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# Model-Based Autotuning Systems with Two-Degree-of-Freedom Control

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**Abstract**—A model-based autotuning system with two-degree-of-freedom (2-df) control is presented. The 2-df control system presented provides capabilities of set-point tracking and load rejection as well in one single system and the two control objectives can be considered independently for design. A closed-loop system is presented to generate an excitation input sequence for system identification. This closed-loop system consists of a control algorithm to eliminate unknown but constant disturbances during identification and guarantee the input to have a zero mean. The identification method is derived from an intermediate stage of the sub-space identification algorithm. From which, an impulse response sequence of the process can be computed and a reduced order model is identified. The effectiveness of this proposed 2-df autotuning system is demonstrated with simulation results.

*Key Words* : Autotuning, Two-degree-of-freedom control, System identification, Impulse response sequence

## INTRODUCTION

Autotuning of PI/PID controllers using relay feedback (Åström and Hägglund, 1984) becomes popular nowaday. It includes estimations of ultimate gain and ultimate frequency or even parameters of transfer function model (Luyben, 1987) to apply to different tuning methods. Regarding the controller design or tuning in these autotuning systems found in literature, the resulting system cannot be optimal for both set-point tracking and disturbance rejection simultaneously in one simple feedback system. Therefore, in the design of a conventional feedback control system, a compromise has to be made between the set-point tracking performance and disturbance rejection performance because these two objectives are conflicting. Unfortunately, the trade-off between them is not easily made due to the lack of clear and simple criterion. To overcome the difficulty and improve the control performance, a control system with two-degree-of-freedom (2-df) can be used. For example, Tian and Gao (1998) proposed a double-controller scheme, where a controller for set-point following and a controller for disturbance rejection can be designed independently. Although their system is theoretically sound, a major drawback of Tian and Gao's system is the complexity of the system in implementation.

In this paper, a new model-based 2-df controller design is presented. The 2-df controllers are similar to the double-controller of Tian and Gao (1998) but simpler and have very clear link to the identification of dynamic model and the objectives of the control. In order to identify the model, an excitation input sequence similar to the pseudo-random binary signal (PRBS) is used to activate the process under closed-loop. The excitation input is generated under a proposed closed-loop, which monitors the mean of the outputs to compensate for unknown but constant disturbances during the identification stage. Using the collected input and output data, the identification algorithm is derived from an intermediate result of the sub-space identification method (van Overschee and de Moor, 1994). From which, a sequence of impulse response of the open-loop process can be obtained and a reduced order model in terms of first-order-plus-dead-time (FOPDT) or second-order-plus-dead-time (SOPDT) dynamics can be identified. Based on the identified model, a 2-df control structure simpler than the double-controller scheme of Tian and Gao (1998) is proposed. The effectiveness of this proposed autotuning system will be demonstrated with simulated examples.

## TWO-DEGREE-OF-FREEDOM CONTROL STRUCTURE

The structure of proposed 2-df control system is as shown in Fig. 1 where $G_p(s)$, $G_c(s)$, and $W(s)$

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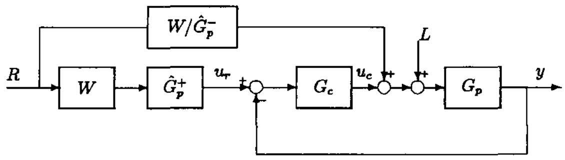



Fig. 1. The two-degree-of-freedom control system.

designate the transfer functions of process, controller and desired set-point response, respectively. Also, $\hat{G}_p(s)$ designates the process model and $\hat{G}_p(s)=\hat{G}_p^+(s)\,\hat{G}_p^-(s)$ where $\hat{G}_p^-(s)$ is the invertible part of model and $\hat{G}_p^+(s)$ is the non-invertible part such as dead time, right-half-plane (RHP) zero.

In the structure of Fig. 1, the set-point $R$ has two paths to the feedback loop. One passes through $W(s)$ and non-invertible part of the model to become the actual set-point of the feedback loop. The other path passes through $W(s)/\hat{G}_p^-(s)$ and then is added to the controller output. Thus, the closed-loop responses for set-point ($R$) and load input ($L$) can be written as:

$$\frac{y(s)}{R(s)}=\frac{W(s)\hat{G}_p^+(s)G_cG_p(s)}{1+G_cG_p(s)}+\frac{W(s)G_p(s)}{[1+G_cG_p(s)]\hat{G}_p^-(s)}, \tag{1}$$

$$\frac{y(s)}{L(s)}=\frac{G_p(s)}{1+G_cG_p(s)}. \tag{2}$$

It can be seen from Eq. (2) that the load response of the closed-loop system is determined only by the controller $G_c$ and has been separated from the set-point response. Therefore, the controller can be designed independently to achieve optimal performance for disturbance rejection (*e.g.*, minimization of performance index such as IAE, ISE, ITAE, $\cdots$).

Regarding the set-point response, from Eq. (1) and with a perfect process model (*i.e.* $G_p(s)=\hat{G}_p^+(s)\hat{G}_p^-(s)$), it becomes:

$$\frac{y(s)}{R(s)}=\frac{W(s)\hat{G}_p^+(s)G_cG_p(s)}{1+G_cG_p(s)}+\frac{W(s)\hat{G}_p^+(s)}{1+G_cG_p(s)}$$

$$=W(s)\hat{G}_p^+(s). \tag{3}$$

Equation (3) clearly indicates that the set-point response is independent of the controller $G_c(s)$ and, except the non-invertible part of model, can be assigned as any desired response by specifying $W(s)$. Theoretically, the desired set-point response, $W(s)$, can be arbitrarily given. However, in order to make $W(s)/\hat{G}_p^-(s)$ be practically realizable, the relative order of $W(s)$ cannot be smaller than that of $\hat{G}_p^-(s)$.

With the proposed control structure, the control of set-point tracking and disturbance rejection can be designed individually, and achieving optimal performance for both objectives becomes possible. Furthermore, with a good process model, the set-point response thus obtained is similar to that resulted from Smith predictor design (Smith, 1957) so that the process dead time can be effectively compensated. An inherent drawback of the Smith predictor is its performance sensitivity to the process model. As we will show below, the proposed 2-df control structure is not only very effective, but also more robust than the Smith predictor.

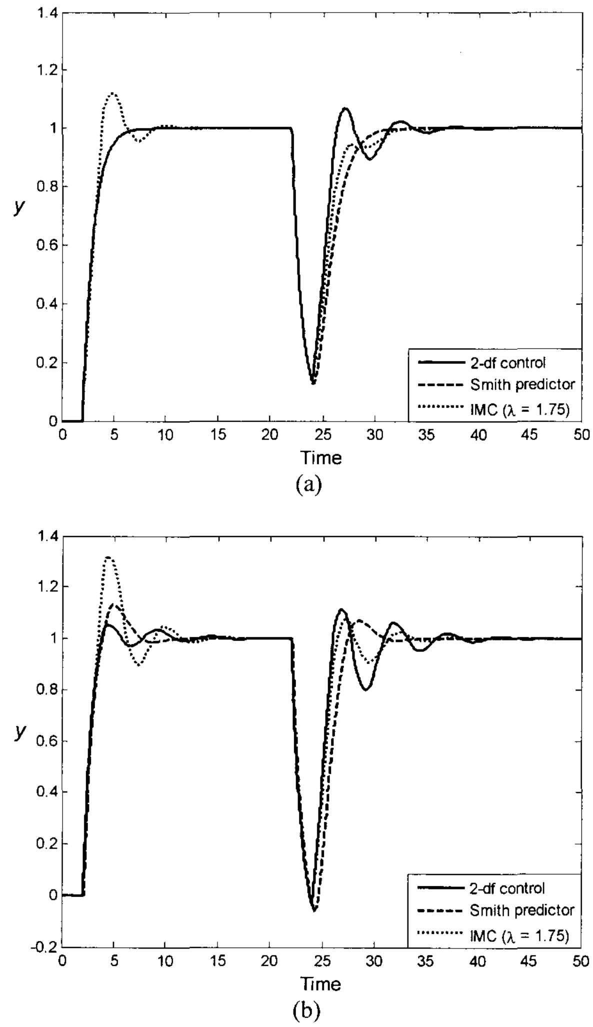



Fig. 2. Comparisons of control performance: (a) perfect process model; (b) process gain deviating from 1 to 1.2.

To demonstrate the effectiveness and robustness of this 2-df control structure, consider a process of $G_p(s)=e^{-2s}/(s+1)$. To design the 2-df control, the desired set-point response is picked as $W(s)=1/(s+1)$. In addition, the controller $G_c$ used in the feedback loop is a series PID controller tuned by minimum IAE formula for disturbance rejection (Shinskey, 1988), which results in $k_c=0.65$, $\tau_R=1.52$, and $\tau_D=0.70$. On the other hand, in the

Smith predictor design using direct synthesis method, this desired set-point transfer function leads to a PI controller with $k_c = 1$ and $\tau_R = 1$. Moreover, a simple feedback structure with IMC-PID controller ($k_c = 0.37$, $\tau_R = 1$, and $\tau_D = 1$) is also used for the comparison of control performance. A unit step change in set-point at $t = 0$ and a unit negative step change in load disturbance at $t = 20$ are introduced as excitation signals. Figure 2(a) shows the responses of these three control schemes with a perfect process model. As the way they have been designed, the set-point responses of the proposed 2-df controller and Smith predictor overlap each other, and better than that of the IMC controller. Moreover, the load response of the 2-df control system is better than that of the Smith predictor. To simulate the process uncertainty, assume the steady-state gain of the process deviate from its nominal value of 1 to 1.2 and all the controller settings are kept unchanged. Figure 2(b) shows the resulting responses of these three control schemes. The 2-df control structure has superior responses for both set-point tracking and disturbance rejection than those of the Smith predictor, indicating that the proposed 2-df control structure is less sensitive to model error.

## IDENTIFICATION OF PROCESS MODEL

To apply the 2-df control structure as shown in Fig. 1 presented in the previous section, a model of the process is required. Usually, high-order processes are represented as reduced order dynamic models such as FOPDT and SOPDT for simplicity. In addition, higher-order process models are not suitable for the proposed 2-df design because, if a high-order model is used, the $W(s)$ has also to be high-order so that the set-point response will be very sluggish. Therefore, a method for the identification of reduced order process model is presented in this section. This proposed method identifies the model through estimation of impulse response sequence of the process, where the algorithm is derived from an intermediate result of the sub-space identification method. The details are described as the following.

### Estimation of impulse response sequence

The identification of a dynamic system is to find a sequence of $h(i)$ so that the output of the system can be expressed as a sum of moving averages from the input as the following.

$$y(k) = \sum_{i=-\infty}^{k} h(k-i)u(i), \tag{4}$$

where $y$ and $u$ denote the system output and input, respectively. This sequence of $h(i)$, $i$ =0, 1, 2,$\cdots$ is called as impulse response sequence or weighting sequence. For open-loop stable system, this sequence will decay to zero after some $i > p$ and the system output can be expressed as:

$$y(k) = \sum_{i=k-p}^{k} h(k-i)u(i). \tag{5}$$

There are a number of obvious advantages of the impulse response sequence model from the viewpoint of system identification (Hsia, 1977). For example, the determination of the impulse response sequence requires less *a priori* knowledge than do the parametric models, and this model can be identified more satisfactorily in the presence of noise. The typical method for identifying the impulse response sequence is the least-squares estimation. First, the input $u(t)$ is used to continuously drive the system and the sampled input sequence $\{u(i)\}$, for $0 \le i \le m+p$ where $m > p$, and output sequence $\{y(i)\}$, for $p \le i \le m+p$, are collected. Then, using the observed input-output data in Eq. (5), one can set up a set of $m$+1 equations written in the vector form as:

$$\mathbf{y} = \mathbf{U}\mathbf{h}, \tag{6}$$

where

$$\mathbf{y} = [y(p) \quad y(p+1) \quad \cdots \quad y(p+m)]^{\mathrm{T}},$$

$$\mathbf{h} = [h(0) \quad h(1) \quad \cdots \quad h(p)]^{\mathrm{T}},$$

$$\mathbf{U} = \begin{bmatrix} u(p) & u(p-1) & \cdots & u(0) \\ u(p+1) & u(p) & \cdots & u(1) \\ \vdots & \vdots & \ddots & \vdots \\ u(p+m) & u(p+m-1) & \cdots & u(m) \end{bmatrix}. \tag{7}$$

Thus, the unknown parameter vector $\mathbf{h}$ can be estimated by the method of least-squares as:

$$\mathbf{h} = (\mathbf{U}^{\mathrm{T}}\mathbf{U})^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{y}. \tag{8}$$

However, before proceeding the estimation, the value of settling time parameter $p$ must be chosen in advance. To obtain the accurate result, $p$ should be picked according to the condition $h(i > p) \approx 0$, which may result in some complexities of computation. Because, usually, we may repeat the solution with progressively increasing $p$ values until a satisfactory fit has been achieved. However, a large $p$ increases the computational difficulties associated with high order matrix inversion in Eq. (8).

To overcome computational difficulties mentioned earlier, an alternative algorithms is then presented to identify the impulse response sequence. The main idea is that the future output can be represented by the past input, past output and future input. First, let the time before and after sampling instant $m$ be referred as past and future, respectively. Then, given recorded process inputs and outputs, the

Hankel matrices of past input ($\mathbf{U}^p$), past output ($\mathbf{Y}^p$), future input ($\mathbf{U}^f$), and future output ($\mathbf{Y}^f$) can be written as:

$$\mathbf{U}^p = \begin{bmatrix} u(0) & u(1) & \cdots & u(n-1) \\ u(1) & u(2) & \cdots & u(n) \\ \vdots & \vdots & \ddots & \vdots \\ u(m-1) & u(m) & \cdots & u(m+n-2) \end{bmatrix}, \quad (9)$$

$$\mathbf{Y}^p = \begin{bmatrix} y(0) & y(1) & \cdots & y(n-1) \\ y(1) & y(2) & \cdots & y(n) \\ \vdots & \vdots & \ddots & \vdots \\ y(m-1) & y(m) & \cdots & y(m+n-2) \end{bmatrix}, \quad (10)$$

$$\mathbf{U}^f = \begin{bmatrix} u(m) & u(m+1) & \cdots & u(m+n-1) \\ u(m+1) & u(m+2) & \cdots & u(m+n) \\ \vdots & \vdots & \ddots & \vdots \\ u(2m-1) & u(2m) & \cdots & u(2m+n-2) \end{bmatrix}, \quad (11)$$

$$\mathbf{Y}^f = \begin{bmatrix} y(m) & y(m+1) & \cdots & y(m+n-1) \\ y(m+1) & y(m+2) & \cdots & y(m+n) \\ \vdots & \vdots & \ddots & \vdots \\ y(2m-1) & y(2m) & \cdots & y(2m+n-2) \end{bmatrix}, \quad (12)$$

where $n >> m$. As a result, the predicted future output can be represented as:

$$\hat{\mathbf{Y}}^f = [\Theta^{\mathbf{U}^p} \quad \Theta^{\mathbf{Y}^p} \quad \Theta^{\mathbf{U}^f}] \begin{bmatrix} \mathbf{U}^p \\ \mathbf{Y}^p \\ \mathbf{U}^f \end{bmatrix}. \quad (13)$$

The parameter matrix $[\Theta^{\mathbf{U}^p} \quad \Theta^{\mathbf{Y}^p} \quad \Theta^{\mathbf{U}^f}]$ is found to minimize the prediction error of $\left\|\mathbf{Y}^f - \hat{\mathbf{Y}}^f\right\|^2$ and its least-squares solution is given as the following:

$$[\Theta^{\mathbf{U}^p} \quad \Theta^{\mathbf{Y}^p} \quad \Theta^{\mathbf{U}^f}] = \mathbf{Y}^f\mathbf{X}^\mathrm{T}(\mathbf{X}\mathbf{X}^\mathrm{T})^{-1} \quad (14)$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{U}^p \\ \mathbf{Y}^p \\ \mathbf{U}^f \end{bmatrix}. \quad (15)$$

It is found that $\Theta^{\mathbf{U}^f}$ thus obtained is a lower block triangular Toeplitz matrix as the following:

$$\Theta^{\mathbf{U}^f} = [\theta_{i,j}^{\mathbf{U}^f}]_{i,j=1,2,\cdots,m} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ z_1 & 0 & 0 & \cdots & 0 \\ z_2 & z_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{m-1} & z_{m-2} & \cdots & z_1 & 0 \end{bmatrix}. \quad (16)$$

In fact, by following the N4SID algorithm (van Overschee and de Moor, 1994) of subspace identification, a state space process model $(\mathbf{A},\mathbf{B},\mathbf{C})$ of the following can be obtained.

$$x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k),$$
$$y(k) = \mathbf{C}x(k) + \mathbf{D}u(k). \quad (17)$$

It is interesting to find that each $z_i$ in Eq. (16) equals $\mathbf{CA}^{i-1}\mathbf{B}$ from N4SID. Notice that the impulse response sequence satisfies $h(i) = \mathbf{CA}^{i-1}\mathbf{B}$, $i = 1, 2, \cdots$, for linear dynamic system. In other words, the sequence of $\{z_i\}$ forms the initial part of impulse response sequence of the system. As a result, the impulse response sequence in Eq. (5) is then taken as the first column of $\Theta^{\mathbf{U}^f}$, that is:

$$\{h(i)\} = \theta_{i+1,1}^{\mathbf{U}^f}; \; i = 0, 1, 2, \cdots, m-1. \quad (18)$$

With the initial portion of the impulse response sequence from Eq. (18), a reduced order transfer function model in terms of FOPDT or SOPDT of the following can be found.

$$\text{FOPDT} \quad \hat{G}_p(s) = \frac{k_p e^{-\theta s}}{\tau s + 1},$$

$$\text{SOPDT} \quad \hat{G}_p(s) = \frac{k_p e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}. \quad (19)$$

For FOPDT model, the impulse response sequence, after transient response, will decay with a constant ratio, designated as $\phi$. Thus, we have:

$$\frac{h(i)}{h(i-1)} = \phi = e^{-T_s/\tau}, \quad (20)$$

where $T_s$ is the sampling interval. For SOPDT model, the impulse response sequence satisfies the following relation after transient response.

$$h(i) = \phi_1 h(i-1) + \phi_2 h(i-2), \quad (21)$$

where

$$\phi_1 = e^{-T_s/\tau_1} + e^{-T_s/\tau_2},$$

$$\phi_2 = -e^{-T_s/\tau_1} \cdot e^{-T_s/\tau_2}. \quad (22)$$

Consequently, the value of $\phi$ or $\phi_1$, $\phi_2$ can be computed from Eq. (20) or Eq. (21) using the initial portion of the impulse response sequence in Eq. (18). Furthermore, the time constant(s) of the model can be calculated by Eq. (20) or Eq. (22) and the remaining portion of the impulse response sequence is estimated using Eq. (20) or Eq. (21). Calculation of the impulse response sequence in this way can efficiently reduce the dimension of the matrix in Eq. (14). After the entire impulse response sequence is obtained, the steady-state process gain is the summation of each weighting value as:

$$k_p = \sum_{i=0}^{p} h(i). \quad (23)$$

Also, the dead time for FOPDT model can be computed by:

$$\theta = \int_0^\infty \left(1 - \frac{y_s(t)}{k_p}\right) dt - \tau \quad (24)$$

or, for SOPDT model,

$$\theta = \int_0^{\infty}\left(1-\frac{y_s(t)}{k_p}\right)\mathrm{d}t - \tau_1 - \tau_2, \tag{25}$$

where $y_s(t)$ is the unit step response of the process and its sampled data is computed from:

$$y_s(i) = \sum_{j=0}^{i} h(j). \tag{26}$$

*Remark 1*

The choice of $m$ and $n$ values needs to consider the trade-off between model accuracy and identification cost (computation effort and experiment time). The value of $m$ should be chosen such that a sufficient number of impulse response coefficients can be obtained for modeling the transfer function of the process. In general, one should have at least three non-zero and constantly decreasing impulse coefficients for FOPDT process. For SOPDT process, it needs about ten non-zero impulse coefficients. Theoretically, the value of $n$ should be much greater than the value of $m$ for accurate estimation. Our experience indicates that a value larger than $7m$ is required, *i.e.*, $n > 7m$.

*Remark 2*

A simple criterion for choosing FOPDT or SOPDT model is to check if the ratio of any two successive impulse response coefficients converges to a constant. Let $\phi(i) = h(i)/h(i-1)$, $i$=1, 2, $\cdots$, $m$–1 and $\bar{\phi}$ is their average value. When, for each $i$, the difference between $\phi(i)$ and $\bar{\phi}$ is smaller than a prescribed tolerance value $\varepsilon$ , *i.e.*, $|\phi(i)-\bar{\phi}| < \varepsilon$, FOPDT model is selected and the value of $\phi$ in Eq. (20) is taken as $\phi = \bar{\phi}$ . Otherwise, SOPDT model shall be used. In that case, $\phi_1$ and $\phi_2$ are obtained to best fit Eq. (21), $i$ =2, 3,$\cdots$, $m$–1, in the least-squares sense.

## Generation of excitation inputs for closed-loop identification

To use this proposed least-squares method for computing the impulse response sequence, the result is accurate only when the input signal, $u(i)$, is uncorrelated. It is straightforward that one can introduce a white random noise to activate the process under open-loop for the identification. However, process activation under closed-loop is usually desirable to prevent process output drifting away from its normal operation range due to unknown disturbances. Therefore, a closed-loop scheme to generate excitation input signal is presented as shown in Fig. 3 for this identification.

In Fig. 3, a pseudo-random binary signal

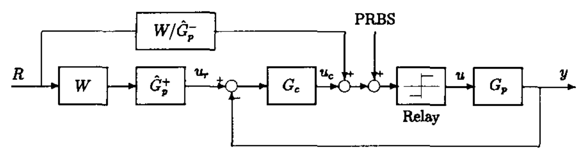



Fig. 3. The closed-loop scheme for identification.

(PRBS) is added to the controller output $u_c$ . The magnitude of this PRBS introduced should be large enough so that its sign will not changed by the controller output. Then, the resulting signal is passed through a relay element before it enters the process. As a result, the process input, $u$, is still similar to a PRBS with magnitude $\pm a$ where $a$ is the height of relay. With this structure, random input to the process can be generated under closed-loop operation and the identification method aforementioned then can be applied.

## Adaptation to unknown disturbance

During the identification stage, unknown disturbance could cause significant error in the identification result. This error will in turn degrade the closed-loop performance. Therefore, a complete autotuning system should include the mechanism to eliminate the identification error caused by unknown disturbance. To eliminate the effect of unknown but constant disturbance, a bias value, $u_b$, is introduced to the process input when the unknown disturbance is detected. In other words, the relay is shifted vertically by this bias value. A feedback method to automatically update the bias value is presented as the following.

Denote the integral of the process output over a period $P$ as $S$, *i.e.*, $S = \int_t^{t+P} y(t)\mathrm{d}t$. Because the process is activated by a PRBS, the average value of $S$, designated as $\bar{S}$ , over several successive periods should approach zero if there is no unknown disturbance. Based on this hypothesis, the Student $t$ statistic is then applied for testing the $\bar{S}$ . Once the value of $t$ statistic falls outside the prescribed significance level, $t_\alpha$ (*e.g.*, $\alpha$=5%), the above hypothesis is rejected and it is recognized that some unknown disturbance has happened to the system. In case of $k_p > 0$, $\bar{S} > 0$ ($\bar{S} < 0$) implies that a positive (negative) disturbance has happened and, hence, a negative (positive) bias has to be introduced. The adverse results can be concluded for the case of $k_p < 0$, . According to the analysis, the introduced bias value has to be updated to eliminate the effect of disturbance by the following rule:

$$u_b^i = u_b^{i-1} - \mathrm{sign}(k_p)\gamma\bar{S}\ , \tag{27}$$

where $\gamma > 0$ is the convergence rate. This adaptive

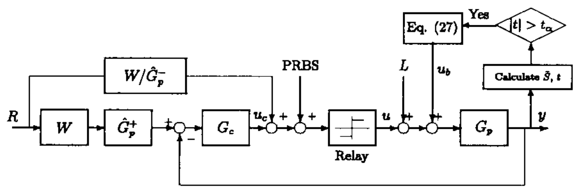



Fig. 4. The adaptive scheme for identification under unknown disturbance.

mechanism is shown graphically in Fig. 4. Such adaptation of $u_b$ can make the process output oscillate around its steady-state value automatically under unknown but constant disturbance so that the proposed autotuning can be proceeded successfully.

## Adaptive 2-df scheme for model error of dead time

Although the 2-df control structure has been shown less sensitive to model error than Smith predictor, the model error of dead time can seriously deteriorate the system performance and robustness. To eliminate the impact from model error of dead time, in the 2-df scheme of Fig. 1, the implementation of dead time in $\hat{G}_p^+(s)$ (*i.e.*, $e^{-\hat{\theta}s}$) is modified to an online adaptive scheme. When the set-point changes, the signal $u_r$ is set to zero until the output $y$ responses to the value of $y_s(\theta)$, where $y_s(\theta)$ designates the value of process output due to a step input with the magnitude of set-point change after a duration of $\theta$. Since the step response of the process, $y_s$, and dead time in the model, $\theta$, have been identified, $y_s(\theta)$ can be easily computed. As the signal $u_r$ is on-line determined by the output $y$, the model error of dead time can be effectively eliminated and, hence, the set-point performance is significantly improved.

## ILLUSTRATIVE EXAMPLES

### Example 1. FOPDT process

To show the procedures of proposed identification method, consider the same FOPDT process aforementioned, *i.e.*, $G_p(s) = e^{-2s}/(s+1)$. By the scheme of Fig. 3, a PRBS with large magnitude is introduced to excited the system and the relay height is set as 1 so that the process input $u$ is similar to a PRBS with magnitude ±1. Meanwhile, the process input and output are collected with sampling interval $T_s = 0.5$. The parameters for estimating the impulse response sequence are chosen as $m$=8 and $n$=61, which means that the data from $t$ =0 to $t$ =37 are used for identification. The initial portion of impulse response sequence estimated is **h**=[0 0 0 0 0 0.3935 0.2387 0.1447], which implies that the process belongs to FOPDT dynamics with $\phi = 0.6065$ (or $\tau = -T_s/\ln\phi = 1$). Then, the entire impulse response sequence is calculated and the model is identified as $\hat{G}_p = e^{-2.02s}/(s+1)$, which is almost identical to the real process.

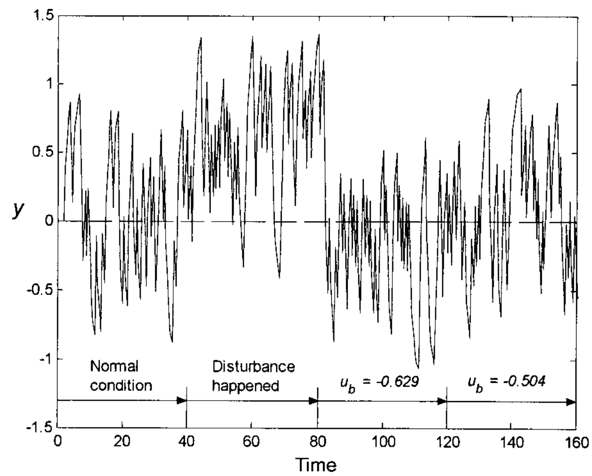



Fig. 5. Process output in example 1.

Assume, at $t$ =40, a step disturbance of magnitude 0.5 happened to the system. According to the scheme shown in Fig. 4, this disturbance is detected from the $t$ statistic of $\bar{S}$ and then the bias $u_b$ has to be updated using Eq. (27). Choosing $\gamma = 0.15$, the value of $u_b$ is converged after two iterations and its final value is –0.504. The whole process output in this experiment is as shown in Fig. 5. Then, the data collected after $t$ =120 are used for identification again and the resulting model is $\hat{G}_p(s) = 1.002e^{-2.025s}/(1.004s+1)$. This result indicates that the proposed identification method performs well even under the presence of unknown disturbance.

### Example 2. Third-order process with dead time

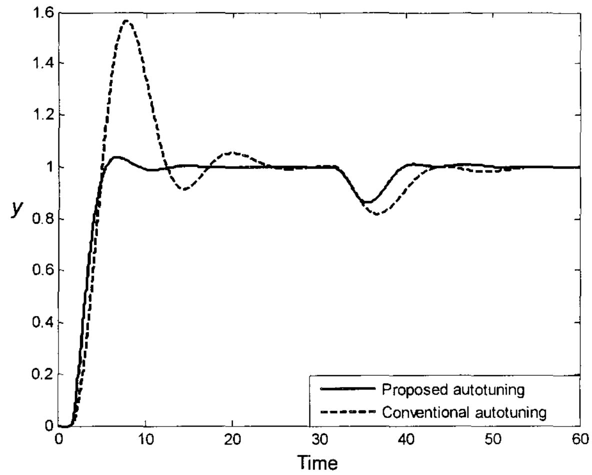
Consider a third-order process of the following:

$$G_p(s) = \frac{e^{-1.5s}}{(s^2+10s+1)(2s+1)}$$

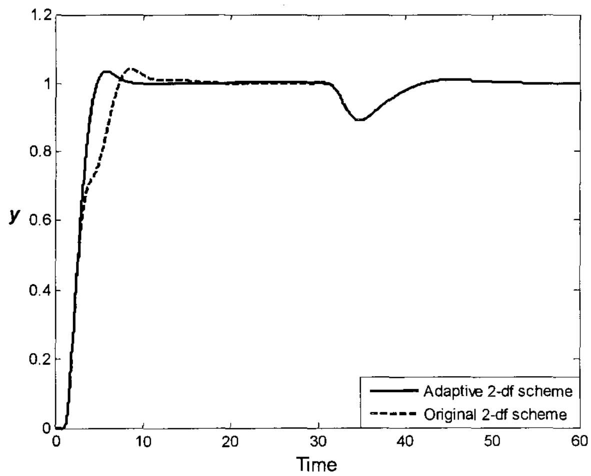
For system identification, the same experiment as that in example 1 is conducted. In addition, the parameters for estimation of impulse response sequence are chosen as $m$=18 and $n$=140. As a result, the initial portion of impulse response sequence estimated is:

$$h = [0\ 0\ 0\ 0\ 0.0044\ 0.0137\ 0.0209\ 0.0260\ 0.0295\ 0.0317\ 0.0330\ 0.0336\ 0.0337\ 0.0334\ 0.0328\ 0.0320\ 0.0310\ 0.0300]$$

It is found that this process can be represented by a SOPDT model with $\phi_1 = 1.727$ and $\phi_2 = -0.738$, or $\tau_1 = 9.88$ and $\tau_2 = 1.98$. Then, the




(a)




(b)

Fig. 6. Closed-loop responses in example 2: (a) nominal case; (b) process dead time deviating.

whole impulse response sequence is calculated and the model is identified as :

$$\hat{G}_p(s) = \frac{0.998\,e^{-1.57s}}{(9.88s+1)(1.98s+1)}$$

Based on this identified model, the 2-df control structure is applied accordingly. The desired set-point response is picked as $W(s) = 1/(s^2 + 1.6s + 1)$ which is slightly underdamped to speed the response. For disturbance rejection, an ideal PID controller is used in the feedback loop and is tuned according to minimum ITAE formula (Sung *et al.*, 1996), which gives $k_c = 8.18$, $\tau_R = 4.38$, and $\tau_D = 1.70$ . Figure 6(a) shows the closed-loop responses of this 2-df control system and also the conventional PID autotuning system of Åström and Hägglund (1984) for comparison. It can be seen that both performances for set-point tracking and disturbance rejection of the proposed autotuning system are satisfactory. Moreover, to simulate the model error of dead time, assume the dead time of the process deviate from its nominal value of 1.5 to 1.0. The closed-loop responses using original and adaptive 2-df schemes are shown in Fig. 6(b) for comparison. It is seen that the set-point performance of adaptive scheme under model error of dead time is superior to that of original scheme and similar to that at the nominal case.

## CONCLUSION

A model-based autotuning system with 2-df control has been proposed in this paper. For system identification, a closed-loop scheme is devised to generate an excitation input similar to a PRBS to estimate the impulse response sequence of the process and then its low-order model is identified accordingly. This identification can be done under closed-loop operation as well as under the presence of unknown but constant disturbances. Based on the identified model, a 2-df control structure is presented to separate the controller design for disturbance rejection from that for set-point tracking in a closed-loop system. The inevitable compromise between these two performances in the conventional feedback system is no longer necessary. The simulation results have shown that this proposed autotuning system is efficient and self-contained.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## NOMENCLATURE

| | |
|---|---|
| $a$ | height of relay |
| $G_c(s)$ | transfer function of controller |
| $G_p(s)$ | transfer function of process |
| $\hat{G}_p(s)$ | transfer function of process model |
| $\hat{G}_p^+(s)$ | non-invertible part of $\hat{G}_p(s)$ |
| $\hat{G}_p^-(s)$ | invertible part of $\hat{G}_p(s)$ |
| $h$ | impulse response sequence |
| $\mathbf{h}$ | impulse response vector |
| $k_c$ | controller gain |
| $k_p$ | process gain |
| $L$ | load disturbance |
| $m, n$ | parameters for identification |
| $P$ | a period for computing $S$ |
| $p$ | settling time parameter |
| $R$ | set-point |
| $S$ | integral of process output over a period $P$ |
| $\bar{S}$ | average value of $S$ |
| $T_s$ | sampling interval |

$t_a$ significance level
$\mathbf{U}$ process input matrix
$\mathbf{U}^f$ Hankel matrix of future input
$\mathbf{U}^p$ Hankel matrix of past input
$u$ process input
$u_b$ bias input
$u_c$ controller output
$W(s)$ transfer function of desired set-point response
$\mathbf{Y}^f$ Hankel matrix of future output
$\hat{\mathbf{Y}}^f$ Hankel matrix of predicted future output
$\mathbf{Y}^p$ Hankel matrix of past output
$y$ process output
$\mathbf{y}$ process output vector
$y_s$ process step response

### Greek symbols

$\gamma$ convergence rate
$\Theta^{\mathbf{U}^f}$ parameter matrix for $\mathbf{U}^f$
$\Theta^{\mathbf{U}^p}$ parameter matrix for $\mathbf{U}^p$
$\Theta^{\mathbf{Y}^p}$ parameter matrix for $\mathbf{Y}^p$
$\theta$ process dead time
$\tau$ time constant of FOPDT process
$\tau_1, \tau_2$ time constants of SOPDT process
$\tau_D$ controller derivative time
$\tau_R$ controller integral time
$\phi$ ratio of FOPDT impulse response sequence
$\phi_1, \phi_2$ ratios of SOPDT impulse response sequence

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# 以模式為基礎且具有雙自由度控制之自動調諧系統


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## 摘　要

本文提出一個以模式為基礎且具有雙自由度控制功能之自動調諧系統。此雙自由度控制系統同時兼具設定點追蹤與負荷干擾排除的能力，而且此兩控制目的可獨立的來進行設計。此外，文中亦提出一個閉環路架構用以產生系統識別時所需要的輸入訊號，該閉環系統可消除在識別時有未知干擾發生的影響並確保程序輸入之平均值為零。識別方法乃是由次空間識別演算法之中間過程演導而得，首先計算出程序之脈衝響應序列，進而識別出其低階模式。模擬結果顯示此雙自由度自動調諧系統具有良好之性能。