I&EC research
Industrial & Engineering Chemistry Research

Article

A System of Procedures for Identification of Simple Models Using Transient Step Response

Hsiao-Ping Huang, Ming-Wei Lee, and Cheng-Liang Chen

Ind. Eng. Chem. Res., **2001**, 40 (8), 1903-1915 • DOI: 10.1021/ie0005001

Ind. Eng. Chem. Res. **2001,** *40,* 1903−1915

A System of Procedures for Identification of Simple Models Using Transient Step Response

Hsiao-Ping Huang,* Ming-Wei Lee, and Cheng-Liang Chen

Department of Chemical Engineering, National Taiwan University, Taipei 10617, Taiwan, Republic of China

In this paper, a system of procedures for identification using the transient step response is presented. The identification aims at modeling dynamic processes with simple models that have general second-order dynamics. The procedures consist of three parts, including classification of the response, selection of the model structure, and the estimation of parameters. Technically, this proposed system provides a strategy based on quantitative measures to identify the model structure and extends the identification of models to include possible RHP/LHP zeros and dead time. One important aspect of this proposed system of procedures is the minimization of the need for human decisions to be made during the identification process. With this advantage, it is then possible to facilitate the development of software to perform the identification automatically.

1. Introduction

In the past, numerous papers have been published on the topic of model identification using on-line and off-line methods. The terminology concerning the on-line methods is used to refer to those methods that (1) infer the model at the same time as data collection, (2) update the model using a recursive algorithm at each time instant when new data are collected, and (3) employ the model to support decisions to be taken *on-line* for purposes of control,[1] such as adaptive control or filtering. Extensive developments of such methods can be found in many books.[1−3] There are also a number of papers regarding developments of off-line methods. This category of methods includes those that use batch-wise data to perform identification for either time-domain or frequency-domain models. In those cases, there is no need for decisions when data are collected, but the model development requires special inputs for open-loop experiments. A good summary of such methods can be found in the book by Juang.[4] Each of these two categories of methods has its advantages and disadvantages, and the justification for which method to be employed is determined by the application of the model, in other words, the objective of the identification.

One of the developments in common to both categories of methods is the use of estimations with recursive least squares, which are directly applied to the measured input and output data. Regarding the method of recursive least squares, there are some major disadvantages in contrast to the off-line methods of nonleast squares. First, such methods require an a priori decision on what model structure to use before the procedure the procedure. This means that the estimation procedures have to be repeated on models with several different structures. Second, within a presumed model structure, the estimation of dead time can be done in two ways. If linear least squares are to be formulated for identification, then estimation of the dead time has to be carried out by trials, which means repetitive estimations for parameters are needed. Otherwise, estimations with nonlinear least squares might be encountered, and the algorithms would become much more complicated and fragile. In general, these methods do not give as accurate a description of the model as those off-line methods using nonleast squares.[1] More difficulties that these methods might encounter include the choice of sampling interval, selection of step size for iterations, bias of the results, convergence of the algorithms, nonunique conversion from discrete-time to continuous-time models,[5] etc. Recently, developments of least-squares identification methods that apply transformed input and output data for continuous-time models have also been reported.[6,7] Nevertheless, the difficulties mentioned have not been resolved.

By the facts described above, it seems that an off-line method of nonleast squares that uses transient step response for identification is more convenient for industrial uses, because the experiments are easy to perform and the development of models for designs does not involve decisions or instant needs for a model during the identification process. In literature, nonleast-squares methods using step responses have been used to develop simple models such as FOPDT (first-order plus dead time) or SOPDT (second-order plus dead time)[8−18] for process control. However, these identification methods usually need to presume that the model structure has been decided a priori, and there seems to be no quantitative criteria for making such decisions. The identification methods thus focused on parameter estimations and ignored the issue of determining the model structure. Furthermore, there is in lack of non-least-squares methods that use step response to develop models that have RHP/LHP zeros. These kind of models will find more and more uses in performing advanced control, such as in tuning advanced PID controllers,[19] tuning the controllers in a cascaded control loop, and tuning controllers in a multi-loop system.

To address the deficiencies just mentioned, a system of procedures for modeling dynamic systems with simple models of the following forms is presented

* Corresponding author. E-mail: huanghpc@ccms.ntu.edu.tw. Tel.:886-2-23638999. Fax: 886-2-3623040.

10.1021/ie0005001 CCC: $20.00
Published on Web 03/27/2001

$$\frac{y(s)}{u(s)} = G(s) = \begin{cases} \dfrac{k_p(1+as)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1} & 0 < \zeta < 1 \quad \text{(model I)} \\ \dfrac{k_p(1+as)e^{-\theta s}}{(\tau s + 1)(\eta\tau s + 1)} & 0 < \eta \le 1 \quad \text{(model II)} \end{cases} \tag{1}$$

where model I is assigned with $\zeta < 1$ and model II with $\zeta \geq 1$.

This system of procedures consists of three parts. In the first part, the transient dynamic response of a given process is classified with its appearance into four classes. Second, a key parameter (i.e., ζ or η) of the system is determined to select a model structure. Third, estimation of the parameters of the model in accord with the selected model structure is performed. Finally, the resulting model is validated by comparing the predicted step response with the experimental one. Technically, this proposed system provides a strategy based on quantitative measures to identify the model structure and extends the identification of models to include possible RHP/LHP zeros. The estimation of dead time is also included in each of models. One important aspect of this proposed system of procedures is that it minimizes the need for human decisions during the identification process. Thus, with the development of computer software, the identification process can be automated.

2. Strategies for Model Structure Identification

As is well-known, a higher-order dynamic process can be described with models in the form of model I or model II in eq 1. Because the steady-state gain, k_p, can be obtained from the changes in the input and output at steady state, the k_p value is assumed to be known and is not included in the study that follows.

By considering $\bar{s} = \tau s$ and $\bar{\theta} = \theta/\tau$ in eq 1, $G(s)$ can be changed into the dimensionless form

$$\frac{y(\bar{s})}{k_p u(\bar{s})} = \frac{\bar{y}(\bar{s})}{u(\bar{s})} = \bar{G}(\bar{s}) = \begin{cases} \dfrac{(1+\bar{a}\bar{s})e^{-\bar{\theta}\bar{s}}}{\bar{s}^2 + 2\zeta\bar{s} + 1} & \text{(model I)} \\ \dfrac{(1+\bar{a}\bar{s})e^{-\bar{\theta}\bar{s}}}{(\bar{s}+1)(\eta\bar{s}+1)} & \text{(model II)} \end{cases} \tag{2}$$

where $\bar{y} = y/k_p$ and $\bar{a} = a/\tau$.

The unit step response resulting from eq 2 is given in the equation

$$\bar{y}(\bar{t}) = \begin{cases} 1 - \left[\dfrac{\zeta - \bar{a}}{\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2}\,\bar{t}) + \cos(\sqrt{1-\zeta^2}\,\bar{t})\right] & \text{for model I} \\ 1 - \dfrac{1-\bar{a}}{1-\eta} e^{-\bar{t}} - \dfrac{\eta - \bar{a}}{\eta - 1} e^{-\bar{t}/\eta} & \text{for model II} \end{cases} \tag{3}$$

where $(\bar{t} - \theta/\tau)$. Thus, excluding the true time delay, the dynamics of a general second-order system can be characterized by two additional dimensionless parameters. They are ζ and $\bar{a}$ for model I and η and $\bar{a}$ for model II. Based on many simulations with various combinations of these key parameters, some typical step responses are shown in Table 1. These typical step responses are grouped into four categories according to their features such as the existence of oscillation, overshoot, inverse response, etc. These four groups are: group A, those that are oscillatory; group B, those that are nonoscillatory and do not have overshoot at all times, nor do they respond in the reverse direction; group C, those that are nonoscillatory and have overshoot; and group D, those that are nonoscillatory and have an inverse response.

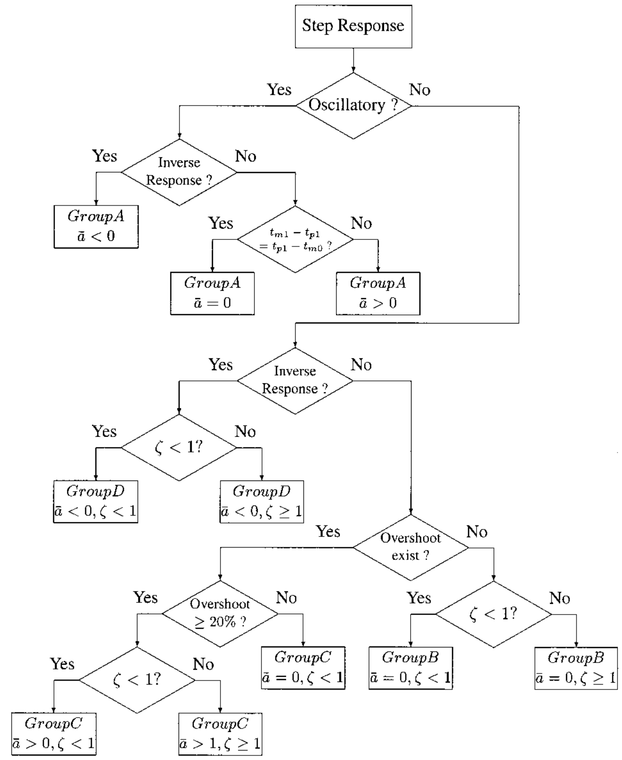

Figure 1. Flowchart for identification of model structure.

Table 2 depicts the important features of the step response of these four categories. In table 2, an entry with the digit 1 designates that the indicated feature relating to the status of the corresponding parameters is feasible, and an entry with a + designates that the feature is conditionally feasible. A strategy for identifying the statuses of ζ and $\bar{a}$ is thus presented in Figure 1. Notably, groups B, C, and D include some ζ values less than 1. It is desirable to devise an index from the step responses to identify the status of ζ. For this purpose, $R_1(x)$ is defined as

$$R_1(x) \equiv \frac{t_x - t_{(x-0.2)}}{t_{(x-0.2)} - t_{(x-0.4)}} \tag{4}$$

where t_x is the time when $y(t_x)/y_\infty = x$, $0 \leq x \leq 1$. Notably, the computation of $R_1(x)$ in eq 4 does not need a model in the form of either model I or model II.

By the definition of $\bar{t}$, $R_1(x)$ can also be written as

$$R_1(x) = \frac{\bar{t}_x - \bar{t}_{(x-0.2)}}{\bar{t}_{(x-0.2)} - \bar{t}_{(x-0.4)}} \tag{5}$$

where $\bar{t}_x = (t_x - \theta)/\tau$ is the dimensionless time derived from t_x and $\bar{y}(\bar{t}_x) = x$.

Table 1. Some Typical Unit Step Responses for Model I and Model II with Various Parametric Values

$y(\frac{t-\theta}{\tau})/k_p$			a/τ = −1.0	0	0.5	2.0
Model I	ζ	0.3				
		0.5				
		0.9				
Model II	η (ζ)	0.9 (1.00)				
		0.5 (1.06)				
		0.3 (1.19)				

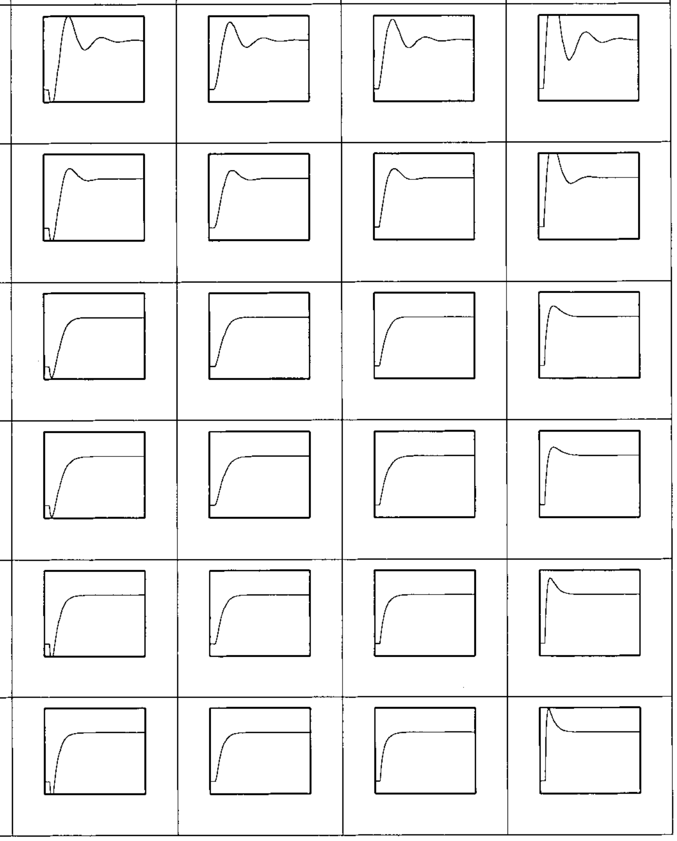

Table 2. Categories of Features for Step Responses

		oscillatory			nonoscillatoy	
		$\Delta = P/2$[a]	$\Delta \neq P/2$[a]	inverse response[b]	inverse response[b]	overshoot[b]
	$a = 0$	1	0	0	0	0
$\zeta < 1$	$a > 0$	0	1	0	0	+
	$a < 0$	0	1	1	1	0
	$a = 0$	0	0	0	0	0
$\zeta \geq 1$	$a > 0$	0	0	0	0	+
	$a < 0$	0	0	0	1	0

[a] $\Delta = t_{p,1} - t_{m,0}$. P = period of oscillations. [b] 1 = feasible; 0 = infeasible; + = conditionally feasible.

With eq 5, theoretical values of $R_1(x)$ can be calculated if models as well as their parameters of eq 1 are given. To understand why this quantity can be used to identify the status of ζ, we can resort to Figure 2. In that figure, values of $R_1(0.7)$ and $R_1(0.9)$ have been calculated extensively for model I and model II with different parameters. Domain maps for $\zeta < 1$ and for $\zeta \geq 1$ have been traced and plotted in the figure with $R_1(0.7)$ and $R_1(0.9)$ as the coordinates. Notice that the circle in the middle of two dark and dotted areas belongs to an FOPDT system. Thus, if we have a step response and R_1 values from experiment, by referring to Figure 2, we will be able to make a choice among model I, model II, or FOPDT models for identification.

Algorithms to estimate the parameters using the step response in each group are depicted as in the following section.

3. Algorithms for Parameters Estimation

In the previous section, we explained that the identification of models depends on whether $\zeta < 1$ or $\zeta \geq 1$,

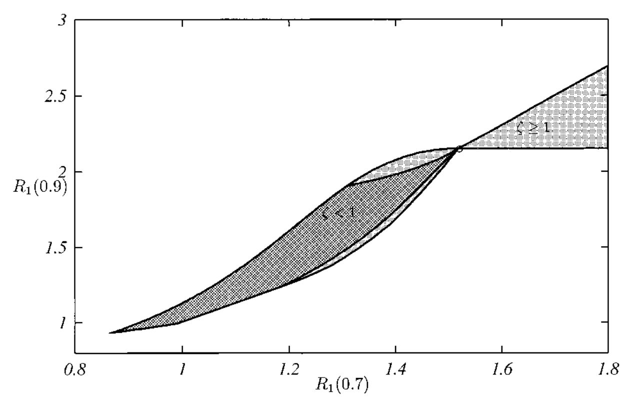

Figure 2. $R_1(0.7)$ vs $R_1(0.9)$.

and, on whether $\bar{a} < 0$ or $\bar{a} \geq 0$. Thus, the parametric estimation algorithms are devised according to the status of these key parameters. In the following, four algorithms are presented for estimating model parameters for the four categories of models.

3.1. Algorithm A for Oscillatory Step Responses. From Table 2, a feasible model for an oscillatory step response is model I with $\zeta < 1$ and $\bar{a} \in (-\infty, \infty)$. For further identification, it is desirable to differentiate $\bar{a}$ into three cases, i.e., $\bar{a} < 0$, $\bar{a} = 0$, and $\bar{a} > 0$. It is easy to conclude that $\bar{a} < 0$ when an inverse response occurs. However, to differentiate between a response with $\bar{a} = 0$ and one with $\bar{a} \neq 0$ the following lemma is needed:

Lemma 1. Consider a system of model I having $\zeta < 1$ and $\bar{a} \in (-\infty, \infty)$. Let $t_{p,i}$ and $t_{m,i}$ be the time instants when the output, y, reaches its ith peak and ith valley, respectively. Then, if and only if $\bar{a} = 0$, we have

$$t_{\mathrm{p},1} - \theta = t_{\mathrm{p},i+1} - t_{\mathrm{m},i} = t_{\mathrm{m},i} - t_{\mathrm{p},i} = \frac{P}{2} = \frac{\pi\tau}{\sqrt{1-\zeta^2}}\ \forall\ i \geq 1, j > i$$

Notably, the value of $t_{\mathrm{p},1} - \theta$ is taken as $t_{\mathrm{p},1} - t_{\mathrm{m},0}$ from Figure 3. Thus, the criterion to establish that $\bar{a} = 0$ is to determine whether $t_{\mathrm{p},1} - t_{\mathrm{m},0}$ is equal to $t_{\mathrm{p},2} - t_{\mathrm{m},1}$. Then, the estimation of the parameters can be obtained by carrying computations of the following quantities:

(1) the time constant, τ, of the system

$$\tau = \frac{P\sqrt{1-\zeta^2}}{2\pi} = \frac{P}{\sqrt{4\pi^2 + P^2\chi^2}} \tag{6}$$

where

$$\chi = \begin{cases} \dfrac{1}{t_{\mathrm{p},1} - t_{\mathrm{p},2}} \ln\left(\dfrac{\bar{y}_{\mathrm{p},2} - 1}{\bar{y}_{\mathrm{p},1} - 1}\right) & \forall \bar{a} \geq 0 \\ \dfrac{1}{t_{\mathrm{m},1} - t_{\mathrm{p},1}} \ln\left(\dfrac{\bar{y}_{\mathrm{p},1} - 1}{1 - \bar{y}_{\mathrm{m},1}}\right) & \forall \bar{a} < 0 \end{cases} \tag{7}$$

and

$$\bar{y}_{\mathrm{p},i} = \bar{y}(t_{\mathrm{p},i}),\ i = 1, 2, ...$$

(2) the damping ratio, ζ, of the system

$$\zeta = \begin{cases} \sqrt{\dfrac{\ln^2(\bar{y}_{\mathrm{p},1} - 1)}{\pi^2 + \ln^2(\bar{y}_{\mathrm{p},1} - 1)}} & \text{for } \bar{a} = 0 \\ \dfrac{P\chi}{\sqrt{4\pi^2 + P^2\chi^2}} & \forall \bar{a} \neq 0 \end{cases} \tag{8}$$

(3) the parameters $\bar{a}$ and θ of the system

(a) for $\bar{a} = 0$

$$\theta = t_{\mathrm{p},1} + \frac{\tau}{\zeta} \ln(y_{\mathrm{p},1} - 1) \tag{9}$$

(b) for $\bar{a} \neq 0$

using the following two equations to calculate $\bar{a}$ and θ sequentially

$$\bar{a} = \begin{cases} \zeta + \sqrt{\zeta^2 + \left[1 - \left(\dfrac{y_{\mathrm{p},1} - 1}{\mathrm{e}^{-\zeta \bar{t}_{\mathrm{p},1}}}\right)^2\right]} & \forall \bar{a} > 0 \\ \zeta - \sqrt{\zeta^2 + \left[1 - \left(\dfrac{1 - \bar{y}_{\mathrm{m},1}}{\mathrm{e}^{-\zeta \bar{t}_{\mathrm{m},1}}}\right)^2\right]} & \forall \bar{a} < 0 \end{cases} \tag{10}$$

and

$$\theta = \begin{cases} t_{\mathrm{p},1} - \dfrac{P}{2\pi}\left(\pi - \tan^{-1} \dfrac{\bar{a}\sqrt{1-\zeta^2}}{1 - \bar{a}\zeta}\right) & \forall \bar{a} > 0 \\ t_{\mathrm{m},1} + \dfrac{P}{2\pi}\left(\tan^{-1} \dfrac{\bar{a}\sqrt{1-\zeta^2}}{1 - \bar{a}\zeta}\right) & \forall \bar{a} < 0 \end{cases} \tag{11}$$

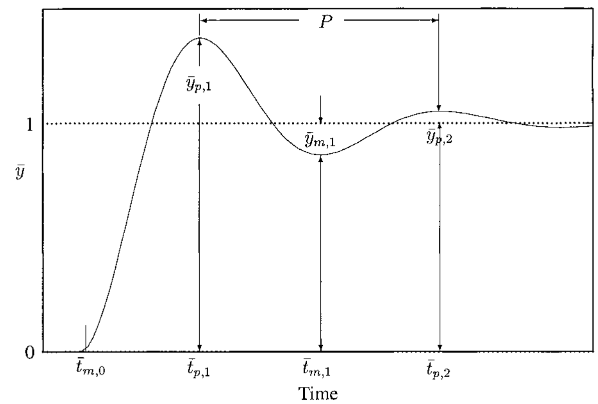

Figure 3. Step response for underdamped system with $\bar{a} \geq 0$.

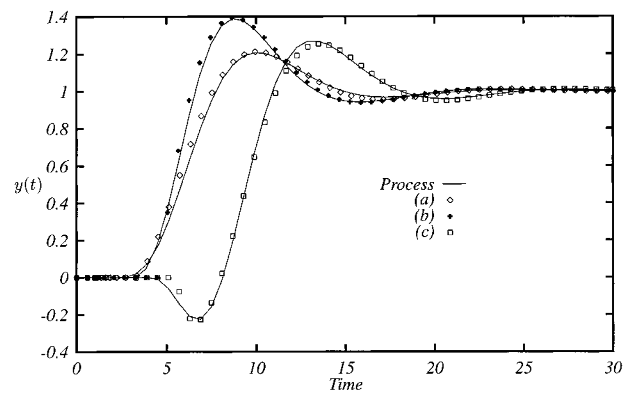

Figure 4. Step responses of example A.

The use of algorithm A for identification is illustrated with the following example.

Example A. Consider the following three processes:

$$G_{\mathrm{p}}(s) = \begin{cases} \dfrac{\mathrm{e}^{-s}}{(4s^2 + 2s + 1)(s^2 + s + 1)} & \text{(a)} \\ \dfrac{(2s + 1)\mathrm{e}^{-2s}}{(4s^2 + 2s + 1)(s^2 + s + 1)} & \text{(b)} \\ \dfrac{(-2s + 1)\mathrm{e}^{-3s}}{(4s^2 + 2s + 1)(s^2 + s + 1)} & \text{(c)} \end{cases}$$

The step responses of these G_{p}'s are as shown in Figure 4. All three responses are oscillatory, and response c has an inverse response. It is found that only response a has equal values for $t_{\mathrm{p},2} - t_{\mathrm{m},1}$ and $t_{\mathrm{p},1} - t_{\mathrm{m},0}$. The identification results based on these observations are given in Table 3. The unit step responses of the resulting models are given in Figure 4.

3.2. Algorithm B for Nonoscillatory Step Response without Overshoot or Inverse Response. The feasible model for this class of response is either model I or model II having $\bar{a} \in [0,1]$. However, it seems trivial to find a nonzero $\bar{a}$ for this case, because it is always possible to find a model in terms of model I or model II with $\bar{a} = 0$ that fits the given step response. Thus, models for this category are assumed to be represented by eq 1 with $\bar{a} = 0$. By this assumption, we obtain advantages in managing the derivations of the algorithm.

By assuming that $\bar{a} = 0$, the $\bar{t}_x$ value at any given x is a function of ζ or η. This value can be easily obtained by solving eq 3 with a single MATLAB instruction. For later uses, the values of $\bar{t}_x$ at $x = 0.3$, 0.5, 0.7, and 0.9

Table 3. Simulation Results for Example A

process	(a) $e^{-s}/(4s^2+2s+1)(s^2+s+1)$	(b) $(2s+1)e^{-2s}/(4s^2+2s+1)(s^2+s+1)$	(c) $(-2s+1)e^{-3s}/(4s^2+2s+1)(s^2+s+1)$
$t_{p,1}$, $y_{p,1}$	9.03, 1.41	6.79, 1.40	12.19, 1.27
$t_{p,2}$, $y_{p,2}$[a]	23.54, 1.06	21.12, 1.01	5.70, −0.23
$t_{m,0}$, $t_{p,1} - t_{m,0}$	1.75, 7.28	2.21, 4.58	–, –
P	14.51	14.33	13.80
χ	0.25	0.26	0.23
τ	2.00	1.97	1.96
ζ	0.44	0.51	0.45
$\bar{a}$	0	1.33	−0.90
θ	2.03	3.50	4.50
resulting model	$e^{-2.03s}/(4.00s^2+1.76s+1)$	$(2.62s+1)e^{-3.50s}/(3.88s^2+2.01s+1)$	$(-1.76s+1)e^{-4.50s}/(3.84s^2+1.76s+1)$

[a] $t_{m,1}$ and $y_{m,1}$ for $G_{p,c}$.

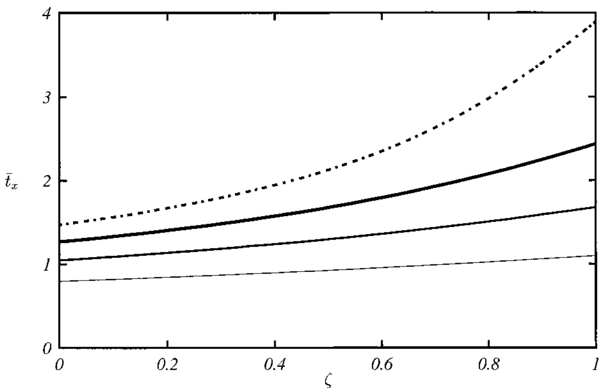

Figure 5. $\bar{t}_x$ for underdamped process with $\bar{a} = 0$ ($\bar{t}_{0.3}$, thin line; $\bar{t}_{0.5}$, normal line; $\bar{t}_{0.7}$, thick line; and $\bar{t}_{0.9}$, dotted line).

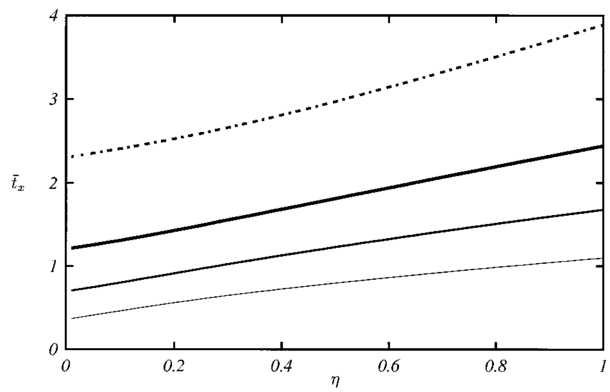

Figure 6. $\bar{t}_x$ for overdamped process with $\bar{a} = 0$ ($\bar{t}_{0.3}$, thin line; $\bar{t}_{0.5}$, normal line; $\bar{t}_{0.7}$, thick line; and $\bar{t}_{0.9}$, dotted line).

have been calculated and correlated with ζ (or η) into a functional form as follows (also see Figures 5 and 6):

(1) for model I

$$\bar{t}_{0.3} = 0.7954 + 0.2204\zeta + 0.0631\zeta^2 + 0.0184\zeta^3$$
$$\bar{t}_{0.5} = 1.0472 + 0.3952\zeta + 0.1577\zeta^2 + 0.0784\zeta^3$$
$$\bar{t}_{0.7} = 1.2662 + 0.6045z + 0.2834\zeta^2 + 0.2868\zeta^3$$
$$\bar{t}_{0.9} = 1.4655 + 0.9862\zeta - 0.1236\zeta^2 + 1.5732\zeta^3 \tag{12}$$

(2) for model II

$$\bar{t}_{0.3} = 0.3548 + 1.1211\eta - 0.5914\eta^2 + 0.2145\eta^3$$
$$\bar{t}_{0.5} = 0.6862 + 1.1682\eta - 0.1704\eta^2 - 0.0079\eta^3$$
$$\bar{t}_{0.7} = 1.1988 + 1.0818\eta + 0.4043\eta^2 - 0.2501\eta^3$$
$$\bar{t}_{0.9} = 2.3063 + 0.9017\eta + 1.0214\eta^2 + 0.3401\eta^3 \tag{13}$$

Next, we define the quantity $R_2(x)$ as

$$R_2(x) = \frac{M_\infty - t_{(x-0.2)}}{t_x - t_{(x-0.2)}} = \frac{\bar{M}_\infty - \bar{t}_{(x-0.2)}}{\bar{t}_x - \bar{t}_{(x-0.4)}} \tag{14}$$

where M designates the integrations of $y_\infty - y(t)$ with respect to time t.

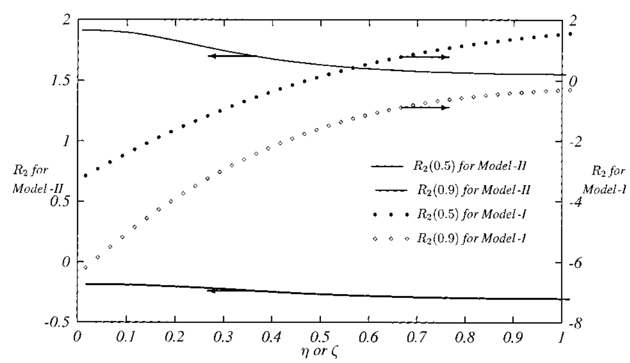

Figure 7. $R_2(x)$ for model I and model II with $\bar{a} = 0$.

$$M(t) = \int_0^t [y_\infty - y(\tau)]\,d\tau \tag{15}$$

$\bar{M}(\bar{t})$ is the normalized value of $M(t)$

$$\bar{M}(\bar{t}) = \int_0^{\bar{t}} [1 - \bar{y}(\tau)]\,d\tau \tag{16}$$

It is straightforward to show that

$$M(t) = [\bar{M}(t)\tau + \theta]k_p \tag{17}$$

Values of $R_2(x)$ at $x = 0.5$ and at $x = 0.9$ are plotted in Figure 7. It is found that, for each value of $R_2(x)$ at a given x, there is only one value of ζ (or η) in correspondence. Thus, from the two curves in Figure 7, we have two values of ζ (or η). As a result, an average of these two values can be taken as the estimate for ζ (or η). For later uses, the values of $R_2(x)$ at $x = 0.5$ and $x = 0.9$ have been calculated and correlated with ζ and η

Table 4. Simulation Results for Example B

process	(a) $1/(s+1)^5$	(b) $e^{-0.5s}/(2s+1)(s+1)(1/2s+1)$	(c) $e^{-0.2s}/(s+1)^3$
$t_{0.3}$, $t_{0.5}$	3.63, 4.67	2.75, 3.48	2.11, 2.87
$t_{0.7}$, $t_{0.9}$	5.89, 7.99	4.67, 7.01	3.82, 5.52
M_∞	5.00	4.00	3.20
$R_1(0.7)$, $R_1(0.9)$	1.18, 1.72	1.31, 1.95	1.24, 1.81
$R_2(0.5)$, $R_2(0.9)$	1.32, −0.42	1.57, −0.29	1.43, −0.36
ζ or η	0.85	0.74	0.92
τ	2.02	1.80	1.41
θ	1.53	0.86	0.61
resulting model	$e^{-1.53s}/(4.08s^2+3.43s+1)$	$e^{-0.86s}/(1.80s+1)(1.33s+1)$	$e^{-0.61s}/(1.99s^2+2.59s+1)$

into a functional form as follows:

(1) for model I ($\zeta < 1$)

$$R_2(0.5) = -3.1623 + 9.3343\zeta - 5.7804\zeta^2 + 1.1588\zeta^3$$

$$R_2(0.9) = -6.1991 + 14.6087\zeta - 12.1250\zeta^2 + 3.4080\zeta^3 \quad (18)$$

(2) for model II ($\zeta \geq 1$)

$$R_2(0.5) = 1.9108 + 0.2275\eta - 5.5504\eta^2 + 12.8123\eta^3 - 11.8164\eta^4 + 3.9735\eta^5$$

$$R_2(0.9) = -0.1871 + 0.0736\eta - 1.2329\eta^2 + 2.1814\eta^3 - 1.5317\eta^4 + 0.3937\eta^5 \quad (19)$$

Thus, by using eqs 18 and 19 and the experimental values of R_2, the value of ζ or η can be obtained. With this obtained value, the values of $\bar{t}_x$ at $x = 0.3$, 0.5, 0.7, and 0.9 can be calculated. Then, τ can be estimated by the following equation:

$$\tau = \frac{t_{x,i} - t_{x,j}}{\bar{t}_{x,i} - \bar{t}_{x,j}} \quad (20)$$

where i and j are any integers. Consequently, model parameters for group B can be estimated by conducting the following procedures:

(1) Calculate $R_2(x)$ at $x = 0.5$ and $x = 0.9$ from the experimental step response, and determine ζ or η from Figure 7 or from eq 18 or 19.

$$\zeta = \frac{\zeta_{0.5} + \zeta_{0.9}}{2} \quad (21)$$

and

$$\eta = \frac{\eta_{0.5} + \eta_{0.9}}{2} \quad (22)$$

(2) Calculate τ (or τ_1) as follows:

$$\tau = \frac{1}{3}\left[\frac{t_{0.9} - t_{0.7}}{\bar{t}_{0.9} - \bar{t}_{0.7}} + \frac{t_{0.7} - t_{0.5}}{\bar{t}_{0.7} - \bar{t}_{0.5}} + \frac{t_{0.5} - t_{0.3}}{\bar{t}_{0.5} - \bar{t}_{0.3}}\right] \quad (23)$$

where $\bar{t}_{0.3}$, $\bar{t}_{0.5}$, $\bar{t}_{0.7}$, and $\bar{t}_{0.9}$ are given in eq 12 or eq 13.

(3) Estimate θ as

$$\theta = \frac{t_{0.9} + t_{0.7} + t_{0.5} + t_{0.3}}{4} - \frac{\bar{t}_{0.9} + \bar{t}_{0.7} + \bar{t}_{0.5} + \bar{t}_{0.3}}{4} \quad (24)$$

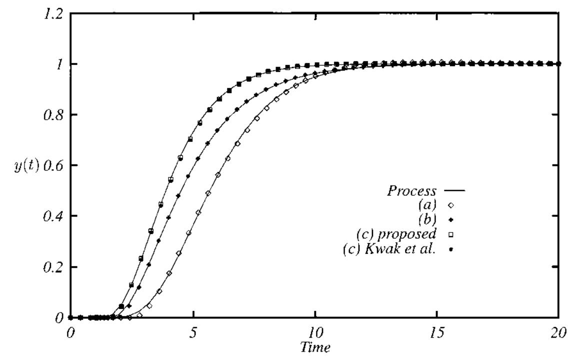

Figure 8. Step responses of example B.

An illustration of the above algorithm is given in example B.

Example B. Consider the following two processes:

$$G_p(s) = \begin{cases} \dfrac{1}{(s+1)^5} & \text{(a)} \\ \dfrac{e^{-0.5s}}{(2s+1)(s+1)\left(\frac{1}{2}s+1\right)} & \text{(b)} \\ \dfrac{e^{-0.2s}}{(s+1)^3} & \text{(c)} \end{cases}$$

The step responses, as shown in Figure 8, are nonoscillatory. Also, there are neither overshoots nor inverse responses in those responses. Some important values related to the identification and the resulting model parameters are given in Table 4. The predicted step responses for each of the models compared with those from the real process are shown in Figure 8.

The example in part c is used for comparison with the method of Kwak et al.[6] According to their work, the high-order model for part c is

$$G(s) = \frac{0.00002s^4 - 0.0007s^3 + 0.0144s^2 - 0.1764s + 1}{-0.0007s^3 + 0.0317s^4 + 1.0986s^3 + 3.1005s^2 + 3.0325s + 1}$$

In their work, this high-order model is used to provide the required frequency response data for finding a reduced second-order model. This reduced model for approximation is

$$G(s) = \frac{e^{-0.60s}}{2.10s^2 + 2.61s + 1}$$

which is to be compared with the model from the proposed method

$$G(s) = \frac{e^{-0.61s}}{1.99s^2 + 2.57s + 1}$$

Figure 8 shows that both models fit the step response equally well.

3.3. Algorithm C for Nonoscillatory Step Response withOvershoot. The step responses of this category can result from a dynamic system of model I or model II with an LHP zero.

As before, according to $R_1(x)$ values and Figure 1, the status of ζ can be identified at the very beginning. Thus, the estimation procedure is divided into two parts: one for $\zeta \geq 1$ and one for $\zeta < 1$.

In the following, let t_p (or $\bar{t}_p$) designate the duration of time (or the normalized time) that is required for the output of the system to reach its maximum. Also, let t_R (or $\bar{t}_R$) designate that for rising time of the output. To derive algorithm C, we need expressions for $\bar{t}_p$ and $\bar{t}_R$. We also need some expressions for $\bar{y}$ and $\bar{M}$ at these time instants. These expressions are given below.

(1) Useful results concerning $\bar{t}_p$, $\bar{y}(\bar{t}_p)$, $\bar{t}_R$, and $\bar{y}(\bar{t}_R)$:

(a) for step response from model I

$$\bar{t}_p = \frac{1}{\sqrt{1-\zeta^2}}\left[\pi + \tan^{-1}\left(\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right)\right] \tag{25}$$

$$\bar{y}_p = 1 + (\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2})e^{-\zeta\bar{t}_p} \tag{26}$$

$$\bar{t}_R = \frac{1}{1-\zeta^2}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\bar{a}-\zeta}\right) \tag{27}$$

(b) for step response from model II

$$\bar{t}_p \text{ (or } \bar{t}_m) = \ln\frac{1-\bar{a}}{1-\bar{y}_p} = \frac{\eta}{\eta-1}\ln\frac{\eta(1-\bar{a})}{\eta-\bar{a}} \tag{28}$$

$$\bar{y}_p \text{ (or } \bar{y}_m) = 1 - (1-\bar{a})\left[\frac{\eta(1-\bar{a})}{\eta-\bar{a}}\right]^{\eta/\eta-1} \tag{29}$$

$$\bar{t}_R = \left(\frac{\eta}{\eta-1}\right)\ln\left(\frac{1-\bar{a}}{\eta-\bar{a}}\right) \tag{30}$$

(2) Useful results concerning $\bar{M}(\bar{t})$ at $\bar{t}_p$ and $\bar{t}_R$:

(a) for underdamped systems

$$\bar{M}(\bar{t}_p) = 2\zeta(\bar{y}_p - 1) + \bar{M}_\infty \tag{31}$$

where

$$\bar{M}_\infty = \lim_{t\to\infty}\bar{M}(\bar{t}) = 2\zeta - \bar{a} \tag{32}$$

(b) for overdamped systems

$$\bar{M}(\bar{t}_p) = (\eta + 1)(\bar{a} - 1)e^{-\bar{t}_p} + \bar{M}_\infty \tag{33}$$

where

$$\bar{M}_\infty = \lim_{t\to\infty}\bar{M}(\bar{t}) = 1 + \eta - \bar{a} \tag{34}$$

The proofs of this property can be found in the Appendix.

With all of the aforementioned results, the estimation of the parameters for this class of step responses can be conducted by the following procedures.

(1) For model I ($\zeta < 1$)

(a) It can be shown that, for an underdamped system, the following equalities hold:

$$\bar{t}_p - \bar{t}_R = \frac{1}{\zeta}\ln\left(2\zeta\frac{\bar{M}_R - \bar{M}_\infty}{\bar{M}_p - \bar{M}_\infty}\right) \tag{35}$$

and

$$\bar{M}_p - \bar{M}_\infty = 2\zeta(\bar{y}_p - 1) \tag{36}$$

Thus, from the measured values t_p, t_R, M_p, M_R, M, and, y_p, the values of ζ and τ can be estimated from the following equations:

$$\frac{t_p - t_R}{M_p - M_\infty} = \frac{\frac{1}{\zeta}\ln\left(2\zeta\frac{M_R - M_\infty}{M_p - M_\infty}\right)}{2\zeta(\bar{y}_p - 1)} \tag{37}$$

and

$$\tau = \frac{M_p - M_\infty}{2\zeta(\bar{y}_p - 1)} \tag{38}$$

(b) The estimate of $\bar{a}$ is thus obtained from the solution of the following equation:

$$\frac{t_p - t_R}{\tau} = \left(-\frac{1}{\zeta}\ln\frac{\bar{y}_p - 1}{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}}\right) - \left[\frac{1}{\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\bar{a}-\zeta}\right)\right] \tag{39}$$

(c) Having obtained these values, the dead time, θ, can be calculated as

$$\theta = \frac{1}{2}\left\{\left[t_p + \frac{\tau}{\zeta}\ln\frac{\bar{y}_p - 1}{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}}\right] + [M_\infty - (2\zeta + \bar{a})\tau]\right\} \tag{40}$$

Proofs of the above equations are given in the Appendix.

(2) For model II ($\zeta \geq 1$):

(a) to estimate η and τ, we use the following equations:

$$\frac{t_p - t_R}{M_p - M_\infty} = \frac{\ln\left[(\eta + 1)\frac{M_R - M_\infty}{M_p - M_\infty}\right]}{(\eta + 1)(\bar{y}_p - 1)} \tag{41}$$

and

$$\tau = \frac{M_p - M_\infty}{(\eta + 1)(y_p - 1)} \tag{42}$$

From the measured quantities M_p, M_∞, t_p, t_R, and $\bar{y}_p$, we can calculate η and τ from the above two equations.

(b) The value of $\bar{a}$ can be solved from the following equation:

$$\frac{t_p - t_R}{\tau} = \ln\left(\frac{1-\bar{a}}{1-\bar{y}_p}\right) - \left(\frac{\eta}{\eta-1}\right)\ln\left(\frac{1-\bar{a}}{\eta-\bar{a}}\right) \tag{43}$$

so that a becomes $a = \tau\,\bar{a}$.

Table 5. Simulation Results for Example C

	(a)	(b)
process	$3s+1/(s+1)^5$	$(5s+1)e^{-2s}/(2s+1)(s+1)(1/2s+1)$
$t_{0.3}$, $t_{0.5}$	1.93, 2.47	2.44, 2.62
$t_{0.7}$, $t_{0.9}$	3.03, 3.73	2.81, 3.01
t_R, M_R	4.24, 2.48	3.13, 2.62
t_p, y_p	6.00, 1.12	4.67, 1.51
M_p, M_∞	2.34, 2.00	2.06, 0.50
$R_1(0.7)$, $R_1(0.9)$	1.04, 1.25	1.01, 1.10
ζ or η	0.56	1.00
τ	1.37	1.52
$\bar{a}$	–	3.22
θ	0.65	2.40
resulting model	$e^{-0.65s}/(1.88s^2+1.53s+1)$	$(4.89s+1)e^{-2.40s}/(1.52s+1)(1.52s+1)$

(c) The dead time, θ, can be estimated as

$$\theta = \frac{1}{2}\left\{\left[t_p - \tau \ln\left(\frac{1-\bar{a}}{1-\bar{y}_p}\right)\right] + [M_\infty - (1+\eta-\bar{a})\tau]\right\} \tag{44}$$

Notably, the nonlinear algebraic equations (such as eqs 37, 39, 41, and 43 can be solved by using any simple strategy for a one-dimensional search.

In addition to the estimations described above, underdamped systems that have very small decay ratios (say, less than 0.05) will give step responses that could be classified into this category. Thus, when the overshoot is less than 20%, an alternative modeling procedure can be taken as follows:

(1) Estimate the damping factor as

$$r = \frac{y_p - y_\infty}{y_\infty}$$

$$\zeta = \sqrt{\frac{(\ln r)^2}{\pi^2 + (\ln r)^2}}$$

(2) Compute t_x values for $x = 0.3$, 0.5, 0.7, and 0.9 according to eq 12

(3) Estimate τ with eq 23.

(4) Estimate θ with eq 24.

An illustration of algorithm C is provided in example C.

Example C. Consider the following two processes:

$$G_p(s) = \begin{cases} \dfrac{(3s+1)}{(s+1)^5} & \text{(a)} \\[2ex] \dfrac{(5s+1)e^{-2.0s}}{(2s+1)(s+1)\left(\frac{1}{2}s+1\right)} & \text{(b)} \end{cases}$$

The step responses of the processes, as well as those of the models, are shown in Figure 9. Notice that, in part a of the example, the response has a small overshoot of 0.12, which corresponds to a decay ratio of 0.014. Thus, the feasible model is taken as model I with $a = 0$. Parameters are thus estimated according to the algorithm just mentioned above. Details of the results are shown in the Table 5. The good fit between the process and the model is also shown in Figure 9.

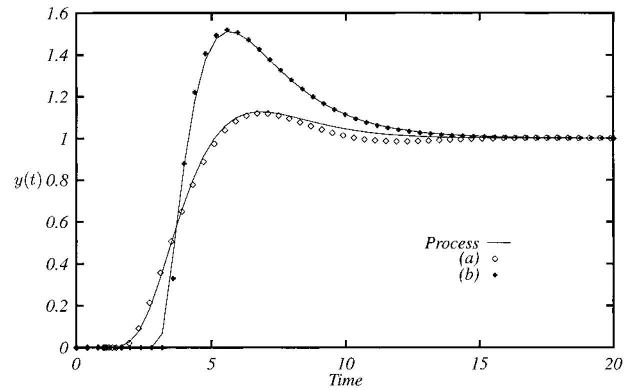

Figure 9. Step responses of example C.

3.4. Algorithm D for Nonoscillatory Step Response with Inverse Response. The feasible model for this category is either model I or model II with $a < 0$. The status of ζ is identified according to the calculated values of $R_1(x)$ at the beginning step. Thus, having this status, the estimation procedure is divided into two parts: one for $\zeta \geq 1$ and another for $\zeta < 1$. Unlike the previous algorithm, an iterative procedure is adopted for the estimation. The parametric estimation algorithm goes as follows:

(1) For $\zeta < 1$

(a) At the beginning, take $\zeta^* = 0.85$ as an initial guess.

(b) The time constant, τ, is calculated using eq 31

$$\tau = \frac{M_m - M_\infty}{2\zeta(\bar{y}_m - 1)} \tag{45}$$

where $\bar{y}_m$ is the lowest height of the response beneath zero in the reverse direction.

(c) The value of $\bar{a}$ can be solved from the equation

$$-\frac{1}{\zeta}\ln\frac{1-\bar{y}_m}{\sqrt{1-2\bar{a}\zeta+\bar{a}^2}} = \frac{1}{\sqrt{1-\zeta^2}}\tan^{-1}\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta-1} \tag{46}$$

(d) Using the results obtained from the last two steps, the dead time, θ, can be calculated from

$$\frac{M_c - M_\infty}{M_m - M_\infty} = \frac{\bar{a}}{2\zeta}\frac{\sin\left(\sqrt{1-\zeta^2}\,\frac{t_c-\theta}{\tau}\right)}{\sin\left(\sqrt{1-\zeta^2}\,\frac{t_m-\theta}{\tau}\right)}e^{-\zeta(t_c-t_m)/\tau} - \frac{2\zeta-\bar{a}}{(2\zeta)(\bar{y}_m-1)} \tag{47}$$

where $\bar{t}_m$ is the time instant when $\bar{y}$ reaches its lowest position in the inverse response and $\bar{t}_c$ is the time instant when $\bar{y}$ return to zero from an inverse response.

Table 6. Simulation Results for Example D

process	(a) $-2s+1/(s+1)^5$	(b) $(-2s+1)e^{-0.5s}/(2s+1)(s+1)(1/2s+1)$
$t_{0.3}$, $t_{0.5}$	5.37, 6.24	4.17, 4.98
$t_{0.7}$, $t_{0.9}$	7.35, 9.34	6.12, 8.41
t_c, M_c	4.10, 4.43	3.16, 3.59
t_m, y_m	2.67, −0.16	1.79, −0.27
M_m, M_∞	2.85, 7.00	1.99, 6.00
$R_1(0.7)$, $R_1(0.9)$	1.27, 1.81	1.40, 2.02
ζ or η	0.88	0.70
τ	2.03	1.86
$\bar{a}$	−0.91	−1.10
θ	1.60	0.84
resulting model	$(-1.85s+1)e^{-1.60s}/(4.12s^2+3.57s+1)$	$(-2.05s+1)e^{-0.84s}/(1.86s+1)(1.30s+1)$

(e) Finally, the new ζ can be calculated from eq 32.

$$\zeta = \frac{M_\infty - \theta + \bar{a}\tau}{2\tau} \tag{48}$$

(f) Update the assumed ζ, and repeat the procedure starting from step 2 until ζ converges.

(2) For $\zeta \geq 1$

(a) At the beginning, take θ^* as an initial guess.

(b) Compute $\bar{a}$, τ, η, and θ according to the following steps:

(i) for zero time constant, a

$$a = \frac{M_m - M_\infty}{\bar{y}_m - 1} - (M_\infty - \theta) \tag{49}$$

(ii) the time constant τ

$$\frac{t_m - \theta}{\tau} = \ln\left(\frac{1 - \frac{a}{\tau}}{1 - \bar{y}_m}\right) \tag{50}$$

(iii) the parameter η

$$\eta = \frac{\bar{M}_m - M_\infty}{\bar{y}_m - 1} - 1 \tag{51}$$

(iv) a new value of θ

$$\theta = t_c + \tau \ln\left(\frac{\bar{M}_c - \bar{M}_\infty + \eta}{\bar{a} - 1}\right) \tag{52}$$

(c) Update the assumed θ with the new value, and repeat the procedure starting from step 2 until θ converges.

In example D, an illustration of algorithm D for identification is provided.

Example D. Consider the following two processes:

$$G_p(s) = \begin{cases} \dfrac{(-2s+1)}{(s+1)^5} & \text{(a)} \\ \dfrac{(-2s+1)e^{-0.5s}}{(2s+1)(s+1)\left(\frac{1}{2}s+1\right)} & \text{(b)} \end{cases}$$

The step responses of the parts of this example are shown in Figure 10, and the estimation results are given in Table 6.

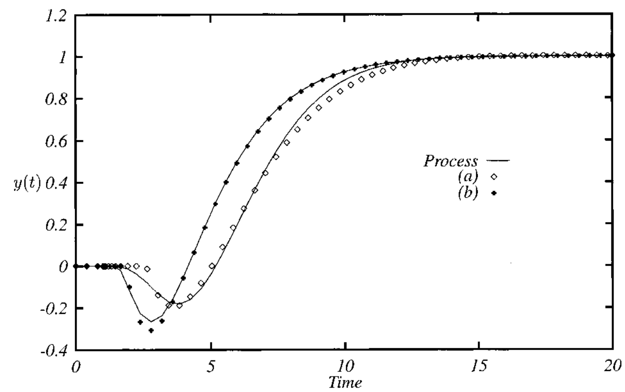

Figure 10. Step responses of example D.

Table 7. Estimated Ultimate Properties of the Examples

example		K_u true value	K_u estimated	ω_c true value	ω_c estimated	feasible region for prediction
A	(a)	1.059	1.117	0.577	0.592	$\omega \leq 1.07$
	(b)	0.747	0.711	0.624	0.628	$\omega \leq 0.82$
	(c)	0.657	0.678	0.367	0.366	$\omega \leq 1.12$
B	(a)	2.885	2.795	0.727	0.735	$\omega \leq 0.85$
	(b)	4.524	4.342	1.168	1.175	$\omega \leq 1.51$
	(c)	5.152	4.851	1.408	1.437	$\omega \leq 1.45$
C	(a)	2.635	2.632	1.234	1.229	$\omega \leq 1.69$
	(b)	0.713	0.675	1.051	1.043	$\omega \leq 1.43$
D	(a)	1.245	1.314	0.508	0.502	$\omega \leq 0.99$
	(b)	1.278	1.253	0.681	0.679	$\omega \leq 1.45$

4. Remarks on Results

The modeling of step responses with models I and II has been described in the above sections. In fact, these identified models have reduced dynamic orders and are treated as approximations to the real processes. Because of the use of step input for exciting dynamic modes and the use of reduced-order models, a few points regarding the results of this identification are required.

(1) In all cases, the fitting to the transient step responses resulting from the identified models is remarkable.

(2) In addition to the good fit to the step responses, justification for the resulting models can have different views. For control design, the capability of the model in predicting the ultimate gain and the ultimate frequency is a major concern. For this, the estimated and true values of the ultimate properties for all four of the examples are given in Table 7. Feasible prediction regions in terms of frequencies are also given to emphasize the regions in which the prediction errors will be less than 10% of their true values.

(3) The predictions of the phases in each of the examples are very close to their true values. Thus, all of the examples give predicted ultimate frequencies with reasonable accuracy (see Table 7).

(4) Nevertheless, good prediction of the amplitude ratio is limited to a low-frequency region (see Table 7). This limitation is common to any modeling or identification methods that uses reduced-order approximations and is mainly caused by the reduction of the dynamic order. In the low-frequency region, the accuracy of the predictions from the model is reasonably good. For the high-frequency region, however, the accuracy degrades as the frequency increases.

(5) The use of the model structure and the magnitude of the dead time also affect the width of the reasonable prediction region. For the former, it happens in the cases when models with or without an LHP zero are all feasible. In general, models with LHP zeros have a better fit to the step response and a narrower region of accurate prediction. The controllers thus designed will have more conservative gains. The decision on which model to use should be made according to the usage of the models.

5. Conclusions

In this paper, we present a system of complete procedures for identifying dynamic models in terms of simple models of eq 1 for general dynamic systems. Dynamic step responses are classified into four groups. The type of model to be used is identified with a defined index, which determines the status of a key parameter in the second-order dynamics. Algorithms for estimating the parameters in the chosen model are then derived according to the step response of each type of model.

To start the procedures, there is no need to presume the model structure to be used. The estimation does not use least squares and is to be performed after a complete set of step response data has been collected. Because of the progress in computer software, the proposed procedures provide a possibility of developing automation for identification. The key parts of the software for this automation would be the development of programs to do the following:

(1) characterize the step responses with specific time instants, peaks, valleys, oscillations or nonoscillations, inverse or monotonic responses, etc.

(2) map the calculated $R_1(0.7)$ and $R_1(0.9)$ values to the diagram in Figure 2 and return the value of the damping factor ζ

(3) classify a dynamic response according to the flowchart in Figure 1

(4) project the value of ζ or η using Figure 6 to obtain $\bar{t}_x$

(5) collect the items and compute the required quantities that are needed for estimations in each different class of dynamics

(6) perform, step-by-step, the estimation algorithms of that class

The resulting models would be useful for the applications of advanced control, such as tuning advanced PID controllers, tuning PID controllers in cascaded loop or in multi-loop systems, etc.

Appendix

I. Proof of Lemma 1. From eq 3, the angular velocity of the response in terms of $\bar{t}$ is $\bar{\omega} = \sqrt{1-\zeta^2}$, so that

$$\omega = \frac{\bar{\omega}}{\tau} = \frac{\sqrt{1-\zeta^2}}{\tau} \tag{A-I-1}$$

Thus, the period of oscillation becomes

$$P = \frac{2\pi\,\tau}{\sqrt{1-\zeta^2}} \tag{A-I-2}$$

By differentiating eq 3, we also have

$$\frac{\mathrm{d}\bar{y}(\bar{t})}{\mathrm{d}\bar{t}} = \left[\left(\frac{1-\bar{a}\zeta}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}) + \bar{a}\cos(\sqrt{1-\zeta^2}\,\bar{t})\right]\mathrm{e}^{-\zeta\bar{t}} \tag{A-I-3}$$

By setting this first derivative to zero, we have the extreme points at $\bar{t}_n^*$ satisfying the equation

$$\frac{\sin(\sqrt{1-\zeta^2}\,\bar{t}_n^*)}{\cos(\sqrt{1-\zeta^2}\,\bar{t}_n^*)} = \tan(\sqrt{1-\zeta^2}\,\bar{t}_n^*) = \frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1} \tag{A-I-4}$$

or

$$\bar{t}_n^* = \frac{1}{\sqrt{1-\zeta^2}}\left[n\pi + \tan^{-1}\left(\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right)\right] \quad \text{for} \quad n = 1, 2, ... \tag{A-I-5}$$

Thus, we have

$$\bar{t}_{\mathrm{p},i} = \frac{1}{\sqrt{1-\zeta^2}}\left[(2i-1)\pi + \tan^{-1}\left(\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right)\right] \quad \text{for} \quad i = 1, 2, ... \tag{A-I-6}$$

$$\bar{t}_{\mathrm{m},i} = \frac{1}{\sqrt{1-\zeta^2}}\left[2i\pi + \tan^{-1}\left(\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right)\right] \quad \text{for} \quad i = 1, 2, ... \tag{A-I-7}$$

and

$$t_{\mathrm{p},1} - \theta = \frac{\tau}{\sqrt{1-\zeta^2}}\left[\pi + \tan^{-1}\left(\frac{\bar{a}\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right)\right] \tag{A-I-8}$$

and

$$t_{\mathrm{p},2} - t_{\mathrm{m},1} = \frac{\pi\tau}{\sqrt{1-\zeta^2}} = \frac{P}{2} = t_{\mathrm{p},i} - t_{\mathrm{m},i-1} \quad \text{for} \quad i \geq 2 \tag{A-I-9}$$

Thus, it is obvious that eq A-I-8 equals eq A-I-9 if and only if $\bar{a}$ equals zero.

II. Derivations for Eqs 26–30. (1) For eq 26, by substituting eq A-I-6 into eq 3, we have

$$\bar{y}_{\mathrm{p}} = 1 - \left[\left(\frac{\zeta - \bar{a}}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}_{\mathrm{p}}) + \cos(\sqrt{1-\zeta^2}\,\bar{t}_{\mathrm{p}})\right]\mathrm{e}^{-\zeta\bar{t}_{\mathrm{p}}} = 1 + (\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2})\mathrm{e}^{-\zeta\bar{t}_{\mathrm{p}}} \tag{A-II-1}$$

Q.E.D.

(2) For eq 27, at $\bar{t} = \bar{t}_R$, $\bar{y}(\bar{t}_R) = 1$; thus

$$\bar{y}(\bar{t}_R) = 1 - \left[\left(\frac{\zeta - \bar{a}}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}_R) + \cos(\sqrt{1-\zeta^2}\,\bar{t}_R)\right]e^{-\zeta\bar{t}_R} \quad \text{(A-II-2)}$$

so that

$$\frac{\sin(\sqrt{1-\zeta^2}\,\bar{t}_R)}{\cos(\sqrt{1-\zeta^2}\,\bar{t}_R)} = \tan(\sqrt{1-\zeta^2}\,\bar{t}_R) = \frac{\sqrt{1-\zeta^2}}{\bar{a}-\zeta} \quad \text{(A-II-3)}$$

and

$$\bar{t}_R = \frac{1}{\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\bar{a}-\zeta}\right) \quad \text{(A-II-4)}$$

Q.E.D.

(3) For eq 28, from eq 3, we have

$$\left.\frac{d\bar{y}(\bar{t})}{d\bar{t}}\right|_{\bar{t}=\bar{t}_p} = \left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}_p} + \frac{1}{\eta}\left(\frac{\eta-\bar{a}}{\eta-1}\right)e^{-\bar{t}_p/\eta} = 0 \quad \text{(A-II-5)}$$

so that

$$\left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}_p} = \frac{1}{\eta}\left(\frac{\eta-\bar{a}}{1-\eta}\right)e^{-\bar{t}_p/\eta} \quad \text{(A-II-6)}$$

and

$$\bar{t}_p = \frac{\eta}{\eta-1}\ln\frac{\eta(1-\bar{a})}{\eta-\bar{a}} \quad \text{(A-II-7)}$$

Q.E.D.

(4) For eq 29, by substituting eq A-II-7 into eq 3, eq 29 results, i.e.

$$\begin{aligned}\bar{y}_p &= 1 - \left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}_p} - \left(\frac{\eta-\bar{a}}{\eta-1}\right)e^{-\bar{t}_p/\eta} \\ &= 1 - (1-\bar{a})e^{-\bar{t}_p} \\ &= 1 - (1-\bar{a})\left[\frac{\eta(1-\bar{a})}{\eta-\bar{a}}\right]^{\eta/(\eta-1)}\end{aligned} \quad \text{(A-II-8)}$$

Q.E.D.

(5) For eq 30, at $\bar{t} = \bar{t}_R$, $\bar{y}(\bar{t}_R) = 1$; thus

$$\bar{y}(\bar{t}_R) = 1 - \left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}_R} - \left(\frac{\eta-\bar{a}}{\eta-1}\right)e^{-\bar{t}_R/\eta} = 1 \quad \text{(A-II-9)}$$

so that

$$\left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}_R} = \left(\frac{\eta-\bar{a}}{1-\eta}\right)e^{-\bar{t}_R/\eta} \quad \text{(A-II-10)}$$

and

$$\bar{t}_R = \left(\frac{\eta}{\eta-1}\right)\ln\left(\frac{1-\bar{a}}{\eta-\bar{a}}\right) \quad \text{(A-II-11)}$$

Q.E.D.

III. Derivations for Eqs 31, 33, and 35. (1) For eq 31

$$\begin{aligned}\bar{M}(\bar{t}) &= \frac{M(t)-\theta}{\tau} \\ &= \int_0^{\bar{t}}[1-\bar{y}(\bar{t})]\,d\bar{t} \\ &= \int_0^{\bar{t}}\left[\left(\frac{\zeta-\bar{a}}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t})e^{-\zeta\bar{t}} + \cos(\sqrt{1-\zeta^2}\,\bar{t})e^{-\zeta\bar{t}}\right]d\bar{t} \\ &= \left[\left(\frac{1+\zeta\bar{a}-2\zeta^2}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}) - (2\zeta-\bar{a})\cos(\sqrt{1-\zeta^2}\,\bar{t})\right]e^{-\zeta\bar{t}} + \bar{M}_\infty\end{aligned} \quad \text{(A-III-1)}$$

where

$$\bar{M}_\infty = \lim_{\bar{t}\to\infty}\bar{M}(\bar{t}) = 2\zeta - \bar{a} \quad \text{(A-III-2)}$$

When $\bar{t} = \bar{t}_p$, we have

$$\bar{M}(\bar{t}_p) = 2\zeta(\bar{y}_p - 1) \quad \text{(A-III-3)}$$

Q.E.D.

(2) For eq 33

$$\begin{aligned}\bar{M}(\bar{t}) &= \frac{M(t)-\theta}{\tau} \\ &= \int_0^{\bar{t}}[1-\bar{y}(\bar{t})]\,dt \\ &= \int_0^{\bar{t}}\left[\left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}} + \left(\frac{\eta-\bar{a}}{\eta-1}\right)e^{-\bar{t}/\eta}\right]d\bar{t} \\ &= -\left(\frac{1-\bar{a}}{1-\eta}\right)e^{-\bar{t}} - \eta\left(\frac{\eta-\bar{a}}{\eta-1}\right)e^{-\bar{t}/\eta} + \bar{M}_\infty\end{aligned} \quad \text{(A-III-4)}$$

where

$$\bar{M}_\infty = \lim_{\bar{t}\to\infty}\bar{M}(\bar{t}) = 1 + \eta - \bar{a} \quad \text{(A-III-5)}$$

When $\bar{t} = \bar{t}_p$, we have

$$\bar{M}_p = \int_0^{\bar{t}_p}[1-\bar{y}(\bar{t})]\,d\bar{t} = (\eta+1)(\bar{y}_p - 1) + \bar{M}_\infty \quad \text{(A-III-6)}$$

Q.E.D.

(3) For eq 35

$$\frac{\bar{M}_R - \bar{M}_\infty}{\bar{M}_p - \bar{M}_\infty} = \frac{(\sqrt{1-2\bar{a}\zeta+\bar{a}^2})e^{-\zeta\bar{t}_R}}{(2\zeta)(\bar{y}_p - 1)} \quad \text{(A-III-7)}$$

so that

$$\ln\left(\frac{M_R - M_\infty}{M_p - M_\infty}\right) = \ln\left(\frac{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}}{\bar{y}_p - 1}\right) - \ln(2\zeta) - \bar{t}_R\zeta \quad \text{(A-III-8)}$$

As a result, by substituting eq A-II-4 into eq A-III-8, we have

$$\ln\left(\frac{M_R - M_\infty}{M_p - M_\infty}\right) = \bar{t}_p\zeta - \ln(2\zeta) - \bar{t}_R\zeta = (\bar{t}_p - \bar{t}_R)\zeta - \ln(2\zeta) \quad \text{(A-III-9)}$$

so that

$$\frac{t_p - t_R}{\tau} = \frac{1}{\zeta}\ln\left[2\zeta\left(\frac{M_R - M_\infty}{M_p - M_\infty}\right)\right] \quad \text{(A-III-10)}$$

Q.E.D.

IV. Derivations for eq 47. From eq 3, we have

$$\bar{y}(\bar{t}_c) = 1 - \left[\left(\frac{\zeta - \bar{a}}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}_c) + \cos(\sqrt{1-\zeta^2}\,\bar{t}_c)\right]e^{-\zeta\bar{t}_c} = 0 \quad \text{(A-IV-1)}$$

so that

$$1 - \left(\frac{\zeta - \bar{a}}{\sqrt{1-\zeta^2}}\right)\sin(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} = \cos(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} \quad \text{(A-IV-2)}$$

As a result, by substituting eq A-IV-2 into eq A-III-2 and letting $\bar{t} = \bar{t}_c$, we have

$$\bar{M}_c = \int_0^{\bar{t}_c}[1 - \bar{y}(\bar{t})]\,d\bar{t} = \frac{1 - 2\bar{a}\zeta + \bar{a}^2}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} - (2\zeta - \bar{a}) + \bar{M}_\infty \quad \text{(A-IV-3)}$$

There also exists some time t_m such that $dy(t)/dt|_{t=t_m} = 0$. Then, the following relations hold:

$$\bar{t}_m = \frac{1}{\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{a\sqrt{1-\zeta^2}}{\bar{a}\zeta - 1}\right) = -\frac{1}{\zeta}\ln\frac{\bar{y}_m - 1}{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}} \quad \text{(A-IV-4)}$$

and

$$\bar{M}_m = \int_0^{\bar{t}_m}[1 - \bar{y}(\bar{t})]\,d\bar{t} = (2\zeta)(\bar{y}_m - 1) + \bar{M}_\infty \quad \text{(A-IV-5)}$$

From eq A-IV-3 and eq A-IV-5, we have

$$\frac{M_c - M_\infty}{M_m - M_\infty} = \frac{\dfrac{1 - 2\bar{a}\zeta + \bar{a}^2}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} - (2\zeta - \bar{a})}{(2\zeta)(\bar{y}_m - 1)} \quad \text{(A-IV-6)}$$

As a result, by substituting eq A-IV-4 into eq A-IV-6, we have

$$\frac{M_c - M_\infty}{M_m - M_\infty} = \frac{1}{2\zeta}\frac{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}}{\sqrt{1-\zeta^2}}\frac{\sqrt{1 - 2\bar{a}\zeta + \bar{a}^2}}{\bar{y}_m - 1}\sin(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} - \frac{2\zeta - \bar{a}}{(2\zeta)(\bar{y}_m - 1)}$$
$$= \frac{1}{2\zeta}\frac{\bar{a}}{\sin(\sqrt{1-\zeta^2}\,\bar{t}_m)}\frac{1}{e^{-\zeta\bar{t}_m}}\sin(\sqrt{1-\zeta^2}\,\bar{t}_c)e^{-\zeta\bar{t}_c} - \frac{2\zeta - \bar{a}}{(2\zeta)(\bar{y}_m - 1)}$$
$$= \frac{\bar{a}}{2\zeta}\frac{\sin\left(\sqrt{1-\zeta^2}\dfrac{t_c - \theta}{\tau}\right)}{\sin\left(\sqrt{1-\zeta^2}\dfrac{t_m - \theta}{\tau}\right)}e^{-\zeta(t_c - t_m)/\tau} - \frac{2\zeta - \bar{a}}{(2\zeta)(\bar{y}_m - 1)} \quad \text{(A-IV-7)}$$

Q.E.D.

V. Derivations for Eq 52. From eq 3, we have

$$\bar{y}(\bar{t}_c) = 1 - \left(\frac{1 - \bar{a}}{1 - \eta}\right)e^{-\bar{t}_c} - \left(\frac{\eta - \bar{a}}{\eta - 1}\right)e^{-\bar{t}_c/\eta} = 0 \quad \text{(A-V-1)}$$

so that

$$\left(\frac{\eta - \bar{a}}{\eta - 1}\right)e^{-\bar{t}_c/\eta} = 1 - \left(\frac{1 - \bar{a}}{1 - \eta}\right)e^{-\bar{t}_c} \quad \text{(A-V-2)}$$

As a result, by substituting eq A-V-2 into eq A-III-5 and letting $\bar{t} = \bar{t}_c$, we have

$$\bar{M}_c = \int_0^{\bar{t}_c}[1 - \bar{y}(\bar{t})]\,d\bar{t} = (\bar{a} - 1)e^{-\bar{t}_c} - \eta + \bar{M}_\infty \quad \text{(A-V-3)}$$

There also exists time t_m such that $dy(t)/dt|_{t=t_m} = 0$. Then

$$\bar{t}_m = \ln\left(\frac{1 - \bar{a}}{\bar{y}_m - 1}\right) \quad \text{(A-V-4)}$$

Thus, according to the definition of $\bar{t}_m$, eq 50 results and

$$\bar{M}_m = \int_0^{\bar{t}_m}[1 - \bar{y}(\bar{t})]\,d\bar{t} = (\eta + 1)(\bar{y}_m - 1) + \bar{M}_\infty \quad \text{(A-V-5)}$$

Then

$$\frac{\bar{M}_m - \bar{M}_\infty}{\bar{y}_m - 1} = \eta + 1 \quad \text{(A-V-6)}$$

Q.E.D.

Literature Cited

(1) Ljung, L.; Söderström, T. *Theory and Practice of Recursive Identification*; The MIT Press: Cambridge, MA, 1983.

(2) Goodwin, G. C.; Payne, R. L. *Dynamic System Identification: Experiment Design and Data Analysis*; Academic Press: New York, 1977.

(3) Landau, I. D. *System Identification and Control Design*; Prentice Hall: New York, 1990.

(4) Juang, J. N. *Applied System Identification*; Prentice Hall: New York 1994.

(5) Ljung, L. *System Identification Toolbox for Use with MATLAB*; The Math Works, Inc.: Novi, MI, 1993.

(6) Kwak, H. J.; Sung, S. W.; Lee, I. B. Process Identification and Autotuning for Integrating Processes. *Ind. Eng. Chem. Res.* **1997**, *36*, 5329.

(7) Sung, S. W.; Lee, I. B.; Lee, J. New Process Identification Method for Automatic Design of PID Controllers. *Automatica* **1998**, *34*, 513.

(8) Chen, C. L. A Closed-Loop Reaction-Curve Method for Controller Tuning. *Chem. Eng. Commun.* **1991**, *104*, 87.

(9) Harriott, P. *Process Control*; McGraw-Hill: New York, 1964.

(10) Huang, C. T.; Chou, C. J. Estimation of the Underdamped Second-Order Parameters from the System Transient. *Ind. Eng. Chem. Res.* **1994**, *33*, 174.

(11) Huang, C. T.; Clements, W. C., Jr. Parameter Estimation for the Second-Order-Plus-Dead-Time Model. *Ind. Eng. Chem. Process Des. Dev.* **1982**, *21*, 601.

(12) Huang, C. T.; Huang, M. F. Estimating of the Second-Order Parameters from Process Transient by Simple Calculation. *Ind. Eng. Chem. Res.* **1993**, *32*, 228.

(13) Oldenbourg, R. C.; Sarorious, H. Dynamics of Automatic Control. *Trans. ASME* **1948**, 228.

(14) Smith, C. L. *Digital Computer Process Control*; Intext: Scranton, PA, 1972.

(15) Smith, S. A.; Corripio, A. B. *Principles and Automatic Process Control*; John Wiley & Sons: New York, 1997.

(16) Sundaresan, K. R.; Krishnaswamy, P. R. Estimation of Time Delay, Time Constant Parameters in Time Domain, Frequency, and Laplace Domains. *Can. J. Chem. Eng.* **1977**, *56*, 257.

(17) Sundaresan, K. R.; Prasad, C. C.; Krishnaswamy P. R. Evaluating Parameters from Process Transients. *Ind. Eng. Chem. Process Des. Dev.* **1978**, 7, 237.

(18) Yuwana, M.; Seborg, D. E. A New Method for On-line Controller Tuning. *AIChE J.* **1982**, *28*, 434.

(19) Huang, H. P.; Lee, M. W.; Chen, C. L. Inverse-Based Design for a Modified PID controller. *J. Chin. Inst. Chem. Eng.* **2000**, *31*, 225.

Received for review May 19, 2000
Revised manuscript received November 24, 2000
Accepted January 26, 2001

IE0005001