Int J Adv Manuf Technol (2001) 18:303–311

The International Journal of Advanced Manufacturing Technology

Manufacturing-to-Sale Planning Model for Fuel Oil Production

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The fuel oil refinery production industry in Taiwan has entered a completely competitive free open market. In such an environment there exist a variety of uncertain factors, and a traditional production planning model will not be able to deal with the new situation. This study develops a responsive and flexible manufacturing-to-sale planning system to deal with uncertain manufacturing factors. The major objective of this study is to model the problem of the uncertain nature faced by the Chinese Petroleum Corporation (CPC) and to establish a manufacturing-to-sale planning model for solving the problem. A linear programming technique is suggested for developing the optimal strategy for use in production plans. Fuzzy theory is adopted for dealing with demand/cost uncertainties. A possibilistic linear programming model is thus formulated in this study. The possible uncertain fluctuations on demand/cost are included in the model. Therefore, the strategy for creating maximum profit for the company can be obtained via the proposed modelling procedures.

Keywords: Aggregate planning; Fuel oil refinement; Imprecise natures; Possibilistic linear programming model

1. Introduction

A conventional oil refinery is usually a closed operation system. The oil refining process from the crude oil to the final product is self-sustained. Thus, the quality of the oil refining operation process affects the profit component of sales. Moreover, the amount of each refined oil product is in proportion to the amount of crude oil being refined. None of an extra specific product can be produced from a specific amount of a crude oil by refining extraction. Also, two energy crises have occurred and the cost of crude oil is fluctuating all the time. Because of the pressure of environmental protectionists, the oil refining processes have recently been forced to adopt an open operation. The import/export types of oils are no longer crude oil only. Semi-products (for use as secondary mixing materials) and final-products are also included as import/export items. The goal of maximum profit can be achieved if not only final fine products, but also various semi-products and crude oils are included in the objective function for optimisation. Both the high degree of vertical integration in the petroleum industries and the high degree of variable correlation in refinery processes, further complicate the whole production-to-marketing planning system. Linear programming software packages such as the GRTMPS package of the Chinese Petroleum Corporation (CPC) are currently being used widely for making short-term, mid-term, or long-term oil refinement–storage–sale plans by many existing large international oil companies. However, none of these software packages and the oil refinement–storage–sale plans thus produced consider the uncertain market fluctuations. Therefore, further study of the accuracy and reliability of the planning models is required.

The CPC was a monopoly business organisation in Taiwan before 1999. Since January 1999, the government has allowed several types of oil fractions, including liquefied petroleum gas, aeronautical fuel, and bunker fuels to be imported. In June 1999, the six-chain plant of the Taiwan Plastics Company began to produce and sell oil products. From January 2000, all crude oil and oil product fractions could be imported by private companies. A complete free open fuel oil market in Taiwan is to be driven purely by the economic demands of the oil industry. The production environment contains many uncertainty factors. How to deal with such a new situation and to develop a complete production-to-sale strategy model are important immediate tasks for the CPC in Taiwan.

Mathematical linear programming is one of the important branches in operational research which originated in World War II. The primary objective of the method is to find the best allocation for a finite number of resources so that the best distribution policy can be made. The first applications of this technique in crude-oil production were in the 1950s. Charnes et al. [1] developed one of the first applications of linear programming to study the blending process in a refinery operation. Garvin and Crandall [2] described applications of linear programming in the oil industry. Lee and Aronofsky [3] formulated a linear programming model to schedule oil production from a single field to achieve maximum net profit. In 1962, Aronofsky and Williams [4] extended the model to compute

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the production schedule for many sources and over a series of time periods. Bohannon [5] proposed a mixed-integer linear programming model for the development of multireservoir pipeline systems. Sullivan [6] described a gasfield-planning model using mixed integer programming to find a development strategy that maximises the economic worth of a partially depleted offshore gas field. Cain and Shehata [7] developed an integrated computer system, the selective production scheduling (SPS) system, for the management of the Kuwait Oil Co. by linear programming to schedule oil and gas production in daily operations. In 1992, Duffuaa et al. [8] studied the impact of oil production on the gas supply to vital industries in Saudi Arabia and determined the minimum level of oil production to sustain its industries. Panne and Cornelis [9] dealt with the design of interactions between two refineries that have complementary demand patterns by formulating a decentralised solution of a linear programming problem. Bopp et al. [10] established a stochastic optimisation model to solve the problem of managing natural gas purchases under conditions of uncertain demand and frequent price change. Lee et al. [11] developed a mixed-integer optimisation model to describe the inventory management problem for a refinery that imports several types of crude oil by different vessels. Iyer et al. [12] formulated a multiperiod mixed-integer linear programming model for the simultaneous capacity planning and scheduling of well and facility operations in offshore oil fields.

Traditional mathematical programming techniques adopted deterministic variables and coefficients for modelling. In fact, most real systems are not deterministic. Traditional modelling techniques may sometimes disagree with real situations. Zadeh [13] presented a fuzzy set theory to describe systems of an imprecise nature. Based on this theory, Bellman and Zadeh [14] derived a fuzzy-set-based decision-making method. Zimmerman [15,16] proposed a fuzzy linear programming (FLP) method to deal with problems with fuzzy natures. Zadeh [17] used the fuzzy sets as a basis to derive a theory of possibility. Possibility theory focuses primarily on problems of an imprecise nature that is intrinsic in natural languages and is assumed to be "possibilistic" rather than "probabilistic". Lai and Hwang [18,19] used a triangular membership function to describe the possible distribution of total production cost. A scheme to maximise the possibility of obtaining a higher profit and minimising the risk of obtaining a lower profit is proposed. Bopp et al. [10] established a stochastic model to handle uncertain demands and frequent price changes. However, stochastic approaches cannot describe the sharp transition from membership to non-membership that exists in real problems.

The major objective of this study is to model the uncertainty problems faced by CPC and to establish a manufacturing-to-sale planning model for solving the problem. A linear programming technique is employed for searching for the optimal strategy for use in production plans. Fuzzy theory is adopted for dealing with demand/cost uncertainties. The possibilistic linear programming formulation is apparently the best approach and so it is proposed for dealing with this problem. The possible uncertain fluctuations in market demand and manufacturing cost can be included in the model. Therefore a strategy for creating maximum company profit can be obtained via the proposed modelling procedures.

2. Aggregate Possibilistic Linear Programming Models

Linear programming is a powerful analytical tool. Based upon the theory presented by Zimmermann [15,16], the theory can be progressively extended to operate for aggregate possibilistic linear programming planning problems. Lai and Hwang have proposed such a modelling method [18]. The modelling procedure is described as follows:

Step 1. Construct a crisp linear programming model for profit maximization. By assuming that the aggregate production plan includes variables $x_1, x_2, x_3, \ldots, x_n$, a variable vector,

$$X = [x_1\ x_2\ x_3\ \cdots\ \cdots\ x_n]^T,\ X \in R^{n\times 1}$$

can represent them. In general, the objective function is

$$\text{Max}\ \ c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$$
$$\text{s.t. } Ax \le D, \text{ and } x \ge 0$$

where $c_1, c_2, \ldots, c_n$ = cost coefficients for each variable

$$A \in R^{r\times n}, \quad D \in R^{r\times 1}$$

Step 2. Decide the extended range parameters, β_D and β_C, of the triangular possibility functions for resources and cost coefficients. A triangular possibility resource is given as $\tilde{D} = (D^m, D^p, D^o)$. If the extended range parameter of a triangular possibility resource is given as β_D, then $D^m = \mathbf{D}$, $D^p = (1 - \beta_D)\,\mathbf{D}$, $D^o = (1 + \beta_D)\,\mathbf{D}$. A triangular possibility cost coefficient is given as $\tilde{C}_x = (C_x^m, C_x^p, C_x^o)$. If the extended range parameter of a triangular possibility cost coefficient is given as β_C, then $C_x^m = C_x$, $C_x^p = (1 - \beta_C)\,C_x$, $\quad C_x^o = (1 + \beta_C)\,C_x$.

Step 3. Construct possibilistic constraints. Resources such as demands in the production system are imprecise. Therefore, some elements in constraint vector $\mathbf{D}$ could be possibilistic coefficients. Then the constraints can be modified as

$$AX \le \tilde{D} \text{ where } \tilde{D} = (D^m, D^p, D^o)$$

Correspondingly, the membership function of $\mathbf{D}$ can be derived.

$$\pi_D = \begin{cases} 0 & \text{if} \quad Ax \le D^p \\ (Ax - D^p)/(D^m - D^p) & \text{if} \quad D^p \le Ax \le D^m \\ (D^o - Ax)/(D^o - D^m) & \text{if} \quad D^m \le Ax \le D^o \\ 0 & \text{if} \quad Ax \ge D^o \end{cases}$$

According to α-cut, a variable λ, $\pi_D \ge \lambda = \alpha$ is introduced. Then, the constraint function can be written as

$$Ax + (D^o - D^m)\,\lambda \le D^o$$
$$Ax - (D^m - D^p)\,\lambda \ge D^p$$

Step 4. Construct possibilistic objectives function and expand the objective function to multiple crisp objective functions. Cost coefficients in the production system are sometimes imprecise. Therefore, some cost coefficients can be triangular possibility numbers, i.e. $\tilde{C}_x = (C_x^m, C_x^p, C_x^o)$. Then the objective function can be modified as

$$\text{Max } \tilde{c}_1x_1 + \tilde{c}_2x_2 + \tilde{c}_3x_3 + \ldots + \tilde{c}_nx_n$$

After rearranging the objective function, we can expand it to multiple crisp objective functions as follows:

$$\text{Max } c_1^m x_1 = c_2^m x_2 + c_3^m x_3 + \dots + c_n^m x_n$$
$$\text{Max } c_1^p x_1 = c_2^p x_2 + c_3^p x_3 + \dots + c_n^p x_n$$
$$\text{Max } c_1^o x_1 = c_2^o x_2 + c_3^o x_3 + \dots + c_n^o x_n$$

Step 5. Define the optimistic/pessimistic value of the objective function, Z_i^{PIS}/Z_i^{NIS} $(i = 1, 2, 3)$, and estimate the membership function of the objective function.

$$Z_1^{PIS} = \text{Max } c_1^m x_1 + c_2^m x_2 + c_3^m x_3$$
$$Z_2^{PIS} = \text{Max } c_1^p x_1 + c_2^p x_2 + c_3^p x_3$$
$$Z_3^{PIS} = \text{Max } c_1^o x_1 + c_2^o x_2 + c_3^o x_3$$
$$Z_1^{NIS} = \text{Min } c_1^m x_1 + c_2^m x_2 + c_3^m x_3$$
$$Z_2^{NIS} = \text{Min } c_1^p x_1 + c_2^p x_2 + c_3^p x_3$$
$$Z_3^{NIS} = \text{Min } c_1^o x_1 + c_2^o x_2 + c_3^o x_3$$

$\forall\ Z_i \quad (i = 1, 2, 3),\ Z_i \in [Z_i^{NIS}, Z_i^{PIS}]$ The membership function can thus be obtained as

$$\pi_{Z_i} = \begin{cases} 0 & \text{if} \quad Z_i \leq Z_i^{NIS} \\ (Z_i - Z_i^{NIS})/(Z_i^{PIS} - Z_i^{NIS}) & \text{if} \quad Z_i^{PIS} \leq Z_i \leq Z_i^{NIS} \\ 0 & \text{if} \quad Z_i \geq Z_i^{PIS} \end{cases}$$

Step 6. Transform the multi-objective linear programming model into a single objective fuzzy linear programming model. According to α-cut, $\pi_{Z_i} \geq \lambda = \alpha$ is introduced. After the maximum–minimum operation, the model becomes

$$\begin{aligned} &\text{Max } \lambda \\ &\text{s.t.} \\ &c_1^m x_1 + c_2^m x_2 + c_3^m x_3 - (Z_1^{PIS} - Z_1^{NIS})\ \lambda \geq Z_1^{NIS} \\ &c_1^p x_1 + c_2^p x_2 + c_3^p x_3 + (Z_2^{NIS} - Z_2^{PIS})\ \lambda \leq Z_2^{NIS} \\ &c_1^o x_1 + c_2^o x_2 + c_3^o x_3 - (Z_3^{PIS} - Z_3^{NIS})\ \lambda \geq Z_3^{NIS} \\ &Ax + (D^o - D^m)\ \lambda \leq D^o \\ &Ax - (D^m - D^p)\ \lambda \geq D^p \\ &x \geq 0 \end{aligned}$$

Step 7. Solve the single objective function linear programming model using software packages such as Lindo.

3. CPC Manufacturing-to-Sale Planning Model

As shown in Fig. 1, the operation flow of the oil stocking sequences of CPC is closely interconnected, including crude oil purchase, transport, unloading to storage tanks, transferring to refinery plant for mixing and refining, and oil product storage, and distribution to local service stations. Moreover, a complete free open market is to be driven by economic demand in the Taiwan oil industry. In order to take into consideration the complicated operation processes, uncertain management environment, and global competitive marketing in a comprehensive model, several imprecise features will have to be investigated.

3.1 Investigation on Imprecise Natures

CPC has to face the following imprecise features:

1. Imprecise market demand: Since adopting the open door national policy, CPC has no longer been a monopoly enterprise. Thus, the market demand for CPC is imprecise.
2. Varying unit purchase cost: The unit cost of imported oils is determined by the transitory international oil price. The factors that will increase the oil price include international political crises, mid-east military tension, seasonal demand, especially in cold climate areas, and oil crises. The factors that will bring down the oil price are the discovery of new oil sources, the increase in supply, the decrease in demand, and over-supply of the market. Therefore, the purchase cost of crude oil fluctuates.
3. Varying unit management cost: The management cost is the cost including inventory, depreciation, taxation, and accrual loss on all items of oil products, semi-products, and raw materials in stock. Since the required amount of oils to be stocked and the unit purchase cost of the oils in stock are uncertain, the unit management cost is uncertain.
4. Varying unit refinery operation cost: As mentioned before, the oil refining process is so complicated that many process factors are of an uncertain nature. The crude oil from different wells produces different percentages of petroleum products and by-products. Therefore, the unit refinery operation cost is uncertain.
5. Varying unit transportation cost: Oil tankers, oil tanker trains, oil tanker trucks and oil pipelines are used to deliver oil or oil products from the harbour to the refinery, from production plants to petrol stations, or from city *A* to city *B*. Therefore, the unit transportation cost is uncertain.
6. Varying unit sale price: The unit sale price of oil products is based on the oil-product price estimation formula sanctioned by the administration of the Taiwan government. Because of variation of the dollar exchange rate and imported oil cost fluctuation, the unit sale price is adjusted dynamically to reflect purchase costs or excessive sale surplus. Therefore, the unit sale price of the refined oil is uncertain.

A triangular possibility distribution function is adopted here to represent these uncertain factors. The possible value of a unit cost coefficient or a unit sale price coefficient, $\tilde{C}$, can be defined geometrically by three corner points, (C^m, C^p, C^o). The possible amount of market demand for oil products, $\tilde{D}$, can be defined geometrically by three corner points, (D^m, D^p, D^o).

3.2 Case Study

A simplified but comprehensive model is employed to illustrate the effectiveness of the possibilistic linear programming procedure presented in this paper. To simplify the model without losing generality, we assume that CPC has two refinery plants, Taoyuan (tao) and Kaohsiung (kao) plants. The produced petroleum products are also confined to gasoline (Gas) and diesel (Dsl). The imported crude oils are limited to Alaska north Sl (O1), Upper Zakum (O2), Djeno (O3) and Oseberg (O4). The

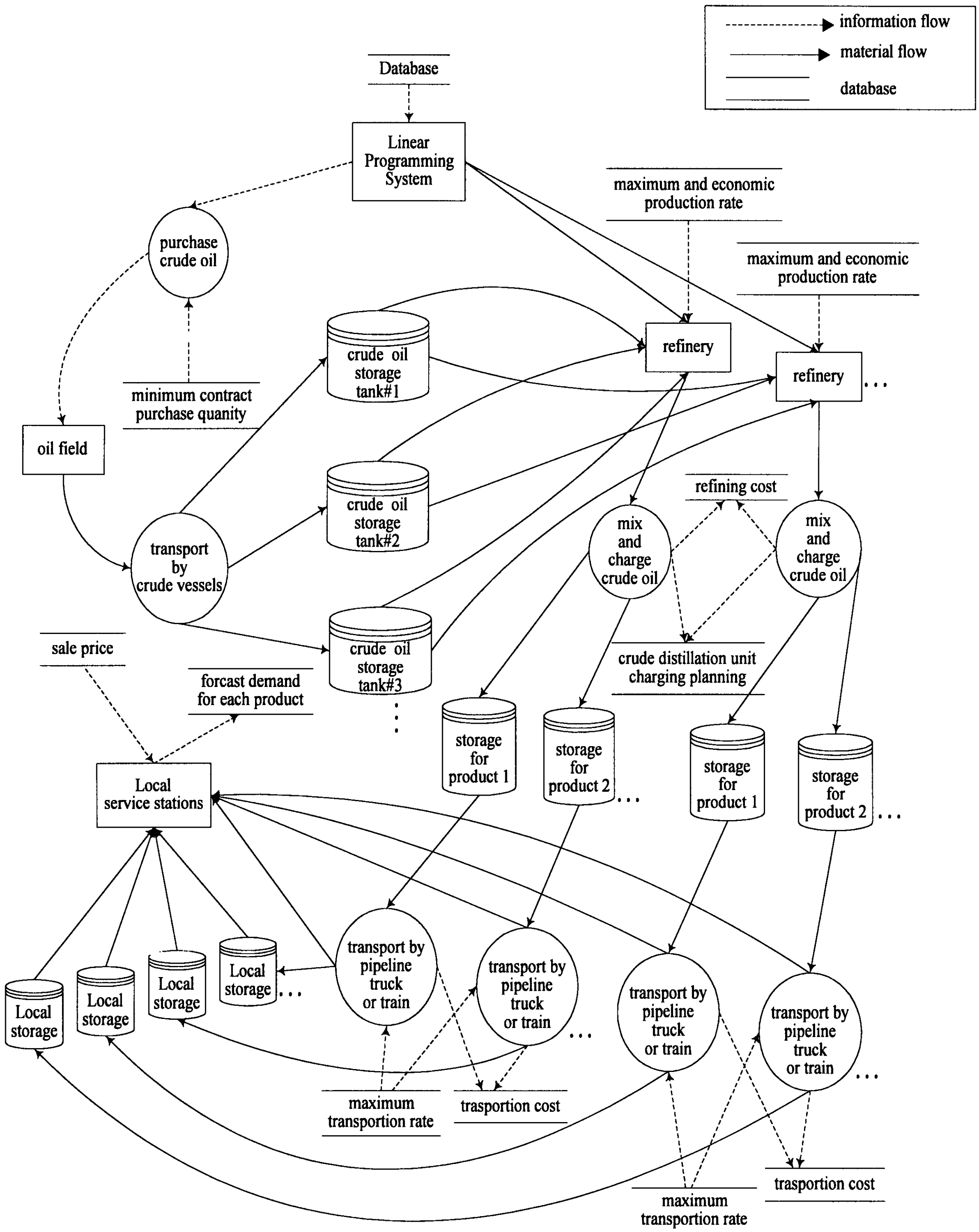

Fig. 1. The operational flow of oil stocking sequences.

gasoline and diesel are supplied to five cities, Taipei (tai), Hsinchu (his), Taichung (tch), Chiayi (chi) and Kaohsiung (kao). Numerous oil pipelines and oil tanks are available for CPC to deliver/store adequate oil products to/in these five cities.

3.2.1 Problem Definitions

Given:

1. T: planned production period, $T = 12$ months.

2. The refining percentages of gas from O1, O2, O3, and O4 are: 26.3%, 18.95%, 10.20%, and 19.82%, respectively. The refining percentages of Dsl from O1, O2, O3, and O4 are 3.45%, 15.73%, 18.30%, and 23.03%, respectively.
3. $D_{city,t}^{product}$: the average amount of gasoline (diesel) demand in month t in a specific area, unit: litres. $D_{tai,t}^{Gas} = 80 \times 10^6$, $D_{hsi,t}^{Gas} = 24 \times 10^6$, $D_{tch,t}^{Gas} = 38 \times 10^6$, $D_{chi,t}^{Gas} = 18.5 \times 10^6$, $D_{kao,t}^{Gas} = 50 \times 10^6$, $D_{tai,t}^{Dsl} = 21 \times 10^6$, $D_{hsi,t}^{Dsl} = 7.5 \times 10^6$, $D_{tch,t}^{Dsl} = 18 \times 10^6$, $D_{chi,t}^{Dsl} = 9 \times 10^6$, $D_{kao,t}^{Dsl} = 19.5 \times 10^6$.
4. $E_{refinery}^{product}$: minimum monthly economic gasoline (diesel) production quantity for the refinery plant, unit: litres. $E_{tao}^{Gas} = 43 \times 10^6$, $E_{kao}^{Gas} = 129 \times 10^6$, $E_{tao}^{Dsl} = 21.55 \times 10^6$, $E_{kao}^{Dsl} = 64.65 \times 10^6$.
5. $M_{refinery}^{product}$: maximum monthly gasoline (diesel) production quantity for the refinery plant, unit: litres. $M_{tao}^{Gas} = 65.5 \times 10^6$, $M_{kao}^{Gas} = 196.5 \times 10^6$, $M_{tao}^{Dsl} = 53.5 \times 10^6$, $M_{kao}^{Dsl} = 160.5 \times 10^6$.
6. The minimum monthly purchase amount of crude oil by contract (unit: litres):

   k1: minimum monthly purchase amount of O1, k1 = 125.5×10^6.

   k2: minimum monthly purchase amount of O2, k2 = 155.5×10^6.

   k3: minimum monthly purchase amount of O3, k3 = 175.5×10^6.

   K4: minimum monthly purchase amount of O4, k4 = 135.5×10^6.
7. $P_{\max, refinery \to city}^{Product}$: maximum monthly delivered amount of gasoline (diesel) from the refinery to the city, unit: litres. $P_{\max,tao \to tai}^{Gas} = 58 \times 10^6$, $P_{\max,tao \to hsi}^{Gas} = 40 \times 10^6$, $P_{\max,tao \to tch}^{Gas} = 35 \times 10^6$, $P_{\max,tao \to chi}^{Gas} = 32 \times 10^6$, $P_{\max,tao \to kao}^{Gas} = 30 \times 10^6$, $P_{\max,kao \to tai}^{Gas} = 74 \times 10^6$, $P_{\max,kao \to hsi}^{Gas} = 55 \times 10^6$, $P_{\max,kao \to tch}^{Gas} = 60 \times 10^6$, $P_{\max,kao \to chi}^{Gas} = 45 \times 10^6$, $P_{\max,kao \to kao}^{Gas} = 95 \times 10^6$, $P_{\max,tao \to tai}^{Dsl} = 55 \times 10^6$, $P_{\max,tao \to hsi}^{Dsl} = 15 \times 10^6$, $P_{\max,tao \to tch}^{Dsl} = 18 \times 10^6$, $P_{\max,tao \to chi}^{Dsl} = 20 \times 10^6$, $P_{\max,tao \to kao}^{Dsl} = 22 \times 10^6$, $P_{\max,kao \to tai}^{Dsl} = 20 \times 10^6$, $P_{\max,kao \to hsi}^{Dsl} = 23 \times 10^6$, $P_{\max,kao \to tch}^{Dsl} = 30 \times 10^6$, $P_{\max,kao \to chi}^{Dsl} = 28 \times 10^6$, $P_{\max,kao \to kao}^{Dsl} = 45 \times 10^6$l.
8. Cost coefficients
   a. $C_{sell}^{product}$: unit sale price of gasoline (diesel), unit: dollars/litre. $C_{sell}^{Gas} = 17.2$, $C_{sell}^{dsl} = 12.2$.
   b. C_{purch}^{Oj}, j = 1, 2, 3, 4: unit purchase price of crude oil, unit: dollars/litre. $C_{purch}^{O1} = 0.2$, $C_{purch}^{O2} = 0.235$, $C_{purch}^{O3} = 0.19$, $C_{purch}^{O4} = 2.05$.
   c. $C_{manuf,refinery}^{Product,Oj}$: unit refinery cost of gasoline (diesel) out of Oj in the refinery plant, unit: dollars/litre. $C_{manuf,tao}^{Gas,O1} = 4.7$, $C_{manuf,tao}^{Gas,O2} = 5.1$, $C_{manuf,tao}^{Gas,O3} = 5.6$, $C_{manuf,tao}^{Gas,O4} = 5.06$, $C_{manuf,kao}^{Gas,O1} = 3.5$, $C_{manuf,kao}^{Gas,O2} = 3.42$, $C_{manuf,kao}^{Gas,O3} = 3.33$, $C_{manuf,kao}^{Gas,O4} = 3.155$, $C_{manuf,tao}^{Dsl,O1} = 4.23$, $C_{manuf,tao}^{Dsl,O2} = 4.59$, $C_{manuf,tao}^{Dsl,O3} = 5.04$, $C_{manuf,tao}^{Dsl,O4} = 4.554$, $C_{manuf,kao}^{Dsl,O1} = 3.15$, $C_{manuf,kao}^{Dsl,O2} = 3.08$, $C_{manuf,kao}^{Dsl,O3} = 2.995$, $C_{manuf,kao}^{Dsl,O4} = 2.84$.
   d. $C_{invent,refinery}^{Product}$: unit inventory cost of gasoline (diesel) in the refinery plant,u nit: dollars/litre, month. $C_{invent,tao.}^{Gas} = 0.86$, $C_{invent,tao}^{Dsl} = 0.61$, $C_{invent,kao}^{Gas} = 0.86$, $C_{invent,kao}^{Dsl} = 0.61$.
   e. $C_{trans,refinery \to city}^{Product}$: unit transportation cost of gasoline (diesel) from the refinery to the city, unit: dollars/litre. $C_{trans,tao \to tai}^{Gas} = 1.75$, $C_{trans,tao \to hsi}^{Gas} = 1.85$, $C_{trans,tao \to tch}^{Gas} = 2.15$, $C_{trans,tao \to chi}^{Gas} = 2.45$, $C_{trans,tao \to kao}^{Gas} = 2.75$, $C_{trans,kao \to tai}^{Gas} = 2.95$, $C_{trans,kao \to hsi}^{Gas} = 2.65$, $C_{trans,kao \to tch}^{Gas} = 2.35$, $C_{trans,kao \to chi}^{Gas} = 1.95$, $C_{trans,kao \to kao}^{Gas} = 0.96$, $C_{trans,tao \to tai}^{Dsl} = 1.25$, $C_{trans,tao \to hsi}^{Dsl} = 1.35$, $C_{trans,tao \to tch}^{Dsl} = 1.65$, $C_{trans,tao \to chi}^{Dsl} = 1.98$, $C_{trans,tao \to kao}^{Dsl} = 2.28$, $C_{trans,kao \to tai}^{Dsl} = 2.58$, $C_{trans,kao \to hsi}^{Dsl} = 2.38$, $C_{trans,kao \to tch}^{Dsl} = 2.08$, $C_{trans,kao \to chi}^{Dsl} = 1.78$, $C_{trans,kao \to kao}^{Dsl} = 0.93$.

Variables:

$Oj_t^{refinery}$: the amount of Oj (j = 1, 2, 3, 4) purchased for the refinery in month t, unit: litres.

$S_{refinery,t}^{product,Oj}$: the amount of gasoline (diesel) produced from Oj (j = 1, 2, 3, 4), by the refinery in month t, unit: litres.

$L_{refinery,t}^{product}$: the amount of gasoline (diesel) stored in the refinery in month t, unit: litres.

$T_{refinery \to city,t}^{product}$: the amount of gasoline (diesel) delivered from a refinery to a city in month t, unit: litres.

Objective function notations:

$TC_{sell,t}$: Total dollar amount from selling gasoline and diesel in month t

$$TC_{sell,t} = C_{sell}^{Gas} \cdot D_t^{Gas} + C_{sell}^{Dsl} \cdot D_t^{Dsl}$$

$TC_{purch,t}$: Total cost for purchasing crude oils in month t

$$TC_{purch,t} = \sum_j C_{purch}^{Oj} \cdot Oj_t$$

$TC_{purch,t}$: Total production cost of gasoline (diesel) in month t

$$TC_{manuf,t} = \sum_j \sum_{refinery} (C_{manuf,refiner}^{Gas,Oj} \cdot S_{refinery,t}^{Gas,Oj} + C_{manuf,refinery}^{Dsl,Oj} \cdot S_{refinery,t}^{Dsl,Oj})$$

$TC_{invent,t}$: Total cost for stocking gasoline (diesel) in month t.

$$TC_{invent,t} = C_{invent,tao}^{Gas} \cdot L_{tao,t}^{Gas} + C_{invent,tao}^{Dsl} \cdot L_{tao,t}^{Dsl} + C_{invent,kao}^{Gas} \cdot L_{kao,t}^{Gas} + C_{invent,kao}^{Dsl} \cdot L_{kao,t}^{Dsl}$$

$TC_{trans,t}$: total transportation cost of gasoline (diesel) in month t

$$TC_{trans,t} = TC_{trans,t}^{Gas} + TC_{trans,t}^{Dsl}$$

where $TC_{trans,t}^{Gas} = \sum_{refinery} \sum_{city} C_{trans,refinery \to city}^{Gas} \cdot T_{refinery \to city,t}^{Gas}$

$$TC_{trans,t}^{Dsl} = \sum_{refinery} \sum_{city} C_{trans,refinery \to city}^{Dsl} \cdot T_{refinery \to city,t}^{Dsl}$$

Constraints:

1. Monthly amount of oil produced must be greater or equal to the amount of oil demanded by the market.
2. The production amount of each oil product should be greater than or equal to the economic production quantity.
3. The production quantity of each plant is restricted by its maximum production capacity.

4. The total amount of crude oils purchased according to contracts is at least 75% of the total amount of crude oils required by CPC.
5. The amount of gasoline (diesel) delivered from a refinery to a city by pipelines is less or equal to the maximum allowable quantity of the pipelines.
6. The total amount of gasoline (diesel) delivered from a refinery should be less than or equal to the total amount of gasoline (diesel) produced by the refinery.
7. The total amount of gasoline (diesel) delivered to each city should be greater than or equal to the gasoline (diesel) demanded by the city.

3.2.2 Model Establishment

The procedure to establish a linear possibilistic programming model for CPC can be accomplished by the following steps:

Step 1. Construct a crisp linear programming model.

$$\text{Max} \sum_{t=1}^{12} TC_{sell,t} - TC_{purch,t} - TC_{manuf,t} - TC_{invent,t} - TC_{trans,t} \tag{1}$$

Subject to:

$$S_t^{Gas} + L_{t-1}^{Gas} - L_t^{Gas} \geq D_t^{Gas};\ S_t^{Dsl} + L_{t-1}^{Dsl} - L_t^{Dsl} \geq D_t^{Dsl}; \tag{2}$$

$$L_t^{Gas} = L_{tao,t}^{Gas} + L_{kao,t}^{Gas};\ L_t^{Dsl} = L_{tao,t}^{Dsl} + L_{kao,t}^{Dsl};$$

$$D_t^{Gas} = D_{tai,t}^{Gas} + D_{hsi,t}^{Gas} + D_{tch,t}^{Gas} + D_{chi,t}^{Gas} + D_{kao,t}^{Gas};$$

$$D_t^{Dsl} = D_{tai,t}^{Dsl} + D_{hsi,t}^{Dsl} + D_{tch,t}^{Dsl} + D_{chi,t}^{Dsl} + D_{kao,t}^{Dsl};$$

$$S_{tao,t}^{Gas} \geq E_{tao,t}^{Gas};\ S_{tao,t}^{Dsl} \geq E_{tao,t}^{Dsl};\ S_{kao,t}^{Gas} \geq E_{kao,t}^{Gas};\ S_{kao,t}^{Dsl} \geq E_{kao,t}^{Dsl}; \tag{3}$$

$$S_{tao,t}^{Gas} \leq M_{tao,t}^{Gas};\ S_{tao,t}^{Dsl} \leq M_{tao,t}^{Dsl};\ S_{kao,t}^{Gas} \leq M_{kao,t}^{Gas};\ S_{kao,t}^{Dsl} \leq M_{kao,t}^{Dsl}; \tag{4}$$

$$O1_t \geq K1_t;\ O2_t \geq K2_t;\ O3_t \geq K3_t;\ O4_t \geq K4_t; \tag{5}$$

$$K1_t + K2_t + K3_t + K4_t \geq 0.75 \cdot O_t^{Total} \tag{6}$$

$$T_{refinery \to city,t}^{product} \leq P_{\max,\ refinery \to city}^{product}; \tag{7}$$

$$\sum_{city} T_{refinery \to city,t}^{\text{Product}} \leq S_{refinery,t}^{\text{Product}}; \tag{8}$$

$$\sum_{refinery} T_{refinery \to city,t}^{\text{Product}} \geq D_{city,t}^{\text{Product}}; \tag{9}$$

where $S_t^{Gas} = S_{tao,t}^{Gas} + S_{kao,t}^{Gas}$; $S_t^{Dsl} = S_{tao,t}^{Dsl} + S_{kao,t}^{Dsl}$,

$S_{tao,t}^{Gas} = 01_t^{tao} \cdot 0.2603 + O2_t^{tao} \cdot 0.1895 + O3_t^{tao} \cdot 0.102 + O4_t^{tao} \cdot 0.1982$,

$S_{kao,t}^{Gas} = 01_t^{kao} \cdot 0.2603 + O2_t^{kao} \cdot 0.1895 + O3_t^{kao} \cdot 0.102 + O4_t^{kao} \cdot 0.1982$,

$S_{tao,t}^{Dsl} = 01_t^{tao} \cdot 0.1345 + O2_t^{tao} \cdot 0.1573 + O3_t^{tao} \cdot 0.183 + O4_t^{tao} \cdot 0.2303$,

$S_{kao,t}^{Dsl} = 01_t^{kao} \cdot 0.1345 + O2_t^{kao} \cdot 0.1573 + O3_t^{kao} \cdot 0.183 + O4_t^{kao} \cdot 0.2303$.

$D_t^{Gas} = D_{tai,t}^{Gas} + D_{hsi,t}^{Gas} + D_{tch,t}^{Gas} + D_{chi,t}^{Gas} + D_{kao,t}^{Gas}$; $D_t^{Dsl} = D_{tai,t}^{Dsl} + D_{hsi,t}^{Dsl} + D_{tch,t}^{Dsl} + D_{chi,t}^{Dsl} + D_{kao,t}^{Dsl}$,

$O_t^{Total} = 01_t + O2_t + O3_t + O4_t$; $L_t^{Gas} = L_{tao,t}^{Gas} + L_{kao,t}^{Gas}$; $L_t^{Dsl} = L_{tao,t}^{Dsl} + L_{kao,t}^{Dsl}$

Step 2. Decide the extended range parameters, β_D and β_C, of the triangular possibility functions for demand and the cost coefficients based on CPC past statistic data and the seasonal forecast. In this example, $\beta_D = 0.1$ and $\beta_C = 0.1$.

Step 3. Construct possibilistic constraints.

Eq. (2) becomes

$$S_t^{Gas} + L_{t-1}^{Gas} - L_t^{Gas} \geq (D_t^{Gas,m}, D_t^{Gas,p}, D_t^{Gas,o}) \text{ and}$$

$$S_t^{Dsl} + L_{t-1}^{Dsl} - L_t^{Dsl} \geq (D_t^{Dsl,m}, D_t^{Dsl,p}, D_t^{Dsl,o}); \tag{10}$$

Eq. (9) becomes

$$\sum_{refinery} T_{refinery \to city,t}^{Gas} \geq (D_{city,t}^{Gas,m}, D_{city,t}^{Gas,p}, D_{city,t}^{Gas,o}) \text{ and} \tag{11}$$

$$\sum_{refinery} T_{refinery \to city,t}^{Dsl} \geq (D_{city,t}^{Dsl,m}, D_{city,t}^{Dsl,p}, D_{city,t}^{Dsl,o})$$

$$D_t^{product,p} = D_t^{product,m} \times (1 - \beta_D),$$

$$D_t^{product,o} = D_t^{product,m} \times (1 + \beta_D)$$

Step 4. Construct multiple objective-functions.

Expand a single objective function, Eq. (1), to three objective functions.

$$\text{Max } Z_1 = \sum_{t=1}^{12} \tag{12}$$

$$(TC_{sell,t}^m - TC_{purch,t}^m - TC_{manuf,t}^m - TC_{invent,t}^m - TC_{trans,t}^m)$$

$$\text{Max } Z_2 = \sum_{t=1}^{12} \tag{13}$$

$$(TC_{sell,t}^p - TC_{purch,t}^p - TC_{manuf,t}^p - TC_{invent,t}^p - TC_{trans,t}^p)$$

$$\text{Max } Z_3 = \sum_{t=1}^{12} \tag{14}$$

$$(TC_{sell,t}^O - TC_{purch,t}^O - TC_{manuf,t}^O - TC_{invent,t}^{)} - TC_{trans,t}^O)$$

Step 5. Define the optimistic/pessimistic value of the objective function,

Z_i^{PIS}/Z_i^{NIS} (i = 1, 2, 3), and estimate the membership function of the objective function.

The positive and negative ideal solutions are obtained using Lindo,

$Z_1^{NIS} = 0.25682 \times 10^{11}$ $Z_1^{PIS} = 0.32074 \times 10^{11}$

$Z_2^{NIS} = 0.23114 \times 10^{11}$ $Z_2^{PIS} = 0.28867 \times 10^{11}$

$Z_3^{NIS} = 0.28250 \times 10^{11}$ $Z_3^{PIS} = 0.35282 \times 10^{11}$

Step 6. Transform the multi-objective linear programming model into a single objective linear programming model as follows:

$$\text{Max } \lambda$$

Subject to

$$\sum_{t=1}^{12} (TC_{sell,t}^m - TC_{purch,t}^m - TC_{manuf,t}^m - TC_{invent,t}^m - TC_{trans,t}^m)$$

$$- (Z_1^{PIS} - Z_1^{NIS})\ \lambda \geq Z_1^{NIS}$$

$$\sum_{t=1}^{12} (TC_{sell,t}^p - TC_{purch,t}^p - TC_{manuf,t}^p - TC_{invent,t}^p - TC_{trans,t}^p)$$

$$- (Z_2^{PIS} - Z_2^{NIS})\ \lambda \geq Z_2^{NIS}$$

$$\sum_{t=1}^{12} (TC^O_{sell,t} - TC^O_{purch,t} - TC^O_{manuf,t} - TC^O_{invent,t} - TC^O_{trans,t})$$

$$- (Z_3^{PIS} - Z_3^{NIS})\, \lambda \leq Z_3^{NIS}$$

and plus Eqs (3) to (11) except Eq. (9).

Step 7. Solve using Lindo. Plans of monthly purchase amount of each type of crude oils for Taoyuan and Kaohsiung plants, amount of gasoline (diesel) produced by Taoyuan and Kaohsiung plants, the amount of gasoline (diesel) in stock for each plant, and the amount of gasoline (diesel) distributed to each major city are obtained. Owing to the length of the paper, these plans are not given here.

4. Demand and Cost Forecast Error Sensitivity Analyses

Errors in forecasts are inevitable. Environmental conditions and resources are time-variant. In order to characterise the influence of uncertainty on the possibilistic linear programming models, several numerical experiments are conducted. By perturbing both cost coefficients and demand, the imprecise nature of the real world can be simulated, and the evaluation of the possibilistic linear models with different ranges of triangular possibility distribution on the cost coefficients and the demand can thus be investigated.

4.1 Sensitivity Analysis of Imprecise Demands

First, the cost coefficients are assumed to have no errors at all. The predicted demand and the actual sales, however, are different with sixteen forecast error conditions, namely, 0%, ±5%, ±10%, 15%, ±20%, ±25%, ±30%, ±35%. When the actual demand exceeds the supply, oil products are imported. Imported gasoline is 2 dollars more than the domestic sale price per kilolitre. Imported diesel is 1.5 dollars more than the domestic sale price per kilolitre. When the actual demand is less than the supply, the surplus oil products are exported. Exported gasoline is 2 dollars less than the domestic sale price per kilolitre. Exported diesel is 1.5 dollars less than the domestic sale price per kilolitre.

By following the procedure mentioned in the previous section, the optimal solutions of the model with β_D = 0, 0.02, 0.04, …, 0.2 are obtained. By substituting in the actual amount of sales, the profit can be obtained. The results are recorded and plotted in Fig. 2. In Fig. 2, it can be seen that the profit from product sales is higher for β_D = 0.14 or 0.20 if the demand forecast error is positive and is greater than or equal to 25%. Whereas in negative demand forecast error conditions, the profit is higher for a β_D = 0.10. However, if the market demand can be precisely forecast, the crisp planning model (when β_D = 0) provides a good profit.

4.2 Sensitivity Analysis of Imprecise Cost Coefficients

This part of the study is to investigate the influence of different values of β_D on profit. Several basic assumptions are made.

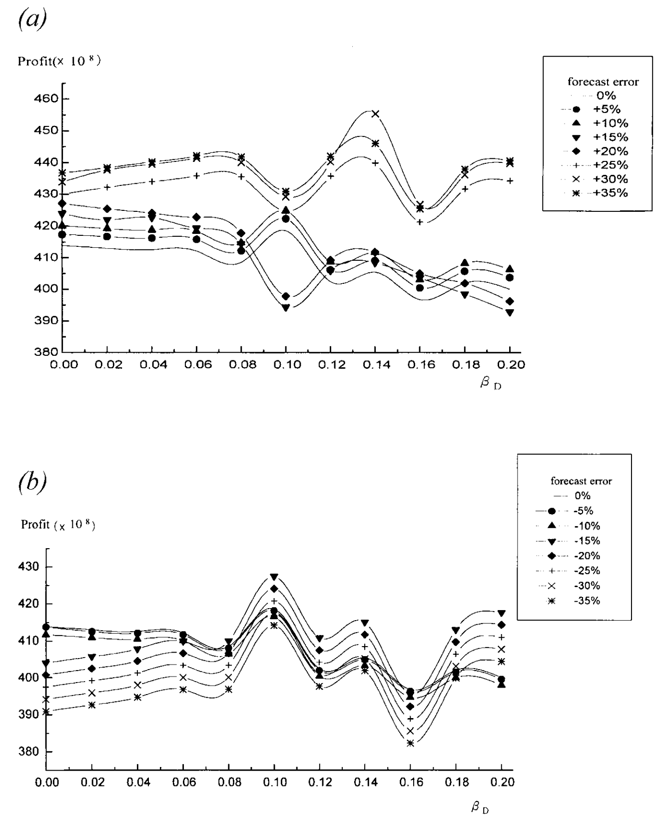

Fig. 2. Demand sensitivity on β_D for (*a*) positive and (*b*) negative demand forecast errors.

A forecast error of 15% on demand is used and β_D = 0.08 is used. Three different cost forecast errors, 5%, 10%, 15%, are used in this study. Three cases are considered here. Case 1: assume the sale price is imprecise while other costs are precise. Case 2: assume the purchase unit cost is imprecise while the sale price and other costs are precise. Case 3: assume both the sale price and purchase costs are imprecise while other costs are precise. To realise the sensitivity of β_c on profit, different values of β_c, 0.02, 0.04, 0.06, …, 0.2, are used for experiments. The resultant profits for the three cases with varying β_c are computed and presented in Figs 3, 4, and 5.

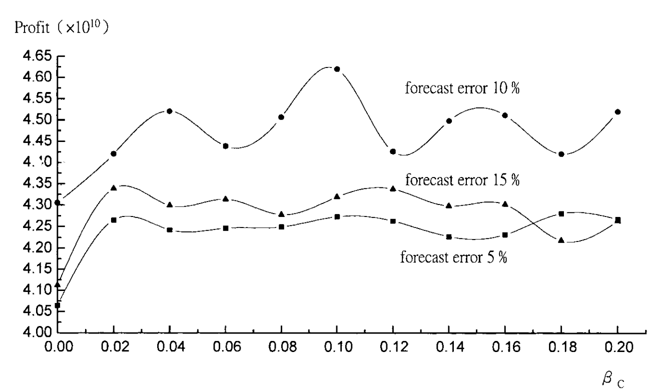

Fig. 3. Sensitivity analysis of β_c assuming that the sale price is imprecise.

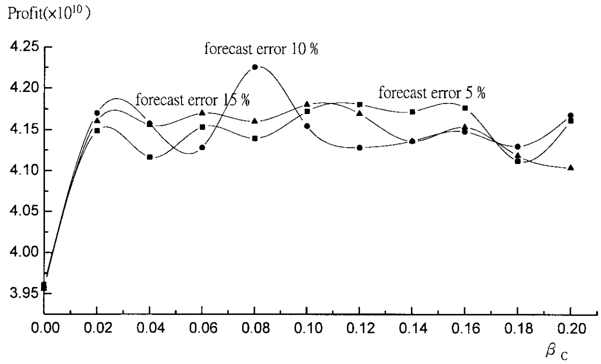

Fig. 4. Sensitivity analysis of β_c assuming that the purchase cost coefficient is imprecise.

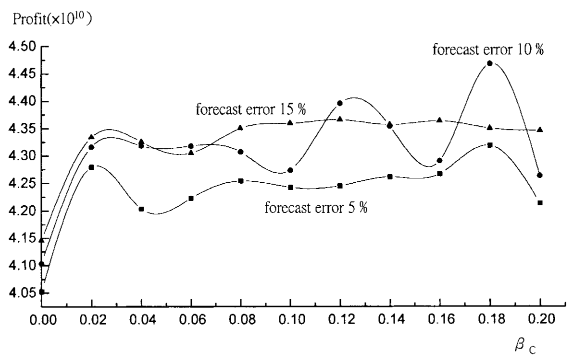

Fig. 5. Sensitivity analysis of β_c assuming that both the sale price and the purchase cost are imprecise.

Generally speaking, these three figures present very similar results. These models with $\beta_c \neq 0$ provide better results than the model with $\beta_c = 0$. The profit curve of 10%-forecast error has more fluctuations than the other two. However, no matter what is the value of β_c, the possibilistic model with a 10% forecast error provides good profit when compared with the models with 5% or 15% forecast error.

4.3 Summary

From the results of case studies presented above, several conclusions are made:

1. If the future market is very buoyant, the system with a large β_D to absorb the uncertain demand fluctuations is better than the system with a small β_D. The profit made out of product sales is also higher for a large β_D.
2. If the future market is depressed, the system with $\beta_D = 0.1$ to absorb uncertain demand fluctuations is best.
3. If the market demand can be precisely forecast, the crisp model is acceptable.
4. The possibility model possesses the ability to absorb uncertain factors appearing in the sale price or purchase cost. Therefore, the possibilistic model provides better planning than the crisp one.
5. The ability of the possibilistic model to absorb the imprecise sale price and purchase cost with mutual interaction is as good as its ability to absorb the imprecise sale price or the imprecise purchase cost coefficient.

5. Conclusion

The petrochemical and oil refinery production industry in Taiwan has entered a completely competitive free open market. In this study, we proposed a responsive and flexible manufacturing-to-sale planning model for CPC to deal with a variety of uncertain factors existing in the CPC planning environment. Fuzzy theory is adopted to formulate a possibilistic linear programming model. Several uncertainty factors such as demand and/or cost are included in the model. A possibilistic modelling procedure for searching for maximum profit is established in this paper. A manufacturing-to-sale planning case study is used to demonstrate this procedure. A complete plan, including purchase, production, inventory, and sales, can be obtained by the proposed method.

In the situation of imprecise market demand and varying production cost in the production environment, using demand and cost forecast error sensitivity analyses, the proposed possibilistic model can greatly enhance the profit using properly assigned values of β_c and β_D. The numerical simulation results indicate that the possibilistic model is superior when compared with conventional crisp models in a highly uncertain production environment. If the proposed model is used with the GRTMPS package currently used by the CPC, a full-scale production-to-marketing planning model can be constructed and used to create maximum operation profit for the company in the diversified and capricious future market environment.

Acknowledgement

The authors would like to thank the Chinese Petroleum Corporation and the National Science Council of the Republic of China for financially supporting this research under Contract no. 87CPC E 002 002.

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