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Demand and cost forecast error sensitivity analyses in aggregate production planning by possibilistic linear programming models

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A production management system contains many imprecise natures. The conventional deterministic and/or stochastic model in a computer integrated production management system (CIPMS) may not capture the imprecise natures well. This study examines how the imprecise natures in the CIPMS affect the planning results. Possibilistic linear programming models are also proposed for the aggregate production planning problem with imprecise natures. The proposed model can adequately describe the imprecise natures in a production system and, in doing so, the CIPMS can adapt to a variety of non-crisp properties in an actual system. For comparison, the classic aggregate production planning problem given by Holt, Modigliani, and Simon (HMS) is solved using the proposed possibilistic model and the crisp model of Hanssmann and Hess (HH). Perturbing the cost coefficients and the demand allows one to simulate the imprecise natures of a real world and evaluate the effect of the imprecise natures to production plans by both the possibilistic and the crisp HH approaches. Experimental results indicate that the possibilistic model does provide better plans that can tolerate a higher spectrum of imprecise properties than those obtained by the crisp HH model.

Keywords: Possibilistic linear programming, aggregate production planning, computer integrated production management system, imprecise properties, sensitivity study

1. Introduction

Communication within a production system must quickly react to fluctuations in market demand. Therefore, the computer integrated production management system (CIPMS) has gradually replaced the manual production management method. Although a highly effective tool for production management and control, CIPMS has several limitations before practical applications. For instance, many managers interested in implementing CIPMS are unable to achieve the expected performance, particularly for the aggregate production planning problem, because they must input many tasks, material resources and a considerable amount of time. This unsatisfactory performance is largely owing to that many fuzzy and/or imprecise natures are in the CIPMS environment, which the conventional approach cannot adequately describe.

Any plan affecting the outcomes of production rate and work force appear to be either a good or poor decision, depending on the market demand and/or product cost after the product is made and/or sold. A plan is not good or bad in itself, but only relative to the global economy during the period in which the influence of the plan is felt. Obviously, the global economy cannot be precisely known, necessitating that the plan be made under imprecise conditions.

An aggregate production planning problem, similar to many other real life problems, has many imprecise properties. This problem was set forth and solved by quadratic programming by Holt, Modigliani, and Simon [HMS] (1955), and Holt, Modigliani, and Muth (1956). Later, Hanssmann and Hess [HH] (1960) remodeled the problem by using a linear programming approach. Since then, the linear programming (LP) technique has been extensively applied to solve related problems. A standard LP model for the aggregate production-planning problem was also developed. At that time, fluctuations within a production system were much slower and, therefore, the imprecise natures did not significantly affect the LP planning results. Thus, the LP model was satisfactory for existing production systems. In addition to accelerating enterprise growth, advances in computer technology have also increased the imprecise natures of a production environment. Therefore, increasing inadequacy in the ability of the LP model to describe a system is reflected by its less satisfactory results.

The stochastic approach has been applied to study the imprecise nature of decision models since the late 1950s in which input data were given probability function distributions (Dantzig, 1955; Beale, 1955; Charnes, Cooper and Symonds, 1958; Charnes and Cooper, 1959). Since then, investigators are still solving aggregate production-planning problems by using stochastic control models (Wets, 1989). Love and Turner (1993) and Shen (1994) applied stochastic control models to solve the HMS problem. Bellman and Zadeh (1970) tacitly dealt the imprecise nature of an aggregate production planning problem by using equivalent randomness. However, the imprecise nature may be associated with classes in which a sharp transition from membership to non-membership does not exist. This circumstance does not reflect a real problem. In addition, applying random theory to certain optimization problem solving reduces computational efficiency. After the pioneering work of Zadeh (1978), possibility theory has found gradual acceptance in this type of research. Moreover, possibilistic decision-making models play an increasingly important role in resolving many related problems.

Possibilistic linear programming models possess many advantages over other stochastic linear programming models with respect to computational efficiency and flexible doctrines. Possibility theory focuses primarily on imprecision, which is intrinsic in natural languages and generally assumed to be possibilistic than probabilistic. Therefore, the term variable is frequently used in a more linguistic sense than in a strictly mathematical one. Possibilistic decision-making models have played a vital role in resolving practical decision-making problems. Yazenin (1987) and Buckley (1990) compared fuzzy/possibilistic and stochastic programming. Tanaka, Ichihashi and Asai (1984) formulated fuzzy linear programming problem based on the weighted average of the upper and lower limits and, in doing so, treated this average as a new objective function. Using the objective function as a triangular possibility distribution, Tanaks and Asai (1984) considered this objective function for a fuzzy constraint. Later, Rommelfanger, Hanuscheck and Wolf (1989) used $\gamma\beta$-level sets and established membership functions of the upper and lower bounds for each β-level set. This problem became Multiple Objective Linear Programming (MOLP) with $2 \times \gamma$ objectives, if nested types of membership function are considered. Rommelfanger (1989) also provided another auxiliary crisp satisfactory model with initially given fuzzy goals. Luhandjula (1987) adopted the β-level set concept and obtained a single objective semi-infinite linear programming problem. That investigation also proposed a cutting plane method to execute this semi-infinite program. Delgado, Verdegay and Vila (1987) considered a convex set with extreme points defined by the lower or upper bounds of the n β-level sets of the fuzzy coefficients. That investigation also obtained an auxiliary MOLP with 2^n objectives. Lai and Hwang (1992) maximized the possibility of obtaining a higher profit and minimized the risk of obtaining a lower profit for the MOLP problem with imprecise objective and/or constraint coefficients.

Although possibilistic models have been widely applied to modeling, performance evaluations have seldom been performed on these models as compared to the outcomes of crisp models. Hanssmann and Hess (1960) indicated that although their approach did not exactly minimize the expected cost when the D_i's have unbiased forecasts, the deviations from exact solutions are insignificant for any reasonably accurate forecast. Although such deviations might not have been significant previously, the deviations could possibly cause serious problems in the CIPMS.

In light of above discussion, this study examines how the previously neglected imprecise properties affect today's planning outcome in CIPMS. The possibility theory is also employed herein to establish a possibilistic MOLP model for solving problems involving imprecise demands and costs in aggregate production planning. In addition, the possibilistic model and the HH crisp model are used to model the classical HMS paint factory problem for a comparative study. By perturbing the cost coefficient and demand, the real world imprecise nature is simulated and the actual production cost for the plans obtained by the possibilistic model and the HH crisp model are estimated, respectively. Also studied herein is the extent to which the possibilistic model can absorb the imprecise nature of the real world.

2. Possibilistic multiple objective linear programming

In Zimmermann (1991), the classic model of linear programming can be stated as

$$\begin{aligned} &\min z = cx \\ &\text{s.t. } Ax \le b \\ &\qquad x \ge 0 \end{aligned} \tag{1}$$

The ''possibilistic'' version of the above linear programming model is given as

$$\begin{aligned} &\tilde{c}x \le z \\ &\tilde{A}x \le \tilde{b} \\ &x \ge 0 \end{aligned} \tag{2}$$

By substituting $\binom{c}{A} = B$ and $\binom{z}{b} = d$ into the above equation, it can be rewritten as

$$\begin{aligned} &\tilde{B}x \le \tilde{d} \\ &x \ge 0 \end{aligned} \tag{3}$$

Each of the $(m+1)$ rows in Equation (3) shall be represented by a possibilistic set. The membership functions, $\pi_i(x)$, and the possibilistic linear programming model are given as

$$\max \min\{\pi_i(x)\} = \max_{x \ge 0} \pi_{\tilde{D}}(x) \tag{4}$$

where

$$\pi_i(x) = \begin{cases} 1 & \text{if } B_i x \le d_i \\ 1 - \frac{B_i x - d_i}{p_i} & \text{if } d_i < B_i x \le d_i + p_i \\ & \quad i = 1, \ldots, m+1 \\ 0 & \text{if } B_i x > d_i + p_i \end{cases}$$

Introducing the variable λ to resemble $\pi_{\tilde{D}}$, one can obtain

$$\begin{aligned} &\max \lambda \text{ s.t. } \quad \lambda p_i + B_i x \le d_i + p_i \quad i = 1, \ldots, m+1 \\ &\qquad x \ge 0 \end{aligned} \tag{5}$$

If the optimal solution of Equation (5) is the vector (λ, x_0), x_0 is the maximum solution of Equation (4) for the model Equation (2).

2.1. *Imprecise constraints*

By considering the objective function as a crisp one, $Ax \le \tilde{b}$ as a set of possibilistic constraints, and adding another set of crisp constraints $Dx \le b'$ into the system, the aforementioned linear programming model becomes (Zimmermann, 1991):

$$\begin{aligned} &\min f(x) = c^T x \le z \\ &\text{s.t. } Ax \le \tilde{b} \\ &\quad Dx \le b' \\ &\quad x \ge 0 \end{aligned} \tag{6}$$

Since $\tilde{b}_i$ is an imprecise value, it can be simply represented by a triangular possibility distribution function. In other words, $\tilde{b}_i$ can be defined geometrically by three corner points including $((b_i^m)^T, 1)$, $((b_i^p)^T, 0)$ and $((b_i^o)^T, 0)$. Obviously, in this distribution function b_i^m is the most possible value (possiblity-1 if normalized). b_i^p (the most pessimistic value) and b_i^o (the most optimistic value) are both the least possible values. The membership functions of the possibilistic constraints (Kaufmann and Gupta, 1985) are defined as

$$\pi_{bi}(x) = \begin{cases} 0 & \text{if } A_i x \le b_i^p \\ \frac{A_i x - b_i^p}{b_i^m - b_i^p} & \text{if } b_i^p < A_i x \le b_i^m \\ \frac{b_i^o - A_i x}{b_i^o - b_i^m} & \text{if } b_i^m < A_i x \le b_i^o \\ 0 & \text{if } b_i^o < A_i x \end{cases} \tag{7}$$

where $\pi_{bi}(x)$ can be interpreted as the degree to which x fulfills (satisfies) the fuzziness unequally to $A_i x \le b_i$.

2.2. *Imprecise objective coefficients*

A linear programming model with imprecise objective coefficients can be stated as

$$\begin{aligned} &\min f(x) = \tilde{c}^T x = \sum_{i=1}^{n} \tilde{c}_i x_i \\ &\text{s.t. Ax} \le b \quad \text{and} \quad x \ge 0 \end{aligned} \tag{8}$$

where $\tilde{c}_i = (c_i^m, c_i^p, c_i^o)$ are the imprecise cost coefficients which can be represented by triangular possibility distributions. c_i^m is the most possible value (possibility of value one if normalized). c_i^p (the most pessimistic value) and c_i^o (the most optimistic value) are both the least possible values.

The possibility distribution functions, π_i, can be expressed in terms of the occurrence frequency of an event and can be taken as the analogous to the probability distribution functions. Thus, the objective function becomes

$$\min_{x\in X}\sum_{i-1}^{n}((c^m)^T x, (c^p)^T x, (c^O)^T x) \tag{9}$$

where $c^m = (c_1^m, c_2^m, \ldots, c_n^m)^T$, $c^p = (c_1^p, c_2^p, \ldots, c_n^p)^T$ and $c^o = (c_1^o, c_2^o, \cdots, c_n^o)^T$. This turns out to be a MOLP problem. Therefore, Equation (9) can be expressed as

$$\begin{aligned} &\min z_1 = (c^p)^T x\\ &\min z_2 = (c^m)^T x\\ &\min z_3 = (c^o)^T x\\ &\text{s.t. } Ax \le b \text{ and } x \ge 0\\ &\qquad x \in X \end{aligned} \tag{10}$$

To solve the above equations, both the fuzzy programming method (Zimmermann, 1991) and the normalization procedure (Lai and Hwang, 1992) are proposed for use. Positive and negative ideal solutions (PIS and NIS) of three objective functions (Hwang and Yoon, 1981) are constructed and given as

$$z_1^{PIS} = \min(c^p)^T x \quad z_1^{NIS} = \max(c^p)^T x, \tag{11}$$

$$z_2^{PIS} = \min(c^m)^T x \quad z_2^{NIS} = \max(c^m)^T x, \tag{12}$$

$$z_3^{PIS} = \min(c^o)^T x \quad z_3^{NIS} = \max(c^o)^T x, \tag{13}$$

The linear membership function of these objective functions can be computed as

$$\pi_{z1} = \begin{cases} 1 & \text{if } z_1 < z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & \text{if } z_1^{PIS} \le z_1 \le z_1^{NIS} \\ 0 & \text{if } z_1 > z_1^{NIS} \end{cases} \tag{14}$$

$$\pi_{z2} = \begin{cases} 1 & \text{if } z_2 > z_2^{PIS} \\ \frac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}} & \text{if } z_2^{PIS} \ge z_2 \ge z_2^{NIS} \\ 0 & \text{if } z_2 < z_2^{NIS} \end{cases} \tag{15}$$

$$\pi_{z3} = \begin{cases} 1 & \text{if } z_3 > z_3^{PIS} \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}} & \text{if } z_3^{PIS} \ge z_3 \ge z_3^{NIS} \\ 0 & \text{if } z_3 < z_3^{NIS} \end{cases} \tag{16}$$

Introducing the variable λ, we obtain the following model:

$$\begin{aligned} &\max \lambda\\ &\text{s.t. } \pi_{Zi}(x) \ge \lambda, \;\; i = 1, 2, 3\\ &Ax \le b \quad \text{and} \quad x \ge 0 \end{aligned} \tag{17}$$

If the optimal solution of Equation (17) is the vector (λ, x_0), x_0 is the minimum solution of the model Equation (8).

2.3. *Imprecise objective coefficients and constraints*

As stated above, an arbitrary possibilistic model of linear programming can be stated as

$$\begin{aligned} &\max f(x) = \tilde{c}^T x\\ &\text{s.t. } Ax \le \tilde{b}\\ &\quad Dx \le b'\\ &\quad x \ge 0 \end{aligned} \tag{18}$$

Equations (7) and (14)–(16) can specify the membership functions of objective functions and the constraints respectively. By employing Equations (5) and (17), (18) can be expressed by an equivalent single-objective linear programming model as

$$\begin{aligned} &\max \lambda\\ &\text{s.t. } \pi_{zi} \ge \lambda, i = 1, 2, 3\\ &\quad \pi_{bi} \ge \lambda, i = 1, 2, \ldots\\ &\quad Dx \le b'\\ &\quad x \ge 0\\ &\quad x \in X \end{aligned} \tag{19}$$

3. Possibilistic MOLP model of HMS problem

To model the HMS problem, the problem is restated again. Given monthly forecast demands for the product manufactured by a paint factory, of relevant interest is the monthly production rate and the work force level that minimizes the total cost of regular payroll, overtime, hiring, layoffs, inventory and backorders incurred during a given planning interval of several months.

Hanssmann and Hess (1960) proposed a linear programming model to solve the HMS problem. After introducing variables of hiring, layoff, regular manpower, overtime manpower, and backorder, the model can be expressed as

$$\min \sum_{t=1}^{T} C_I I_t + C_H H_t + C_F F_t + C_O O_t + C_B B_t + C_P P_t$$
$$\text{s.t. } R_t + I_{t-1} - B_{t-1} - I_t + B_t = D_t,$$
$$kR_t = P_t + O_t,$$
$$P_t - P_{t-1} = H_t - F_t,$$
$$I_t, H_t, F_t, O_t, B_t, P_t, R_t, D_t \geq 0,$$
$$t = 1, \ldots, T \qquad (20)$$

Notation used in this representation including variables, parameters, and constants are listed below for reference.

(1) model variables
- P_t = regular work force of month t,
- H_t = hiring work force of month t,
- F_t = layoff work force of month t,
- O_t = overtime work force of month t,
- D_t = demand of month t,
- I_t = inventory level of month t,
- B_t = backorder quantity of month t (in gallon/month),
- R_t = production quantity of month t (in gallon/month),

(2) parameters and constants
- C_P = regular labour cost per man month,
- C_H = hiring cost per man,
- C_F = layoff cost per man,
- C_O = overtime labor cost per man month,
- C_I = inventory cost per product,
- C_B = backorder cost per gallon,
- T = planning time periods (in months)
- $k = R_t/(P_t + O_t)$, oxtransformation constant from production quantity to labor required.

Coefficients in objective functions and constraints in the above model must be constants or precise numbers. In a real system, however, this is generally not the case. The above linear programming model does not exactly minimize the expected cost if demands have unbiased forecasts. If these cost coefficients in objective functions and product demands are taken as triangular possibility distribution functions, the possibilistic linear programming model can be established and presented as

$$\min \sum_{t=1}^{T} \tilde{C}_I I_t + \tilde{C}_H H_t + \tilde{C}_F F_t + \tilde{C}_O O_t + \tilde{C}_B B_t + \tilde{C}_P P_t$$
$$\text{s.t. } R_t + I_{t-1} - B_{t-1} - I_t + B_t = \tilde{D}_t,$$
$$kR_t = P_t + O_t,$$
$$P_t - P_{t-1} = H_t - F_t,$$
$$I_t, H_t, F_t, O_t, B_t, P_t, R_t, D_t \geq 0,$$
$$t = 1, \ldots, T \qquad (21)$$

where

$$\tilde{C}_I = (C_I^m, C_I^p, C_I^o) \quad \tilde{C}_H = (C_H^m, C_H^p, C_H^o),$$
$$\tilde{C}_F = (C_F^m, C_F^p, C_F^o) \quad \tilde{C}_O = (C_O^m, C_O^p, C_O^o),$$
$$\tilde{C}_B = (C_B^m, C_B^p, C_B^o) \quad \tilde{C}_P = (C_P^m, C_P^p, C_P^o),$$
$$\tilde{D}_t = (D_t^m, D_t^p, D_t^o)$$

By using the notion in Equations (1)–(19), the above model can be rewritten as

$$\max \lambda$$
$$\text{s.t. } \sum_{t=1}^{T} (C_I^m - C_I^p) I_t + (C_H^m - C_H^p) H_t + (C_F^m - C_F^p) F_t + (C_O^m - C_O^p) O_t + (C_B^m - C_B^p) B_t + (C_P^m - C_P^p) P_t + (Z_1^{NIS} - Z_1^{PIS})\lambda \leq Z_1^{NIS},$$
$$\sum_{t=1}^{T} C_I^m I_t + C_H^m H_t + C_F^m F_t + C_O^m O_t + C_B^m B_t + C_P^m P_t + (Z_2^{NIS} - Z_2^{PIS})\lambda \leq Z_2^{NIS},$$
$$\sum_{t=1}^{T} (C_I^o - C_I^m) I_t + (C_H^o - C_H^m) H_t + (C_F^o - C_F^m) F_t + (C_O^o - C_O^m) O_t + (C_B^o - C_B^m) B_t + (C_P^o - C_P^m) P_t] + (Z_3^{NIS} - Z_3^{PIS})\lambda \leq Z_3^{NIS},$$
$$R_t + I_{t-1} - B_{t-1} - I_t + B_t - (D_t^m - D_t^p)\lambda \geq D_t^p,$$
$$R_t + I_{t-1} - B_{t-1} - I_t + B_t + (D_t^o - D_t^m)\lambda \leq D_t^o,$$
$$kR_t = P_t + O_t,$$
$$P_t - P_{t-1} = H_t - F_t$$
$$I_t, H_t, F_t, O_t, B_t, P_t, R_t, D_t^o, D_t^m, D_t^p \geq 0,$$
$$t = 1, \ldots, T \qquad (22)$$

Surprisingly, the resulting possibilistic model is a purely linear model. A linear programming package LINDO (*L*inear *IN*teractive and *D*iscrete *O*ptimizer; Schrage, 1989) can, therefore, be used to solve the problem.

4. Sensitivity analyses

Having successfully presented the possibilistic linear programming model for solving the HMS problem in the above section, next relevant task is to assess the effectiveness of this model in accommodating the imprecise variations of a real system. Assume that the triangular possibility distributions of the cost coefficients are within $\pm\beta_C C^m$ and the distributions of the demands are within $\pm\beta_D D_t^m$, where β_C and β_D are any arbitrary numbers between 0 and 0.5. Tables 1 and 2 list the monthly forecast demands obtained from Mellichamp and Love (1978) and cost coefficients approximated by Kolenda (Barman and Tersine, 1991) for the crisp linear model, respectively. The initial conditions are given as $B_{t=0} = 57$ and $P_{t=0} = 81$. The planning time period, T, is set to 12 months. Both imprecise demands and imprecise unit cost are used for sensitivity analysis. The results are presented below.

4.1. *Sensitivity analysis of imprecise demands*

Numerical experiments were performed by using the demand forecast errors of 0%, 5%, 10%, 15%, 20%, 25%, 30%, 35%, and 40%, respectively. Actual twelve-month demands are generated following the given steps. Table 3 summarizes those results.

Step 1: Estimation of the mean, $\hat{\mu}_D$, of monthly demands.

$$\hat{\mu}_D = \sum_{t=1}^{T} D_t \Big/ T$$

where T is the length of the planning periods, and D_t, $t = 1, \ldots, T$, is the forecast demand of month t.

Step 2: Estimation of the standard deviation, $\hat{\sigma}_D$, of ε% forecast error in monthly demand

$$\hat{\sigma}_D = \varepsilon\% \times \hat{\mu}_D$$

Step 3: Generation of actual demand, S_t, of month t,

$$S_t = D_t + Z_t \hat{\sigma}_D$$

where Z_t is a random number from a standard normal distribution.

For comparison purpose, we solve the HH crisp model ($\beta_C = 0$ and $\beta_D = 0$) and the possibilistic model with $\beta_C = 0.1$ and $\beta_D = 0.05$ by LINDO, and tabulate the planned regular work force and regular production capacity for the two models in Table 4. The cost analyses for these two approaches at both 0% and 5% demand forecast error conditions are then tabulated in Table 5.

We assume that the given cost coefficients reflect the real one. In other words, we assume that the estimation errors of cost coefficients are 0%. By setting $\beta_C = 0.1$ and letting β_D to vary from 0 to 0.50 in the increment of 0.025, the aggregate production plans for different β_D's are evaluated by LINDO. For planned regular production capacity, the real production cost can be estimated by adjusting monthly overtime work force, inventory level and backorder quantity. The effects of various β D values (0 to 0.5) and percentages of demand forecast errors (0% to 40%) on production cost are estimated and plotted in Fig. 1.

Based on the above numerical results, we conclude the following:

(1) The production cost of a possibilistic plan is somewhat higher than that of the HH crisp plan when the demand forecast errors are not involved.

Table 1. Most likely monthly demand (D_t^m) for the planning period (Mellichamp and Love, 1978)

Month	1	2	3	4	5	6	7	8	9	10	11	12
Demand	430	447	440	316	397	375	292	458	400	350	284	400

Table 2. Most possible cost coefficients (C_t^m) for the planning period

Cost coeff.	C_I	C_F	C_H	C_O	C_B	C_P	K
Value	5.51	208.9	208.9	630	5.51	340	5.67

Table 3. Generated actual demands

Demand forecast error *Month*	*5%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*	*40%*
1	434	438	442	446	450	454	458	462
2	419	391	363	335	307	279	251	223
3	415	390	365	340	315	290	265	240
4	299	282	265	248	231	214	197	180
5	454	511	568	625	682	739	796	853
6	386	397	408	419	430	441	452	463
7	275	258	241	224	207	190	173	156
8	459	460	461	462	463	464	465	466
9	397	394	391	388	385	382	379	376
10	338	326	314	302	290	278	266	254
11	293	302	311	320	329	338	347	356
12	402	404	406	408	410	412	414	416

(2) When the demand forecast errors are relatively small (0% and 5%), the HH crisp model can generally provide a good plan.

(3) When the demand forecast errors are median (10% to 20%), the possibilistic model with a reasonable β_D value (0.1 to 0.3) can provide a good plan.

(4) When the demand forecast errors are relatively large (more than 25%), the possibilistic model with largest β_D value (0.5) can provide a good plan.

(5) In practice, a possibilistic model has a higher capacity to accommodate the imprecise nature of demands than that of the HH crisp model.

4.2. *Sensitivity analysis of imprecise cost coefficients*

To characterize the contribution of each cost item with respect to overall production cost, one specific cost item is selected to generate its actual cost coefficient with (% forecast error, and the remaining cost items are treated as no forecast errors involved. Real cost coefficients are generated by the following steps:

(1) Estimation of forecast cost coefficients, C_X, here X could be I, H, F, O, B or P.

(2) Estimation of the standard deviation, $\hat{\sigma}_{CX}$, of ε% forecast error for the cost coefficient

$$\hat{\sigma}_{CX} = \varepsilon\% \times C_X$$

(3) Generation of the real cost coefficient, C'_{Xt}, of month t,

$$C'_{Xt} = C_X + Z_t \hat{\sigma}_C$$

where Z_t is a random number from a standard normal distribution.

Table 4. Planned regular work force and production capacity using the HH crisp and possibilistic models for $\beta_C = 0.1$ and $\beta_D = 0.05$

	Month	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
HH crisp	Work force	81	81	74	66	66	66	66	66	61	61	61	61
	Production capacity	459	459	419	375	375	375	375	375	344	344	344	344
Possibilistic	Work force	81	80	70	64	64	64	64	64	64	64	64	64
	Production capacity	459	455	399	364	364	364	364	364	364	364	364	364

Table 5. Cost analyses using the HH crisp and possibilistic models for 0% and 5% demand forecast errors

Cost Items	*0% Demand forecast error*		*5% Demand forecast error*	
	HH crisp	*Possibilistic*	*HH crisp*	*Possibilistic*
Regular labor cost	275,236	275,226	275,236	275,226
Overtime labor cost	0	0	0	0
Hiring/layoff cost	4234	3509	4234	3509
Inventory cost	1989	3366	2160	2762
Backorder cost	0	0	0	0
Total cost	281,454	282,102	281,631	281,498

Numerical experiments are performed for the following cases including

(1) 20% of demand-forecast error, (2) $\beta_D = 0.2$, and (3) $\beta_C = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5$, and (4) cost coefficient forecast error $=$ 5% to 30%, respectively. Numerical results in Figs. 2–5, indicate that the effects of various values of β_C and different degrees of labor, layoff, inventory, and backorder cost coefficient forecast errors (from 5% to 30%) on production cost are insignificant. Herein, these two items are neglected since overtime and hiring work forces are unnecessary for these analyses.

From above numerical results, we conclude that the forecast errors of cost coefficient do not affect the production cost as significantly as those of the forecast errors of demand. This finding also indicates that the imprecise nature of the cost coefficients does not affect the plans obtained from the possibilistic MOLP.

5. Summary

Numerical results in this study demonstrate that a plan obtained by using a possibilistic approach tolerates a wider range of imprecise demands and also offers a lower production cost than the results obtained by using the HH crisp approach. In that respect, we can infer that the possibilistic approach can more effectively handle imprecise demands encountered in real world than the conventional approach. The imprecise cost coefficient, however, does not significantly affect the production cost at least in the HMS problem. This finding is possibly owing to that the effect from a single cost item is too small to be observed. More experiments should be performed for perturbing more than one cost item at one time. Barman and Tersine (1991) performed sensitivity study on imprecise cost coefficients of the same problem. According to their results, only two cost

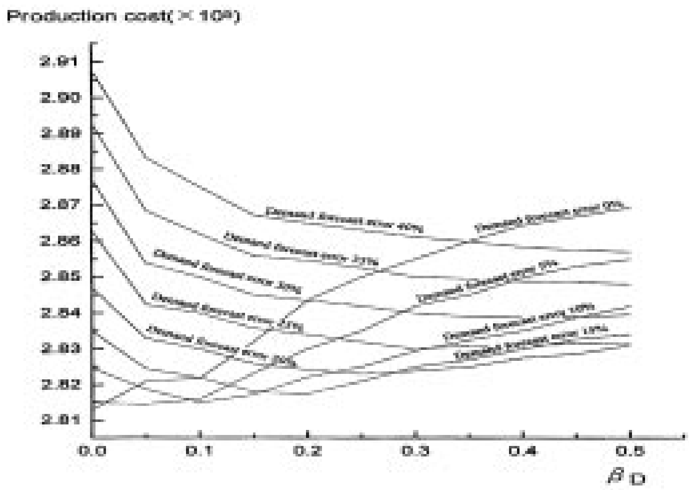

Fig. 1. Effects of various β_D and demand forecast errors on production cost.

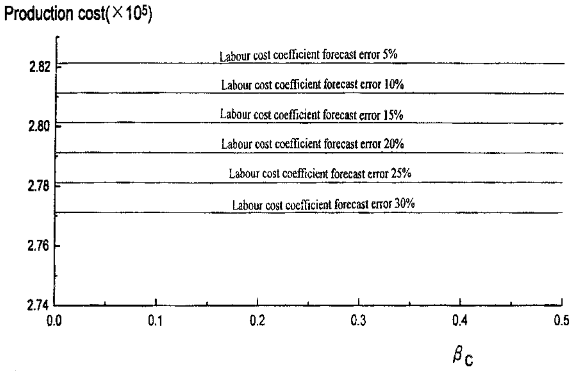

Fig. 2. Effects of various β_C and forecast errors of labor cost on production cost.

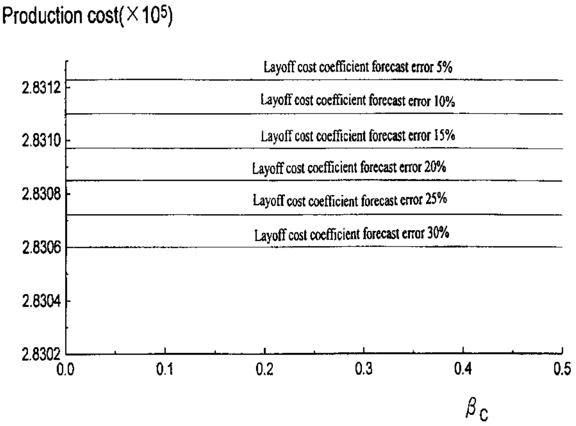

Fig. 3. Effects of various β_C and forecast errors of layoff cost on production cost.

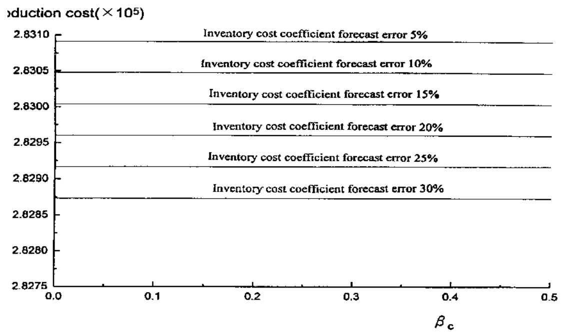

Fig. 4. Effects of various β_C and forecast errors of inventory cost on production cost.

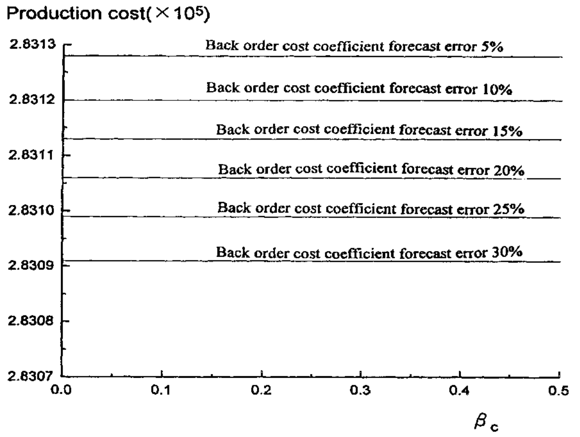

Fig. 5. Effect of various β_C and cost forecast errors of back orders on production cost.

parameters appear to be crucial to the production cost: (1) the production capacity per employee, and (2) the ideal level of ending inventory. The first one is taken as a fixed value of 5.67 (*K* value) in this study here. The second one is related to the demands. These results are correlate well with our modeling results.

6. Conclusions

CIPMS focus mainly on efficient management/administration to gain maximum profit. However, today's enterprise environment is full of fuzzy/imprecise natures. The performance of CIPMS is thus unsatisfactory by conventional approach. This study largely concentrates on the imprecise properties in the aggregate production planning and, in doing so, presents a possibilistic MOLP model to adapt to today's fast changes. The optimal production planning can therefore be obtained at the moment that the actual demand or the real cost is uncertain. For comparison, the proposed model and the HH crisp model are used to solve the HMS problem, respectively. Results in this study demonstrate that the possibilistic MOLP model provides better plans that can tolerate a higher spectrum of imprecise properties than the crisp model. Therefore, the proposed model enhances the ability of CIPMS.

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