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An observer-based state feedback control of parametrically excited systems

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Abstract

In the study of adaptive parameter identification, the parameter error dynamics resulting from the gradient algorithm represents a very special class of parametrically excited system, whose stability condition has been studied thoroughly. By utilizing this stability condition, one can develop a new active control design for a parametrically excited system which exhibits sustained but bounded oscillations due to the time-varying system matrix. The resultant observer-based state feedback control guarantees exponential decay of the state oscillation given that the system is both uniformly controllable and uniformly observable. The advantage of the proposed design is that it requires neither information of time derivatives of the parametric excitations, nor predicting future information of the parametric excitations. © 1997 Elsevier Science B.V.

Keywords: Linear time-varying system; Parametric excitation; Gradient algorithm; State feedback control; Observer

1. Introduction

In a parametrically excited system [10], the time-varying coefficients in the governing equation act as internal excitations that can cause sustained oscillations or even instability of the system. The parametric excitation is commonly categorized into two types: one is called the stationary excitation, and the other nonstationary excitation. For stationary excitation, the amplitudes and frequencies of the internal excitations are constant. Typical examples are systems with periodically time-varying parameters in the governing equation. For nonstationary excitation, the amplitudes and frequencies of the internal excitations vary as time evolves.

For systems with stationary parametric excitations, particularly for those with periodic parametric excitations, there have been a variety of control design approaches developed. Calico and Wiesel [2] propose an active modal control that can arbitrarily shift the position of one characteristic multiplier of the periodic system. Sinha and Joseph [14] use the Lyapunov–Floquet transformation to obtain a new system representation with a constant system matrix. However, the proposed control input is equal to the desired stabilizing control only in the least-squares sense. Other approaches include the optimal control using a periodic Riccati equation by Kano and Nishimura [6] and Bittanti et al. [1].

For systems with nonstationary parametric excitations, the pole-placement like control proposed by Wolovich [17], Follinger [5], and Valasek and Olgac [15] provides a sound theoretical solution to the control problem. However, their control designs require time derivatives of the parametric excitations up to the order of the system dimension. Since it is impossible to have "clean" measurement of the parametric excitations, the measurement noise will inevitably be amplified in the time differentiation process, and

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cause noticeable state oscillations in the closed-loop response. Other controls for systems with nonstationary parametric excitations are the LQ optimal control [7] and the controllability-grammian-based control [12]. However, they suffer from the disadvantage that the control engineers must be able to predict how the amplitudes and frequencies of the internal excitations vary in the future in order to calculate the desired control input at any present instant.

In this paper, a new approach is presented for the vibration control of a parametrically excited system whose state exhibits sustained but bounded oscillations. In this new approach, one uses the open-loop state transition matrix to transform the system into one with a *zero* system matrix, and then utilizes the "gradient algorithm" in adaptive parameter identification [13] to synthesize a stabilizing control. The same approach also leads to an observer design that asymptotically recovers the system state from the system output when full state measurement is not available. The resultant observer-based state feedback control can be applied to systems with stationary or nonstationary parametric excitations. Most importantly, it offers two advantages over other controls: (1) There is no need to predict *future* information of the parametric excitations. The control design requires only *past* and *present* information of the parametric excitations. (2) There is no need to take time derivatives of the parametric excitations. This avoids the noise amplification problem in the pole-placement like control.

2. Problem formulation

Consider a multivariable linear time-varying system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \quad x(0) = x_0, \\ y(t) &= C(t)x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the system output, the system matrix $A(t) \in \mathbb{R}^{n\times n}$, the input matrix $B(t) \in \mathbb{R}^{n\times m}$, and the output matrix $C(t) \in \mathbb{R}^{p\times n}$ are time-varying matrices whose elements are bounded, piecewise continuous functions of time. Notice that the system matrix $A(t)$, the input matrix $B(t)$ and the output matrix $C(t)$ are not restricted to be periodically time varying. Hence, the parametric excitations can be of the nonstationary type. It is assumed that due to the excitations of the time-varying parameters in the system matrix, the open-loop state trajectory $x(t)$ exhibits sustained but bounded oscillations. In other words, the state transition matrix $\Phi(t,t_0) \in \mathbb{R}^{n\times n}$ of the open-loop system remains bounded and nonzero for all t and t_0. Therefore, there exist two positive constants m_1 and m_2 such that for all t and t_0,

$$m_1 \leqslant \sigma_i[\Phi(t,t_0)] \leqslant m_2, \quad \forall i = 1,2,\ldots,n \tag{2}$$

where σ_i denotes the singular value [8] of a matrix, and the state transition matrix $\Phi(t,t_0)$ is defined by [4]

$$\begin{aligned} &\frac{\partial \Phi(t,t_0)}{\partial t} = A(t)\Phi(t,t_0), \\ &\Phi(t,t) = I, \quad \Phi(t_0,t) = \Phi^{-1}(t,t_0). \end{aligned} \tag{3}$$

The objective of this paper is to find an active vibration control to suppress the oscillatory behavior of the open-loop system under the condition that the only accessible signal in the system is the system output $y(t)$. In particular, an observer-based state feedback control

$$u(t) = -K(t)\hat{x}(t) \tag{4}$$

is considered, where $\hat{x}(t)$ is an estimate of the state $x(t)$. For the existence of a stabilizing control as in Eq. (4), it is assumed that the system (1) is uniformly controllable as well as uniformly observable as defined below [12].

Definition 1. The pair $(A(t),B(t))$ is uniformly controllable if there exist Δ, β_1 and $\beta_2 \in \mathbb{R}^+$ such that for all $t > 0$,

$$\beta_1 I \leqslant P_c(t) \leqslant \beta_2 I, \tag{5}$$

where $P_c(t) \in \mathbb{R}^{n\times n}$ is the controllability grammian defined by

$$P_c(t) \triangleq \int_{t-\Delta}^{t} \Phi(t-\Delta,\tau)B(\tau)B^{\mathrm{T}}(\tau)\Phi^{\mathrm{T}}(t-\Delta,\tau)\,\mathrm{d}\tau,$$

in which $\Phi(t,\tau)$ is the state transition matrix defined in Eq. (3).

Definition 2. The pair $(A(t),C(t))$ is uniformly observable if there exist Δ, γ_1 and $\gamma_2 \in \mathbb{R}^+$ such that for all $t > 0$,

$$\gamma_1 I \leqslant P_0(t) \leqslant \gamma_2 I, \tag{6}$$

where $P_0(t) \in \mathbb{R}^{n\times n}$ is the observability grammian defined by

$$P_0(t) \triangleq \int_{t-\Delta}^{t} \Phi^{\mathrm{T}}(\tau,t-\Delta)C^{\mathrm{T}}(\tau)C(\tau)\Phi(\tau,t-\Delta)\,\mathrm{d}\tau.$$

3. Preliminaries

The control and observer designs, in this paper, will utilize the well-known "gradient algorithm" originally developed in the adaptive parameter identification problem [9]. To study the convergence property of the gradient algorithm, one would need the following lemma, which states that the uniform observability property is invariant under output injection.

Lemma 1. *Consider the pair $(F(t),H(t))$, and its output-injected pair $(F(t)+L(t)H(t),H(t))$ with an output injection gain $L(t)$, where $F(t)\in\mathbb{R}^{n\times n}$, $H(t)\in\mathbb{R}^{p\times n}$, and $L(t)\in\mathbb{R}^{n\times p}$. Assume that for some $\Delta>0$, there exists a $\mu>0$ such that, for all $t>0$,*

$$\int_{t-\Delta}^{t}\|L(\tau)\|^2\,\mathrm{d}\tau\leqslant\mu.$$

If the observability grammian $P_0(t)$ of $(F(t),H(t))$, defined over time interval $[t-\Delta,t]$, satisfies $\eta_1 I\leqslant P_0(t)\leqslant\eta_2 I$, then the observability grammian $P_L(t)$ of $(F(t)+L(t)H(t),H(t))$ satisfies

$$P_L(t)\geqslant\sigma I\quad\text{where }\sigma=\frac{\eta_1}{(1+\sqrt{\mu\eta_2})^2}.\tag{7}$$

Proof. See Lemma 2.5.2 in [13]. □

Consider the parameter estimation error dynamics of the gradient algorithm:

$$\dot z(t)=-\gamma w(t)w^{\mathrm{T}}(t)z(t),\quad z(t)\in\mathbb{R}^n,\tag{8}$$

where γ is any positive constant, $z(t)$ represents the parameter estimation error, and $w(t)\in\mathbb{R}^{n\times m}$ is usually called the "regressor vector". The following theorem gives a sufficient condition on the exponential stability of the system (8).

Theorem 1. *If the regressor vector $w(t)$ is "persistently exciting" in the sense that there exist positive constants Δ, α_1, and α_2 such that*

$$\alpha_1 I\leqslant\int_{t-\Delta}^{t}w^{\mathrm{T}}(\tau)w(\tau)\,\mathrm{d}\tau\leqslant\alpha_2 I,\quad\forall t>0,\tag{9}$$

then the system (8) is exponentially stable in the sense that

$$\|z(k\Delta)\|^2\leqslant\rho^k\|z(0)\|^2,\quad k=1,2,\ldots,\tag{10}$$

where $0<\rho<1$, and

$$\rho=1-2\sigma\gamma,\quad\sigma=\frac{\alpha_1}{(1+\gamma\alpha_2\sqrt{n})^2}.\tag{11}$$

Proof. Firstly, note that the persistent excitation condition (9) is equivalent to the uniform observability condition of the pair $[0,w^{\mathrm{T}}(t)]$ according to Definition 2, and the observability grammian of $[0,w^{\mathrm{T}}(t)]$ satisfies

$$\alpha_1 I\leqslant P_0(t)\leqslant\alpha_2 I.$$

Secondly, observe that the pair $[-\gamma w(t)w^{\mathrm{T}}(t),w^{\mathrm{T}}(t)]$ is an output-injected pair from $[0,w^{\mathrm{T}}(t)]$ with an injection gain $L(t)=-\gamma w(t)$. Since this gain satisfies

$$\begin{aligned}\int_{t-\Delta}^{t}\|L(\tau)\|^2\,\mathrm{d}\tau&=\int_{t-\Delta}^{t}\|\gamma w(\tau)\|^2\,\mathrm{d}\tau\\&=\gamma^2\,\mathrm{tr}\left[\int_{t-\Delta}^{t}w(\tau)w^{\mathrm{T}}(\tau)\,\mathrm{d}\tau\right]\\&\leqslant\gamma^2\,\mathrm{tr}[\alpha_2 I]=n\gamma^2\alpha_2,\end{aligned}$$

where the inequality is due to Eq. (9), it follows from Eq. (7) in Lemma 1 that the observability grammian of $[-\gamma w(t)w^{\mathrm{T}}(t),w^{\mathrm{T}}(t)]$ satisfies

$$P_L(t)\geqslant\sigma I,\quad\sigma=\frac{\alpha_1}{(1+\gamma\alpha_2\sqrt{n})^2}.\tag{12}$$

One can now prove the exponential stability of the system (8) by choosing a Lyapunov function candidate $V(t)=\|z(t)\|^2$. Calculating the time derivative of V along the trajectory (8) yields

$$\dot V=-2\gamma z^{\mathrm{T}}(t)w(t)w^{\mathrm{T}}(t)z(t).$$

Integrating the above equation from $t-\Delta$ to t gives

$$\begin{aligned}&\|z(t)\|^2-\|z(t-\Delta)\|^2\\&\quad=-2\gamma\int_{t-\Delta}^{t}z^{\mathrm{T}}(\tau)w(\tau)w^{\mathrm{T}}(\tau)z(\tau)\,\mathrm{d}\tau.\end{aligned}\tag{13}$$

Let the state transition matrix of the system (8) be $\Psi(t,\tau)$; then $z(\tau)=\Psi(\tau,t-\Delta)z(t-\Delta)$. Substituting this relationship into Eq. (13) results in

$$\begin{aligned}&\|z(t)\|^2-\|z(t-\Delta)\|^2\\&\quad=-2\gamma z^{\mathrm{T}}(t-\Delta)\left(\int_{t-\Delta}^{t}\Psi^{\mathrm{T}}(\tau,t-\Delta)w(\tau)w^{\mathrm{T}}(\tau)\right.\\&\qquad\left.\times\Psi(\tau,t-\Delta)\,\mathrm{d}\tau\right)z(t-\Delta)\\&\quad=-2\gamma z^{\mathrm{T}}(t-\Delta)P_L(t)z(t-\Delta)\\&\quad\leqslant-2\sigma\gamma\|z(t-\Delta)\|^2,\end{aligned}$$

where the inequality is due to Eq. (12). Rearranging the equation, one obtains

$$\|z(t)\|^2 \leqslant \rho \|z(t-\Delta)\|^2, \quad \forall t \geqslant 0, \tag{14}$$

where ρ is as defined in Eq. (11). It is straightforward to verify that $0 < \rho(\gamma) < 1$ for all $\gamma > 0$. □

4. State feedback control design

In this section it is temporarily assumed that the system state $x(t)$ is accessible, and the goal is to find a state feedback control

$$u(t) = -K(t)x(t) \tag{15}$$

such that the closed-loop system (1), (15) is exponentially stable. The proposed control design starts with a coordinate transformation

$$x(t) = \Phi(t, t_0)z(t), \tag{16}$$

where $z(t)$ is the new state coordinate, and the transformation matrix is exactly the open-loop state transition matrix $\Phi(t, t_0)$ in Eq. (3). Note that $\Phi(t, t_0)$ is always invertible by Theorem 4-2 in [4]. Since $\Phi(t, t_0)$ is uniformly bounded as assumed in Eq. (2), the transformation (16) converts the stabilization problem of the system state $x(t)$ into that of the new state $z(t)$. According to Eqs. (1), (3) and (16), the governing equation of the new state $z(t)$ is given by

$$\dot{z}(t) = \Phi(t, t_0)^{-1}B(t)u(t). \tag{17}$$

Note carefully that in this new coordinate, the system matrix is identically zero, and this is due to the fact that the open-loop state transition matrix is used as the coordinate transformation matrix. Such an approach, called the *variation of parameter*, is commonly used in solving a nonhomogeneous linear differential equation [11]. For a system with a zero system matrix as in Eq. (17), Theorem 1 in the previous section immediately suggests that the control $u(t)$ be chosen as

$$\begin{aligned} u(t) &= -\gamma B^{\mathrm{T}}(t)\Phi^{-\mathrm{T}}(t, t_0)z(t) \\ &= -\gamma B^{\mathrm{T}}(t)\Phi^{-\mathrm{T}}(t, t_0)\Phi^{-1}(t, t_0)x(t), \end{aligned} \tag{18}$$

where γ can be any positive constant. With this choice, the transformed closed-loop dynamics becomes

$$\dot{z}(t) = -\gamma\Phi^{-1}(t, t_0)B(t)B^{\mathrm{T}}(t)\Phi^{-\mathrm{T}}(t, t_0)z(t), \tag{19}$$

which has exactly the same structure as that in Eq. (8) with $\Phi^{-1}(t, t_0)B(t)$ acting as the regressor vector.

The following lemma shows that the persistent excitation condition on the regressor vector $\Phi^{-1}(t, t_0)B(t)$, which is required for the exponential stability of the system (19), is guaranteed by the *uniform controllability* condition of the system (1).

Lemma 2. *If $(A(t), B(t))$ of the system (1) is uniformly controllable as shown by Eq. (5), the observability grammian $P_1(t)$ of the pair $(0, B^{\mathrm{T}}(t)\Phi^{-\mathrm{T}}(t, t_0))$ satisfies*

$$\begin{aligned} \frac{\beta_1}{m_2^2} I &\leqslant P_1(t) \\ &= \int_{t-\Delta}^{t} \Phi^{-1}(\tau, t_0)B(\tau)B^{\mathrm{T}}(\tau)\Phi^{-\mathrm{T}}(\tau, t_0)\,\mathrm{d}\tau \\ &\leqslant \frac{\beta_2}{m_1^2} I, \end{aligned} \tag{20}$$

where β_i's are as in Eq. (5) and m_i's as in Eq. (2).

Proof. Comparing the observability grammian $P_1(t)$ in Eq. (20) with the controllability grammian $P_c(t)$ of the system (1) in Eq. (5) shows that $P_1(t) = \Phi(t_0, t-\Delta)P_c(t)\Phi^{\mathrm{T}}(t_0, t-\Delta)$. Hence, given any constant vector x, one has

$$\beta_1 \|\Phi^{\mathrm{T}}(t_0, t-\Delta)x\|^2 \leqslant x^{\mathrm{T}}P_1(t)x \leqslant \beta_2 \|\Phi^{\mathrm{T}}(t_0, t-\Delta)x\|^2,$$

due to Eq. (5). Since $\Phi(t_0, t-\Delta) = \Phi^{-1}(t-\Delta, t_0)$, it follows from Eq. (3) that

$$\frac{1}{m_2}\|x\| \leqslant \|\Phi^{\mathrm{T}}(t_0, t-\Delta)x\| \leqslant \frac{1}{m_1}\|x\|.$$

Therefore,

$$\frac{\beta_1}{m_2^2}\|x\|^2 \leqslant x^{\mathrm{T}}P_1(t)x \leqslant \frac{\beta_2}{m_1^2}\|x\|^2. \qquad \square$$

The following theorem, which is the main result of this paper, shows that the proposed control (18) ensures exponential stability of the closed-loop system.

Theorem 2. *Consider the system (1) and the state feedback control (18). If the system (1) is uniformly controllable, and the open-loop state transition matrix satisfies the inequalities (2), the closed-loop system state $x(t)$ converges to zero exponentially.*

Proof. Based on Lemma 2 and Theorem 1, one concludes immediately from Eq. (19) that the transformed state $z(t)$ decays to zero exponentially in the sense that

$$\|z(k\Delta)\|^2 \leqslant \rho^k \|z(0)\|^2, \quad k = 1, 2, \ldots, \tag{21}$$

where $0 < \rho < 1$, and

$$\rho = 1 - 2\sigma\gamma, \quad \sigma = \frac{m_1^4 \beta_1}{m_2^2(m_1^2 + \gamma\beta_2\sqrt{n})^2}, \tag{22}$$

in which Eq. (22) is obtained from Eq. (12) with $\alpha_1 = \beta_1/m_2^2$ and $\alpha_2 = \beta_2/m_1^2$. Since $\|x(t)\| \leqslant m_2\|z(t)\|$ due to Eqs. (2) and (16), $x(t)$ also converges to zero exponentially. □

Remark. According to Theorem 2, the closed-loop system (1), (18) is exponentially stable. Therefore, there exists a quadratic Lyapunov function for the closed-loop system (see [16, Theorem 64]). Then, by quoting Theorem 121 in Callier and Desoer [3], one can prove that the proposed control (18) is robust with respect to small parametric uncertainties. In other words, even if there exist small errors in on-line measuring or identifying the system matrices $A(t)$ and $B(t)$, the proposed control (18) and (3) can still stabilize the system.

5. Observer design

In this section, it is assumed that the only accessible signal in the system (1) is the system output $y(t)$. In order to estimate the system state $x(t)$, a conventional Luenberger-type observer is adopted:

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + L(t)(y(t) - C(t)\hat{x}(t)),$$
$$\hat{x}(0) = \hat{x}_0, \tag{23}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is an estimate of the system state $x(t)$, and $L(t) \in \mathbb{R}^{n\times p}$ is the observer feedback gain matrix to be determined so that $\hat{x}(t)$ approach $x(t)$ exponentially. Denote the state estimation error by $\tilde{x} = \hat{x} - x$, and subtract Eq. (23) from Eq. (1) to yield the state estimation error dynamics

$$\dot{\tilde{x}} = [A(t) - L(t)C(t)]\tilde{x}. \tag{24}$$

To design the observer feedback gain $L(t)$, the same coordinate transformation as in the control design is applied to the estimation error dynamics (24):

$$\tilde{x}(t) = \Phi(t, t_0)\tilde{z}(t),$$

where $\tilde{z}(t)$ defines the new coordinate, and $\Phi(t, t_0)$ as before is the state transition matrix of $A(t)$. The governing equation of $\tilde{z}(t)$ then becomes

$$\dot{\tilde{z}} = -\Phi^{-1}(t, t_0)L(t)C(t)\Phi(t, t_0)\tilde{z}.$$

Theorem 1 suggests that the observer feedback gain $L(t)$ be chosen as

$$L(t) = v\Phi(t, t_0)\Phi^{\mathrm{T}}(t, t_0)C^{\mathrm{T}}(t), \quad v > 0, \tag{25}$$

where v can be any positive constant. The resultant estimation error dynamics becomes

$$\dot{\tilde{z}}(t) = -v\Phi^{\mathrm{T}}(t, t_0)C^{\mathrm{T}}(t)C(t)\Phi(t, t_0)\tilde{z}(t), \tag{26}$$

which again has the same structure as the parameter estimation error dynamics of the gradient algorithm in Eq. (8) with now the regressor vector being $\Phi^{\mathrm{T}}(t, t_0)C^{\mathrm{T}}(t)$.

The following lemma shows that this regressor vector $\Phi^{\mathrm{T}}(t, t_0)C^{\mathrm{T}}(t)$ is persistently exciting if the system (1) is *uniformly observable.*

Lemma 3. *If $(A(t), C(t))$ of the system (1) is uniformly observable as shown in Eq. (6), the observability grammian $P_2(t)$ of the pair $(0, C(t)\Phi(t, t_0))$ satisfies*

$$\gamma_1 m_1^2 I \leqslant P_2(t) = \int_{t-\Delta}^{t} \Phi^{\mathrm{T}}(\tau, t_0)C^{\mathrm{T}}(\tau)C(\tau)\Phi(\tau, t_0)\,\mathrm{d}\tau$$
$$\leqslant \gamma_2 m_2^2 I, \tag{27}$$

where γ_i's are as in Eq. (6) and m_i's as in Eq. (2).

The proof of Lemma 3 duplicates that of Lemma 2 by noticing that $P_2(t) = \Phi^{\mathrm{T}}(t-\Delta, t_0)P_0(t)\Phi(t-\Delta, t_0)$. With the persistent excitation condition (27), one can now conclude the exponential stability of the state estimation error dynamics (24) by quoting Theorem 1.

Theorem 3. *Consider the state estimation error dynamics (24) and (25). If the system (1) is uniformly observable, and the open-loop state transition matrix satisfies the inequalities (2), the state estimation error $\hat{x}(t) - x(t)$ converges to zero exponentially.*

Proof. The proof follows exactly that of Theorem 2, and is omitted. □

Finally, it is remarked that when the only accessible signal is the system output $y(t)$, the state feedback control in Section 4 should be replaced by an observer-based state feedback control

$$u(t) = -K(t)\hat{x}(t), \tag{28}$$

where $K(t)$ is as designed in Eq. (18), and $\hat{x}(t)$ is obtained from the observer (23). Under the uniform

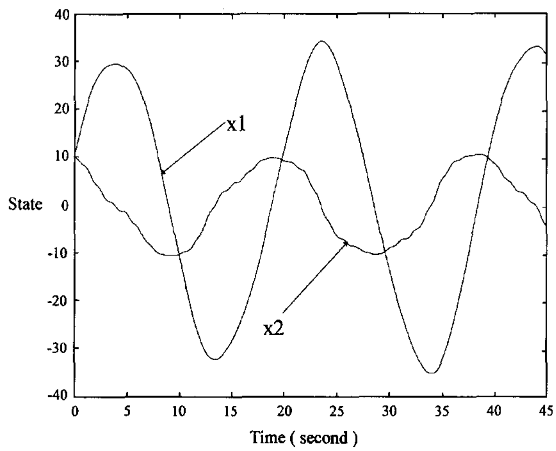

Fig. 1. Time history of open-loop response.

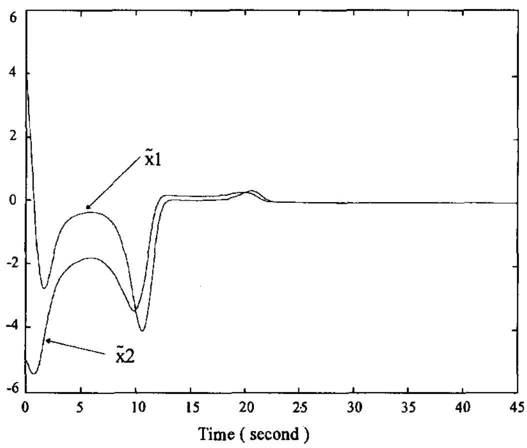

Fig. 2. Time history of state estimation errors.

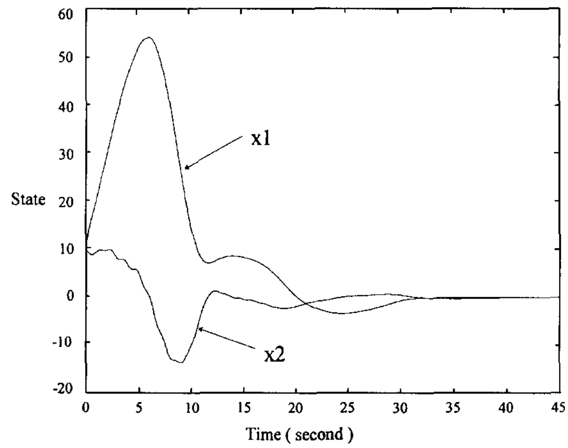

Fig. 3. Time history of system states.

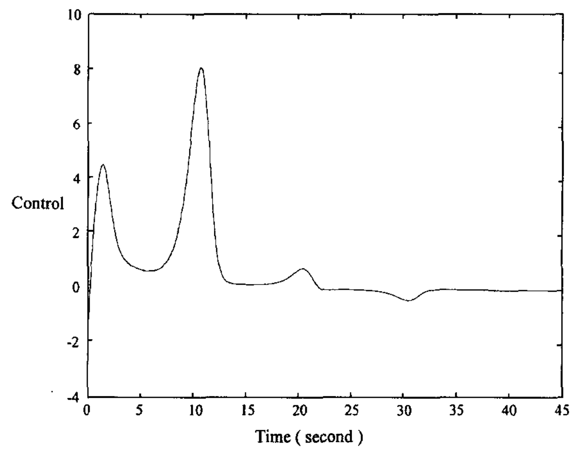

Fig. 4. Time history of control input.

controllability and observability assumptions and by quoting Theorems 2, 3, and the well-known Separation Property [7], it can be shown that the closed-loop system state under the control (28) converges to zero exponentially.

6. Simulation example

Consider the system (1) with

$$A(t)=\begin{bmatrix} 0 & 1 \\ 0.1-0.1\sin(\pi t)\cos(2t) & 0 \end{bmatrix},$$

$$B(t)=\begin{bmatrix} 0 \\ 1 \end{bmatrix},\quad C(t)=[1 \;\; 0].$$

Fig. 1 shows the open-loop state response ($u=0$), which exhibits bounded and sustained oscillation. The system is simulated under the proposed observer-based state feedback control with $\gamma=0.5$ in Eq. (18), $v=0.5$ in Eq. (25), and the initial conditions $x^{\mathrm{T}}(0)=[10,10]$, $\hat{x}^{\mathrm{T}}(0)=[15,5]$. Fig. 2 shows the time history of the state estimation error, and Fig. 3 the closed-loop system state. Both converge to zero as predicted by the analysis in this paper. Fig. 4 shows the time history of the control input.

7. Conclusions

This paper proposes a new control design approach for a class of parametrically excited systems whose state exhibits sustained but bounded oscillations. The new approach utilizes the gradient algorithm, which is originally used in the parameter identification problem, to find stabilizing control and observer feedback gains. It is worth mentioning that conventionally for a system with time-varying parameters, the observer feedback gain depends on past and present information of the time-varying parameters, and the control feedback gain, which is obtained as a dual result of the observer design, depends on *future* information of the time-varying parameters. Notice that for the proposed design in this paper, both the observer feedback gain in Eq. (25) and the control feedback gain in Eq. (18) depend only on past and present information of the time-varying parameters.

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