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Static Output Feedback Control for Periodically Time-Varying Systems

Min-Shin Chen and Yong-Zhi Chen

***Abstract*—Most control designs for periodically time-varying systems use either full-state feedback or observer-based state feedback. In this paper, it is shown that *static* output feedback is sufficient for the exponential stabilization of a periodical system under both the controllability and observability assumptions. In fact, by incorporating a new generalized hold function in the control design, one is able to arbitrarily shift all the Poincaré exponents of the periodical system. Most importantly, the control signal is guaranteed to be *continuous* in time while the control signal from previous designs may be discontinuous.**

***Index Terms*—Continuous-time system, generalized hold function, periodically time-varying system, Poincaré exponents, static output feedback.**

I. Introduction

An important class of linear time-varying systems in the physical world is the class of periodical systems, in which the system parameters vary periodically. Analysis for such systems has been done thoroughly in the past [1], [2]. One of the most important results is summarized in the Floquet theory, which states that the stability property of a linear periodical system can be determined by n constant numbers called the Poincaré exponents, where n is the dimension of the system. As in the time-invariant case, if all the Poincaré exponents are in the open left-half plane, the periodical system is exponentially stable. If at least one of the Poincaré exponents is in the open right-half plane, the system is unstable.

For the stability synthesis of periodical systems, most control designs are based on the assumption that all the state variables are accessible for measurement. Among these, the earliest approach is the LQ optimal control, in which one solves a periodical Riccati equation to obtain a stabilizing state feedback control [3], [4]. Another approach is the modal control proposed in [5] which can arbitrarily shift only one of the Poincaré exponents of the system. Later, a layer of modal controllers is suggested to shift *all* the Poincaré exponents [6]. Recently, the generalized hold function design, originally developed in [7], is applied to the state feedback control of a periodical system [8]. However, the resultant control signal may have large discontinuities in time. In practice, such large discontinuities are either unacceptable under the actuator constraint or undesirable due to the possible excitation of high-frequency unmodeled dynamics. Even though an attempt has been made to make the control signal continuous, its success is obstructed by a singularity problem [8].

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The authors are with the Department of Mechanical Engineering, National Taiwan University, Taipei 106, Taiwan, R.O.C. (e-mail: mschen@ccms.ntu.edu.tw).

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In this paper, a new design is proposed to avoid discontinuities in the control signal. Furthermore, it is shown that when the periodical system is both controllable and observable, simple *static output* feedback control is sufficient for the arbitrary assignment of all the Poincaré exponents (note that full *state* feedback is required in [8]). The key elements in the new control design are the well-known Floquet transformation [5] and a new generalized hold function design. This paper is arranged as follows. In Section II, the definition of Poincaré exponents for a periodical system is presented. In Section III, a *discontinuous* output feedback control is developed to assign the Poincaré exponents of the closed-loop system, and the control design is further modified in Section IV in order to remove discontinuities in the control signals.

II. Stability Analysis for Periodical Systems

Consider the stability analysis of the following system:

$$\dot{x}(t) = A(t)x(t) \tag{1}$$

where $x(t) \in R^n$ is the state vector and the system matrix $A(t) \in R^{n\times n}$ is T-periodic in the sense that

$$A(t+T) = A(t), \qquad \forall t > 0.$$

In the famous Floquet theory [1], the stability property of (1) is studied through a state transformation into a new coordinate, on which the system matrix becomes time invariant. Such a transformation, called the *Floquet transformation*, is given by

$$z(t) = P(t)x(t), \qquad P(t) = e^{Jt}\Phi^{-1}(t,0) \tag{2}$$

where $\Phi(t,0)$ is the state transition matrix [2] of (1), satisfying

$$\frac{\partial \Phi(t,\tau)}{\partial t} = A(t)\Phi(t,\tau), \qquad \Phi(t,t) = I, \qquad \Phi(\tau,t) = \Phi^{-1}(t,\tau) \tag{3}$$

and J is a constant matrix given by

$$J = \frac{1}{T}\ln \Phi(T,0). \tag{4}$$

From (1)–(3), the periodical system (1) has a *constant* representation in the new coordinate

$$\dot{z}(t) = Jz(t). \tag{5}$$

One can verify (see [2]) that the state transformation matrix $P(t)$ in (2) is also T-periodic and remains uniformly bounded and nonsingular. The stability property of the periodical system (1) can then be inferred from that of the constant system (5). In the literature, the eigenvalues of the constant matrix J in (5) are referred to as the *Poincaré (or characteristic) exponents*

$$\text{P.E.} \triangleq \lambda_i(J) = \frac{1}{T}\ln \lambda_i[\Phi(T,0)] \tag{6}$$

where the second equality results from (4). The condition for exponential stability of (1) is thus

$$\mathrm{Re}[\lambda_i(J)] < 0 \tag{7}$$

or equivalently

$$|\lambda_i[\Phi(T,0)]| < 1$$

due to (7), where $\Phi(T,0)$ is called the *monodromy matrix* [8]. To end this section, note that from (2)–(4) one can derive the identity

$$\Phi(T,t) = e^{J(T-t)}P(t) \tag{8}$$

which will be used in subsequent sections.

III. Stability Synthesis by Discontinuous Control

Consider now a periodical system with control

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \qquad x(0) = x_0 \\ y(t) &= C(t)x(t) \end{aligned} \tag{9}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ the control input, and $y(t) \in R^q$ the system output. It is assumed that the only accessible signal is the system output $y(t)$ and that $A(t) \in R^{n\times n}, B(t) \in R^{n\times p}, C(t) \in R^{q\times n}$ are all T-periodic; i.e.,

$$(A(t+T), B(t+T), C(t+T)) = (A(t), B(t), C(t)) \quad \forall t > 0.$$

The objective in this section is to find a stabilizing *static output* feedback control $u(t)$ that can arbitrarily assign the locations of the Poincaré exponents of the closed-loop system. For this purpose, the following assumptions are required.

A1) The pair $(A(t), B(t))$ is controllable in the sense that its controllability grammian [1] W defined on the time interval $[0, T]$ is positive definite, where

$$W = \int_0^T \Phi(T,\tau)B(\tau)B^T(\tau)\Phi^T(T,\tau)\,d\tau \tag{10}$$

and $\Phi(t,\tau)$ is the open-loop state transition matrix of (1).

A2) The constant pair $(\Phi(T,0), C_0)$ is observable [9], where $\Phi(T,0)$ is the open-loop monodromy matrix and $C_0 \triangleq C(0) = C(kT)$.

The proposed control design proceeds as follows.

Step I: Choose n Poincaré exponents ω_i (real or in complex conjugate pairs) with $\mathrm{Re}(\omega_i) < 0$. Calculate the eigenvalues for the closed-loop monodromy matrix based on (6)

$$\lambda_i^c = e^{T\omega_i} \tag{11}$$

and then use the pole placement algorithm [9] to find a constant matrix $L \in R^{n\times q}$ so that

$$\lambda_i(\Phi(T,0) + LC_0) = \lambda_i^c \tag{12}$$

where $\Phi(T,0)$ and C_0 are as in Assumption A2). Note that Assumption A2) guarantees the existence of L in (12) for any choice of the value λ_i^c.

Step II: Construct a T-periodic generalized hold function $G(t) \in R^{p\times q}$

$$G(t) = B^T(t)\Phi^T(T,t)W^{-1}L, \;\; t \in [0,T) \quad \text{and} \quad G(t+T) = G(t) \tag{13}$$

where $\Phi(T,t)$ and W are as in (10). Note that Assumption A1) guarantees the invertibility of W in (13).

Step III: Set the control input to be

$$u(t) = G(t)y(kT), \qquad t \in [kT, kT+T) \tag{14}$$

where $y(kT)$ is the sampled system output with a sampling period T and $G(t)$ is in (13).

Notice that in the above control design (13), one needs to calculate the open-loop state transition matrix $\Phi(T,\tau)$ over the entire period $\tau \in [0,T]$. Such computation becomes difficult when the period is long or when the system dimension is large. For an efficient and accurate numerical algorithm in calculating the state transition matrix, one may refer to [10] and [11].

Theorem 1: Under Assumptions A1) and A2), the static output feedback control (14) stabilizes (9) exponentially. Furthermore, the closed-loop Poincaré exponents are located as specified in *Step I* in the design procedure.

Proof: Apply the Floquet transformation (2) to the controlled system (9), obtaining

$$\begin{aligned} \dot{z}(t) &= Jz(t) + \bar{B}(t)u(t) \\ y(t) &= \bar{C}(t)z(t) \end{aligned} \tag{15}$$

where J is given by (4), and both the new input matrix $\bar{B}(t) = P(t)B(t)$ and output matrix $\bar{C}(t) = C(t)P^{-1}(t)$ are T-periodic since $P(t)$, $B(t)$, and $C(t)$ are all T-periodic.

Discretizing the transformed system (15) with a sampling period T yields

$$z((k+1)T) = \Phi(T,0)z(kT) + \mathbf{R}[u(\tau)] \tag{16}$$

where $\Phi(T,0)$ is the open-loop monodromy matrix and $\mathbf{R}[\cdot]$ the controllability map [2] defined by

$$\begin{aligned} \mathbf{R}[u(\tau)] &= \int_{kT}^{(k+1)T} e^{J[(k+1)T-\tau]}\bar{B}(\tau)u(\tau)\,d\tau \\ &= \int_0^T e^{J(T-\tau)}P(\tau)B(\tau)u(\tau+kT)\,d\tau \\ &= \int_0^T \Phi(T,\tau)B(\tau)u(\tau+kT)\,d\tau \end{aligned} \tag{17}$$

where the identity (8) has been used to obtain the last equality. Since $P(kT) = I$ for all $k \geq 0$, it follows from (2) that $x(kT) = z(kT)$. Hence, (16) becomes

$$x((k+1)T) = \Phi(T,0)x(kT) + \mathbf{R}[u(\tau)]. \tag{18}$$

Substituting the control (14) and (13) into (18) gives

$$\begin{aligned} x((k+1)T) &= \Phi(T,0)x(kT) + \mathbf{R}[G(\tau)C_0x(kT)] \\ &= (\Phi(T,0) + \mathbf{R}[G(\tau)]C_0)x(kT). \end{aligned} \tag{19}$$

One can verify that the T-periodic generalized hold function $G(t)$ in (13) satisfies

$$\begin{aligned} \mathbf{R}[G(\tau)] &= \int_0^T \Phi(T,\tau)B(\tau)G(\tau+kT)\,d\tau \\ &= \int_0^T \Phi(T,\tau)B(\tau)G(\tau)\,d\tau = L. \end{aligned} \tag{20}$$

Hence, the discretized closed-loop dynamics (19) becomes

$$x((k+1)T) = (\Phi(T,0) + LC_0)x(kT).$$

The closed-loop monodromy matrix $\Phi_c(T,0)$ is thus given by $\Phi(T,0) + LC_0$, and the Poincaré exponents are now successfully relocated as desired

$$\begin{aligned} \text{P.E.} &= \frac{1}{T}\ln\lambda_i[\Phi_c(T,0)] = \frac{1}{T}\ln\lambda_i(\Phi(T,0)+LC_0) \\ &= \frac{1}{T}\ln\lambda_i^c = \frac{1}{T}\ln e^{T\omega_i} = \omega_i \end{aligned}$$

where the third and fourth equations are obtained via (11) and (12). Consequently, the discretized state $x(kT)$ converges to zero exponentially.

To examine the intersampling behavior of the system state $x(t)$, one can discretize the closed-loop system (14) and (15), from $t = kT$ to $t = kT + s, s \in (0,T)$, to obtain

$$x(s) = P^{-1}(s)\left[\Phi(s,0) + \int_0^s \Phi(s,\tau)B(\tau)G(\tau)d\tau \cdot C_0\right]x(kT)$$

where (2) and (8) have been used in deriving the equation, and $P(s)$ is the T-periodic transformation matrix in (2). Since $x(kT)$

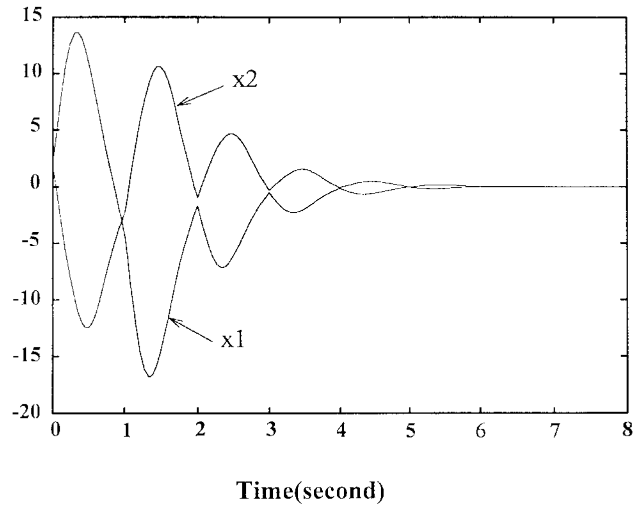

Fig. 1. State response with discontinuous control (14).

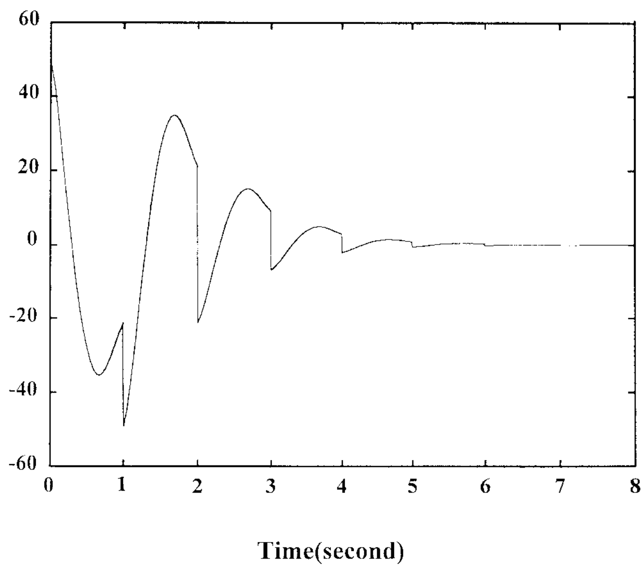

Fig. 2. Control signal with discontinuous control (14).

converges to zero exponentially, and all the other terms on the right-hand side of the above equation are uniformly bounded functions of $s, s \in (0, T)$, one concludes that the intersampling state $x(s)$ remains uniformly bounded and converges to zero exponentially as k approaches infinity. □

A simple simulation example is provided below to verify the proposed control design.

Example 1: Consider a periodic system, which is open-loop unstable

$$\dot{x}(t) = \begin{bmatrix} 1+\cos 2\pi t & 0 \\ 0 & 2+\sin 2\pi t \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$
$$y(t) = [2+\sin 4\pi t \quad -9+\sin 4\pi t] x(t).$$

The period of the system is 1 s. The proposed control (14) is applied to the system with the initial condition $x^T(0) = [2, 2]$. The design parameters in Step I are $\omega_1 = \ln 0.1$ and $\omega_2 = \ln 0.3$. Fig. 1 shows the time history of the system state, which converges exponentially as predicted by Theorem 1, and Fig. 2 shows the control input.

IV. Stability Synthesis by Continuous Control

Fig. 2 in Example 1 reveals a problem with the control design based on the generalized hold function: the control signal $u(t)$ in (14) has large discontinuities at the sampling instants $t = kT$. Since control with large discontinuities is either not implementable in practice or unacceptable from the robustness consideration, the objective of this section is to modify the previous control design so that assignment of the Poincaré exponents can be achieved by a *continuous* control input.

The approach adopted here is to find a new generalized hold function $\bar{G}(t)$ to replace $G(t)$ in the discontinuous control (14), with $\bar{G}(t)$ now satisfying

$$\bar{G}(kT) = 0, \qquad \forall k = 0, 1, 2, \cdots. \tag{21}$$

This will force the control input $u(t)$ to be continuous at any time instant $t = kT$; in fact, according to (14) and (21), one has

$$u(t^-) = u(t^+) = 0 \quad \text{at} \quad t = kT. \tag{22}$$

Equation (21) can be achieved by the following modified design procedure.

Step I and Step II: They are the same as in the previous section.

Step III: Choose any two T-periodic matrix functions $G_1(t) \in R^{p\times q}$ and $G_2(t) \in R^{p\times q}$ that are continuous on $(0, T)$. Calculate $\mathbf{R}[G_1(\tau)]$ and $\mathbf{R}[G_2(\tau)]$, where $\mathbf{R}[\cdot]$ is the controllability map in (17). Denote the two resultant constant matrices by

$$L_1 \triangleq \mathbf{R}[G_1(\tau)] \in R^{n\times q}, \qquad L_2 \triangleq \mathbf{R}[G_2(\tau)] \in R^{n\times q}. \tag{23}$$

Step IV: Construct the following two T-periodic matrix functions from $G_1(t)$ and $G_2(t)$:

$$G_{01}(t) = G_1(t) - B^T(t)\Phi^T(T,t)W^{-1}L_1, \quad t \in [0, T) \tag{24}$$
$$G_{02}(t) = G_2(t) - B^T(t)\Phi^T(T,t)W^{-1}L_2, \quad t \in [0, T) \tag{25}$$

where $B(t)$, $\Phi(T, t)$, and W are as in (13) and L_1 and L_2 in (23).

Step V:

Case a) When the number of inputs is no less than that of outputs $(p \geq q)$, determine two constant matrices α_1 and $\alpha_2 \in R^{p\times p}$ from the equations

$$G(0^+) + \alpha_1 G_{01}(0^+) + \alpha_2 G_{02}(0^+) = 0 \tag{26}$$
$$G(T^-) + \alpha_1 G_{01}(T^-) + \alpha_2 G_{02}(T^-) = 0 \tag{27}$$

where $G(t)$ is given by (13) and $G_{01}(t)$ and $G_{02}(t)$ by (24) and (25). Solutions α_1 and α_2 in (26) and (27) exist if the following matrix is full rank:

$$\text{rank}\begin{vmatrix} G_{01}(0^+) & G_{01}(T^-) \\ G_{02}(0^+) & G_{02}(T^-) \end{vmatrix} = 2q.$$

If this condition is not satisfied for the $G_1(t)$ and $G_2(t)$ chosen in Step III, one can simply choose a different pair of $G_1(t)$ and $G_2(t)$ until the required rank condition is satisfied. Notice that there are *infinite* degrees of freedom in choosing $G_1(t)$ and $G_2(t)$ for no constraint is imposed on them except continuity on the interval $(0, T)$. Hence, choosing $G_1(t)$ and $G_2(t)$ to meet the above rank condition is in general quite easy.

Case b) When the number of inputs is no more than that of outputs $(p \leq q)$, determine two constant matrices β_1 and $\beta_2 \in R^{q\times q}$ from the equations

$$G(0^+) + G_{01}(0^+)\beta_1 + G_{02}(0^+)\beta_2 = 0$$
$$G(T^-) + G_{01}(T^-)\beta_1 + G_{02}(T^-)\beta_2 = 0$$

where $G(t)$, $G_{01}(t)$, and $G_{02}(t)$ are as in (26). Similarly, it will be assumed that in solving the above equations for

β_1 and β_2, the following matrix (note that it is different from the previous one) is full rank:

$$\text{rank}\begin{vmatrix} G_{01}(0^+) & G_{02}(0^+) \\ G_{01}(T^-) & G_{02}(T^-) \end{vmatrix} = 2p.$$

Step VI: Set the control input to be

$$u(t) = \bar{G}(t)y(kT), \qquad t \in [kT, kT+T) \tag{28}$$

where $y(kT)$ is the sampled system output and $\bar{G}(t) = \bar{G}(t+T)$ is the new generalized hold function

$$\begin{aligned} \bar{G}(t) &= G(t) + \alpha_1 G_{01}(t) + \alpha_2 G_{02}(t), \quad \text{if } p \ge q \\ \bar{G}(t) &= G(t) + G_{01}(t)\beta_1 + G_{02}(t)\beta_2, \quad \text{if } p \le q \end{aligned} \tag{29}$$

in which $G(t)$, $G_{01}(t)$, $G_{02}(t)$ are as in (26) and α_i and β_i from Step V.

The following theorem shows that the new control (28) will shift the Poincaré exponents to the *same* desired locations as the discontinuous control (14) in the previous section; furthermore, the new control input (28) is now continuous for all $t > 0$.

Theorem 2: The closed-loop system (9) with the *continuous* control (28) is exponentially stable. Furthermore, the closed-loop Poincaré exponents are located as specified in Theorem 1.

Proof: Continuity of the control (28) can be verified by noticing that (26) and (27) ensure that the new generalized hold function $\bar{G}(t)$ in (29) satisfies (21). Hence, the control input is continuous for all time instants $t = kT$ as suggested in (22).

To prove that the Poincaré exponents are relocated as desired, observe that $G_{01}(t)$ in (24) is in fact in the null space of the controllability map

$$\begin{aligned} \mathbf{R}[G_{01}(t)] &= \mathbf{R}[G_1(t)] - \mathbf{R}[B^T(t)\Phi^T(T,t)W^{-1}L_1] \\ &= L_1 - L_1 = 0 \end{aligned}$$

and so is $G_{02}(t)$ in (25). As a result, the new generalized hold function $\bar{G}(t)$ in (29) (the case $p \le q$ can be shown similarly) satisfies, recalling (20)

$$\begin{aligned} \mathbf{R}[\bar{G}(t)] &= \mathbf{R}[G(t)] + \alpha_1\mathbf{R}[G_{01}(t)] + \alpha_2\mathbf{R}[G_{02}(t)] \\ &= L + \alpha_1 \cdot 0 + \alpha_2 \cdot 0 = L. \end{aligned}$$

Following (19), with $G(\tau)$ replaced by the new $\bar{G}(\tau)$, one obtains

$$\begin{aligned} x((k+1)T) &= (\Phi(T,0) + \mathbf{R}[\bar{G}(\tau)]C_0)x(kT) \\ &= (\Phi(T,0) + LC_0)x(kT) \end{aligned}$$

where it is seen that the same closed-loop monodromy matrix $\Phi(T,0) + LC_0$ is obtained as in Theorem 1. Hence, the discretized system state $x(kT)$ converges to zero exponentially. The same argument as in Theorem 1 can show that the intersampling state $x(t)$ also converges to zero exponentially. □

Example 2: In this example, the *continuous* control (28) is simulated for the same system as in Example 1. The control design parameters in Step I are as before, and the T-periodic functions ($T = 1$ s) in Step III are chosen to be $G_1(t) = t$, $G_2(t) = 1 - t$, where $t \in [0, T)$. Fig. 3 shows the time history of the controlled system state and Fig. 4 the control input. Observe that the control input now becomes continuous in time while the state convergence rate remains the same as in Example 1.

In Fig. 4 the control signal has become continuous; however, there is a spike taking place at $t = 1$ s, which may still excite high-frequency unmodeled dynamics. Observe that such a spike results from the discontinuity of the *time derivative* of the control signal. One can easily avoid such undesirable spikes by a further modification of the generalized hold function. For example, to ensure continuity of the first-order *time derivative* of the control signal at the time instants $t = kT$, one can design the T-periodic generalized hold function $\bar{G}(t)$ to satisfy, in addition to (21)

$$\left.\frac{d\bar{G}(t)}{dt}\right|_{t=kT} = 0, \qquad \forall k = 0, 1, 2, \cdots. \tag{30}$$

This will force the time derivative of the control input to be continuous at $t = kT$ since, from (28)

$$\dot{u}(t^-) = \dot{u}(t^+) = 0, \quad \text{at} \quad t = kT.$$

In order to satisfy (21) and (30) at the same time, the generalized hold function will have to be in the form (say $p \ge q$)

$$\bar{G}(t) = G(t) + \alpha_1 G_{01}(t) + \alpha_2 G_{02}(t) + \alpha_3 G_{03}(t) + \alpha_4 G_{04}(t)$$

where the constant matrices $\alpha_i's, i = 1, 2, 3, 4$ are chosen to satisfy the four conditions in (21) and (30). The detailed design procedure is omitted here since it is a straightforward extension of the proposed design in this section.

Finally, to conclude the paper, it is mentioned that in Step III of the design procedure, the matrix function $G_1(t)$ and $G_2(t)$ may

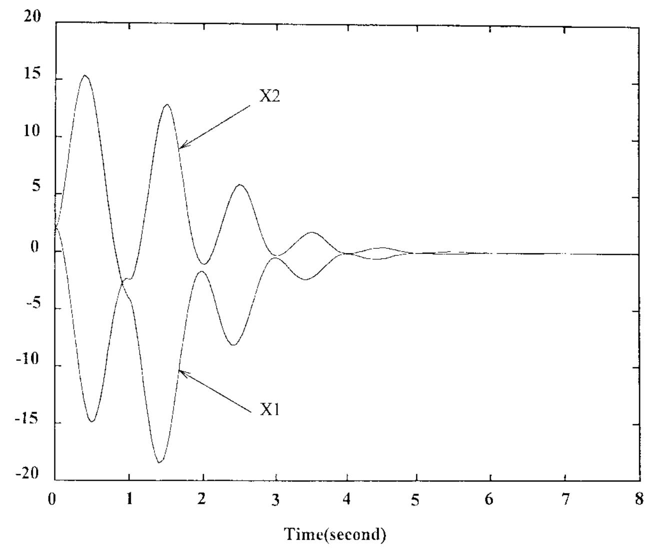

Fig. 3. State response with continuous control (28).

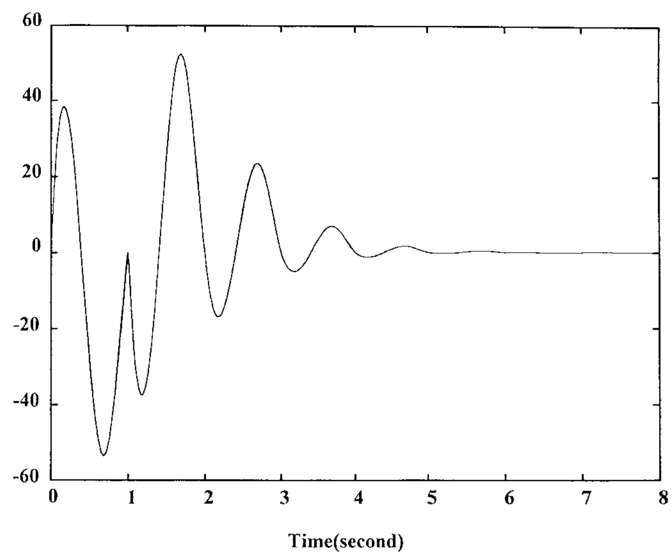

Fig. 4. Control signal with continuous control (28).

be arbitrarily chosen. However, their choice affects the maximum amplitude of the control input [i.e., $\max \|\bar{G}(t)\|$, $t \in [0, T]$ in (28)]. Hence, a future direction of research is to perform an optimization (minimization) on a certain norm of $\bar{G}(t)$ subject to the constraints (21) and/or (30), so that the control objective is achieved with minimum control effort.

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From Singular to Nonsingular Filtering of Periodic Systems: Filling the Gap with the Spectral Interactor Matrix

Sergio Bittanti, Patrizio Colaneri, and Marco Fabio Mongioví

***Abstract*— With reference to periodic discrete-time systems, in this paper the concept of the periodic spectral interactor matrix operator (PSIMO) is introduced. Such an interactor enables one to transform the singular system into a nonsingular one with identical spectral properties. This concept is useful to characterize the family of periodic solutions of the singular periodic Riccati equation arising from the singular filtering problem. The PSIMO is also a synthetic tool to represent the invariant zeros structure at infinity (delays) of periodic systems in an operatorial fashion.**

***Index Terms*— Interactor matrix, periodic systems, Riccati equation, singular filtering.**

I. INTRODUCTION

Singular prediction and control problems for time-invariant systems have been studied by a number of authors; see, e.g., [10], [14], [15], [19], and [21]. However, in the realm of periodic systems, such problems have not been given the attention they deserve. Singular problems arise for example in the LQG/LTR framework or in multirate sampled data control systems; see [3], [11], and [13]. In this paper, we consider the problem of singular filtering for discrete-time periodic systems, and we reduce it to a nonsingular one. In this way, one can take advantage of the well-established results concerning periodic nonsingular filtering and control problems; see, e.g., [6]. The basic tool which will be used for the reduction of the periodic singular problem to the nonsingular one is the so-called spectral interactor matrix (SIM), defined in [8] and [10] as a particular characterization of the class of interactor matrices; see [16], [17], [20], and [23].

In this way, it is possible to clearly point out the different roles played by the finite zeros and by the delays of the system in determining the optimal filter.

The paper organization is as follows. In Section II, we introduce the *periodic spectral interactor matrix* (PSIMO); see also [9]. The PSIMO is an anticausal periodic system, with a polynomial-like input–output representation, which preserves the spectral properties and acts on the system as a delay eraser. The determination of the PSIMO can be carried out by making reference to the lifted time-invariant representation as discussed in Section III. The relation between the invariant zeros of the original system and its delay-free image (obtained by applying the PSIMO to the original system) is addressed in Section IV. This sets the basis for the study of the singular Riccati equation (Section V), leading to the solution of the prediction problem as stated in Section VI.

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The authors are with the Dipartimento di Elettronica del Politecnico di Milano, 20133 Milano, Italy (e-mail: colaneri@elet.polimi.it).

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