selection of the interaction exponent does not greatly affect the prediction.

2. Although the application of random vibration theory in the prediction of fatigue damage and fatigue life has been questioned by some fatigue experimenters, the present study indicates that a general formula, or its equivalent, derived from random vibration theory can really offer us rapid prediction. The prediction is even more accurate as compared with cycle-by-cycle calculation. Under this circumstance, the tedious and time-consuming cycle-by-cycle fatigue damage calculation process can be avoided. In general, if the statistical properties of the random loading are known in advance, then not only the fatigue damage, but also the fatigue life, can be predicted very easily and accurately by the derived formulas. Although Rayleigh distribution is assumed for the random stress amplitudes in the present study, it is projected that the derived formulas can still be applied to other kinds of variable-amplitude or random loading so long as the statistical properties are known in advance.
3. When comparison is made among different sets of random loading, it is found that fatigue damage prediction based on Miner's rule becomes less conservative as the average stress level becomes lower and the number of cycles to failure becomes larger. The improvement of damage prediction based on Morrow's plastic work interaction damage rule becomes more significant under this circumstance. This is quite reasonable because there are more interaction effects ignored in the application of Miner's rule.

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