0021-8502(94)00123-5

AXISYMMETRIC THERMOPHORETIC MOTION OF TWO SPHERES

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Abstract—An exact analytical study is presented for the thermophoretic motion of two spheres in a uniform prescribed temperature gradient along their line of centers. The particles may differ in radius, in thermal conductivity and in surface properties, but the Knudsen numbers are assumed to be small so that the fluid flow is described by a continuum model with a thermal creep and a hydrodynamic slip at the particle surfaces. The appropriate energy and momentum equations are solved in the quasisteady situation using spherical bipolar coordinates and the thermophoretic velocities of the particles are calculated for various cases. The interaction between particles can be significant when the surface-to-surface spacing approaches zero. The influence of the interaction, in general, is stronger on the smaller particle than on the larger one. For the thermophoresis of two identical spheres, both migrate faster than the velocity they would possess if isolated. The thermophoretic motion of a sphere in the direction normal to a plane wall is also studied for the case that the gas-solid surfaces may have different properties. In general, the particle-particle and particle-wall interaction effects in thermophoresis are much weaker than for sedimentation.

1. INTRODUCTION

When a particle is suspended in a gaseous medium possessing a temperature gradient, it will move in the direction of decreasing temperature. This motion is known as thermophoresis and has been the subject of considerable study for many years (Waldmann and Schmitt, 1966; Derjaguin et al., 1976; Friedlander, 1977). The physical explanation of this phenomenon is based on kinetic-theory arguments and depends upon the fact that the particle receives a greater number of molecular impacts on its hotter side than on its colder one, thus leading to a net rate of change of momentum in the direction opposite to the temperature gradient. Theoretical analyses yield the following expression for the thermophoretic velocity of an isolated particle in a constant temperature gradient ∇T_∞:

$$\mathbf{U}^{(0)} = -K \frac{\eta}{\rho} \frac{1}{T_\infty(0)} \nabla T_\infty, \tag{1}$$

where η is the fluid viscosity, ρ is the fluid density, and $T_\infty(0)$ is the bulk gas absolute temperature at the particle center in the absence of the particle (or the mean gas temperature in the vicinity of the particle). The thermophoretic coefficient K depends upon the magnitude of the Knudsen number, l/a, where a is the radius of the particle and l is the mean free path of the gas molecules.

In the limit of small Knudsen numbers where the particle is large compared with the mean free path, the fluid flow may be described by a continuum model and the thermophoretic force arises from an induced "thermal creep" along the particle surface due to the existence of a tangential temperature gradient at the particle-fluid interface. Utilizing gas kinetic theory, Maxwell (1879) predicted that a tangential temperature gradient $\nabla_s T$ at a gas-solid surface would cause a thin layer of gas adjacent to the surface to move, with the relative velocity at the outer edge of the layer being

$$\mathbf{v}_s = C_s \frac{\eta}{\rho} \frac{1}{T} \nabla_s T, \tag{2}$$

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where T is the local gas temperature. The thermal slip coefficient C_s was found to be $\frac{3}{4}$ by Maxwell on the assumption that the distribution function in the bulk of the gas held all the way to the solid wall. The thermal creep velocity v_s is directed toward the high temperature side.

By using the Maxwellian creep velocity equation (2), which gives the coupling between temperature and velocity fields, as a slip velocity boundary condition and solving the equation of continuum fluid motion (at low Reynolds number) incorporating the heat conduction in the gas and particle, Epstein (1929) derived the following equation for the thermophoretic coefficient:

$$K = 2C_s \frac{k}{2k + \hat{k}}, \tag{3}$$

where k and $\hat{k}$ are the thermal conductivities of the gas and the particle, respectively. The thermophoretic velocity predicted by equation (3), which is independent of particle size, is in fair agreement with experimental data when the ratio $\hat{k}/k$ is not too high, less than about 10, as for oil droplets. But it gives much too small thermophoretic velocities (by a factor 30 and more) for particles with large thermal conductivity, for instance, NaCl (sodium chloride) particles with $\hat{k}/k \approx 100$. Indeed, in the limit $\hat{k} \to \infty$, equation (3) yields $U^{(0)} = 0$, whereas Schadt and Cadle (1961) distinctly observed and measured a thermophoresis with NaCl particles.

Equation (3) for the thermophoretic velocity of an aerosol particle was improved by Brock (1962) using the low Knudsen number effects of temperature jump at the gas–solid surface as well as frictional (isothermal) gas slippage (in addition to the thermal creep velocity) along the particle surface. The resulting expression for the thermophoretic coefficient of a suspended aerosol sphere is

$$K = \frac{2C_s\left(k + \hat{k}C_t\dfrac{l}{a}\right)}{\left(1 + 2C_m\dfrac{l}{a}\right)\left(2k + \hat{k} + 2\hat{k}C_t\dfrac{l}{a}\right)}, \tag{4}$$

where the dimensionless coefficients C_t and C_m (numerical factors of order unity) account for the temperature jump and hydrodynamic slip, respectively, at the particle surface and must be determined experimentally for each gas–solid system. Both Epstein and Brock assumed the value of C_s in equations (3) and (4) to be equal to the Maxwell value ($C_s = \frac{3}{4}$). Note that equation (4) is applicable to the range of finite Knudsen number and reduces to equation (3) when $l/a = 0$. For large particles and $\hat{k} \to \infty$, equation (4) yields $K = 2C_sC_t(l/a)$, whereas Epstein formula (3) gives no particle migration. Satisfactory agreement of the prediction by equation (4) with experiments (Schadt and Cadle, 1961) has been obtained. The best kinetic-theory values for complete thermal and momentum accommodations appear to be $C_s = 1.17$, $C_t = 2.18$ and $C_m = 1.14$ (Talbot et al., 1980).

Equation (4) is valid only for a spherical particle suspended in gaseous media that extend to infinity in all directions. However, in practical applications of thermophoresis, aerosol particles usually are not isolated and might interact with nearby particles and/or boundaries. Through an exact representation in spherical bipolar coordinates, Reed and Morrison (1975) studied the thermophoretic motion of two identical aerosol spheres (with equal sizes, conductivities, coefficients C_t and C_m) along their line of centers using the Maxwell value for C_s. They also examined the thermophoresis of a sphere normal to an infinite plane wall with the same coefficients C_t and C_m; the value of C_s was taken to be $\frac{3}{4}$ for the particle and zero for the wall. In their calculations, they failed to obtain any numerical result when the gap thickness between the surfaces is less than 9% of the particle radius. In the present work, our objective is to obtain an exact solution to the quasisteady problems of the thermophoresis of two arbitrary spherical particles along their line of centers and of the thermophoresis of an arbitrary spherical particle normal to a large plane wall. In the former problem, the particles may differ in radius, in thermal conductivity, and in coefficients C_s, C_t and C_m; in

the latter problem, the plane wall may have different values in C_s, C_t and C_m from the particle. The energy and momentum equations applicable to the system are solved by using bipolar coordinates and the fluid streamlines for various cases are presented. Our numerical results for the particle–particle and particle–wall interaction parameters, which are convergent even when the surface-to-surface distance is as small as 0.02% of the particle radius, agree well with those obtained by Reed and Morrison in the situations considered by them.

2. ANALYSIS FOR THE THERMOPHORESIS OF TWO SPHERES

We consider the thermophoretic motion of two spherical particles along their line of centers in an infinite gaseous medium, which is assumed to be Newtonian and incompressible. A linear temperature field $T_\infty(\mathbf{x})$ with a uniform thermal gradient $E_\infty \mathbf{e}_z$ (equal to ∇T_∞) is prescribed in the fluid far away from the pair of aerosol spheres; $\mathbf{e}_z$ is the axial unit vector in the cylindrical polar coordinate system (ρ, ϕ, z). The particles may differ in radius and in physical properties. Gravitational effects are ignored. Our purpose here is to determine the correction to equation (4) for one particle due to the presence of the other in the temperature and flow fields.

For conveniently satisfying the boundary conditions at particle surfaces, an orthogonal curvilinear coordinate system (ψ, ξ, ϕ), known as spherical bipolar coordinates (shown in Fig. 1), is utilized to solve this problem. This coordinate system is related to cylindrical polar coordinates by the following relation in any meridian plane ϕ = constant (Morse and Feshbach, 1953; Happel and Brenner, 1983):

$$\rho = \frac{c \sin \psi}{\cosh \xi - \cos \psi} \tag{5a}$$

and

$$z = \frac{c \sinh \xi}{\cosh \xi - \cos \psi}, \tag{5b}$$

where c is a characteristic length in the bipolar coordinate system which is positive. The coordinate surfaces ξ = constant correspond to a family of nonintersecting spheres whose
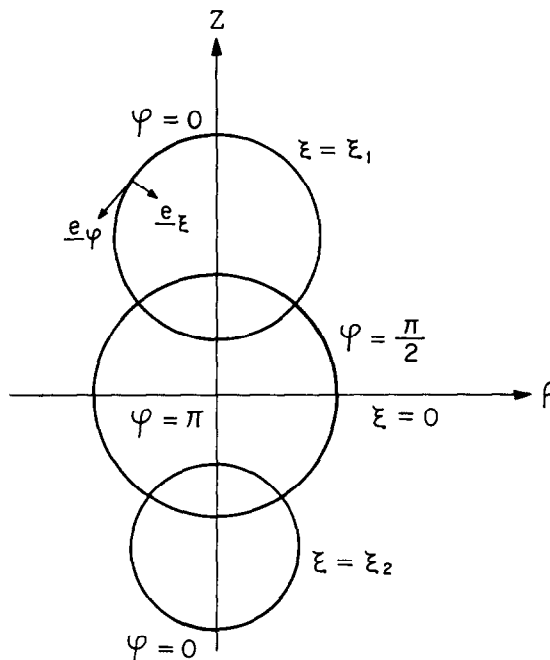

Fig. 1. Geometrical sketch for spherical bipolar coordinates.

centers lie along the z-axis. The value $\xi = 0$ is a sphere of infinite radius and equivalent to the entire plane $z = 0$. Two spheres external to each other are chosen to be $\xi = \xi_1$ (with $\xi_1 > 0$) and $\xi = \xi_2$ (with $\xi_2 < 0$), and the sphere radii a_1 and a_2 as well as the distances of their centers from the origin d_1 and d_2 are given by

$$a_i = c \operatorname{cosech} |\xi_i| \tag{6}$$

and

$$d_i = c \coth |\xi_i|, \tag{7}$$

for $i = 1$ or 2. The center-to-center distance between the particles, r_{12}, equals $(d_1 + d_2)$.

To determine the thermophoretic velocities of the two aerosol spheres, it is necessary to ascertain the temperature and velocity distributions.

2.1. Temperature distribution

In our problem of thermophoresis of two spheres, the transport of momentum and energy is inherently unsteady. However, the problem can be considered quasisteady if the Peclet and Reynolds numbers are small (the effects of convection are neglected). The energy equation governing the temperature distribution $T(\mathbf{x})$ for the fluid phase of constant thermal conductivity k is Laplace's equation:

$$\nabla^2 T = 0. \tag{8a}$$

For the two particles one has

$$\nabla^2 T_i = 0, \quad i = 1 \text{ or } 2, \tag{8b}$$

where $T_i(\mathbf{x})$ is the temperature field inside particle i. The boundary conditions at the particle surfaces require that the normal heat fluxes be continuous and a temperature jump which is proportional to the normal temperature gradient (Kennard, 1938) occur. Also, the fluid temperature must approach the linear prescribed field far from the particles and the temperature inside each particle is finite everywhere. Thus, one has

$$\xi = \xi_i: \qquad\qquad k\frac{\partial T}{\partial \xi} = k_i\frac{\partial T_i}{\partial \xi}, \tag{9a}$$

$$T - T_i = -\frac{\xi_i}{|\xi_i|}C_{ti}l\frac{\cosh \xi - \cos \psi}{c}\frac{\partial T}{\partial \xi}, \tag{9b}$$

$$|\xi| \to \infty: \qquad\qquad T_i \text{ is finite}, \tag{9c}$$

$$(\rho^2 + z^2)^{1/2} \to \infty: \qquad\qquad T \to T_\infty = T_0 + E_\infty z \tag{9d}$$

for $i = 1$ or 2; k_1 and k_2 are the thermal conductivities of the particles, which are assumed to be independent of temperature. In formula (9b), l is the mean free path of the surrounding fluid and C_{ti} is the dimensionless temperature jump coefficient of particle i. The undisturbed temperature at the plane $z = 0$ has been set equal to T_0.

A general solution to the Laplace equation (8a) suitable for satisfying these boundary conditions is (Morse and Feshbach, 1953)

$$T = cE_\infty(\cosh \xi - \mu)^{1/2}\sum_{n=0}^{\infty}[A_n\cosh(n + \tfrac{1}{2})\xi + B_n\sinh(n + \tfrac{1}{2})\xi]P_n(\mu)$$
$$+ T_0 + E_\infty z, \tag{10a}$$

where P_n is the Legendre polynomial of order n and for brevity we have put $\mu = \cos \psi$. Boundary condition (9d) is immediately satisfied by a solution of this form. The solution to equation (8b), satisfying the boundary condition (9c), can be expressed as

$$T_i = cE_\infty(\cosh \xi - \mu)^{1/2}\sum_{n=0}^{\infty}C_{in}\exp[-(n + \tfrac{1}{2})|\xi|]P_n(\mu) + T_0 + E_\infty z \tag{10b}$$

for $i = 1$ or 2. The coefficients A_n, B_n, C_{1n} and C_{2n} in equation (10) are to be determined using equations (9a) and (9b).

Utilizing the expansion, which can be derived using the generating function of the Legendre polynomials,

$$\frac{\cosh\xi}{(\cosh\xi - \mu)^{1/2}} - \frac{\sinh^2\xi}{(\cosh\xi - \mu)^{3/2}} = \sqrt{2}\sum_{n=0}^{\infty}\exp[-(n+\tfrac{1}{2})|\xi|]$$
$$\times[\cosh\xi - (2n+1)\sinh|\xi|]P_n(\mu) \quad (11)$$

and the recurrence relations of the Legendre polynomials, one can apply the boundary conditions (9a) and (9b) at the particle–fluid interfaces to the general solution (10) to yield the following four algebraic recursion formulas:

$$-\frac{C_{ii}l}{2c}(n+1)[A_{n+1}\sinh(n+\tfrac{3}{2})\xi_i + B_{n+1}\cosh(n+\tfrac{3}{2})\xi_i]$$

$$+\left[\frac{\xi_i}{|\xi_i|} + \frac{C_{ii}l}{2c}\sinh\xi_i\right][A_n\cosh(n+\tfrac{1}{2})\xi_i + B_n\sinh(n+\tfrac{1}{2})\xi_i]$$

$$+\frac{C_{ii}l}{2c}(2n+1)\cosh\xi_i[A_n\sinh(n+\tfrac{1}{2})\xi_i + B_n\cosh(n+\tfrac{1}{2})\xi_i]$$

$$-\frac{\xi_i}{|\xi_i|}C_{in}\exp[-(n+\tfrac{1}{2})|\xi_i|]$$

$$-\frac{C_{ii}l}{2c}n[A_{n-1}\sinh(n-\tfrac{1}{2})\xi_i + B_{n-1}\cosh(n-\tfrac{1}{2})\xi_i]$$

$$=\sqrt{2}\frac{C_{ii}l}{c}[(2n+1)\sinh|\xi_i| - \cosh\xi_i]\exp[-(n+\tfrac{1}{2})|\xi_i|], \quad (12a)$$

$$-(n+1)[A_{n+1}\sinh(n+\tfrac{3}{2})\xi_i + B_{n+1}\cosh(n+\tfrac{3}{2})\xi_i]$$

$$-\frac{\xi_i}{|\xi_i|}k_i^*(n+1)C_{i(n+1)}\exp[-(n+\tfrac{3}{2})|\xi_i|]$$

$$+\sinh\xi_i[A_n\cosh(n+\tfrac{1}{2})\xi_i + B_n\sinh(n+\tfrac{1}{2})\xi_i]$$

$$+(2n+1)\cosh\xi_i[A_n\sinh(n+\tfrac{1}{2})\xi_i + B_n\cosh(n+\tfrac{1}{2})\xi_i]$$

$$-k_i^*\left[\sinh\xi_i - \frac{\xi_i}{|\xi_i|}(2n+1)\cosh\xi_i\right]C_{in}\exp[-(n+\tfrac{1}{2})|\xi_i|]$$

$$-n[A_{n-1}\sinh(n-\tfrac{1}{2})\xi_i + B_{n-1}\cosh(n-\tfrac{1}{2})\xi_i]$$

$$-\frac{\xi_i}{|\xi_i|}k_i^*nC_{i(n-1)}\exp[-(n-\tfrac{1}{2})|\xi_i|]$$

$$=2\sqrt{2}(k_i^*-1)[\cosh\xi_i - (2n+1)\sinh|\xi_i|]\exp[-(n+\tfrac{1}{2})|\xi_i|] \quad (12b)$$

for $i = 1$ or 2. In (12), $k_i^* = k_i/k$ is the ratio of thermal conductivity between the particle and the surrounding fluid. The four formulas represent a group of infinite coupled equations for the unknown coefficients A_n, B_n, C_{1n} and C_{2n}. Because these coefficients should individually approach zero as $n \to \infty$ for the temperature field (10) to remain bounded, they can be determined by solving the first m sets of the four recursion equations, provided that m is sufficiently large that all A_{m+1}, B_{m+1}, $C_{1(m+1)}$ and $C_{2(m+1)}$ are negligible.

2.2. Fluid velocity distribution

With knowledge of the solution for the temperature field, we can now proceed to find the fluid velocity distribution. Due to the low Reynolds numbers encountered in thermo-

phoretic motions, the fluid flow is governed by the quasisteady fourth-order differential equation for viscous axisymmetric flows:

$$E^4 \Psi = 0, \tag{13}$$

where Ψ is the Stokes stream function. The Stokes operator E^2 assumes the following form in spherical bipolar coordinates:

$$E^2 \equiv \frac{\cosh \xi - \mu}{c^2} \left\{ \frac{\partial}{\partial \xi} \left[(\cosh \xi - \mu) \frac{\partial}{\partial \xi} \right] + (1 - \mu^2) \frac{\partial}{\partial \mu} \left[(\cosh \xi - \mu) \frac{\partial}{\partial \mu} \right] \right\}. \tag{14}$$

The stream function Ψ is related to the velocity field $\mathbf{v}$ by

$$v_\xi = -\frac{(\cosh \xi - \mu)^2}{c^2} \frac{\partial \Psi}{\partial \mu} \tag{15a}$$

and

$$v_\psi = -\frac{(\cosh \xi - \mu)^2}{c^2 (1 - \mu^2)^{1/2}} \frac{\partial \Psi}{\partial \xi}. \tag{15b}$$

Owing to the thermal creep velocity given by equation (2) and the frictional slip velocity along the particle surfaces as well as the fluid at rest far from the particles, the boundary conditions for the fluid velocity are

$$\xi = \xi_i: \qquad \mathbf{v} = \mathbf{U}_i - \frac{\xi_i}{|\xi_i|} \frac{C_{mi} l}{\eta} (\mathbf{I} - \mathbf{e}_\xi \mathbf{e}_\xi) \mathbf{e}_\xi : \tau + C_{si} \frac{\eta}{\rho} \frac{1}{\overline{T}_i} \nabla_s T, \tag{16a}$$

$$(\rho^2 + z^2)^{1/2} \to \infty: \qquad \mathbf{v} \to \mathbf{0} \tag{16b}$$

for $i = 1$ or 2. Here, $\overline{T}_i = T_0 + cE_\infty \coth \xi_i$, which is the prescribed temperature at the position of the center of particle i; $\tau (= \eta [\nabla \mathbf{v} + (\nabla \mathbf{v})^T])$ is the viscous stress tensor for the fluid; $\mathbf{e}_\xi$ is the unit vector in ξ-direction; $\mathbf{I}$ is the unit dyadic; C_{mi} and $C_{si} (i = 1$ or 2) are the hydrodynamic slip and thermal slip coefficients, respectively, about the surface of particle i; $\mathbf{U}_1 (= U_1 \mathbf{e}_z)$ and $\mathbf{U}_2 (= U_2 \mathbf{e}_z)$ are the instantaneous thermophoretic velocities of the two aerosol particles to be determined. The tangential temperature gradient, $\nabla_s T$, can be obtained from equation (10a). The validity of the expression for the thermal creep velocity in equation (16a) is based on the assumption that the fluid is only slightly nonuniform in the undisturbed temperature on the length scale of the particle radii.

Because the particles are freely suspended in the surrounding fluid, the net force exerted by the fluid on the surface of each particle must vanish:

$$\mathbf{F} = \iint_{\substack{\text{particle} \\ \text{surface}}} \mathbf{n} \cdot \mathbf{\Pi} \, dS = \mathbf{0}, \tag{17}$$

where $\mathbf{n}$ is the unit normal vector at the particle surface pointing into the surrounding fluid and $\mathbf{\Pi}$ is the total stress tensor. For the axisymmetric motion considered in this work, one can evaluate U_1 and U_2 by merely satisfying constraint (17) after solving equations (13) and (16).

A general solution of equation (13) satisfying the boundary condition (16b) is (Stimson and Jeffery, 1926; Happel and Brenner, 1983)

$$\Psi = c^2 (\cosh \xi - \mu)^{-3/2} \sum_{n=1}^{\infty} \left[a_n \cosh(n - \tfrac{1}{2})\xi + b_n \sinh(n - \tfrac{1}{2})\xi \right.$$

$$\left. + c_n \cosh(n + \tfrac{3}{2})\xi + d_n \sinh(n + \tfrac{3}{2})\xi \right] G_{n+1}^{-1/2}(\mu). \tag{18}$$

Here, $G_{n+1}^{-1/2}(\mu)$ is the Gegenbauer polynomial of order $n + 1$ and degree $-\frac{1}{2}$, which is related to the Legendre polynomials by

$$G_{n+1}^{-1/2}(\mu) = \frac{P_{n-1}(\mu) - P_{n+1}(\mu)}{2n + 1}. \tag{19}$$

The coefficients a_n, b_n, c_n and d_n are to be determined from the boundary conditions given by equation (16a) using equation (11) and the recurrence relations of the Legendre polynomials. The procedure is straightforward but tedious and the results, which consist of four algebraic recursion formulas, are

$$a_n \cosh(n - \tfrac{1}{2})\xi_i + b_n \sinh(n - \tfrac{1}{2})\xi_i + c_n \cosh(n + \tfrac{3}{2})\xi_i + d_n \sinh(n + \tfrac{3}{2})\xi_i$$

$$= -\frac{\sqrt{2}}{2} U_i n(n + 1) \left\{ \frac{\exp[-(n - \tfrac{1}{2})|\xi_i|]}{2n - 1} - \frac{\exp[-(n + \tfrac{3}{2})|\xi_i|]}{2n + 3} \right\}, \qquad (20a)$$

$$\frac{C_{mi} l}{c} \frac{n}{2n + 3} \{ (n + \tfrac{1}{2})^2 [a_{n+1} \cosh(n + \tfrac{1}{2})\xi_i + b_{n+1} \sinh(n + \tfrac{1}{2})\xi_i]$$

$$+ (n + \tfrac{5}{2})^2 [c_{n+1} \cosh(n + \tfrac{5}{2})\xi_i + d_{n+1} \sinh(n + \tfrac{5}{2})\xi_i] \}$$

$$- \frac{C_{mi} l}{c} \cosh \xi_i \{ (n - \tfrac{1}{2})^2 [a_n \cosh(n - \tfrac{1}{2})\xi_i + b_n \sinh(n - \tfrac{1}{2})\xi_i]$$

$$+ (n + \tfrac{3}{2})^2 [c_n \cosh(n + \tfrac{3}{2})\xi_i + d_n \sinh(n + \tfrac{3}{2})\xi_i] \}$$

$$- \frac{\xi_i}{|\xi_i|} \{ (n - \tfrac{1}{2})[a_n \sinh(n - \tfrac{1}{2})\xi_i + b_n \cosh(n - \tfrac{1}{2})\xi_i]$$

$$+ (n + \tfrac{3}{2})[c_n \sinh(n + \tfrac{3}{2})\xi_i + d_n \cosh(n + \tfrac{3}{2})\xi_i] \}$$

$$+ \frac{C_{mi} l}{c} \frac{n + 1}{2n - 1} \{ (n - \tfrac{3}{2})^2 [a_{n-1} \cosh(n - \tfrac{3}{2})\xi_i + b_{n-1} \sinh(n - \tfrac{3}{2})\xi_i]$$

$$+ (n + \tfrac{1}{2})^2 [c_{n-1} \cosh(n + \tfrac{1}{2})\xi_i + d_{n-1} \sinh(n + \tfrac{1}{2})\xi_i] \}$$

$$= \frac{\sqrt{2}}{4} U_i n(n + 1) \left(\frac{3}{2} \frac{C_{mi} l}{c} \sinh |\xi_i| - 1 \right) \{ \exp[-(n - \tfrac{1}{2})|\xi_i|] - \exp[-(n + \tfrac{3}{2})|\xi_i|] \}$$

$$- \frac{\sqrt{2}}{4} U_i \frac{C_{mi} l}{c} n(n + 1) \cosh \xi_i \left\{ \frac{\exp[-(n - \tfrac{1}{2})|\xi_i|]}{2n - 1} - \frac{\exp[-(n + \tfrac{3}{2})|\xi_i|]}{2n + 3} \right\}$$

$$+ \frac{\xi_i}{|\xi_i|} C_{si} \frac{\eta E_\infty}{\rho \bar{T}_i} \left\{ \frac{n(n + 1)}{2} [A_{n+1} \cosh(n + \tfrac{3}{2})\xi_i + B_{n+1} \sinh(n + \tfrac{3}{2})\xi_i] \right.$$

$$- n(n + 1) \cosh \xi_i [A_n \cosh(n + \tfrac{1}{2})\xi_i + B_n \sinh(n + \tfrac{1}{2})\xi_i]$$

$$+ \frac{n(n + 1)}{2} [A_{n-1} \cosh(n - \tfrac{1}{2})\xi_i + B_{n-1} \sinh(n - \tfrac{1}{2})\xi_i] \Big\}$$

$$- \sqrt{2} C_{si} \frac{\eta E_\infty}{\rho \bar{T}_i} n(n + 1) \{ \exp[-(n - \tfrac{1}{2})|\xi_i|] - \exp[-(n + \tfrac{3}{2})|\xi_i|] \} \qquad (20b)$$

for $i = 1$ or 2. Because the unknown coefficients a_n, b_n, c_n and d_n become small with large n, simultaneous solution of these four recursion equations for the first m sets yields $4m$ coefficients, thereby determining the Stokes stream function for the fluid according to equation (18).

By integration of the stress vector on the particle surface using the first part of equation (17), the drag force opposing the thermophoretic motion of the particle at $\xi = \xi_1$ is (Stimson and Jeffery, 1926)

$$F_1 = -2\sqrt{2}\pi\eta c \sum_{n=1}^{\infty} (a_n + b_n + c_n + d_n), \qquad (21a)$$

and for the particle at $\xi = \xi_2$,

$$F_2 = -2\sqrt{2}\pi\eta c \sum_{n=1}^{\infty} (a_n - b_n + c_n - d_n). \qquad (21b)$$

2.3. *Particle velocities*

Since the net force acting on each particle vanishes to fulfill the requirement of (17), we have

$$F_1 = 0 \qquad\qquad (22a)$$

and

$$F_2 = 0. \qquad\qquad (22b)$$

To determine the instantaneous thermophoretic velocities U_1 and U_2 of the particles, the above two equations incorporated with equation (21) must be solved. The result can be expressed as

$$U_1 = M_{11} U_1^{(0)} + M_{12} U_2^{(0)} \qquad\qquad (23a)$$

and

$$U_2 = M_{21} U_1^{(0)} + M_{22} U_2^{(0)}, \qquad\qquad (23b)$$

where $U_1^{(0)}$ and $U_2^{(0)}$ are the particle velocities that would exist in the absence of the other and are computed from equations (1) and (4). The numerical results for the dimensionless mobility parameters M_{11}, M_{12}, M_{21} and M_{22} as a function of k_1^*, k_2^*, $C_{t1} l/a_1$, $C_{t2} l/a_2$, $C_{m1} l/a_1$, $C_{m2} l/a_2$, a_2/a_1 and $(a_1 + a_2)/r_{12}$ have been determined and are presented in the following section. Note that $M_{11} = M_{22} = 1$ and $M_{12} = M_{21} = 0$ when the two particles are separated by an infinite distance (i.e. $r_{12} \to \infty$).

3. RESULTS FOR THE THERMOPHORESIS OF TWO SPHERES

The coefficients of the temperature distribution (10) and the stream function (18) in the present quasisteady problem have been calculated for various values of k_1^*, k_2^*, $C_{t1} l/a_1$, $C_{t2} l/a_2$, $C_{m1} l/a_1$, $C_{m2} l/a_2$, a_2/a_1 and $(a_1 + a_2)/r_{12}$ using a DEC VAX 6520 digital computer. For the difficult case of $a_2/a_1 = 10$ and $(a_1 + a_2)/r_{12} = 0.9999$, m equal to 3000 was sufficiently large that the $(m + 1)$th terms of these coefficients are negligible and increases in m do not alter the calculated values appreciably.

3.1. *Streamlines*

A sketch of the streamline pattern for the fluid surrounding an isolated aerosol sphere undergoing thermophoresis was exhibited by Keh and Yu (1994). The velocity field caused by thermophoresis was found to be a potential dipole, decaying with distance r from the center of the particle as r^{-3}. This velocity field decays much faster than that for a Stokeslet (motion driven by a body force) which decays as r^{-1}.

The distortion of the fluid flow due to interactions between two thermophoretic spheres is illustrated in Figs 2–4. In each case, streamlines in a meridian plane are depicted. As both the governing equations and the boundary conditions in the system are linear, the streamline patterns (and also the particle velocities U_i or the mobility parameters M_{ij}) are independent of which sphere is in front. Figures 2 and 3 picture the situation when the two particles have identical radii and the distance between the surfaces is equal to the sum of their radii. The streamlines for a case of two identical spheres ($a_1 = a_2 = a$, $k_1^* = k_2^* = k^*$, $C_{t1} = C_{t2} = C_t$, $C_{m1} = C_{m2} = C_m$, $U_1^{(0)} = U_2^{(0)} = U^{(0)}$) are drawn in Fig. 2. Since the convective effects in the energy and momentum equations have been neglected, the contour pattern shows equivalent local recirculations in the vicinity of each sphere and a global recirculation pattern with fore-and-aft symmetry far away from the particles. These balanced recirculations will be distorted if the two spheres differ in thermal conductivity (or in surface properties), as shown in Fig. 3. It can be found that the spacing between streamlines near the sphere with smaller thermal conductivity is narrower and the local fluid recirculation is stronger than that for the other sphere. Thus, the sphere with lower thermal conductivity shows a larger migration velocity. These results are consistent with the prediction of equation (4).
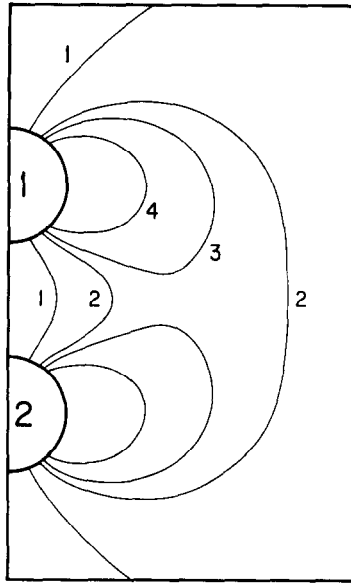
Fig. 2. Streamlines for the thermophoretic motion of two identical spheres with $k^* = 100$, $C_t l/a = 0.2$, $C_m l/a = 0.1$ and $2a/r_{12} = 0.5$. $\Psi/c^2 U^{(0)}$: (1) 0.02; (2) 0.05; (3) 0.06; (4) 0.08.
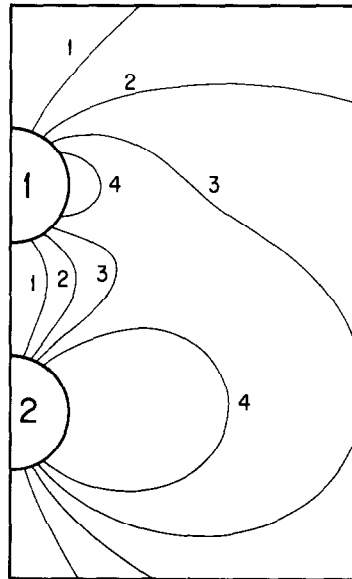

Fig. 3. Streamlines for the thermophoretic motion of two spheres of identical radii and surface properties with $k_1^* = 100$, $k_2^* = 1$, $C_t l/a = 0.2$, $C_m l/a = 0.1$ and $2a/r_{12} = 0.5$. $\Psi/c^2 U_1^{(0)}$: (1) 0.02; (2) 0.05; (3) 0.08; (4) 0.12.

The situation of two spheres with identical thermal conductivities and surface properties but unequal radii is considered in Fig. 4, which corresponds to a case of $a_2/a_1 = 2$ and $(a_1 + a_2)/r_{12} = 0.5$. It is noted that the local recirculation in the vicinity of the larger sphere is substantial, whereas it is constrained in the vicinity of the smaller sphere. As the radius ratio becomes larger, the fluid flow will be dominated by the larger particle, with the smaller one introducing only local perturbations. The flow patterns of thermophoresis are a sharp contrast to those of the motion of aerosol spheres driven by a body-force field (Chen and
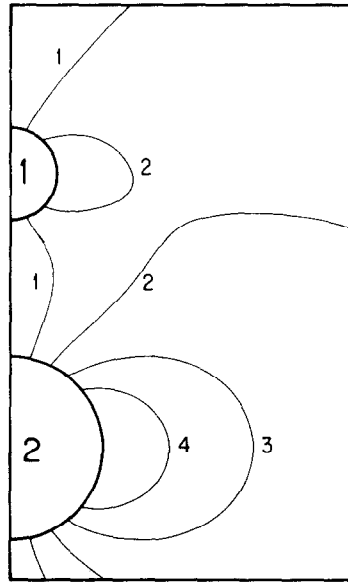
Fig. 4. Streamlines for the thermophoretic motion of two spheres of identical thermal conductivities and surface properties with $a_2/a_1 = 2$, $k^* = 100$, $C_t l/a_1 = 0.2$, $C_m l/a_1 = 0.1$ and $(a_1 + a_2)/r_{12} = 0.5$. $\Psi/c^2 U_1^{(0)}$: (1) 0.01; (2) 0.04; (3) 0.08; (4) 0.12.

Keh, 1994), such as gravity, in which no local recirculations in the vicinity of a particle can be observed.

3.2. Thermophoretic velocities

The numerical solutions of the mobility parameters M_{11}, M_{12}, M_{21} and M_{22}, defined by equation (23), for the case of two aerosol spheres of identical physical properties with various values of k^*, $C_t l/a_1$, $C_m l/a_1$, a_2/a_1 and $(a_1 + a_2)/r_{12}$ are presented in Tables 1 and 2. All of the listed results converge to at least five significant figures. As expected, these results illustrate that the particles' interaction decreases rapidly, for all values of physical properties and a_2/a_1, with an increase in the gap between them (i.e., decreasing $(a_1 + a_2)/r_{12}$). However, the interaction between particles can be significant when the surface-to-surface spacing gets close to zero.

Numerical results of the mobility parameters for the thermophoresis of two identical spheres are given in Table 1. For this symmetric case, the two particles will move at the same velocity because $M_{12} = M_{21}$, $M_{11} = M_{22}$ and $U_1^{(0)} = U_2^{(0)}$. It can be seen that the inter-action effect makes each particle move faster than its undisturbed velocity $U^{(0)}$. This enhancement in particle velocity is similar to the observation when the driving force for the migration of aerosol spheres is gravity (Chen and Keh, 1994). However, the particle interaction effect in thermophoresis is much weaker than for motion under a body-force field. Reed and Morrison (1975) calculated the values of $(M_{11} + M_{12})$ for the thermophor-esis of two identical spheres for the range $2a/r_{12} < 0.957$. Our solutions in Table 1 are in excellent agreement with their calculations.

In Table 2, results of mobility parameters for the thermophoresis of two unequal-sized spheres with the same physical properties are given for the case $k^* = 100$, $C_t l/a_1 = 0.2$ and $C_m l/a_1 = 0.1$. Also, the normalized migration velocities of the particles and the ratio of the particle velocities for this case are plotted versus $(a_1 + a_2)/r_{12}$ in Figs 5 and 6, respectively, with a_2/a_1 as a parameter. It can be seen that the effect of particle interaction, in general, is greater on the smaller of the two spheres than on the larger one (M_{12} or $U_1/U_1^{(0)}$ increases and M_{21} or $U_2/U_2^{(0)}$ decreases significantly with the increasing a_2/a_1 for a given value of $(a_1 + a_2)/r_{12}$). The larger particle is hardly influenced by the presence of the smaller one for

Table 1. The mobility parameters for the thermophoresis of two identical spheres

k^*	$2a/r_{12}$	$C_t l/a = 0.2$ $C_m l/a = 0.1$			$C_t l/a = 0.02$ $C_m l/a = 0.01$		
		M_{11}	M_{12}	$M_{11}+M_{12}$	M_{11}	M_{12}	$M_{11}+M_{12}$
1	0.2	0.9999	0.0010	1.0009	1.0000	0.0010	1.0010
	0.4	0.9986	0.0080	1.0067	0.9994	0.0080	1.0074
	0.6	0.9919	0.0280	1.0200	0.9937	0.0280	1.0216
	0.8	0.9602	0.0770	1.0372	0.9592	0.0800	1.0392
	0.9	0.9101	0.1351	1.0452	0.8962	0.1509	1.0471
	0.95	0.8553	0.1935	1.0488	0.8201	0.2306	1.0507
	0.99	0.7505	0.3010	1.0514	0.6756	0.3776	1.0533
	0.999	0.6672	0.3848	1.0520	0.5904	0.4634	1.0539
	0.9999	0.6263	0.4258	1.0521	0.5632	0.4908	1.0539
10	0.2	1.0009	0.0010	1.0019	1.0014	0.0010	1.0024
	0.4	1.0067	0.0077	1.0143	1.0110	0.0075	1.0185
	0.6	1.0199	0.0234	1.0432	1.0352	0.0213	1.0565
	0.8	1.0363	0.0468	1.0830	1.0810	0.0329	1.1140
	0.9	1.0372	0.0656	1.1028	1.1260	0.0239	1.1499
	0.95	1.0212	0.0906	1.1118	1.1655	0.0057	1.1712
	0.99	0.9387	0.1798	1.1185	1.1652	0.0266	1.1918
	0.999	0.8202	0.2997	1.1198	0.9816	0.2157	1.1973
	0.9999	0.7478	0.3722	1.1200	0.8414	0.3564	1.1979
100	0.2	1.0011	0.0010	1.0021	1.0018	0.0010	1.0028
	0.4	1.0086	0.0075	1.0161	1.0144	0.0070	1.0214
	0.6	1.0270	0.0213	1.0482	1.0485	0.0148	1.0633
	0.8	1.0587	0.0321	1.0908	1.1405	− 0.0232	1.1173
	0.9	1.0818	0.0284	1.1102	1.3093	− 0.1674	1.1419
	0.95	1.0873	0.0308	1.1181	1.5826	− 0.4308	1.1518
	0.99	1.0238	0.0994	1.1232	2.2748	− 1.1177	1.1571
	0.999	0.8893	0.2348	1.1241	2.1496	− 0.9921	1.1575
	0.9999	0.7996	0.3246	1.1242	1.6449	− 0.4874	1.1576
1000	0.2	1.0011	0.0010	1.0021	1.0019	0.0010	1.0029
	0.4	1.0088	0.0075	1.0163	1.0148	0.0068	1.0216
	0.6	1.0278	0.0210	1.0488	1.0505	0.0116	1.0621
	0.8	1.0615	0.0300	1.0915	1.1569	− 0.0511	1.1059
	0.9	1.0877	0.0228	1.1105	1.3796	− 0.2642	1.1153
	0.95	1.0963	0.0218	1.1181	1.7690	− 0.6582	1.1108
	0.99	1.0357	0.0870	1.1227	2.8542	− 1.7595	1.0947
	0.999	0.8990	0.2245	1.1235	2.8003	− 1.7132	1.0871
	0.9999	0.8067	0.3168	1.1236	2.0914	− 1.0053	1.0861

the situation of $a_2/a_1 \lesssim 0.2$ or $\gtrsim 5$. Although these results illustrate that the effect of interactions between the spheres, in general, increases as the separation distance is decreased, they show that the thermophoretic velocity of either sphere is not necessarily a monotonic increasing or decreasing function of the separation parameter $(a_1 + a_2)/r_{12}$. These complex results are generated from the combined effects of particle interactions on local temperature gradient and fluid velocity field.

It can be seen in Fig. 6 that the smaller sphere migrates considerably faster than the larger sphere for various values of $(a_1 + a_2)/r_{12}$. This indicates that (1) a stable distance will be developed between the spheres for the situation where the smaller sphere is ahead; (2) for the case where the smaller sphere is following, its thermophoretic velocity even at a distance with $(a_1 + a_2)/r_{12} = 0.9999$ is still greater than for the big particle and both spheres will therefore collide. Note that, for the case of two touching spheres $[(a_1 + a_2)/r_{12} = 1]$ undergoing thermophoresis, they migrate as a single particle and hence their velocities U_1 and U_2 are identical. This understanding explains why there exists a maximum or minimum for each curve with $a_2/a_1 \neq 1$ in Fig. 6. For conciseness, we do not present here the numerical results of thermophoretic velocities for the case of two spheres with different physical properties.

Table 2. The mobility parameters for the thermophoresis of two spheres with the same physical properties ($k^* = 100$, $C_t l/a_1 = 0.2$ and $C_m l/a_1 = 0.1$)

$\dfrac{a_2}{a_1}$	$\dfrac{a_1 + a_2}{r_{12}}$	M_{11}	M_{12}	M_{21}	M_{22}
0.1	0.2	1.0000	0.0000	0.0060	1.0067
	0.4	1.0000	0.0000	0.0481	1.0535
	0.6	0.9998	0.0001	0.1626	1.1803
	0.8	0.9995	0.0002	0.3900	1.4236
	0.9	0.9991	0.0003	0.5758	1.5848
	0.95	0.9988	0.0006	0.7224	1.6408
	0.99	0.9982	0.0015	0.9695	1.5054
	0.999	0.9977	0.0031	1.1224	1.1588
	0.9999	0.9976	0.0043	1.1495	0.8931
0.5	0.2	1.0002	0.0003	0.0024	1.0026
	0.4	1.0013	0.0021	0.0188	1.0208
	0.6	1.0031	0.0053	0.0615	1.0683
	0.8	1.0027	0.0070	0.1367	1.1507
	0.9	1.0003	0.0095	0.1860	1.1888
	0.95	0.9964	0.0183	0.2216	1.1716
	0.99	0.9778	0.0564	0.3228	1.0057
	0.999	0.9465	0.1075	0.4769	0.7616
	0.9999	0.9253	0.1382	0.5795	0.6137
2	0.2	1.0034	0.0024	0.0003	1.0003
	0.4	1.0274	0.0185	0.0019	1.0024
	0.6	1.0915	0.0579	0.0036	1.0072
	0.8	1.2180	0.1004	− 0.0025	1.0165
	0.9	1.3121	0.0637	− 0.0109	1.0322
	0.95	1.3444	− 0.0157	− 0.0114	1.0506
	0.99	1.2032	− 0.0593	0.0227	1.0584
	0.999	0.8921	0.1664	0.0877	1.0110
	0.9999	0.6950	0.3622	0.1284	0.9706
10	0.2	1.0110	0.0060	0.0000	1.0000
	0.4	1.0880	0.0480	0.0000	1.0000
	0.6	1.2970	0.1616	− 0.0001	1.0001
	0.8	1.7045	0.3660	− 0.0003	1.0003
	0.9	1.9915	0.3904	− 0.0003	1.0010
	0.95	2.1081	0.0355	0.0000	1.0026
	0.99	1.8675	− 1.6313	0.0015	1.0076
	0.999	1.1982	− 1.8677	0.0039	1.0074
	0.9999	0.7774	− 1.0659	0.0053	1.0046

4. THERMOPHORESIS OF A SPHERE NORMAL TO A PLANE WALL

In this section, we consider the thermophoresis of a spherical particle of radius a in the direction normal to an infinite plane wall of constant temperature located at a distance d from the sphere center. The fluid is allowed to slip, both thermally and frictionally, and the temperature jump may occur at the particle surface and the plane wall. The relative conductivity and the thermophoretic velocity of the particle are denoted by k^* and $U e_z$, respectively. For this case, the mathematical formulation concerning the temperature and flow fields is the same as that for the case of axisymmetric thermophoresis of two spheres when the radius and thermal conductivity of one sphere (say, sphere 2) are taken to be infinite. Thus, equations (5)–(21a) are still applicable to the sphere–plane system if we let $a = a_1$, $d = d_1$, $U = U_1$, $k^* = k_1^*$, $\xi_2 = 0$ and $U_2 = 0$. Now, C_{s1} (C_{t1}, C_{m1}) and C_{s2}(C_{t2}, C_{m2}) represent the thermal slip coefficients (temperature jump coefficients, hydrodynamic slip coefficients) associated with the particle surface and the plane wall, respectively. Because the wall temperature T_2 is a constant, equations (8b), (9a), (9c), (10b) and (12b) for the case of $i = 2$ become trivial. Thus, equation (12) provides three recurrence formulas for the three sets of unknown constants A_n, B_n and C_{1n}. Also, the temperature $\bar{T}_2$ in equations (16a) and (20b) should be equal to the constant T_2 now. Since the particle is freely suspended in the surrounding fluid, the net force exerted by the fluid on the surface of
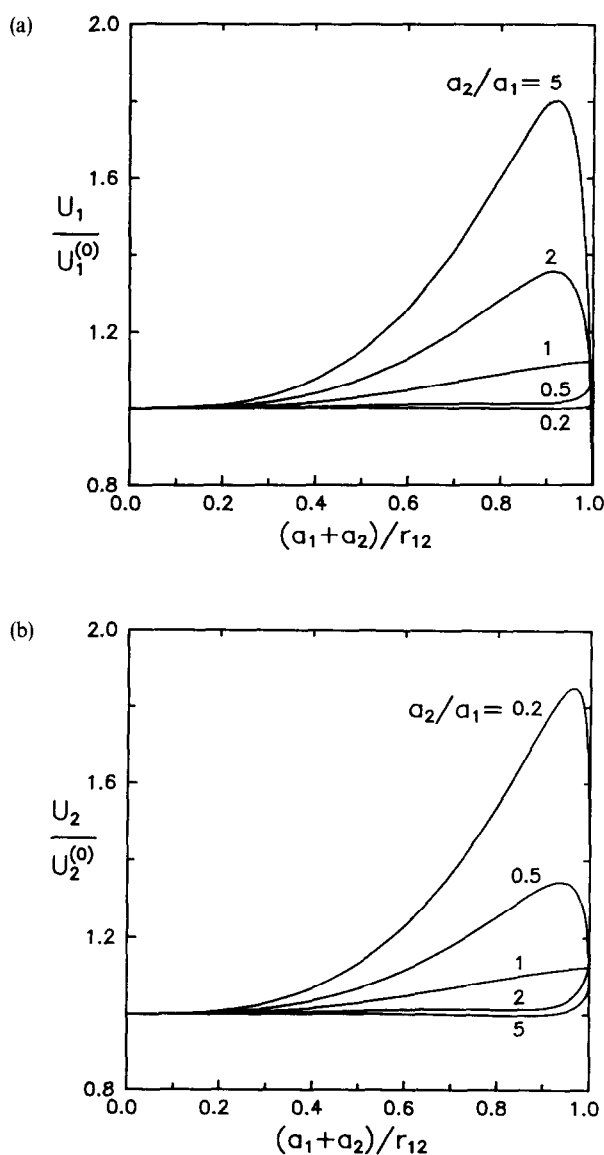
(a)

(b)

Fig. 5. Normalized thermophoretic velocities of two spheres of identical thermal conductivities and surface properties with $k^* = 100$, $C_t l/a_1 = 0.2$ and $C_m l/a_1 = 0.1$ versus the separation parameter $(a_1 + a_2)/r_{12}$ with a_2/a_1 as parameter: (a) sphere 1; (b) sphere 2.

the particle, given by equation (21a), must vanish, and the mobility coefficient $U/U^{(0)}$ is to be determined from this constraint. Here, $U^{(0)}$ is the thermophoretic velocity of the particle in the absence of the plane wall and can be calculated using equations (1) and (4).

The flow pattern for the thermophoretic motion of an aerosol sphere normal to a plane wall is illustrated in Fig. 7. Here, streamlines in meridian section are drawn for the case that the effect of thermal creep is negligible at the surface of the wall ($C_{s2} = 0$). Due to the linearity of the problem, the streamline pattern (and also the particle velocity) is the same whether the sphere moves toward or away from the wall. In addition to the local "inner" recirculations in the vicinity of the particle, the contour pattern also shows an "outer" recirculation region far from the particle. In Fig. 7, a stagnation point other than the origin of the coordinate system appears on the plane wall. In addition, there is one more stagnation point on the axis of symmetry ($\rho = 0$). The dividing streamline with $\Psi = 0$, which separates the two regions of circulating flow, intersects the axis at this point
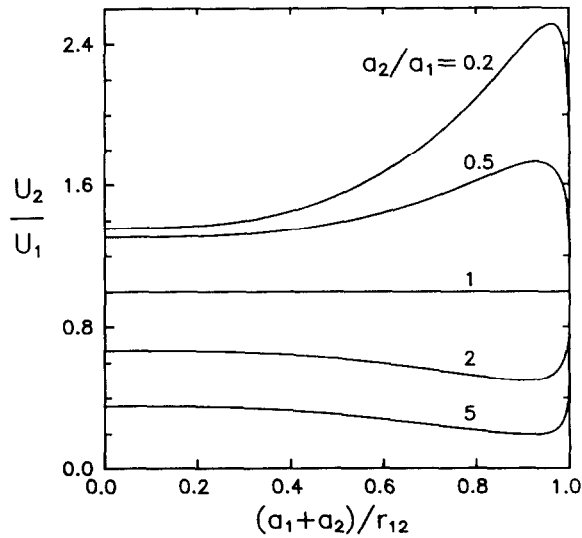
Fig. 6. The ratio of thermophoretic velocities of two spheres of identical thermal conductivities and surface properties with $k^* = 100$, $C_t l/a_1 = 0.2$ and $C_m l/a_1 = 0.1$ versus the separation parameter $(a_1 + a_2)/r_{12}$ with a_2/a_1 as a parameter.
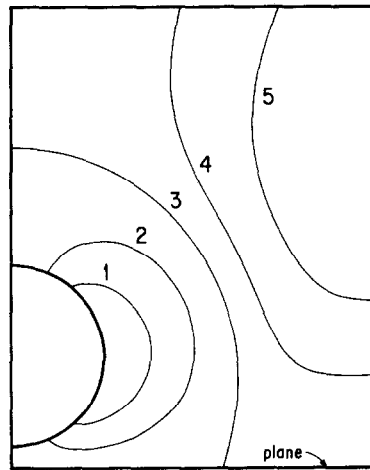

Fig. 7. Streamlines for the thermophoretic motion of a sphere normal to a plane wall with $k^* = 100$, $C_{t1} l/a = C_{t2} l/a = 0.2$, $C_{m1} l/a = C_{m2} l/a = 0.1$, $C_{s2} = 0$ and $a/d = 0.8$. $\Psi/c^2 U^{(0)}$: (1) -0.1; (2) -0.05; (3) 0; (4) 0.1; (5) 0.3.

orthogonally. For any specified values of k^*, $C_t l/a$ and $C_m l/a$, the relative distance of the dividing streamline from the particle decreases with increasing a/d. In comparison with this flow pattern of thermophoresis, the presence of a plane wall causes no dividing streamline in the motion of an aerosol sphere under gravity (Chen and Keh, 1994).

Numerical results of the mobility coefficient $U/U^{(0)}$ of a sphere undergoing thermophoresis in the direction perpendicular to a plane wall for various values of k^*, $C_{t1} l/a$, $C_{t2} l/a$, $C_{m1} l/a$, $C_{m2} l/a$ and a/d are presented in Table 3. As expected, the particle moves with the velocity that would exist in the absence of the wall as $a/d \rightarrow 0$. Then, the velocity of the particle decreases steadily as it approaches the wall (with increasing a/d) going to zero in the limit. A comparison between the mobility results for the sedimentation of an aerosol sphere normal to a slip wall previously obtained (Chen and Keh, 1994) and the results in Table 3 shows that the wall effect on the thermophoresis is much weaker.

Table 3. The mobility coefficient $U/U^{(0)}$ for the thermophoresis of a sphere in normal motion toward an infinite plane wall

k^*	a/d	$C_{t1}l/a = 0.2 \quad C_{m1}l/a = 0.1$			$C_{t1}l/a = 0.02 \quad C_{m1}l/a = 0.01$		
		Case A	Case B	Case C	Case A	Case B	Case C
1	0.2	0.9962	0.9962	0.9959	0.9960	0.9960	0.9960
	0.4	0.9714	0.9709	0.9675	0.9688	0.9688	0.9684
	0.6	0.9057	0.9036	0.8867	0.8915	0.8914	0.8894
	0.8	0.7627	0.7568	0.6976	0.7090	0.7089	0.7002
	0.9	0.6257	0.6173	0.5089	0.5266	0.5264	0.5079
	0.95	0.5120	0.5027	0.3517	0.3790	0.3788	0.3496
	0.99	0.3345	0.3262	0.1228	0.1814	0.1813	0.1338
	0.999	0.2084	0.2026	0.0172	0.0831	0.0830	0.0278
	0.9999	0.1480	0.1438	0.0019	0.0501	0.0500	0.0038
10	0.2	0.9965	0.9970	0.9969	0.9973	0.9975	0.9975
	0.4	0.9702	0.9775	0.9759	0.9780	0.9806	0.9806
	0.6	0.8936	0.9250	0.9183	0.9226	0.9362	0.9360
	0.8	0.7200	0.8078	0.7881	0.7952	0.8452	0.8466
	0.9	0.5584	0.6883	0.6474	0.6702	0.7657	0.7732
	0.95	0.4326	0.5811	0.5031	0.5614	0.6974	0.7133
	0.99	0.2584	0.3956	0.2105	0.3629	0.5297	0.5200
	0.999	0.1535	0.2506	0.0323	0.1975	0.3174	0.1654
	0.9999	0.1079	0.1784	0.0035	0.1233	0.2026	0.0248
100	0.2	0.9964	0.9972	0.9972	0.9971	0.9979	0.9979
	0.4	0.9674	0.9791	0.9780	0.9728	0.9845	0.9846
	0.6	0.8800	0.9310	0.9274	0.8945	0.9556	0.9565
	0.8	0.6806	0.8242	0.8204	0.7073	0.9400	0.9515
	0.9	0.4993	0.7136	0.7057	0.5342	1.0076	1.0583
	0.95	0.3646	0.6108	0.5743	0.4036	1.1284	1.2616
	0.99	0.1944	0.4235	0.2574	0.2214	1.2465	1.5403
	0.999	0.1076	0.2703	0.0408	0.1097	0.9073	0.6654
	0.9999	0.0745	0.1927	0.0045	0.0670	0.6024	0.1069
1000	0.2	0.9963	0.9973	0.9972	0.9968	0.9980	0.9980
	0.4	0.9669	0.9793	0.9783	0.9683	0.9852	0.9853
	0.6	0.8778	0.9317	0.9285	0.8728	0.9610	0.9623
	0.8	0.6744	0.8263	0.8247	0.6392	0.9770	0.9929
	0.9	0.4900	0.7170	0.7138	0.4231	1.1153	1.1868
	0.95	0.3538	0.6148	0.5845	0.2648	1.3341	1.5280
	0.99	0.1843	0.4274	0.2643	0.0785	1.6176	2.0915
	0.999	0.1004	0.2731	0.0420	0.0151	1.2243	0.9529
	0.9999	0.0693	0.1947	0.0046	0.0057	0.8189	0.1547

Case A: $C_{s1} = C_{s2}$, $C_{t2} = C_{t1}$, $C_{m2} = C_{m1}$.
Case B: $C_{s2} = 0$, $C_{t2} = C_{t1}$, $C_{m2} = C_{m1}$.
Case C: $C_{s2} = C_{t2} = C_{m2} = 0$.

Reed and Morrison (1975) evaluated the values of $U/U^{(0)}$ for the thermophoretic migration of a sphere normal to a plane wall for a case of no thermal slip on the wall ($C_{s2} = 0$, $C_{t2} = C_{t1}$, $C_{m2} = C_{m1}$) over the range $a/d < 0.887$. Our results of Case B in Table 3, which are also calculated with $C_{s2} = 0$, $C_{t2} = C_{t1}$ and $C_{m2} = C_{m1}$, are consistent with theirs.

5. SUMMARY

The thermophoretic motion of two spherical particles along their line of centers has been analyzed in this work. The temperature and velocity fields are solved using bipolar coordinates and the particle velocities are obtained for various values of the physical properties, sizes and separation distance of the particles. The results indicate that the interaction between particles can be significant when their gap thickness gets close to zero. The influence of the interaction is greater on the smaller of the two particles. For the special case of two identical spheres, both migrate with the same velocity, which is larger in magnitude than that which would exist in the absence of one of the particles. The

thermophoretic motion of a spherical particle in the direction perpendicular to a plane wall is also studied for various cases. The effect of retardation caused by the plane wall on the particle migration increases steadily as the particle approaches the wall and becomes infinity when the surfaces are in contact. In general, the effect of particle–particle and particle–wall interactions on thermophoresis is much weaker than that on the motion in a gravitational field.

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