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Creeping motions of a composite sphere in a concentric spherical cavity

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Abstract

An analytical study is presented for the quasisteady translation and steady rotation of a spherically symmetric composite particle composed of a solid core and a surrounding porous shell located at the center of a spherical cavity filled with an incompressible Newtonian fluid. In the fluid-permeable porous shell, idealized hydrodynamic frictional segments are assumed to distribute uniformly. In the limit of small Reynolds number, the Stokes and Brinkman equations are solved for the flow field of the system, and the hydrodynamic drag force and torque exerted by the fluid on the particle which is proportional to the translational and angular velocities, respectively, are obtained in closed forms. For a given geometry, the normalized wall-corrected translational and rotational mobilities of the particle decrease monotonically with a decrease in the permeability of its porous shell. The boundary effects of the cavity wall on the creeping motions of a composite sphere can be quite significant in appropriate situations. In the limiting cases, the analytical solutions describing the drag force and torque or mobilities for a composite sphere in the cavity reduce to those for a solid sphere and for a porous sphere.

Keywords: Fluid mechanics; Composite sphere; Porous sphere; Multiphase flow; Creeping flow; Boundary effects

1. Introduction

The area of the moving of colloidal particles in a continuous medium at low Reynolds number has continued to receive much attention from investigators in the fields of chemical, biomedical, and environmental engineering and science. The majority of these moving phenomena are fundamental in nature, but permit one to develop rational understanding of many practical systems and industrial processes such as sedimentation, flotation, spray drying, agglomeration, and motion of blood cells in an artery or vein. The theoretical study of this subject has grown out of the classic work of Stokes' (1851) for a translating rigid sphere in an unbounded viscous fluid.

In most practical applications, particles are not isolated. So, it is important to determine if the presence of neighboring particles and/or boundaries significantly affects the movement of particles. Problems of the hydrodynamic interactions between two or more particles and between particles and boundaries have been treated extensively in the past. Summaries for the useful knowledge in this area and some informative references can be found in Happel and Brenner (1983) and Kim and Karrila (1991).

The surface of a colloidal particle is generally not hard and smooth as assumed in many theoretical models. For instance, surface layers are purposely formed by adsorbing polymers to make the suspended particles stable against flocculation (Napper, 1983). Even the surfaces of model colloids such as silica and polystyrene latex are "hairy" with a gel-like polymeric layer extending a substantial distance into the suspending medium from the bulk material inside the particle (Anderson and Solomentsev, 1996). In particular, the surface of a biological cell is not a hard smooth surface, but rather is a permeable rough surface with various appendages ranging from protein molecules on the order of nanometers to cilia on the order of micrometers (Wunderlich, 1982). Such particles can be modeled as a composite particle having a central solid core and an outer porous shell (Sasaki, 1985).

The creeping flow of an incompressible Newtonian fluid past a spherically symmetric composite particle was solved by Masliyah et al. (1987) using the Brinkman equation (which may be regarded as an extension of Darcy's law) for the flow field inside the fluid-permeable surface layer and the Stokes equations for the flow field external to the particle. An analytical formula for the drag force experienced

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by the particle was derived as a function of the radius of the solid core, the thickness of the porous shell, and the permeability of the shell. They also measured the settling velocity of a solid sphere with attached threads and found that theoretical predictions for the composite sphere are in excellent agreement with the experimental results. Employing a unit-cell model for the creeping flow relative to an assemblage of identical composite spheres, Prasad et al. (1990) and Keh and Kuo (1997) later obtained analytical solutions for the dependence of the average drag force of this assemblage on the volume fraction of the particles.

Anderson and Solomentsev (1996) analyzed motions of a spherical particle covered by a thin layer of adsorbed polymer near an infinite plane wall and along the centerline of a long cylindrical tube. Using the concept of hydrodynamic thickness of the polymer layer and a method of reflections together with the technique of matched asymptotic expansions, they determined the boundary effects on the particle movement to $O(\lambda^2)$ in increasing powers of ζ up to $O(\zeta^3)$, where λ is the ratio of the polymer-layer length scale to the particle radius and ζ is the ratio of the particle radius to the distance between the particle center and the boundary. On the other hand, the quasisteady motion of a sphere coated with a thin polymer layer normal to an infinite plane, which can be either a solid wall or a free surface, has been investigated (Kuo and Keh, 1999). A combined analytical-numerical exact solution of the hydrodynamic effect exerted by the plane on the moving particle accurate to $O(\lambda^2)$ was also obtained using the method of matched asymptotic expansions and a boundary collocation technique. Recently, the boundary effect on the motion of a composite sphere perpendicular to one or two plane walls has been examined by Chen and Ye (2000). The boundary collocation method was used to study the general case where the porous shell thickness and separation distance between the particle and the wall can be arbitrary, and a lubrication theory was used to analyze the special case of a particle with a thin permeable layer in near contact with a single plane.

The purpose of this work is to obtain insights into the boundary effects on the translational and rotational motions of an arbitrary composite sphere within a small pore. This type of problem is difficult to solve due to the structural difference for hydrodynamics inside and outside the porous surface layer and the complexity of the actual system geometry. In order to avoid the mathematical difficulties encountered in the problem of a sphere in a cylinder (which is a widely used model for particles in pores), we choose to examine the motions of a composite sphere situated at the center of a spherical cavity. Although the geometry of spherical cavity is an idealized abstraction of any real system, the results obtained in this geometry have been shown to be in good agreement with available solutions for the boundary effects on the partition coefficient (Giddings et al., 1968; Glandt, 1981), settling velocity (Bungay and Brenner, 1973; Happel and Brenner, 1983), and electrophoretic mobility (Zydney, 1995; Keh and Chiou, 1996) of a "bare" particle in a cylindrical pore. The spherical symmetry in this model system allows exact analytical solutions to be obtained, and the results show that the boundary effects on the motions of a composite particle can be significant in general situations.

2. Translation of a composite sphere in a spherical cavity

In this section we consider the quasisteady translational motion of a spherical composite particle of radius b in a concentric spherical cavity (or pore) of radius c filled with an incompressible Newtonian fluid of viscosity η, as illustrated in Fig. 1. The composite sphere has a surface layer of homogeneous porous material of constant thickness $b-a$ and permeability k so that the radius of the rigid impermeable core is a. The porous shell is assumed to be non-deformable, and the particle velocity equals U in the positive z (axial) direction. The spherical coordinate system (r, θ, ϕ) is established with its origin at the particle center. The Reynolds number is assumed to be sufficiently small so that the inertial terms in the fluid momentum equation can be neglected, in comparison with the viscous terms. Our purpose here is to determine the hydrodynamic drag force exerted on the particle in the presence of the cavity.

The fluid flow between the particle and the cavity ($b \leqslant r \leqslant c$) is governed by the Stokes equations

$$\eta\nabla^2\mathbf{v} - \nabla p = \mathbf{0}, \tag{1a}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{1b}$$

where $\mathbf{v}$ is the fluid velocity field for the flow outside and relative to the particle and p is the corresponding dynamic pressure distribution. For the fluid flow within the porous surface layer ($a \leqslant r \leqslant b$), the relative velocity $\mathbf{v}^*$ and dynamic pressure p^* are governed by the Brinkman equation,

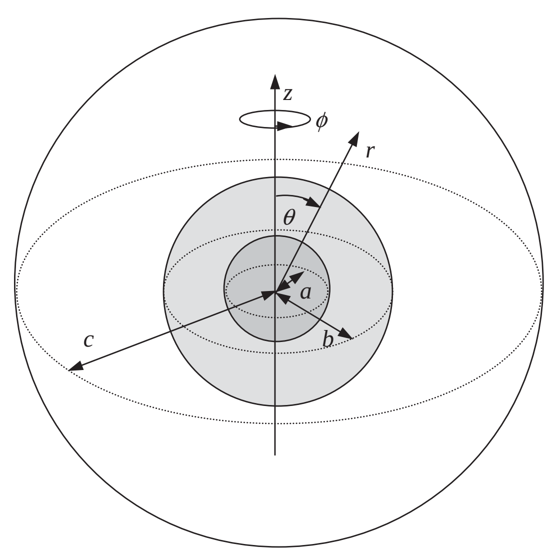

Fig. 1. Geometric sketch for the motion of a composite sphere in a concentric spherical cavity.

which is preferred to the Darcy equation to accommodate the boundary conditions at the particle surface ($r = b$),

$$\eta \nabla^2 \mathbf{v}^* - \frac{\eta}{k} \mathbf{v}^* - \nabla p^* = \mathbf{0} \tag{2a}$$

and

$$\nabla \cdot \mathbf{v}^* = 0, \tag{2b}$$

where the superscript $*$ represents a macroscopically averaged quantity pertaining to the porous layer region. Here, we have assumed that the fluid has the same viscosity inside and outside the permeable layer which is reasonable according to available evidence (Koplik et al., 1983).

Since the flow field is axially symmetric, it is convenient to introduce the Stokes stream function $\Psi(r,\theta)$ which satisfies Eq. (1b) and is related to the velocity components in the spherical coordinate system by

$$v_r = -\frac{1}{r^2 \sin\theta} \frac{\partial \Psi}{\partial \theta}, \tag{3a}$$

$$v_\theta = \frac{1}{r \sin\theta} \frac{\partial \Psi}{\partial r}. \tag{3b}$$

Taking the curl of Eq. (1a) and applying Eq. (3) gives a fourth-order linear partial differential equation for Ψ,

$$E_s^4 \Psi = E_s^2(E_s^2 \Psi) = 0 \quad \text{if } b \leqslant r \leqslant c, \tag{4}$$

where the axisymmetric Stokes operator E_s^2 is given by

$$E_s^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2} \frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right). \tag{5}$$

Accordingly, Eq. (2) can be expressed in terms of the stream function $\Psi^*(r,\theta)$, which is related to the velocity components v_r^* and v_θ^* within the porous layer by Eq. (3), as

$$E_s^4 \Psi^* - \frac{1}{k} E_s^2 \Psi^* = 0 \quad \text{if } a \leqslant r \leqslant b. \tag{6}$$

Due to the continuity of velocity and stress components at the outer surface of the porous shell, which is physically realistic and mathematically consistent for the present problem (Neale et al., 1973; Koplik et al., 1983; Chen and Ye, 2000), and the no-slip requirement at the solid surfaces, the boundary conditions for the flow field are

$$r = a: \quad v_r^* = v_\theta^* = 0, \tag{7}$$

$$r = b: \quad v_r^* = v_r, \quad v_\theta^* = v_\theta, \tag{8a,b}$$

$$\tau_{rr}^* = \tau_{rr}, \quad \tau_{r\theta}^* = \tau_{r\theta}, \tag{9a,b}$$

$$r = c: \quad v_r = -U\cos\theta, \quad v_\theta = U\sin\theta. \tag{10a,b}$$

Here τ_{rr} and $\tau_{r\theta}$ are the normal and shear stresses for the fluid flow relevant to the particle surface. Eqs. (7)–(10) take a reference frame that the composite sphere is at rest and the velocity of the fluid at the cavity wall is the particle velocity in the opposite direction. Since we take the same fluid viscosity inside and outside the porous shell, use the fluid velocity continuity given by Eq. (8), and neglect the possible osmotic effects in the porous shell, Eq. (9a) is equivalent to the continuity of pressure ($p^* = p$ at $r = b$).

A solution to Eqs. (4) and (6) suitable for satisfying boundary conditions on the spherical surfaces is

$$\Psi = \tfrac{1}{2} kU(A\xi^{-1} + B\xi + C\xi^2 + D\xi^4)\sin^2\theta \quad \text{if } \beta \leqslant \xi \leqslant \gamma, \tag{11a}$$

$$\Psi^* = \tfrac{1}{2} kU[E\xi^{-1} + F\xi^2 + G(\xi^{-1}\cosh\xi - \sinh\xi) + H(\xi^{-1}\sinh\xi - \cosh\xi)]\sin^2\theta \quad \text{if } \alpha \leqslant \xi \leqslant \beta, \tag{11b}$$

where the dimensionless variables $\xi = r/k^{1/2}$, $\alpha = a/k^{1/2}$, $\beta = b/k^{1/2}$ and $\gamma = c/k^{1/2}$. The dimensionless constants A, B, C, D, E, F, G and H are found from Eqs. (7)–(10) using Eq. (3). The procedure is straightforward but tedious, and the result is given in Appendix A.

The drag force (in the z direction) exerted by the external fluid on the composite sphere with the spherical boundary $r = b$ can be determined from (Happel and Brenner, 1983)

$$F_d = \pi\eta \int_0^\pi r^3 \sin^3\theta \frac{\partial}{\partial r}\left(\frac{E_s^2 \Psi}{r^2 \sin^2\theta}\right) r \, d\theta. \tag{12}$$

Substitution of Eq. (11a) into the above integral results in the simple relation

$$F_d = 4\pi\eta U B k^{1/2}, \tag{13}$$

where B is given by Eq. (A.10) in Appendix A.

In the limiting case of $\beta/\gamma = b/c = 0$, the above equation becomes

$$F_d^{(0)} = -6\pi\eta b U \frac{R}{\beta S}, \tag{14}$$

where

$$R = W\alpha\cosh\alpha - 3\alpha^2(V + \alpha\sinh\alpha) + \cosh(\beta-\alpha) \times[W(V\alpha - \beta\cosh\alpha) + 3\alpha^2\beta\sinh\alpha] + \sinh(\beta-\alpha)[W\cosh\alpha + 3\alpha^2(V\alpha - \sinh\alpha)], \tag{15a}$$

$$S = (\alpha\sinh\beta - \cosh\alpha)[(W + 3\beta)\cosh(\beta-\alpha) + 3(\alpha^2 - 1)\sinh(\beta-\alpha) - 6\alpha] \tag{15b}$$

with

$$V = \beta\sinh\beta - \cosh\beta, \tag{16a}$$

$$W = 2\beta^3 + \alpha^3 + 3\alpha. \tag{16b}$$

The formula given by Eq. (14) is the reduced result for the translation of an isolated composite sphere in an unbounded fluid obtained by Masliyah et al. (1987).

Through the use of Eqs. (13) and (14), the normalized translational mobility of a composite sphere in a concentric spherical cavity can be expressed as

$$M = \frac{F_d^{(0)}}{F_d} = -\frac{3R}{2BS}. \tag{17}$$

Note that $M = 1$ as $\beta/\gamma = 0$ and $0 \leqslant M < 1$ as $0 < \beta/\gamma \leqslant 1$. The presence of the cavity wall always enhances the hydrodynamic drag on the particle since the fluid flow vanishes at the wall as required by Eq. (10).

When $\beta = \alpha$ or $k \to \infty$, $F_d^{(0)} = -6\pi\eta a U$ (Stokes' law) and Eq. (17) reduces to

$$M = \left(1 - \frac{9}{4}\zeta + \frac{5}{2}\zeta^3 - \frac{9}{4}\zeta^5 + \zeta^6\right)(1-\zeta^5)^{-1}, \tag{18}$$

where $\zeta = a/c$. This is the result for the translation of a solid sphere of radius a in a cavity of radius c.

When $\alpha = \beta$ or $k = 0$, $F_d^{(0)} = -6\pi\eta b U$ and Eq. (17) still reduces to Eq. (18), but with $\zeta = b/c$. This result corresponds to the translation of a solid sphere of radius b in a cavity of radius c.

When $a = 0$, Eqs. (14) and (17) become

$$F_d^{(0)} = -6\pi\eta b U \, \frac{2\beta^2(\beta - \tanh\beta)}{2\beta^3 + 3(\beta - \tanh\beta)}, \tag{19}$$

$$M = \frac{(\beta\cosh\beta - \sinh\beta)(s_{21}\beta\cosh\beta - 3s_{23}\sinh\beta)}{2\gamma[\beta(2\beta^2+3)\cosh\beta - 3\sinh\beta](s_8\sinh\beta - s_5\beta\cosh\beta)}, \tag{20}$$

where the dimensionless parameters s_5, s_8, s_{21} and s_{23} are defined by Eq. (A.18) in Appendix A. The hydrodynamic drag and normalized mobility given by Eqs. (19) and (20) describe the translation of a porous (permeable) sphere of radius b in an unbounded fluid (Neale et al., 1973) and in a cavity of radius c, respectively. In the limiting case of $\beta \to \infty$ (or $k = 0$), Eq. (19) reduces to $F_d^{(0)} = -6\pi\eta b U$, while in the limit of $\beta = 0$ (or $k \to \infty$), it becomes $F_d^{(0)} = 0$.

The variation of the normalized mobility M given by Eq. (20) for the translation of a porous sphere (with $a = 0$ or $\alpha/\beta = 0$) at the center of a spherical cavity with the separation parameter β/γ for various values of β from zero to infinity is presented in Fig. 2. The separation parameter β/γ ($= b/c$), reflecting the extent of closeness between the particle and the cavity wall, ranges from 0 (far apart) to 1 (in contact). The curve with $\beta = 0$ (or $k \to \infty$) represents the result for a porous sphere with no resistance to the fluid flow, while the curve with $\beta \to \infty$ (or $k = 0$) denotes the result for a solid particle. As expected, the normalized mobility M equals unity as $\beta = 0$ for any value of β/γ and is a monotonic decreasing function of β/γ for any given value of $\beta > 0$. Obviously, the boundary effect on the particle mobility (or drag force) is stronger when the permeability k of the particle is smaller (or β is greater). For $\beta < 1$, the particle mobility varies slowly with the separation parameter β/γ, compared with the case of lower permeability (or greater β). This weak interaction can be explained by the fact that, instead of bypassing, the fluid can easily flow through a porous particle with a high permeability, leading to a great reduction in the resistant force. For $\beta > 10$, the value of the particle mobility is quite close to that of a solid particle (with $\beta \to \infty$ and M given by Eq. (18) with $\zeta = b/c$) when β/γ is small, but the difference becomes more significant as the particle gets closer to the wall. This implies that, far from the wall, a porous particle with a low permeability behaves like a solid one with most fluid flowing over it. When the porous particle and cavity wall become sufficiently close together, a large pressure gradient is developed in between to drive more fluid to permeate through the porous medium (Chen and Ye, 2000). Interestingly, for cases with a finite value of β, the particle mobility does not vanish even as the particle touches the cavity wall (i.e., as $\beta/\gamma = 1$).

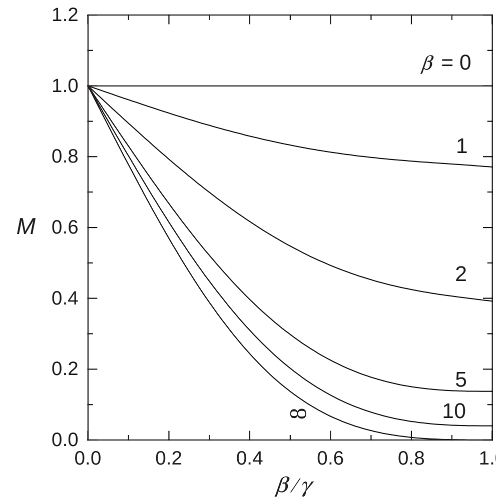

Fig. 2. Plots of the normalized translational mobility M for a porous sphere ($a = 0$) in a concentric spherical cavity versus the separation parameter β/γ for various values of β.

After understanding the boundary effect on the translation of a porous sphere, we now examine the general case of a translating composite sphere in a concentric spherical cavity. Figs. 3a and b show the normalized mobility M as a function of β/γ over the entire ranges of the separation and the parameter α/β for the cases of $\beta = 5$ and 1, respectively. Again, M decreases monotonically with an increase in β/γ for fixed values of α/β and β and with an increase in β for constant values of α/β and β/γ. The curve with $\alpha/\beta = 1$ represents the result for a solid sphere (given by Eq. (18)) and the curve with $\alpha/\beta = 0$ denotes that for a porous sphere (given by Eq. (20)). All the other curves for a composite sphere lie between these lower and upper bounds and M is a monotonic decreasing function of α/β for given values of β and β/γ. Namely, the hydrodynamic drag acting on the particle is reduced as the porous layer becomes thick for a given particle size, permeability, and separation distance. It can be seen that, for the case of $\beta = 5$, the behavior of a composite sphere with $\alpha/\beta = 0.6$ can be roughly approximated by that

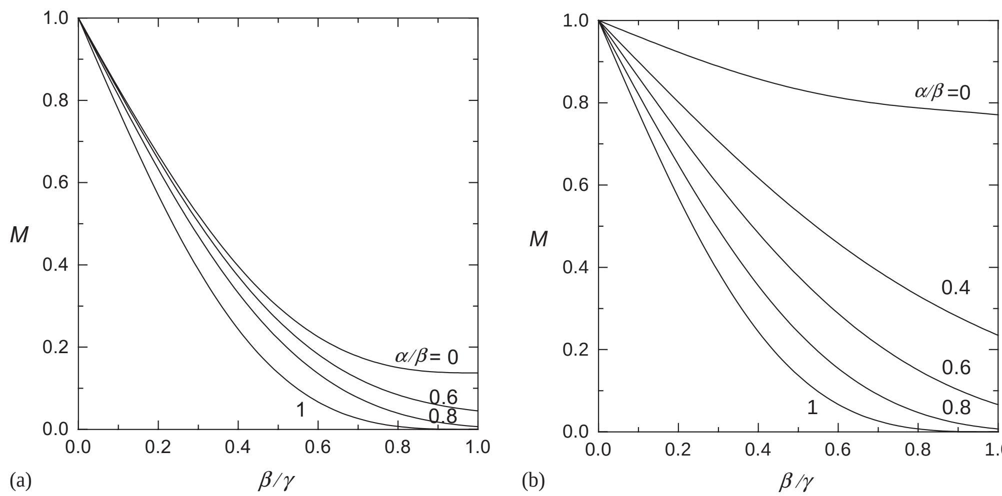

Fig. 3. Plots of the normalized translational mobility M for a composite sphere in a concentric spherical cavity versus the separation parameter β/γ for various values of α/β: (a) $\beta = 5$, (b) $\beta = 1$.

of a porous one of equal size and permeability when $\beta/\gamma < 0.5$. This is because when a porous layer has a low-to-moderate permeability, it is difficult for the fluid to penetrate deep to reach the core surface as long as the layer is sufficiently thick and the cavity wall is not too close. Thus, the solid core hardly feels a relative fluid motion, merely exerting a negligibly small resistant force on the fluid (Masliyah et al., 1987; Chen and Ye, 2000). However, this approximation is no longer valid for a porous layer with a large permeability, as for the case of $\beta = 1$ shown in Fig. 3b. Again, for cases with $\alpha/\beta < 1$, the particle mobility is not necessarily equal to zero as $\beta/\gamma = 1$.

3. Rotation of a composite sphere in a spherical cavity

We now consider the steady rotational motion of a spherical composite particle of radius b located at the center of a spherical cavity of radius c. Again, the composite sphere has a porous surface layer of uniform thickness $b - a$ and permeability k. The angular velocity of the particle is Ω in the positive z (axial) direction and the Reynolds number is vanishingly small. The objective in this section is to obtain the hydrodynamic torque acting on the particle in the presence of the cavity.

The fluid flow fields outside and inside the porous shell of the composite sphere are still governed by Eqs. (1) and (2), respectively, and they must be solved subject to the following boundary conditions resulting from the continuity of velocity and stress components:

$$r = a: \quad v_\phi^* = 0, \tag{21}$$

$$r = b: \quad v_\phi^* = v_\phi, \tag{22}$$

$$\tau_{r\phi}^* = \tau_{r\phi}, \tag{23}$$

$$r = c: \quad v_\phi = -\Omega c \sin\theta, \tag{24}$$

where v_ϕ is the ϕ-component of the fluid velocity field and $\tau_{r\phi}$ is the shear stress for the rotational fluid flow on the particle surface. Obviously, the r and θ components of the fluid velocity disappear and the fluid dynamic pressure is constant everywhere. Eqs. (21)–(24) take a reference frame that the particle is at rest and the angular velocity of the cavity wall is that of the particle in the opposite direction.

A solution to Eqs. (1) and (2) suitable for satisfying the boundary conditions (21)–(24) is

$$v_\phi = k^{1/2}\Omega(A'\xi^{-2} + B'\xi)\sin\theta \quad \text{if } \beta \leqslant \xi \leqslant \gamma, \tag{25a}$$

$$v_\phi^* = k^{1/2}\Omega[C'(\xi^{-2}\cosh\xi - \xi^{-1}\sinh\xi) + D'(\xi^{-2}\sinh\xi - \xi^{-1}\cosh\xi)]\sin\theta \quad \text{if } \alpha \leqslant \xi \leqslant \beta, \tag{25b}$$

where the dimensionless variables ξ, α, β and γ were defined right after Eq. (11). The constants A', B', C' and D' can be determined from Eqs. (21)–(24), and the result is given in Appendix A.

After the fluid velocity field is solved, the torque (in the z direction) exerted on the rotating composite sphere about its center by the external fluid can be obtained as

$$T_d = -8\pi\eta\Omega A' k^{3/2}, \tag{26}$$

where A' is given by Eq. (A.23). In the limit of $\beta/\gamma = b/c = 0$, Eq. (26) becomes

$$T_d^{(0)} = -8\pi\eta b^3 \Omega \frac{R'}{\beta^2 S'}, \tag{27}$$

the reduced result for the rotation of a composite sphere in an unbounded fluid, where

$$R' = (\alpha\beta^2 - 3\beta + 3\alpha)\cosh(\beta - \alpha) + (\beta^2 - 3\alpha\beta + 3)\sinh(\beta - \alpha), \tag{28a}$$

$$S' = \alpha\cosh(\beta - \alpha) + \sinh(\beta - \alpha). \tag{28b}$$

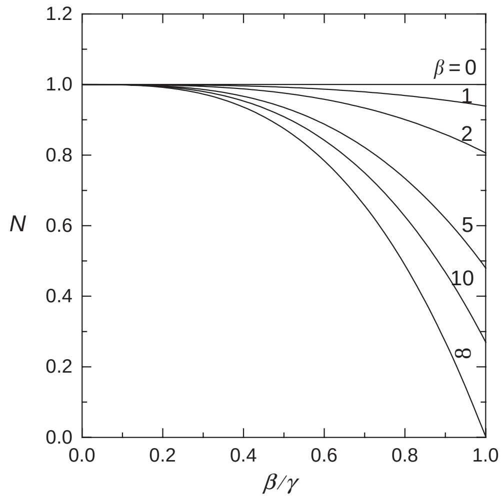

Fig. 4. Plots of the normalized rotational mobility N for a porous sphere ($a = 0$) in a concentric spherical cavity versus the separation parameter β/γ for various values of β.

The normalized rotational mobility of a composite sphere in a concentric spherical cavity can be expressed as

$$N = \frac{T_d^{(0)}}{T_d} = \frac{\beta R'}{A'S'}. \tag{29}$$

The presence of the no-slip cavity wall always enhances the hydrodynamic torque on the rotating particle. Thus, $0 \leqslant N \leqslant 1$ for the entire range of β/γ ($N = 1$ as $\beta/\gamma = 0$).

When $\beta = \alpha$ or $k \to \infty$, $T_d^{(0)} = -8\pi\eta a^3 \Omega$ and Eq. (29) reduces to

$$N = 1 - \zeta^3, \tag{30}$$

where $\zeta = a/c$. This is the result for the rotation of a solid sphere of radius a in a cavity of radius c.

When $\alpha = \beta$ or $k = 0$, $T_d^{(0)} = -8\pi\eta b^3 \Omega$ and Eq. (29) still reduces to Eq. (30) with $\zeta = b/c$. This result corresponds to the rotation of a solid sphere of radius b in a cavity of radius c.

When $a = 0$, Eqs. (27) and (29) become

$$T_d^{(0)} = -8\pi\eta b^3 \Omega (1 + 3\beta^{-2} - 3\beta^{-1} \coth \beta), \tag{31}$$

$$N = 1 - \frac{\beta^3}{\gamma^3}(1 + 3\beta^{-2} - 3\beta^{-1} \coth \beta). \tag{32}$$

The hydrodynamic torque and normalized mobility predicted by Eqs. (31) and (32) describe the rotation of a porous sphere of radius b in an unbounded fluid and in a cavity of radius c, respectively. In the limiting case of $\beta \to \infty$ (or $k = 0$), Eq. (31) reduces to $T_d^{(0)} = -8\pi\eta b^3 \Omega$, while in the limit of $\beta = 0$ (or $k \to \infty$), it results in $T_d^{(0)} = 0$.

Fig. 4 shows the plot of the normalized rotational mobility N for a porous sphere at the center of a spherical cavity versus the separation parameter β/γ for various values of β over the entire ranges. Analogous to the result of the translational mobility M of the particle, the rotational mobility N equals unity as $\beta = 0$ for all values of β/γ and decreases monotonically with an increase in the value of β/γ for a specified value of $\beta > 0$. The boundary effect on the rotational mobility (or hydrodynamic torque) of the permeable particle is stronger when the permeability k is smaller (or β is greater). For $\beta < 1$, the rotational mobility is not a sensitive function of β/γ (except as $\beta/\gamma \to 1$), compared with the result for a lower permeability (or greater β). For $\beta > 10$, the value of the rotational mobility of the porous sphere is close to that of a solid particle (with $k = 0$ or $\beta \to \infty$ and N given by Eq. (30) with $\zeta = b/c$) when β/γ is small, while the difference is more significant as $\beta/\gamma \to 1$. When the particle is in contact with the cavity wall ($\beta/\gamma = 1$), its rotational mobility does not vanish for cases with a finite value of β.

In Figs. 5a and b, we present the results of the normalized rotational mobility N for a composite sphere in a concentric spherical cavity as a function of the parameters β/γ and α/β over the entire ranges for the cases of $\beta = 5$ and 1, respectively. Again, N decreases monotonically with an increase in β/γ for given values of β and α/β and with an increase in β for fixed values of α/β and β/γ. The curves with $\alpha/\beta = 1$ and $\alpha/\beta = 0$ represent the results for the rotation of a solid sphere and a porous sphere, respectively. All the other curves for a composite sphere locate between these lower and upper bounds and N decreases monotonically with an increase in α/β for specified values of β and β/γ. For a particle with $\alpha/\beta > 1$, its rotational mobility does not vanish as $\beta/\gamma = 1$. For the case of $\beta = 5$, the normalized rotational mobility of a composite sphere with $\alpha/\beta = 0.6$ can be well approximated by that of a porous one of equal size, permeability, and separation distance from the cavity wall. As illustrated in Fig. 5b, however, this approximation is no longer valid for a layer with a large permeability.

4. Concluding remarks

The quasisteady translation and steady rotation of a composite sphere (which can reduce to a solid sphere and a porous sphere in the limiting cases) in a concentric spherical cavity filled with an incompressible Newtonian fluid have been theoretically investigated in this study. In the creeping flow regime, the Stokes and Brinkman equations for the fluid flow field applicable to these axisymmetric motions are analytically solved and the hydrodynamic drag force and torque exerted on the particle as functions of the parameters α/β ($=a/b$), β/γ ($=b/c$), and β are obtained in the closed-form expressions (13) and (26). It has been found that, for a specified geometry (fixed values of α/β and β/γ), the wall-corrected translational and rotational mobilities of the particle normalized by their corresponding values in the absence of the cavity wall are monotonic decreasing functions of the parameter β (or increasing functions of the permeability k of the porous surface layer) of the particle.

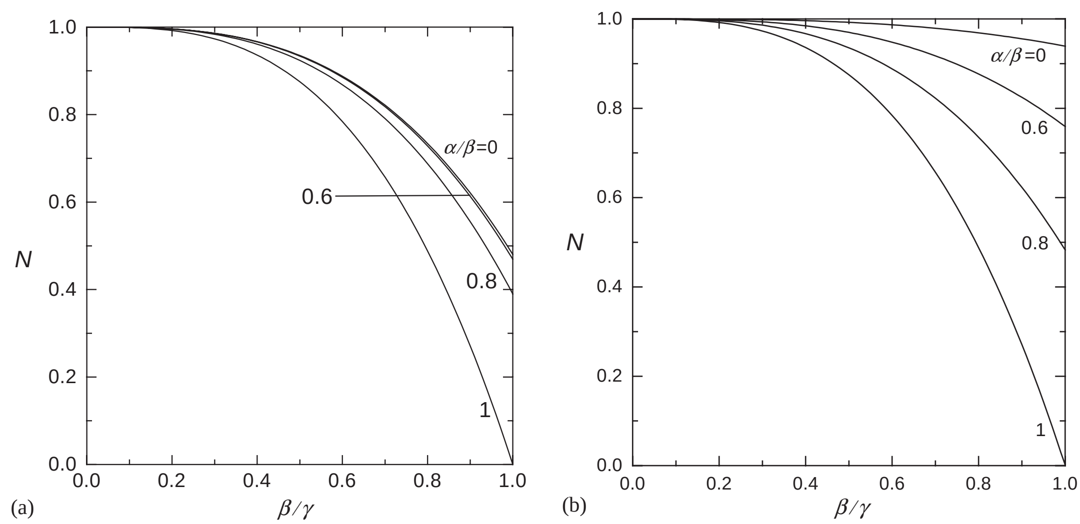

Fig. 5. Plots of the normalized rotational mobility N for a composite sphere in a concentric spherical cavity versus the separation parameter β/γ for various values of α/β: (a) $\beta = 5$, (b) $\beta = 1$.

For given values of α/β and β, these normalized mobilities decrease monotonically with an increase in the separation parameter β/γ. The analysis assumes that the porous shell of the composite sphere is non-deformable. The results would be very different, particularly in the case where the particle just fits within the cavity, if the porous shell were able to deform in response to the flow (as might be expected for a layer composed of entangled polymer).

Our results, which provide useful insights into the actual phenomena regarding the creeping motions of a composite/porous particle in a small pore, show that the boundary effect of the cavity wall on these motions can be significant in appropriate situations. More detailed analyses of the fluid flow for a composite/porous particle in open and closed cylindrical pores will clearly be required to quantify the actual behavior in this system and to determine the overall applicability of the results obtained in this paper for the spherical cavity to more realistic pore geometries.

Notation

a	radius of the solid sphere, m
b	radius of the composite sphere, m
c	radius of the spherical cavity, m
A, B, C, D	coefficients in Eq. (11a) for the external flow field, given by Eqs. (A.9)–(A.12)
A', B', C', D'	coefficients in Eq. (25) for the rotational velocity field, given by Eqs. (A.23)–(A.26)
E, F, G, H	coefficients in Eq. (11b) for the flow field inside the porous shell, given by Eqs. (A.13)–(A.16)
F_d	drag force acting on the particle, N
$F_d^{(0)}$	drag force acting on the particle in the absence of the cavity, N
k	permeability in the porous shell, m^2
M	normalized translational mobility of the particle
N	normalized rotational mobility of the particle
p	dynamic pressure distribution, N m^{-2}
r	radial spherical coordinate, m
T_d	hydrodynamic torque exerted on the particle, N m
$T_d^{(0)}$	hydrodynamic torque exerted on the particle in the absence of the cavity, N m
U	translational velocity of the particle, m s^{-1}
$\mathbf{v}$	fluid velocity field, m s^{-1}
v_r, v_θ, v_ϕ	components of fluid velocity in spherical coordinates, m s^{-1}

Greek letters

α	$=a/k^{1/2}$
β	$=b/k^{1/2}$
γ	$=c/k^{1/2}$
ζ	ratio of the particle radius to the distance between the particle center and the wall
η	viscosity of the fluid, kg m^{-1} s^{-1}
θ, ϕ	angular spherical coordinates
ξ	$=r/k^{1/2}$
$\tau_{rr}, \tau_{r\theta}, \tau_{r\phi}$	fluid stresses relevant to the particle surface, N m^{-2}
Ψ	Stokes stream function of the fluid flow, m^3 s^{-1}
Ω	angular velocity of the particle, s^{-1}

Superscript

*	fluid inside the porous shell

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Appendix A

For conciseness algebraic equations for the determination of the coefficients in Eqs. (11) and (25) as well as their solutions are presented in this appendix.

Applying the boundary conditions given by Eqs. (7)–(10) to the general solution given by Eq. (11) for the translation of a composite sphere in a concentric spherical cavity, one obtains

$$E + F\alpha^3 + G(\cosh\alpha - \alpha\sinh\alpha) + H(\sinh\alpha - \alpha\cosh\alpha) = 0, \tag{A.1}$$

$$E - 2F\alpha^3 - G[\alpha\sinh\alpha - (\alpha^2+1)\cosh\alpha] - H[\alpha\cosh\alpha - (\alpha^2+1)\sinh\alpha] = 0, \tag{A.2}$$

$$A + B\beta^2 + C\beta^3 + D\beta^5 = E + F\beta^3 - G(\beta\sinh\beta - \cosh\beta) + H(\sinh\beta - \beta\cosh\beta), \tag{A.3}$$

$$A - B\beta^2 - 2C\beta^3 - 4D\beta^5 = E - 2F\beta^3 - G[\beta\sinh\beta - (\beta^2+1)\cosh\beta] - H[\beta\cosh\beta - (\beta^2+1)\sinh\beta], \tag{A.4}$$

$$6A + 6D\beta^5 = 6E + 3(\beta^2+2)(G\cosh\beta + H\sinh\beta) - \beta(\beta^2+6)(G\sinh\beta + H\cosh\beta), \tag{A.5}$$

$$2B + 20D\beta^3 = E - 2F\beta^3, \tag{A.6}$$

$$A + B\gamma^2 + C\gamma^3 + D\gamma^5 = \gamma^3, \tag{A.7}$$

$$A - B\gamma^2 - 2C\gamma^3 - 4D\gamma^5 = -2\gamma^3. \tag{A.8}$$

The above simultaneous algebraic equations can be solved to yield the eight unknown constants as

$$A = 2\Delta\beta^3\gamma^3[18\alpha\beta(\gamma^3 - 10\beta^3) + (\alpha\beta s_0 s_1 - 9\alpha^2 s_2 + 2\beta^2 s_3)\cosh(\beta-\alpha) - 3(s_0 s_2 - \alpha^2\beta s_1 + 2\beta s_4)\sinh(\beta-\alpha)], \tag{A.9}$$

$$B = 6\Delta\gamma[60\alpha\beta^2 - (2\beta^4 s_5 - 3\alpha^2 s_6 + \alpha\beta s_0 s_7)\cosh(\beta-\alpha) + (2\beta^3 s_8 + \alpha s_0 s_6 - 3\alpha^2\beta s_7)\sinh(\beta-\alpha)], \tag{A.10}$$

$$C = \Delta\gamma[45\beta^5 s_{10} + (9\alpha^2\beta^2 s_9 - \alpha s_0 s_{10} - 2\beta^3 s_{11})\cosh(\beta-\alpha) + 3(\alpha\beta^2 s_0 s_9 - 9\alpha^2 s_{10} + 6 s_{12})\sinh(\beta-\alpha)], \tag{A.11}$$

$$D = 3\Delta\gamma\{6\alpha\beta^3 + [2\beta^4(s_{13}-6) - 3\alpha^2 s_{14} + \alpha\beta s_0 s_{13}]\times\cosh(\beta-\alpha) + [2\beta^3(s_{14}+6) + 3\alpha^2\beta s_{13} - \alpha s_0 s_{14}]\times\sinh(\beta-\alpha)\}, \tag{A.12}$$

$$E = 12\Delta\alpha\gamma[\beta^3 s_{15} - (3\alpha\beta s_{16} - s_0 s_{17})\cosh(\beta-\alpha) + (3\alpha s_{17} - \beta s_0 s_{16})\sinh(\beta-\alpha)], \tag{A.13}$$

$$F = 6\Delta\gamma[\alpha s_{18} + 2\beta s_{16}\cosh(\beta-\alpha) - 2 s_{17}\sinh(\beta-\alpha)], \tag{A.14}$$

$$G = 6\Delta\gamma[3\alpha^2 s_{18}\cosh\alpha + 6\alpha\beta s_{16}\cosh\beta - (2\beta^3 s_{15} + \alpha s_0 s_{18})\sinh\alpha - 6\alpha s_{17}\sinh\beta], \tag{A.15}$$

$$H = 6\Delta\gamma[(2\beta^3 s_{15} + \alpha s_0 s_{18})\cosh\alpha + 6\alpha s_{17}\cosh\beta - \alpha s_{18}\sinh\alpha - 2\beta s_{16}\sinh\beta], \tag{A.16}$$

where

$$\Delta = [12\alpha s_{22} + (9\alpha^2 s_{19} - \alpha s_0 s_{20} - 2\beta s_{21})\cosh(\beta-\alpha) + 3(2 s_{23} + \alpha s_0 s_{19} - \alpha^2 s_{20})\sinh(\beta-\alpha)]^{-1}, \tag{A.17}$$

$$s_0 = \alpha^2 + 3, \qquad s_1 = \beta^3 + 45\beta - \gamma^3,$$
$$s_2 = 2\beta^3 + 15\beta - \gamma^3,$$
$$s_3 = \beta^5 + 15\beta^3 - \beta^2\gamma^3 - 6\gamma^3,$$
$$s_4 = 2\beta^5 + 5\beta^3 - \beta^2\gamma^3 - 2\gamma^2,$$
$$s_5 = \beta^5 + 15\beta^3 - \gamma^5, \qquad s_6 = 6\beta^5 + 45\beta^3 - \gamma^5,$$
$$s_7 = \beta^5 + 45\beta^3 - \gamma^5, \qquad s_8 = 6\beta^5 + 15\beta^3 - \gamma^5,$$
$$s_9 = \gamma^2 - 15\beta^2, \qquad s_{10} = 4\gamma^5 - 9\beta^5 + 5\beta^3(\gamma^2 - 36),$$
$$s_{11} = 4\gamma^5 - 9\beta^5 + 5\beta^3(\gamma^2 - 24) + 30\beta(\gamma^2 - 9),$$
$$s_{12} = 2\gamma^5 - 15\beta^7 + \beta^5(5\gamma^2 - 72) + 10\beta^3(\gamma^2 - 9),$$
$$s_{13} = \gamma^2 - \beta^2, \qquad s_{14} = 3\beta^2 - \gamma^2,$$
$$s_{15} = 3\beta^5 - 5\beta^3\gamma^2 + 2\gamma^5,$$
$$s_{16} = 6\beta^5 - 5\beta^3(\gamma^2 - 9) - \gamma^5,$$
$$s_{17} = 21\beta^5 - 5\beta^3(\gamma^2 - 9) - \gamma^5,$$
$$s_{18} = 3\beta^5 - 5\beta^3(\gamma^2 - 18) + 2\gamma^5,$$

$$s_{19} = 8\beta^5 - 15\beta^4\gamma + 60\beta^3 + 10\beta^2\gamma^3 - 3\gamma^5,$$

$$s_{20} = 4\beta^6 - 9\beta^5\gamma + 180\beta^4 + 10\beta^3\gamma(\gamma^2 - 18) - 9\beta\gamma^5 + 4\gamma^6,$$

$$s_{21} = 4\beta^8 - 9\beta^7\gamma + 60\beta^6 + 2\beta^5\gamma(5\gamma^2 - 63) - 3\beta^3\gamma(3\gamma^4 - 20\gamma^2 + 90) + 4\beta^2\gamma^2 + 6\gamma^6,$$

$$s_{22} = 20\beta^6 - 27\beta^5\gamma + 5\beta^3\gamma(\gamma^2 - 18) + 2\gamma^6,$$

$$s_{23} = 8\beta^8 - 15\beta^7\gamma + 20\beta^6 + 2\beta^5\gamma(5\gamma^2 - 36) - \beta^3(3\gamma^4 - 20\gamma^2 + 90) + 2\gamma^6. \quad (A.18)$$

Application of the boundary conditions (21)–(24) to the general solution (25) for the rotation of a composite sphere in a concentric spherical cavity yields

$$C'(\alpha^{-2}\cosh\alpha - \alpha^{-1}\sinh\alpha) + D'(\alpha^{-2}\sinh\alpha - \alpha^{-1}\cosh\alpha) = 0, \quad (A.19)$$

$$A'\beta^{-2} + B'\beta = C'(\beta^{-2}\cosh\beta - \beta^{-1}\sinh\beta) + D'(\beta^{-2}\sinh\beta - \beta^{-1}\cosh\beta), \quad (A.20)$$

$$3A' = [C'(\beta^2 + 3) - 3\beta D']\cosh\beta - [3\beta C' - D'(\beta^2 + 3)]\sinh\beta, \quad (A.21)$$

$$A'\gamma^{-2} + B'\gamma = -\gamma. \quad (A.22)$$

The four unknown constants appearing in the above equations can easily be solved, with the result

$$A' = \Delta'\beta\gamma^3 R', \quad (A.23)$$

$$B' = \Delta'\gamma^3 S', \quad (A.24)$$

$$C' = 3\Delta'\beta\gamma^3(\alpha\cosh\alpha - \sinh\alpha), \quad (A.25)$$

$$D' = 3\Delta'\beta\gamma^3(\cosh\alpha - \alpha\sinh\alpha), \quad (A.26)$$

where

$$\Delta' = \{\gamma^3[\cosh(\beta - \alpha) + \sinh(\beta - \alpha)] - \beta R'\}^{-1} \quad (A.27)$$

and R' and S' are defined by Eq. (28).

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