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以線性矩陣不等式及模糊動態模式
設計非線性動態系統之 H^{∞} PI/PID 控制器(2/3)

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Abstract

An LMI approach for designing an H^{∞} fuzzy controller for nonlinear dynamic systems is presented. The entire operating range for a nonlinear system is partitioned into several regimes. A local linear model with parameter uncertainties is identified for each region. These local models are integrated as the norm-bounded Tagaki-Sugeno (T-S) fuzzy model. The output feedback H^{∞} fuzzy controller design procedures are then investigated based on the T-S fuzzy model, theirin the standard H^{∞} design problem is formulated as Linear Matrix Inequalities (LMIs). The necessary and suficient conditions for the existence of an H^{∞} controller is derived. One numerical example is supplied, demonstrating the effectiveness of the proposed design procedures.

1 Introduction

Most real industrial processes are nonlinear in nature. However, the controller design for most practical nonlinear processes is still based on local linear models and linear theories although the adequacy of the model might be questionable for a process operated far away from its original design conditions. Furthermore, the resulting controller is usually conservative if the design is based on a single local linear model because the model uncertainty is significant.

Recently, multiple local linear models have been applied frequently to describe process dynamics. The weighted sum of the multiple local linear models produces a global nonlinear model. A less conservative single controller or a network of local controllers can be designed based on reduced modeling errors when employing multiple local linear models. Among the various weighting methods for local linear models/controllers, the so-called Takagi-Sugeno (T-S) fuzzy model/controller approach [9] has been widely adoped. For example, [12] proposed a parallel distributed compensation (PDC) method where a local state-feedback controller is designed for each of the T-S linear models. The major problem in the previous related works is that no effective method was presented to determine a specific positive definite matrix for the quadratic stability of the overall system. This problem is reduced when a Linear Matrix Inequality (LMI) approach is found applicable as a computational tool for inferring the required symmetrical positive definite matrix in designing a fuzzy feedback controller [11, 14]. Two main drawbacks are still present. First, only state feedback controllers were addressed in most research works. Second, the state feedback controller should be predetermined before checking the closed-loop stability. Thus, [8, 10] presented fuzzy observer design to compensate the measurement problem of state feedback control. [7] addressed the design of output feedback controller. [6, 5, 4] used the H^{∞} control to guarantee the overall stability requirement. Without considering multiple models, [1] has applied an LMI approach for designing the H^{∞} output feedback control.

In this article, the LMI approach for the design of H^{∞} output feedback control studied by [1], is extended to the T-S fuzzy models. Two different design methods are investigated: in method (A) one single H^{∞} controller is designed for the whole T-S fuzzy rule set, in method (B) an H^{∞} fuzzy-logic controller is established based on the local models in the T-S fuzzy dynamic system. The single H^{∞} controller based on one local linear model is also included for comparison. One chemical process, a non-isothermal continuous-stirred-tank reactor (CSTR) with a first-order exothermic reaction in it, is illustrated to demonstrate the effectiveness of the proposed LMI-based H^{∞} fuzzy output feedback control design method for nonlinear dynamic processes.

2 Parameter Uncertainty Fuzzy Dynamic Model

Consider a nonlinear dynamic system whose operating space is partitioned into several regimes accrrding to premise variables $z(t) = \left[z_1(t), z_2(t), \cdots, z_p(t)\right]^T$. The i-th plant local linear model in the T-S fuzzy rule set is,

$$\begin{array}{llll} \textit{IF} & z_1(t) \ \textit{is} \ Z_1^{(i)} \ \textit{and} \ \cdots \ \textit{and} \ z_p(t) \ \textit{is} \ Z_p^{(i)} \\ \textit{THEN} & \dot{x}(t) & = & (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ & y(t) & = & C_i x(t) \qquad i = 1, \cdots, r \end{array} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^\ell$, denote the state, control input, and measured output, respectively; $Z_j^{(i)}, j = 1, \cdots, p$, is the fuzzy term for premise variable $z_j(t)$; A_i, B_i, and C_i designate the model parameters with appropriate dimensions; ΔA_i and ΔB_i are parametric uncertainty terms.

Suppose all elements in the uncertain parameters, (ΔA_i and ΔB_i), are bounded, then a norm-bounded uncertainty form can be reformulated, and can be expressed in a standard state-space formulation,

$$\begin{array}{llll} \textit{IF} & z_1(t) \ \textit{is} \ Z_1^{(i)} \ \textit{and} \ \cdots \ \textit{and} \ z_p(t) \ \textit{is} \ Z_p^{(i)} \\ \textit{THEN} & \dot{x}(t) & = & \overline{A}_i x(t) + \overline{B}_i u(t) + \overline{E}_i v(t) \\ & q(t) & = & F_{i1} x(t) + F_{i2} u(t) \\ & y(t) & = & C_i x(t) \qquad i = 1, \cdots, r \end{array} \tag{2}$$

where $q(t) \in \mathbb{R}^s$ is the fictitious output, and $v(t) \in \mathbb{R}^s$ is the square-integrable disturbance input vector.

The local models can be integrated into a global nonlinear model using a series of fuzzy inference procedures. By using the product as the fuzzy intersection, and the center-of-average method as the defuzzifer, the final output of the global fuzzy dynamic model, Eq.(1), becomes,

$$\begin{aligned} \dot{x}(t) &= \overline{A}(w)x(t) + \overline{B}(w)u(t) + \overline{E}(w)v(t) \\ q(t) &= F_1(w)x(t) + F_2(w)u(t) \\ y(t) &= C(w)x(t) \end{aligned} \tag{3}$$

where

$$\begin{aligned} \overline{A}(w) &= \sum_{i=1}^{r} w_i(z(t))\overline{A}_i & \overline{B}(w) &= \sum_{i=1}^{r} w_i(z(t))\overline{B}_i \\ C(w) &= \sum_{i=1}^{r} w_i(z(t))C_i & \overline{E}(w) &= \sum_{i=1}^{r} w_i(z(t))\overline{E}_i \\ F_1(w) &= \sum_{i=1}^{r} w_i(z(t))F_{i1} & F_2(w) &= \sum_{i=1}^{r} w_i(z(t))F_{i2} \end{aligned} \tag{4}$$

and

$$\begin{aligned} w_i(z(t)) &= \frac{h_i(z(t))}{\sum_{i=1}^{r} h_i(z(t))} \\ h_i(z(t)) &= \prod_{j=1}^{p} Z_j^{(i)}(z_j(t)) \quad \geq \quad 0 \end{aligned} \tag{5}$$

Here, $Z_j^{(i)}(z_j(t))$ denotes the grade of membership of the premise variable $z_j(t)$ for the fuzzy term $Z_j^{(i)}$ in the i-th plant local model; $h_i(z(t))$ is the firing level of the i-th plant model; $w_i(z(t))$ is the weighting. Notably, $\sum_{i=1}^{r} w_i = 1 \ \forall t$ and for all premise state.

3 H^∞ Fuzzy Controller Design

Suppose one H^∞ controller is designed for each local model in Eq.(2).

$$\begin{array}{llll} \textit{IF} & z_1(t) \ \textit{is} \ Z_1^{(\ell)} \ \textit{and} \ \cdots \ \textit{and} \ z_p(t) \ \textit{is} \ Z_p^{(\ell)} \\ \textit{THEN} & \dot{\hat{x}}(t) & = & \hat{A}_\ell \hat{x}(t) + \hat{B}_\ell y(t) \\ & u(t) & = & \hat{C}_\ell \hat{x}(t) + \hat{D}_\ell y(t) \qquad \ell = 1, \cdots, r \end{array} \tag{6}$$

For a given premise state, $z(t)$, the final output of the global fuzzy controller can be inferred as following:

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}(w)\hat{x}(t) + \hat{B}(w)y(t) \\ u(t) &= \hat{C}(w)\hat{x}(t) + \hat{D}(w)y(t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \hat{A}(w) &= \sum_{\ell=1}^{r} w_\ell \hat{A}_\ell & \hat{B}(w) &= \sum_{\ell=1}^{r} w_\ell \hat{B}_\ell \\ \hat{C}(w) &= \sum_{\ell=1}^{r} w_\ell \hat{C}_\ell & \hat{D}(w) &= \sum_{\ell=1}^{r} w_\ell \hat{D}_\ell \end{aligned} \tag{8}$$

Applying the fuzzy controller, Eq.(7), on the global fuzzy process model, Eq.(3), results in the overall closed-loop system,

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} &= \begin{bmatrix} \overline{A}(w) + \overline{B}(w)\hat{D}(w)C(w) & \overline{B}(w)\hat{C}(w) \\ \hat{B}(w)C(w) & \hat{A}(w) \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \overline{E}(w) \\ 0 \end{bmatrix} v(t) \\ q(t) &= \begin{bmatrix} F_1(w) + F_2(w)\hat{D}(w)C(w) & F_2(w)\hat{C}(w) \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \end{aligned} \tag{9}$$

Or in a more compact form,

$$\begin{aligned} \dot{\xi}(t) &= \mathbb{A}(w)\xi(t) + \mathbb{B}(w)v(t) \\ q(t) &= \mathbb{C}(w)\xi(t) \end{aligned} \tag{10}$$

The transfer function of Eq.(10) is,

$$T_{q,v}(s;w) = \mathbb{C}(w)\left(sI - \mathbb{A}(w)\right)^{-1}\mathbb{B}(w) \tag{11}$$

The H^∞ control design problem involves determining a set of controller parameters $\hat{A}_\ell$, $\hat{B}_\ell$, $\hat{C}_\ell$, and $\hat{D}_\ell, \ell = 1, \cdots, r$, such that the infinity norm of the closed-loop transfer function is limited, *i.e.*, $||T_{q,v}(s;w)||_\infty < \gamma$. The following theorem gives the necessary and sufficient conditions for the H^∞ controller design problem.

Theorem 1. *If there exist positive definite matrices R and S simultaneously satisfying the following LMI′s,*

$$\begin{aligned}
\begin{bmatrix} \overline{A}_i R + R\overline{A}_i^T & \overline{E}_i & RF_{i1}^T \\ \overline{E}_i^T & -\gamma I & 0 \\ F_{i1}R & 0 & -\gamma I \end{bmatrix} &< 0 \quad \forall\, i = 1, \cdots, r \\
\begin{bmatrix} \overline{A}_i^T S + S\overline{A}_i & S\overline{E}_i & F_{i1}^T \\ \overline{E}_i^T S & -\gamma I & 0 \\ F_{i1} & 0 & -\gamma I \end{bmatrix} &< 0 \quad \forall\, i = 1, \cdots, r \\
\textit{and} \qquad \begin{bmatrix} R & I \\ I & S \end{bmatrix} &\geq 0
\end{aligned} \tag{12}$$

then there exists the local controllers, Eq.(6), for the system of Eq.(3) or Eq.(2), such that the closed-loop system is quadratically stable.

Proof. See [2]. □

With the R, S matrices, one can compute two full-column-rank matrices M, N such that

$$MN^T = I - RS \tag{13}$$

The required positive definite matrix P can thus be obtained uniquely by solving the following relation,

$$\begin{bmatrix} S & I \\ N^T & 0 \end{bmatrix} = P \begin{bmatrix} I & R \\ 0 & M^T \end{bmatrix} \tag{14}$$

The fuzzy controller parameters Θ_ℓ can then be solved using the LMI equations,

$$\sum_{k=1}^{r} w_k \Pi_k + \left(\sum_{i=1}^{r} w_i \Phi_i\right)^T \left(\sum_{\ell=1}^{r} w_\ell \Theta_\ell\right)^T \left(\sum_{j=1}^{r} w_j \Psi_{Pj}\right) + \left(\sum_{j=1}^{r} w_j \Psi_{Pj}\right)^T \left(\sum_{\ell=1}^{r} w_\ell \Theta_\ell\right)\left(\sum_{i=1}^{r} w_i \Phi_i\right) < 0 \tag{15}$$

or

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} w_i w_j w_\ell \left(\Pi_{k=i\,\text{or}\,j\,\text{or}\,\ell} + \Phi_i^T \Theta_\ell^T \Psi_{Pj} + \Psi_{Pj}^T \Theta_\ell \Phi_i\right) < 0 \tag{16}$$

where

$$\begin{aligned}
\Theta_\ell &= \begin{bmatrix} \hat{A}_\ell & \hat{B}_\ell \\ \hat{C}_\ell & \hat{D}_\ell \end{bmatrix} \\
\Pi_k &= \begin{bmatrix} \mathcal{A}_k^T P + P\mathcal{A}_k & P\mathcal{E}_k & \mathcal{F}_{k1}^T \\ \mathcal{E}_k^T P & -\gamma I & 0 \\ \mathcal{F}_{k1} & 0 & -\gamma I \end{bmatrix} \qquad (k = i \textit{ or } j \textit{ or } \ell) \\
\Phi_i &= \begin{bmatrix} \mathcal{C}_i & 0 & 0 \end{bmatrix} \\
\Psi_{Pj} &= \begin{bmatrix} \mathcal{B}_j^T P & 0 & \mathcal{F}_{j2}^T \end{bmatrix}
\end{aligned}$$

Certainly, one can determine the fuzzy controller coefficients by solving a set of LMI′s. There are two conditions: the first case involves finding parameters for an H^∞ fuzzy controller, $\Theta_\ell, \ell = 1, \cdots, r$; and the second case involves determining parameters for a single H^∞ controller, Θ.

Case 1: fuzzy controller parameters $\Theta_\ell, \ell = 1, \cdots, r$
We can find the H^∞ fuzzy controller by selecting $k = \ell$ in Eq.(15) and then solving the following r sets of LMI′s sequentially.

$$\begin{aligned} \Pi_\ell + \Phi_i^T \Theta_\ell^T \Psi_{Pj} + \Psi_{Pj}^T \Theta_\ell \Phi_i < 0 \\ \forall\, i, j = 1, \cdots, r; \ell = 1, \cdots, r \end{aligned} \tag{17}$$

Case 2: single controller parameter Θ
We can find a single H^∞ controller by selecting $k = i$ (or $k = j$) in Eq.(15) and then solving the following LMI′s simultaneously. Notably, only one controller can be found in this case, *i.e.*, $\Theta_1 = \cdots = \Theta_r \equiv \Theta$. Thus we need to solve these LMI′s once.

$$\begin{aligned} \Pi_i + \Phi_i^T \Theta^T \Psi_{Pj} + \Psi_{Pj}^T \Theta \Phi_i < 0 \\ \forall\, i, j = 1, \cdots, r \end{aligned} \tag{18}$$

Notably, we consider the single H^∞ controller based on the sole local linear model as a special case of these two designs.

4 Illustrative Example

Here we use one example to illustrate the proposed output feedback H^∞ controller design method. Consider a continuous stirred-tank reactor (CSTR), where a first-order exothermic reaction is conducted with the following balance relations. equations [13],

$$\begin{aligned}
V\frac{dC_A(t)}{dt} &= q\left(C_{Af} - C_A\right) - Vk_0 C_A e^{-E/RT} \\
\rho C_p V \frac{dT(t)}{dt} &= \rho C_p q\left(T_f - T\right) + \\
&\quad \rho C_p(-\Delta H)k_0 C_A e^{-E/RT} + \\
&\quad \rho_c C_{pc} q_c\left(1 - e^{-h_A/\rho_c C_{pc} q_c}\right)\left(T_{cf} - T\right)
\end{aligned} \tag{19}$$

The physical meanings and numerical values of these variables are listed in Table 1. In this example, T is the controlled vari-

Table 1: The physical meanings and nominal values of variables in the CSTR example

reactant conc.	C_A	0.1	mol/ℓ
reactor temp.	T	438.54	K
coolant rate	q_c	103.41	ℓ/min
process rate	q	100	ℓ/min
feed conc.	C_{Af}	1	mol/ℓ
feed temp.	T_f	350	K
avv coolant temp.	T_{cf}	350	K
reactor volume	V	100	ℓ
heat trans. coeff.	h_A	7×10^5	$cal/min/K$
reaction rate	k_0	7.2×10^{10}	$1/min$
activation energy	E/R	10^4	K
heat of reac.	ΔH	-2×10^5	cal/mol
liquid dens.	ρ,ρ_c	10^3	g/ℓ
specific heat	C_p, C_{pc}	1	$cal/g/K$

able, q_c is the manipulated variable, and the feed concentration, C_{Af}, is the main disturbance. Based on the linearized dynamic equations, two types of models are used for subsequent controller design, a single local linear model and a global fuzzy nonlinear model:

Model 1: the single local linear model (SM)
Considering a single local linear model,

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned} \qquad (20)$$

Suppose the process will be operated around $q_c(t) \in [93.069, 113.751]$, $C_{Af}(t) \in [.9, 1.1] mol/\ell$, and $T_f(t) \in [345, 355]K$. The local linear model is established around the midpoint of the operating regiem, *i.e.*, $q_c(t) = 103.41\ell/min$, $C_{Af}(t) = 1mol/\ell$, and $T_f(t) = 350K$. One can determine the possible values for elements in the coefficient matrices, as shown in Table 4. According to the varying ranges of the coefficient values, one can determine the standard state-space equivalent of the parametric uncertainty model.

Table 2: The spread of parameters for a single model in the CSTR example

	$q_c(t) \in [93.069, 113.751]$		
	max	nominal	min
a_{11}	-3.6934	-9.9987	-30.836
a_{12}	.63108	.57326	.46446
a_{21}	-43.973	-146.92	-487.12
a_{22}	8.2079	7.3264	5.5305
b_1	0	0	0
b_2	-5.5321	-7.4077	-9.4843

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -17.265 & .54777 \\ -265.55 & 6.8692\end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -7.5082 \end{bmatrix} u(t) + \\ &\quad \begin{bmatrix} .91047 & .7098 & .39252 \\ 15.055 & 11.426 & 7.2477 \end{bmatrix} v(t) \\ q(t) &= \begin{bmatrix} 9.4297 & .60771 \\ 7.4103 & -.51668 \\ -.69793 & -.26304 \end{bmatrix} x(t) + \begin{bmatrix} -6.2076 \\ 5.315 \\ 4.7876 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)\end{aligned} \qquad (21)$$

Model 2: the global fuzzy model (FM)
Considering the same operating ranges as those of *Model 1*. Suppose $C_{Af}(t) = 1mol/\ell$, $T_f(t) = 350K$, and three local linear models are found around the operating points of $q_c(t) \in \{98.234, 103.41, 108.58\}$, respectively. The three local models and the membership functions, shown in Fig. 1, are used to establish the global fuzzy model. According to the given membership functions, it is clear that the three models are responsible for $q_c(t)$ values falling into the range of $[93.069, 103.41]$, $[98.234, 108.58]$ and $[103.41, 113.751]$, respectively. Thus we can determine the possible model coefficient values, such as shown Table 4. The

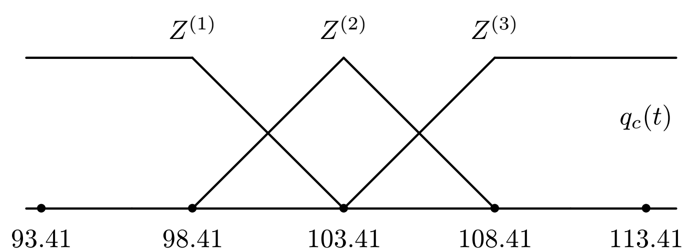

Figure 1: The membership functions for q_c

equivalent norm-bounded state-space models can then be found.

1st Model:

$$\begin{aligned} &IF \quad u(t) \quad is \quad Z^{(1)} \quad THEN \\ \dot{x}(t) &= \begin{bmatrix} -17.3 & .54773 \\ -266.12 & 6.9106 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -7.5431 \end{bmatrix} u(t) + \\ &\begin{bmatrix} .78019 & .95551 & .36745 \\ 12.3327 & 16.032 & 7.4717 \end{bmatrix} v(t) \\ q(t) &= \begin{bmatrix} 6.7136 & -.37994 \\ 8.9759 & .48971 \\ -.75741 & -.25032 \end{bmatrix} x(t) + \begin{bmatrix} 5.5662 \\ -6.3674 \\ 4.7392 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

2nd Model:

$$\begin{aligned} &IF \quad u(t) \quad is \quad Z^{(2)} \quad THEN \\ \dot{x}(t) &= \begin{bmatrix} -15.01 & .54781 \\ -227.99 & 6.8792 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -7.3589 \end{bmatrix} u(t) + \\ &\begin{bmatrix} .40246 & .82821 & .6475 \\ 7.4175 & 13.597 & 10.189 \end{bmatrix} v(t) \\ q(t) &= \begin{bmatrix} 1.3838 & -.18049 \\ 7.9102 & .41248 \\ 6.4984 & -.28867 \end{bmatrix} x(t) + \begin{bmatrix} 4.5533 \\ -5.5398 \\ 4.2557 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

3rd Model:

$$\begin{aligned} &IF \quad u(t) \quad is \quad Z^{(3)} \quad THEN \\ \dot{x}(t) &= \begin{bmatrix} -12.983 & .54841 \\ -196.46 & 6.8792 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -7.1916 \end{bmatrix} u(t) + \\ &\begin{bmatrix} .49974 & .64954 & .38185 \\ 8.3788 & 10.144 & 6.8867 \end{bmatrix} v(t) \\ q(t) &= \begin{bmatrix} 8.2103 & .015641 \\ 6.2823 & .096996 \\ 2.9229 & .031037 \end{bmatrix} x(t) + \begin{bmatrix} .54071 \\ -1.2776 \\ 1.4655 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

Now, we design the k-dimensional output feedback controllers based on the above two types of models. Here, a single H^∞ controller and an H^∞ fuzzy controller will be investigated, respectively. Notably, we choose $\gamma = 3$ in all three design cases.

Controller 1: a single H^∞ controller design based on the single liner model (SM-SC)

The positive definite matrices of R, S and P can be determined according to *Model 1*.

$$R = \begin{bmatrix} .17365 & 2.7366 \\ 2.7366 & 56.746 \end{bmatrix} \quad S = \begin{bmatrix} 65.767 & -3.3015 \\ -3.3015 & .2211 \end{bmatrix}$$
$$P = \begin{bmatrix} 65.767 & -3.3015 & 7.7392 & 0 \\ -3.3015 & .2211 & -2.4664 & -.47594 \\ 7.7392 & -2.4664 & 115.94 & 26.343 \\ 0 & -.47594 & 26.343 & 6.0601 \end{bmatrix}$$

The resulting single H^∞ controller can be obtained by solving

Table 3: The spread of parameters for the three models in the CSTR example

	$q_c(t) \in [93.069, 103.41]$		
	max	nominal	min
a_{11}	-3.7638	-12.074	-30.836
a_{12}	.62725	.57362	.46821
a_{21}	-45.124	-180.8	-487.12
a_{22}	8.2079	7.3837	5.6113
b_1	0	0	0
b_2	-5.6018	-7.7633	-9.4843
	$q_c(t) \in [98.234, 108.58]$		
	max	nominal	min
a_{11}	-3.6941	-9.9987	-26.236
a_{12}	.62988	.57326	.46574
a_{21}	-43.958	-146.92	-412.02
a_{22}	8.2079	7.3264	5.5505
b_1	0	0	0
b_2	-5.5504	-7.4077	-9.1675
	$q_c(t) \in [103.41, 113.751]$		
	max	nominal	min
a_{11}	-3.6837	-8.173	-22.383
a_{12}	.63108	.5702	.46574
a_{21}	-43.816	-117.1	-349.11
a_{22}	8.2079	7.2253	5.5505
b_1	0	0	0
b_2	-5.5279	-7.0259	-8.8552

Eq.(18),

$$\begin{aligned} \dot{\hat{x}}(t) &= \begin{bmatrix} -2.2513 & -.23086 \\ -.091052 & -1.7041 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -.20563 \\ .61407 \end{bmatrix} y(t) \\ u(t) &= \begin{bmatrix} -.66548 & -.1135 \end{bmatrix} \hat{x}(t) + .14592 y(t) \end{aligned} \tag{22}$$

Controller 2: a single H^∞ controller design based on the fuzzy dynamic model (FM-SC)

The positive definite matrices of R, S and P, can be determined according to *Model 2*.

$$R = \begin{bmatrix} .19575 & 3.0918 \\ 3.0918 & 64.675 \end{bmatrix} \quad S = \begin{bmatrix} 68.151 & -3.5933 \\ -3.5933 & .24087 \end{bmatrix}$$
$$P = \begin{bmatrix} 68.151 & -3.5933 & 21.724 & 0 \\ -3.5933 & .24087 & -3.4604 & -.23773 \\ 21.724 & -3.4604 & 156.02 & 15.309 \\ 0 & -.23773 & 15.309 & 1.6048 \end{bmatrix}$$

A single H^∞ controller can be established by solving Eq.(18), where 9 LMIs should be solved simultaneously.

$$\begin{aligned} \dot{\hat{x}}(t) &= \begin{bmatrix} -2.0576 & -.12301 \\ .021401 & -1.7883 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -.14027 \\ 1.0923 \end{bmatrix} y(t) \\ u(t) &= \begin{bmatrix} -.55446 & -.014802 \end{bmatrix} \hat{x}(t) + .081101 y(t) \end{aligned} \tag{23}$$

Controller 3: an H^∞ fuzzy controller design based on the fuzzy dynamic model (FM-FC)

The positive definite matrices of R, S and P are the same as that for *Controller 2*. The elements of the fuzzy controller can be found by solving Eq.(17) sequentially.

$$
\begin{aligned}
&1\textbf{st} \quad \textbf{Controller:}\\
&\textit{IF} \quad u(t) \quad \textit{is} \quad Z^{(1)} \quad \text{THEN}\\
&\dot{\hat{x}}(t) = \begin{bmatrix} -2.0878 & -.13043 \\ .023258 & -1.8483 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -.15082 \\ .89047 \end{bmatrix} y(t)\\
&u(t) = \begin{bmatrix} -.59782 & -.0011671 \end{bmatrix} \hat{x}(t) + .15793 y(t)\\
&2\textbf{nd} \quad \textbf{Controller:}\\
&\textit{IF} \quad u(t) \quad \textit{is} \quad Z^{(2)} \quad \text{THEN}\\
&\dot{\hat{x}}(t) = \begin{bmatrix} -1.9118 & -.092157 \\ .048426 & -1.9329 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -.15523 \\ 1.112 \end{bmatrix} y(t)\\
&u(t) = \begin{bmatrix} -.52867 & -.002886 \end{bmatrix} \hat{x}(t) + .13002 y(t)\\
&3\textbf{rd} \quad \textbf{Controller:}\\
&\textit{IF} \quad u(t) \quad \textit{is} \quad Z^{(3)} \quad \text{THEN}\\
&\dot{\hat{x}}(t) = \begin{bmatrix} -1.8858 & -.083425 \\ .046909 & -1.876 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -.13997 \\ 1.2599 \end{bmatrix} y(t)\\
&u(t) = \begin{bmatrix} .5295 & -.016391 \end{bmatrix} \hat{x}(t) + .057829 y(t)
\end{aligned}
\tag{24}
$$

5 Conclusion

The LMI based H^∞ fuzzy controller design for nonlinear dynamic systems has been investigated in this article. The entire possible operating range for a process was partitioned into several smaller regimes. A set of multiple local linear models with norm-bounded parameter uncertainties was then identified and integrated as the so-called Tagaki-Sugeno (T-S) fuzzy model. The control design problem, based on the norm-bounded T-S fuzzy model, was transformed into a multiple standard H^∞ control problem. The necessary and sufficient conditions for the existence of an H^∞ fuzzy controller was formulated into a set of Linear Matrix Inequalities. An effective computational procedure was also established for determining controller parameters. One chemical process, an exothermic non-isothermal continuous stirred tank reactor, was used to demonstrate the effectiveness and appropriatness of the proposed design method.

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