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On obtaining machine tool stiffness by CAE techniques

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Abstract

In this paper, a single module method and a newly developed hybrid modeling method for analyzing the stiffness of machine tools are introduced in detail. Techniques include building suitable finite element models, determining equivalent loads, simulating the interface between two modules, considering boundary constraints, and interpreting results. By taking a detailed finite element mesh for one of the five modules (the headstock, the column, the table, the saddle and the bed), together with simplified meshes for the other four modules, a hybrid finite element model is assembled. The elastic modulli of the four simplified meshes are kept several orders higher than that of the detailed one. Therefore, the calculated stiffness of the hybrid model is essentially the stiffness of the softer module with the detailed mesh. The stiffness of the five modules can be obtained one after another in the same manner. By supporting the hybrid model only at the middle of the short edge on the bottom surface of the bed, the machine tool can be properly constrained, and its stiffness can be estimated correctly. The controversial issue as to how to simulate properly the boundary condition of the casters under the bed will not occur in this method. A cumbersome procedure to transform the external loads into the equivalent forces as required in SMM is also avoided. There is no local effect due to unevenly distributed nodal forces. It is shown that the hybrid modeling method is better than the single module method in accuracy and efficiency. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Machine tool; Single module method; Hybrid modeling method; Stiffness analysis; Finite element method

1. Introduction

Many researchers have studied the stiffness of machine tools by experimental, analytical or numerical methods in the past decades [1–9]. However, reports which include detailed technical

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know-how in analyzing the stiffness of machine tools by the applied numerical method, namely, the computer aided engineering (CAE) method, are difficult to find in literature.

This report is believed to pioneer the study of technical detail in the CAE method for analyzing the stiffness of machine tools. It investigates two approaches: a Single Module Method (SMM) and a newly developed Hybrid Modeling Method (HMM). The techniques include building suitable finite element models, determining nodal forces, transforming and applying equivalent loads, simulating the interface between two modules, considering boundary constraints, and interpreting results.

The advantage of SMM lies in that only one module needs to be meshed, and hence less effort needs to be put into preparing a finite element model. However, this method has some drawbacks. Firstly, it is necessary to transform the loads applied on the cutting tool, as well as those on the work-piece, into the equivalent forces. They are then applied onto the nodes within the interfacial areas between the two connecting modules. If the mesh is made using an auto-meshing process, densities of the nodes and shapes of the tetrahedral elements are difficult to control. Local deformation and over-stressing may happen at locations with higher nodal density. Secondly, using SMM to analyze the stiffness, the boundary condition of each module is unique. Thus, the method leads to a controversy as to whether boundary constraints can be properly simulated. Thirdly, there is no common base to compare the stiffness of different modules since their boundary constraints are different. Hence, it is not possible to know the influence of the stiffness of each module on the entire strength of the machine tool.

HMM is processed by taking a detailed finite element mesh for one of the five modules (the headstock, the column, the table, the saddle and the bed), together with simplified meshes for the other four modules, to assemble a hybrid model of the machine tool. By supporting the hybrid model only at the middle of the short edge on the bottom surface of the bed, the machine tool can be properly constrained, and its stiffness can be estimated correctly. The controversial issue on the constraint condition of the casters under the bed will not occur in this method. Using HMM, external loading only needs to be applied on the cutting tool and the work-piece. A cumbersome procedure to transform the external loads into the equivalent forces as required in SMM is avoided. There is no local effect due to unevenly distributed nodal forces. Since each hybrid model is constrained at the same point, the stiffness of each module is comparable. The weaker modules in the machine tool can be identified and modified.

The report is organized as follows: In Section 2, a machine tool for analysis is described. In Section 3, the SMM for stiffness analysis is reviewed and illustrated with two examples. In Section 4, the HMM is introduced. In Section 5, results of both methods are presented and compared. Conclusions are given in Section 6.

2. The machine tool structure and the coordinate system

As shown in Fig. 1, the structure of a vertical machining center contains five modules: the headstock, the column, the table, the saddle, and the bed unit. A spindle set which is not shown in the figure is mounted on the flange of the headstock. On analyzing the structural strength of a machine tool, the five modules are primary ones to be considered. The deformation due to self-weight is not counted here.

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Fig. 1. The five major modules and the coordinate system of a vertical machining center.

The coordinate system is defined by the right hand rule. As shown in Fig. 1, the +X-axis is defined pointing from the left to the right, the +Z-axis from the bottom to the top, and the +Y-axis from the foreground to the background. Throughout the paper, this sign convention is followed to position boundary constraints, to direct forces and displacements, and to define stiffness.

3. Stiffness analysis by the single module method (SMM)

Due to the complexity of each module, its finite element mesh is built using the auto-meshing process to save modeling time. Unit forces applied at the tool tip and the work-piece are transformed into equivalent forces acting on each single module. Coupling components which connect two modules, such as the linear guide blocks or the bolted joints, are considered as constraints in SMM, which is illustrated by the following two examples.

3.1. Stiffness analysis of the column

The finite element model of the column is established using the auto-mesh process with two types of elements. The regularly shaped reinforcing ribs and walls of the column are modeled by shell elements, and the irregularly shaped outer structure is modeled by tetrahedral elements. There are two boundaries where the column interfaces with other modules: one at the bottom surface (jointed to the bed by several bolts) and the other at the front surface (connected to the headstock by four linear guide blocks). At the bottom surface, the nodes located at the positions of the holes for bolting are fully constrained to simulate the fixed boundary conditions. Loading is transmitted from the headstock to the column through the four linear guide blocks, shown as rectangular areas with numbers in Fig. 2. On analyzing the stiffness of the column, it is necessary to transform external forces originally applied on the tool tip into equivalent forces and then apply them onto the four block areas. How the equivalent forces are obtained is described below by vector analysis:



Fig. 2. Front view of the column shows positions of four linear guide blocks, which are the interfaces between the head and the column.

3.1.1. (1)

When a unit cutting force is applied in the +X direction on the tool tip, it generates a force, + $F_x \vec{i}$, and a moment, + $M_z \vec{k}$, with respect to the column. The latter results from a cross product of the distance vector $-R_y \vec{j}$ (the horizontal distance from the column to the tool tip, see Fig. 3) and the force + $F_x \vec{i}$. The moment + $M_z \vec{k}$ can be decomposed into a pair of couples, $\pm (M_z/D_c) \vec{j}$, where D_c is the distance between the bearing blocks on the front surface of the column, as shown in Fig. 2. The equivalent forces are then applied to the interfacial areas (areas 1–4 in Fig. 2) between the head and the column. Areas 1 and 3 are subjected to a tangential force, + $(F_x/4)\vec{i}$, and a compressive force, $-(F_x R_y/2D_c)\vec{j}$, while areas 2 and 4 are subjected to a tangential force, + $(F_x/4)\vec{i}$, and a tensile force, + $(F_x R_y/2D_c)\vec{j}$. These forces are obtained as follows:

$$-R_{y}\vec{j}\times F_{x}\vec{i}=M_{z}\vec{k}$$
⁽¹⁾



Fig. 3. Side view of the head and the column.

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$$D_c \vec{\iota} \times (M_z/D_c) \vec{j} = M_z \vec{k}$$
⁽²⁾

Substituting the magnitude of M_z from Eq. (1) into Eq. (2) obtains the magnitude of the couple in terms of the force F_x and the distance R_y as:

$$D_c \vec{i} \times (M_z/D_c) \vec{j} = [D_c \vec{i} \times 2(F_x R_y/2D_c) \vec{j}]$$
(3)

3.1.2.(2)

When a unit cutting force is applied in the +Y direction on the tool tip, it generates a force, + $F_{y}\vec{j}$, and a moment, + $M_x\vec{i}$, with respect to the column. The latter results from a cross product of the distance vector $-R_z\vec{k}$ (the vertical distance from the center of the upper and the lower bearing blocks to the tool tip, see Fig. 3) and the unit force + $F_y\vec{j}$. The moment + $M_x\vec{i}$ can be decomposed into a pair of couples, $\pm(M_x/L_c)\vec{j}$, where L_c is the distance between the centerlines of the upper and the lower bearing blocks, as shown in Fig. 2. The equivalent forces are applied to the interfacial areas (areas 1–4 in Fig. 2) between the head and the column. Areas 1 and 2 are subjected to a tensile force, $+(F_y/4)\vec{j}$, and a compressive force, $-(F_yR_z/2L_c)\vec{j}$; while areas 3 and 4 are subjected to two tensile forces, $+(F_y/4)\vec{j}$ and $+(F_yR_z/2L_c)\vec{j}$. These forces are obtained as follows:

$$-R_z \vec{k} \times F_y \vec{j} = M_x \vec{i}$$
(4)

$$-L_c \vec{k} \times (M_x/L_c) \vec{j} = M_x \vec{i}$$
(5)

Substituting the magnitude of M_x from Eq. (4) into Eq. (5) obtains the magnitude of the couple in terms of the force F_y and the distance R_z as:

$$-L_c\vec{k} \times (M_x/L_c)\vec{j} = -[L_c\vec{k} \times 2(F_vR_z/2L_c)\vec{j}]$$
(6)

3.1.3. (3)

When a unit force is applied in the +Z direction on the tool tip, a force, $+F_z\vec{k}$, is generated on the lower surface of the support of the Z-axis feeding device, and a moment, $-M_x\vec{i}$, on the bearing block areas (areas 1–4, see Fig. 2). The latter results from a cross product of the distance vector $-R_y\vec{j}$ and the unit force $+F_z\vec{k}$. The moment $-M_x\vec{i}$ can be decomposed into a pair of couples, $\pm(M_x/L_c)\vec{j}$. Areas 1 and 2 are subjected to a tensile force, $+(F_zR_y/2L_c)\vec{j}$, while areas 3 and 4 are subjected to a compressive force, $-(F_zR_y/2L_c)\vec{j}$.

After the displacements of the loaded structure are calculated, the stiffness of the column around the interfacial areas can be obtained by taking the inverse of the averaged nodal displacements within the four block areas.



Fig. 4. Top view of the bed shows C_x and C_y . C_x is the distance between the tool tip and the middle of the interfacial areas A2A1 and A4A3; C_y is the distance between the tool tip and the middle of area A2A1.

3.2. Stiffness analysis of the bed

The mesh of the bed is established in the same manner as that the column is built. In the bed module, there are three boundaries, which contact the column, the saddle, and the ground, respectively. Casters, which support the machine tool, are the interface between the ground and the bed. Thus, nodes need to be created to simulate these casters. However, to fully constrain or to release some degrees of freedom of these boundary nodes is a controversy. It depends on the practical condition. In this study, boundary nodes at the locations of casters are assumed fixed.

The unit cutting force applied on the tool tip generates a force and a moment, which act on the interfacial areas (shown in Fig. 4) between the column and the bed. The areas representing the four linear guide blocks of the *Y*-axis feeding system under the saddle (shown in Fig. 5) are projected on the bed and are outlined in Fig. 6. The reactive cutting force applied on the workpiece generates a force and a moment also, which act on these four block areas. The loading on the bed is analyzed as follows:

3.2.1. (1)

When a unit force is applied in the +X direction on the tool tip, it generates a force, $+F_x \vec{i}$, and a moment, $+M_z \vec{k}$, with respect to the bed. The moment results from a cross product of the distance



Fig. 5. Side view of the bed shows the distance B_z between the top surface of the work-piece and that of the bed, and the distance B_x between the centerline of the work-piece and that of the two linear guides on the bed.



Fig. 6. Top view of the bed shows positions of the four linear guide blocks, which are the interfaces between the saddle and the bed.

vector $-C_y \vec{j}$ (the horizontal distance between the tool tip and the middle of area A2A1, see Fig. 4) and the unit force $+F_x \vec{i}$. The moment $+M_z \vec{k}$ can be decomposed into a pair of couples, $\pm (M_z/B_s)\vec{k}$, where B_s is the distance between the two rows of bolts (shown in Fig. 4). These equivalent forces are then applied on the interfacial areas (areas A1-A4 in Fig. 4) between the column and the bed. Areas A1 and A2 are subjected to a tangential force, $+(F_x/4)\vec{i}$, and a compressive force, $-(C_yF_x/2B_s)\vec{j}$; while areas A3 and A4 are subjected to a tangential force, $+(F_x/4)\vec{i}$, and a tensile force, $+(C_yF_x/2B_s)\vec{j}$.

When a unit force is applied in the -X direction on the top of the work-piece, it generates a force, $-F_x \vec{i}$, and a moment, $-M_y \vec{j}$, with respect to the bed. The moment results from a cross product of the distance vector $+B_z \vec{k}$ (the vertical distance between the top surface of the bed to that of the work-piece, see Fig. 5) and the unit force $-F_x \vec{i}$. The moment $-M_y \vec{j}$ can be decomposed into a pair of couples, $\pm (M_y/D_b)\vec{j}$, where D_b is the distance between the two centerlines of the linear guide blocks (shown in Fig. 6). These equivalent forces are then applied on the interfacial areas (areas 1–4 in Fig. 6) between the saddle and the bed. Areas 1 and 3 are both subjected to a tangential force, $-(F_x/4)\vec{i}$, and a compressive force, $-(B_zF_x/2D_b)\vec{k}$; while areas 2 and 4 are both subjected to a tangential force, $-(F_x/4)\vec{i}$, and a tensile force, $+(B_zF_x/2D_b)\vec{k}$.

3.2.2.(2)

When a unit force is applied in the +Y direction on the tool tip, it generates a force, $+F_y\vec{j}$, and a moment, $-M_z\vec{k}$, with respect to the bed. The moment results from a cross product of the distance vector $-C_x\vec{i}$ (the vertical distance between the tool tip and the middle of the two interfacial areas A2A1 and A4A3, see Fig. 4) and the unit force $+F_y\vec{j}$. The moment $-M_z\vec{k}$ can be decomposed into a pair of couples, $\pm(M_z/B_s)\vec{k}$. These equivalent forces are then applied on the interfacial areas (areas A1–A4 in Fig. 4) between the column and the bed. Areas A1 and A2 are both subjected to a tangential force, $+(F_y/4)\vec{j}$, and a tensile force, $+(C_xF_y/2B_s)\vec{j}$; while areas A3 and A4 are both subjected to a tangential force, $+(F_y/4)\vec{j}$, and a compressive force, $-(C_xF_y/2B_s)\vec{j}$.

When a unit force is applied in the -Y direction on the top of the work-piece, it generates a force, $-F_{y}\vec{j}$, distributed on the surface of the support of the Y-axis feeding device (see Fig. 6),

and a moment, $+M_x\vec{i}$, with respect to the bed. The moment results from a cross product of the distance vector $+B_z\vec{k}$ and the unit force $-F_y\vec{j}$. The moment $+M_x\vec{i}$ can be decomposed into a pair of couples, $\pm(M_x/L_b)\vec{i}$, where L_b is the distance between the two blocks of the same linear guide (shown in Fig. 6). These equivalent forces are applied on the interfacial areas (areas 1–4 in Fig. 6) between the saddle and the bed. Areas 1 and 2 are subjected to a tensile force, $+(B_zF_y/2L_b)\vec{k}$; while areas 3 and 4 are subjected to a compressive force, $-(B_zF_y/2L_b)\vec{k}$.

3.2.3. (3)

When a unit force is applied in the +Z direction on the tool tip, it generates a unit force, + $F_z\vec{k}$, and a moment, $-M_x\vec{i}$, with respect to the bed. The latter results from a cross product of the distance vector $-C_y\vec{j}$ and the unit force + $F_z\vec{k}$. The moment $-M_x\vec{i}$ can be decomposed into a pair of couples, $\pm(M_x/B_t)\vec{i}$. These equivalent forces are applied on the interfacial areas (areas A1-A4 in Fig. 4) between the column and the bed. Areas A1 and A3 are both subjected to a tensile force, $+(F_z/4)\vec{k}$, and a compressive force, $-(C_yF_z/2B_t)\vec{k}$; while areas A2 and A4 are both subjected to two tensile forces, $+(F_z/4)\vec{k}$ and $+(C_yF_z/2B_t)\vec{k}$.

When a unit force is applied in the -Z direction on the top of the work-piece, it generates a force, $-F_z \vec{k}$, and a moment, $-M_y \vec{j}$, with respect to the bed. The moment results from a cross product of the distance vector $-B_x \vec{i}$ (the horizontal distance between the centerline of the work-piece and that of the two linear guides on the bed, see Fig. 5) and the unit force $-F_z \vec{k}$. The moment $-M_y \vec{j}$ can be decomposed into a pair of couples, $\pm (M_y/D_b)\vec{j}$, where D_b is the distance between the two centre lines of the linear guide blocks (Fig. 6). These equivalent forces are applied on the interfacial areas (areas 1–4 in Fig. 6) between the saddle and the bed. Areas 1 and 3 are both subjected to two compressive forces, $-(F_z/4)\vec{k}$ and $-(B_xF_z/2D_b)\vec{k}$; while areas 2 and 4 are both subjected to a compressive force, $-(F_z/4)\vec{k}$, and a tensile force, $+(B_xF_z/2D_b)\vec{k}$.

After the displacements of the bed are calculated, the stiffness of the bed around the interfacial areas (areas A1–A4) between the column and the bed can be obtained by taking the inverse of the averaged nodal displacements within the areas. Similarly, the stiffness of the bed around the interfacial areas (areas 1–4) between the saddle and the bed can be obtained.

4. Stiffness analysis by the hybrid modeling method (HMM)

4.1. Building a hybrid model for a machine tool

4.1.1. Step 1

The simplified finite element models of the headstock, the column, the table, the saddle, and the bed, are built by hexahedral elements. Interfacial areas and joints between two modules are modeled. The orientations and origin of the local coordinate system of the bed are defined as those of the global coordinate system of the whole machine tool, as shown in Fig. 7.

4.1.2. Step 2

The detailed finite element meshes of the five modules are established using the auto-mesh process and two types of elements. The regularly shaped reinforcing ribs and walls of the head,



Fig. 7. The hybrid model with detailed mesh of the column and the simplified others.

the column, the table, the saddle and the bed set are modeled by shell elements, while the irregularly shaped outer structures are modeled by tetrahedral elements. A cylinder is built to simulate the spindle. Nodes and elements are built to simulate the ball screw supports of the Y and the Zaxis feeding devices on the bed and the column, respectively. Similarly, the U-shaped mounts of the X and the Y-axis feeding devices, which are under the table and the saddle, respectively, are also modeled.

4.1.3. Step 3

One of the detailed meshes of the five modules which are to be investigated is combined with the other four simplified ones to become a hybrid model. Other hybrid models can be established in the same way. Examples are shown in Figs. 7 and 8. The local coordinates of each module need to be translated and rotated relative to the global coordinates to keep the gaps between two modules equal to the height of the linear guide. True elastic modulus is assigned to the detailed mesh, which is regarded as an elastic body; while greater (100 times higher) elastic modulii are for the simplified ones, which are regarded as rigid bodies. A work-piece is built right at the center of the top surface of the table. The tool tip is kept in contact with the top surface of the work-piece.

4.2. Setting up the boundary conditions

4.2.1. Step 1

If the interfaces between the headstock and the column are linear guides, nodes are established at the locations of the holes on the four linear guide blocks, which are mounted on the back of the headstock, and at the corresponding locations on the linear guides on the column. The normal (Y) and the tangential (X) degrees of freedom of the corresponding nodes on the linear guides and the blocks are coupled. The same skill is implemented to couple the Y and the Z degrees of

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Fig. 8. The hybrid model with detailed mesh of the bed and the simplified others.

freedom of the corresponding nodes on the table and the saddle, and the X and the Z degrees of freedom of the corresponding nodes on the saddle and the bed, for each individual interface.

4.2.2. Step 2

The Z-axis feeding bolt screw is located between the headstock and the column. For simplicity, X, Y and Z degrees of freedom of the nodes on the U-shaped mount of the Z-axis bolt screw housing on the back of the headstock, and the support of the Z-axis feeding device, are coupled, instead of using beam elements to simulate the feeding bolt screw. The same skill is applied to treat the X-axis and the Y-axis feeding devices.

4.2.3. Step 3

All degrees of freedom of the corresponding nodes, which are at the locations of the bolt-holes on the column and the bed, are coupled. As shown in Fig. 7, the origin of the local coordinate system on the bed, which is also the origin of the global coordinate system, is chosen as the constraint point. It can be viewed as if the finite element model of the overall structure were supported at this point.

4.3. Applying the loading

On calculating the nodal displacements of the hybrid model, a unit force is applied on the tool tip in the +X, +Y and +Z direction, respectively. Meanwhile, a reactive unit force is applied on the top of the work-piece in the -X, -Y and -Z direction, respectively. The stiffness of the machine tool varies as relative positions among modules change. Different stiffnesses of machine tool can be obtained by moving the headstock, the work-piece, the table, and the saddle away from their original positions as shown in Fig. 1.

4.4. Interpreting the results

The relative stiffnesses, K_x , K_y and K_z (in kgf/µm) are defined as reciprocals of the nodal displacements of each module. Based on the relative displacements between the tool tip and the cutting point on the top of the work-piece, the contribution of each module to the stiffness of an entire machine can be estimated. Since the elastic moduli of the simplified modules are kept two orders higher than that of the detailed one, the calculated stiffness of the hybrid model is essentially the stiffness of the softer module with detailed mesh. The stiffness of all five modules can be obtained in the same manner.

5. Numerical results and discussion

Based upon a real VMC similar to the one shown in Fig. 1, the static stiffness of each module is analyzed by both SMM and HMM. The stiffness of each module is listed in Tables 1 and 2 and Table 3, respectively. In these tables, stiffness K_{ij} is defined as the reciprocal of the displacement in direction *i* as the structure of concern is subjected to a unit force in direction j(i, j=x, y, z). The stiffness of the column with headstock at two stroke positions, the bottom (Z=0) and the top (Z=505) positions of the Z-axis bolt screw, is investigated. The stiffness of the bed at two interfacial connections, one with the column (CB) and the other with the saddle (SB), is also studied. The ratios listed in the tables are the quotients of K_{ij} (SMM) divided by K_{ij} (HMM). Since the displacement in the direction of loading is larger than that in the other two lateral directions, the stiffness of the module in the direction of loading is the smallest. From Tables 1, 2 and 3, it can be seen that the stiffness of the headstock, the saddle, and the table calculated by SMM is smaller than that calculated by HMM. Differences in stiffness obtained by these two methods vary from -13% to -17% for the headstock, -10% to -13% for the saddle, and -7% to -10% for the table. On the other hand, the stiffness of the column and the bed calculated by SMM is larger

Table 1

_	_	_			_		_	_
K_{xx}	K_{xx}	SMM/	K_{yx}	K_{yx}	SMM/	K_{zx}	K_{zx}	SMM/
SMM	HMM	Ratio	SMM	HMM	Ratio	SMM	HMM	Ratio
11	13	0.85	-324	-372	0.87	330	384	0.86
13.2	12.2	1.08	-257	-238	1.08	-2818	-2635	1.07
8.3	7.5	1.11	-155	-142	1.09	2550	2432	1.09
18.2	16.2	1.12	-466	-427	1.09	714	655	1.09
-17.1	-15.1	1.13	14580	13120	1.11	-2063	-1840	1.12
-78	-88	0.89	2575	2846	0.90	1440	1636	0.88
-55	-60	0.91	1369	1488	0.92	310	336	0.92
		$\begin{array}{cccc} K_{xx} & K_{xx} \\ kgf/\mu m \\ SMM & HMM \\ \hline 11 & 13 \\ 13.2 & 12.2 \\ 8.3 & 7.5 \\ 18.2 & 16.2 \\ -17.1 & -15.1 \\ -78 & -88 \\ -55 & -60 \\ \hline \end{array}$	$\begin{array}{ccccccc} K_{xx} & K_{xx} & \text{SMM} / \\ \text{kgf/} \mu m & \text{kgf/} \mu m & \text{HMM} \\ \text{SMM} & \text{HMM} & \text{Ratio} \\ \hline 11 & 13 & 0.85 \\ 13.2 & 12.2 & 1.08 \\ 8.3 & 7.5 & 1.11 \\ 18.2 & 16.2 & 1.12 \\ -17.1 & -15.1 & 1.13 \\ -78 & -88 & 0.89 \\ -55 & -60 & 0.91 \\ \hline \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				

The stiffness of each module obtained using SMM and HMM. A unit force is applied at the tool tip in the +X direction and at the work-piece in the -X direction, respectively

^a Z=0: The headstock is at the lower end of the Z-axis bolt screw; Z=505: The headstock is at the upper end of the Z-axis bolt screw.

^b cb: The interface between the column and the bed; sb: The interface between the saddle and the bed.

Stiffness Modules	K _{xy} kgf/μm SMM	K _{xy} kgf/μm HMM	SMM/ HMM Ratio	K _{yy} kgf/μm SMM	K _{yy} kgf/μm HMM	SMM/ HMM Ratio	<i>K_{zy}</i> kgf/μm SMM	K _{zy} kgf/μm HMM	SMM/ HMM Ratio
Head stock	-405	-490	0.83	26	31	0.84	-40	-48	0.83
Column $(Z=0)^{a}$	-1055	-984	1.07	85	80	1.08	345	325	1.06
(Z=505)	-607	-558	1.09	33	30	1.10	56	52	1.08
Bed cb ^b	-4854	-4450	1.09	44	39	1.13	39	35	1.11
sb	8352	7520	1.11	-57	-50	1.14	960	854	1.12
Saddle	5642	6365	0.87	-22	-25	0.88	580	656	0.88
Table	2463	2700	0.91	-81	-90	0.90	-1236	-1340	0.92

The stiffness of each module obtained using SMM and HMM. A unit force is applied at the tool tip in the +Y direction and at the work-piece in the -Y direction, respectively

^a Z=0: The headstock is at the lower end of the Z-axis bolt screw; Z=505: The headstock is at the upper end of the Z-axis bolt screw.

^b cb: The interface between the column and the bed; sb: The interface between the saddle and the bed.

Table 3

Table 2

The stiffness of each module obtained using SMM and HMM. A unit force is applied at the tool tip in the +Z direction and at the work-piece in the -Z direction, respectively

Stiffness Modules	<i>K_{xz}</i> kgf/μm SMM	<i>K_{xz}</i> kgf/μm HMM	SMM/ HMM Ratio	K _{yz} kgf/μm SMM	K _{yz} kgf/μm HMM	SMM/ HMM Ratio	<i>K_{zz}</i> kgf/μm SMM	K _{zz} kgf/μm HMM	SMM/ HMM Ratio
Head stock	346	403	0.86	-45	-53	0.85	24	29	0.83
Column (Z=0) ^a	-3378	-3210	1.06	1428	1360	1.06	17.3	16.1	1.07
(Z=505)	-251	-234	1.07	72	66	1.09	14.4	13.2	1.09
Bed cb ^b	2940	2648	1.11	31	28	1.10	19	17	1.12
sb	-20000	-17700	1.13	-3024	-2702	1.12	-32	-28	1.14
Saddle	-9715	-11020	0.88	598	662	0.90	-34	-38	0.89
Table	1474	1600	0.92	-1265	-1362	0.93	-40	-44	0.91

^a Z=0: The headstock is at the lower end of the Z-axis bolt screw; Z=505: The headstock is at the upper end of the Z-axis bolt screw.

^b cb: The interface between the column and the bed; sb: The interface between the saddle and the bed.

than that calculated by HMM. Differences vary from +6% to +11% for the column and from +9% to +14% for the bed. Fig. 9 illustrates the stiffness K_{xx} , K_{yy} and K_{zz} calculated by HMM.

On using SMM, the normal and lateral rigidities of the linear guides on the headstock, the table and the saddle are simulated by spring elements. The structural deformation in these areas may be greater as they are under loading. Moreover, using the auto-meshing process and tetrahedral elements in building the finite element model, the distribution of nodes is less controllable, so are the shape of the elements. Unless an interfacing area stands out from the structural wall, it is not possible to smoothly mesh it with irregularly shaped tetrahedral elements. The four faces of each tetrahedral element are numbered in sequence by the code. However, the sequence may be different for elements next to each other. Thus, it is difficult to apply the equivalent forces as



Fig. 9. The stiffness of the five modules calculated using the hybrid modeling method (HMM).

surface pressure to the faces of the tetrahedral elements within the interfacial areas. The easier alternative is to apply the equivalent forces as nodal point loads. By dividing the equivalent forces by the number of nodes within the interfacial areas, the averaged nodal forces are obtained. If the latter is applied over the unevenly distributed nodes within the interfacial areas, the force distribution will be uneven also. It may cause a local effect in the loading areas. That means a higher local nodal density causes a higher loading, and thus larger local deformation. Due to these factors, the stiffness of the headstock, the table and the saddle calculated by SMM is smaller than that by HMM. However, when SMM is used to calculate the stiffness of the column and the bed, the interface between the column and the bed may be over-constrained, and their effective stiffness may increase. However, the local effect due to non-uniformly distributed loading is insignificant as compared to this over-constraining effect. Thus, the stiffness of the column and the bed calculated by SMM is higher.

In summary, when SMM is used to analyze the stiffness of a machine tool, there are always three issues: The first is how to apply properly the external loading at the right locations on each module. The second is how to model properly the boundary condition of each module, especially the bed. The third is how to find properly suitable local stiffnesses to represent the stiffness of each module. Using SMM, the boundary constraint of each module is unique. Thus, it is not meaningful to compare their stiffnesses since their bases are not the same.

Using HMM, the unit forces applied at the tool tip and the work-piece are transmitted as internal forces through the structure onto the nodes within the interfacial areas. There is no need to transform the external loading into the equivalent forces to be applied at interfacial areas on a single module. Local effect due to unevenly distributed nodal forces can be avoided. Each hybrid finite element model is constrained only at the middle of the bed, as shown in Fig. 7. The constrained condition of each module becomes the same. It avoids each module being over-constrained, and hence its stiffness is estimated properly. The calculated stiffnesses of different modules can be compared on the same base. Thus, the weaker module in the entire chain can be modified.

Between two modules, there exist flexible connecting joints, such as bolt screw feeding devices, linear guides, and bearings. In addition, accessories such as tool magazines and driving motors

are attached to the machine tool. It is not easy to simulate properly the structural characteristics of these flexible joints and accessories. An experiment was conducted to measure the displacements of the VMC by using load cells. The stiffness of the VMC in each coordinate direction, K_{xx} , K_{yy} and K_{zz} , is about 6.2 kgf/µm, 3.3 kgf/µm and 6.7 kgf/µm, respectively. The tested stiffness is even smaller than that of some single module listed in Tables 1–3. Because of the existence of the flexible joints, the stiffness of the entire machine tool becomes much lower.

From the above discussion, it appears that even though a detailed finite element model with five modules, spindle, all flexible joints and accessories is established to obtain the stiffness of the entire machine, it may demand extensive computer resources and yet, the results may not be better than HMM procedure. On the other hand, using HMM to obtain the stiffness of a hybrid model, which consists of one detailed module and other simplified ones, the results provide information not only related to the stiffness of each module, but also about its contribution to the overall stiffness of the machine.

6. Conclusion

A computer aided engineering technique to analyze the stiffness of a machine tool has been studied. Two methods, a simple module method and a hybrid modeling method, are presented. HMM appears to be superior to SMM in five merits:

- 1. The loads applied at the tool tip and the work-piece are transmitted as internal forces through the structure onto the nodes within the interfacial areas, which are between two connecting modules. There is no need to transform the external loading into the equivalent forces to be applied at the interfacial areas on a single module. Possible human errors occurring during transformation of loading is avoided.
- 2. The local deformation effect due to unevenly distributed nodal forces on the interface can be avoided. The accuracy may be improved.
- 3. The hybrid finite element model of the entire machine is much simplified and can be assembled by modules. It doesn't need large computer memory.
- 4. The five hybrid models are all constrained at the same point. It avoids over-estimating the stiffness of the module due to over-constraining the structure.
- 5. Since the boundary conditions of all hybrid models are the same, the stiffness of an individual module can be compared based on the same foundation. The contribution of each module to the stiffness of the entire machine is known. It helps designers to identify weaker parts in the chain and to strengthen them.

The primary contribution of this paper is to introduce a new method for obtaining the stiffness of machine tools and compare this new method with the conventional one. It is shown that HMM is better than SMM in accuracy and efficiency. If a database of stiffness of various machine tools can be established by using HMM as a standard method, it could be a valuable reference for the evaluation of characteristics of a machine tool. In a similar way, HMM can be used to obtain the stiffness of other complex machines.

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