轉換函數法以控制旋轉圓盤之振動

Vibration Control of a Spinning Disk via Transfer Function Technique

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摘要

本計畫以轉換函數法為基礎，提出一套主動控制法，可以控制旋轉圓盤於臨界轉速以上旋轉時的振動。回授控制器包含一個空間中不動的點驅動器及一個感知器。該感知器可同時測得圓盤位移及斜率。當只有位移資訊時我們証明任何控制器均無法在超臨界轉速時，使圓盤趨於穩定。然而當位移及斜率資訊同時存在時，則可設計出適合的控制器以達到控制圓盤振動的目標。

關鍵詞: **轉換函數，主動控制，旋轉圓盤**

Abstract

An active control scheme based on the transfer function model is proposed to suppress the vibration of a disk spinning both in the sub- and super-critical speed ranges. The feedback controller consists of a space-fixed point sensor and a point actuator, which are not on a nodal circle of the freely spinning disk. The point sensor can measure both the displacement and the pitching slope of the spinning disk. In the sub-critical speed range, a controller measuring the displacement information alone works well in suppressing the disk vibration. However, such a controller is not only unable to suppress the disk vibration in the super-critical speed range, it may destabilize the spinning disk instead. In order to control a disk in the super-critical speed range, a controller using both the displacement and slope information is proposed.

Introduction

Spinning disks are the key elements in many mechanical applications, such as gas turbines, grinding wheels, circular saws, and computer disk drives. In general, large amplitude of transverse vibration is undesirable as it can degrade the performance of the devices, and in some cases destroy the machines.

In implementing active control to suppress the spinning disk vibration, Ellis and Mote (1979) developed a proportional-derivative controller aimed at increasing the transverse stiffness and damping of a circular saw with an electromagnetic actuator. Radcliffe and Mote (1982) used the on-line FFT analysis of the disk displacement to identify the dominant mode of the disk vibration. Fung (1997) employed the method of independent modal space control to suppress the vibration in a non-constant rotating disk.

Another approach of active vibration control is to extend the feedback control theory based on lumped system transfer function to the control of distributed systems (Butkovskiy, 1983). Since this approach considers all the vibration modes and the dynamics of actuators and sensors, it can avoid the spillover problems as long as accurate data on the open-loop pole and zero locations can be obtained. In this report we extend Butkovskiy's theory and propose a control scheme which is able to suppress the vibration of a disk spinning both in the sub- and the super-critical speed ranges. The control method requires the measurements of the displacement and pitching slope at a space-fixed point which is not on a nodal circle.

Open-Loop Transfer Function

We consider an elastic circular disk clamped on the inner radius and free on the outer radius. In terms of dimensionless parameters the equation of motion of the spinning disk subject to a space-fixed concentrated impulsive force at position $r =$ ¶, ¯ $=$ ¤ at time $t = {}^{2}$ can be written as

$$w_{,tt} + 2\Omega w_{,\theta t} + \nabla^4 w + \left(\Omega^2 - \frac{\kappa}{r^2}\right) w_{,\theta\theta} - \frac{1}{r}\left(\kappa_{,r} r w_{,r}\right)_{,r} = \frac{1}{r}\delta(t-\tau)\delta(r-\rho)\delta(\theta-\phi) \qquad (1)$$

The initial conditions of the spinning disk are assumed to be $w(r,\theta) = w_0(r,\theta)$ and $w_{,t}(r,\theta) = v_0(r,\theta)$ at time t=0.

For a freely spinning disk, the eigenvalues are purely imaginary and occur in complex conjugate pairs, i.e., $\lambda_{mn} = \pm i\mu_{mn}$, where $i = \sqrt{-1}$ and μ_{mn} is real. The eigenfunction corresponding to λ_{mn} is in general complex and assumes the form

$$w_{mn} = R_{mn}(r)e^{in\theta} \qquad (2)$$

It has been shown in Chen and Bogy (1992) that the eigenfunctions w_{mn} are complete in the associated Hilbert space. Therefore, the solution $w(r,\theta,t)$ of Eq.(1) can be expressed in terms of $w_{mn}(r,\theta)$ as

$$w(r,\theta,t) = \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\left[c_{mn}(t)w_{mn}(r,\theta)\right] \qquad (3)$$

After substituting Eq.(3) into Eq.(1), multiplying both sides of Eq.(1) by $\overline{w}_{pq}$, and integrating over the annular region we can discretize Eq.(1) into a system of decoupled equations for $c_{mn}(t)$,

$$\ddot{c}_{mn} + 2in\Omega\dot{c}_{mn} + \mu_{mn}(\mu_{mn} + 2n\Omega)c_{mn} = R_{mn}(\rho)e^{-in\phi}\delta(t-\tau) \qquad (4)$$

A superposed dot represents derivative with respect to time. The initial conditions for c_{mn} are $c_{mn}(0) = c^0_{mn}$ and $\dot{c}_{mn}(0) = \dot{c}^0_{mn}$, Recall that at a certain rotation speed there are in general two modes with m nodal circles and n nodal diameters. In the sub-critical speed range, one is a forward mode and the other is a backward mode. In the super-critical speed range, one is a forward mode and the other is a reflected mode. In either case, if μ_{mn} is the natural frequency of one (m,n) mode, $(\mu_{mn} + 2n\Omega)$ is the natural frequency of the other (m,n) mode, also denoted by $\mu_{m\bar{n}}$. The solution of Eq.(4) can be written as

$$c_{mn}(t) = \frac{H(t-\tau)}{i\hat{\mu}_{mn}}\left[e^{i\mu_{mn}(t-\tau)} - e^{-i\mu_{m\bar{n}}(t-\tau)}\right]R_{mn}(\rho)e^{-in\phi} + \frac{1}{i\hat{\mu}_{mn}}\left[ic^0_{mn}\left(\mu_{m\bar{n}}e^{i\mu_{mn}t} + \mu_{mn}e^{-i\mu_{m\bar{n}}t}\right) + \dot{c}^0_{mn}\left(e^{i\mu_{mn}t} - e^{-i\mu_{m\bar{n}}t}\right)\right] \qquad (5)$$

where $H(t)$ represents the Heaviside step function, and $\hat{\mu}_{mn} = \frac{1}{2}\left(\mu_{mn} + \mu_{m\bar{n}}\right)$. After substituting Eq.(5) back into Eq.(3) and rearranging the series, we obtain the response of the spinning disk under concentrated impulse and the prescribed initial conditions as,

$$w(r,\theta,t;\rho,\phi,\tau) = G_0(r,\theta,\rho,\phi,t-\tau) + \int_0^{2\pi}\int_b^1 G_0(r,\theta,\rho,\phi,t)\left[2\Omega w_{0,\phi}(\rho,\phi) + v_0(\rho,\phi)\right] + G_{0,t}(r,\theta,\rho,\phi,t)\,w_0(\rho,\phi)\rho\, d\rho\, d\phi \qquad (6)$$

G_0 is the Green's function

$$G_0(r,\theta,\rho,\phi,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\hat{\mu}_{mn}}R_{mn}(\rho)R_{mn}(r)\sin\hat{\mu}_{mn}t\cos[n(\theta-\phi-\Omega t)] \qquad (7)$$

The Laplace transform of Eq.(7) is the open-loop transfer function of the spinning disk

$$G_0^*(r,\theta,\rho,\phi,s) = \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\frac{e^{in(\theta-\phi)}}{4i\hat{\mu}_{mn}}\varepsilon_n R_{mn}(\rho)R_{mn}(r)\left[\frac{1}{s-i\mu_{mn}} - \frac{1}{s+i\mu_{m\bar{n}}}\right] \qquad (8)$$

The superposed (*) denotes the Laplace transform of the function.

Closed-Loop Transfer Functions

To form a feedback control system, we add a control module to the feedback channel. The feedback control module measures the displacement w and the pitching slope $w_{,\theta}$ at a space-fixed point (r_s,θ_s) and exerts a control force at point (r_a,θ_a). By following a procedure similar to Yang and Mote (1991) we can find the closed-loop transfer function

$$G_{cl}^*(r,\theta,\rho,\phi,s) = G_0^*(r,\theta,\rho,\phi,s) + \frac{G_0^*\left[\gamma_1 C_1 G_0^* + \gamma_2 C_2 G_{0,\theta}^*\right]}{1 - \gamma_1 C_1 G_0^* - \gamma_2 C_2 G_{0,\theta}^*} \qquad (9)$$

We assume that transfer functions $C_1(s)$ and $C_2(s)$ can be expressed in the general forms

$$C_1(s) = \frac{N_1(s)}{D_1(s)} \text{ and } C_2(s) = \frac{N_2(s)}{D_2(s)}$$

where polynomials $N_1(s)$ and $D_1(s)$, and $N_2(s)$ and $D_2(s)$ have no common roots, respectively. The associated characteristic equation of the closed-loop system is

$$D_1 D_2 - \alpha_1 N_1 D_2 G_0^* - \alpha_2 N_2 D_1 G_{0,\theta}^* = 0 \quad (10)$$

Controller Design

In the case of an uncontrolled disk the eigenvalues $s_{mn} = i\mu_{mn}$ are purely imaginary. As α_1 and α_2 increase from zero, the eigenvalues may remain purely imaginary or start to have nonzero real parts. In order for the controlled spinning disk to be asymptotically stable, all its eigenvalues must lie in the left-half *s*-plane. One way to estimate the effect of the control gains on the change of the eigenvalues is to calculate the derivatives of the eigenvalues with respect to the control gains. By following the procedure proposed by Yang and Mote (1991) we can derive the following derivatives

$$\left.\frac{\partial s_{mn}}{\partial \alpha_1}\right|_{\alpha_1 \to 0^+} = \frac{w_{mn}(r_s, \theta_s)\overline{w}_{mn}(r_a, \theta_a) N_1(i\mu_{mn})}{2i\hat{\mu}_{mn} D_1(i\mu_{mn})} \quad (11)$$

$$\left.\frac{\partial s_{mn}}{\partial \alpha_2}\right|_{\alpha_2 \to 0^+} = \frac{w_{mn}(r_s, \theta_s)\overline{w}_{mn}(r_a, \theta_a) n N_2(i\mu_{mn})}{2\hat{\mu}_{mn} D_2(i\mu_{mn})} \quad (12)$$

In the case when the actuator and the sensor are at the same location, i.e., $(r_s, \theta_s) = (r_a, \theta_a)$, the first order approximation of the eigenvalues of the controlled disk is then

$$s_{mn} \cong i\mu_{mn} + \frac{R_{mn}^2}{2\hat{\mu}_{mn}}\left[\frac{\alpha_1 N_1}{iD_1} + \frac{\alpha_2 n N_2}{D_2}\right] \quad (13)$$

Case 1: $\alpha_2 = 0$. This is the most intuitive design in which only the transverse displacement of the spinning disk is measured. We can show that it is impossible for the spinning disk rotating in the super-critical speed range to be stabilized asymptotically by this design. Any polynomial of s can be rearranged into the real and the imaginary parts when s is replaced by $i\mu_{mn}$. The real part is a function of μ_{mn}^2, while the imaginary part is a function of μ_{mn}. Therefor we can write the transfer function $\frac{N_1}{D_1}$ as

$$\frac{N_1}{D_1} = \frac{N_{iR}(\mu_{mn}^2) + iN_{iI}(\mu_{mn})}{D_{iR}(\mu_{mn}^2) + iD_{iI}(\mu_{mn})} \quad (14)$$

As a consequence the real part of the derivative (11) becomes

$$\mathrm{Re}\left[\frac{\partial s_{mn}}{\partial \alpha_1}\right]_{\alpha_1 \to 0^+} = \frac{R_{mn}^2}{2\hat{\mu}_{mn}}\left[\frac{D_{1R}(\mu_{mn}^2)N_{1I}(\mu_{mn}) - D_{1I}(\mu_{mn})N_{1R}(\mu_{mn}^2)}{D_{1R}^2(\mu_{mn}^2) + D_{1I}^2(\mu_{mn})}\right] \quad (15)$$

Recall that the term $\hat{\mu}_{mn}$ represents the natural frequency of the spinning disk as seen by a disk-fixed observer and is always positive. The term in the bracket is an odd function of μ_{mn}. In the sub-critical speed range, the natural frequencies of both the forward and the backward wave are positive and there is no difficulty in choosing a controller to stabilize both modes at the same time. In the super-critical speed range, however, the natural frequency of the forward wave remains positive while the natural frequency of the reflected wave becomes negative. Therefore, The controller will always destabilize one mode while stabilize the other. For a specific example we choose a velocity feed-back control,

$$N_1(s) = -s, \quad D_1(s) = 1 \quad (16)$$

Equation (22) then becomes

$$\mathrm{Re}\left[\frac{\partial s_{mn}}{\partial \alpha_1}\right]_{\alpha_1 \to 0^+} = -\frac{\mu_{mn} R_{mn}^2(r_s, \theta_s)}{2\hat{\mu}_{mn}} \quad (17)$$

Therefore, the velocity feedback controller stabilizes both the forward and backward waves in the sub-critical speed range, but destabilize the reflected waves in the super-critical speed range.

Case 2: $\alpha_1 = 0$. In this case only the pitching slope information is used. Following the similar procedure as described in case 1 we can derive the real part of the first order derivative as

$$\mathrm{Re}\left[\frac{\partial s_{mn}}{\partial \alpha_2}\right]_{\alpha_2 \to 0^+} = \frac{nR_{mn}^2}{2\hat{\mu}_{mn}}\left[\frac{D_{2R}N_{2R} - D_{2I}N_{2I}}{D_{2R}^2 + D_{2I}^2}\right] \quad (18)$$

The bracket term in Eq.(18) is an even function of μ_{mn}. The sign of the derivative is then dependent upon the number *n*. Recall that the number n for a forward wave is negative, while n for a backward or a reflective wave is positive. Therefore, such a controller will always destabilize the spinning disk both in the sub- and super-critical speed ranges. For a specific example we examine the case when the control force is proportional to the pitching slope, i.e.,

$$N_2(s) = -1\,,\quad D_2(s) = 1 \qquad (19)$$

Equation (18) becomes

$$\mathrm{Re}\left[\frac{\partial s_{mn}}{\partial \alpha_2}\right]_{\alpha_1=0,\,\alpha_2\to 0^+} = -\frac{nR_{mn}^2(r_s,\theta_s)}{2\hat{\mu}_{mn}} \qquad (20)$$

Apparently this controller always destabilizes the forward modes both in the sub- and the super-critical speed ranges.

Case 3: $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$: We have shown in case 1 that to control a disk spinning in the sub-critical speed range many common controllers using displacement information alone will suffice. The challenge is to control the disk in the super-critical speed range. Although it is impossible to control the disk in the super-critical speed range with the controllers described in Cases 1 and 2, we can design a controller using both the displacement and slope information to suppress the disk vibration. For instance, we can combine the velocity feedback controller (23) and the slope feedback controller (26). The first order approximation of the closed-loop eigenvalue is then

$$s_{mn}(\alpha_1,\alpha_2) \cong i\mu_{mn} - \frac{R_{mn}^2(r_s,\theta_s)}{2\hat{\mu}_{mn}}[\alpha_1\mu_{mn} + \alpha_2 n] \qquad (21)$$

In order for the eigenvalue changes of both the $(m,\pm n)$ modes in the super-critical speed range to be negative real, the control gain parameters must satisfy the following relations,

$$\left|\frac{(\mu_{mn})_f}{n}\right| > \frac{\alpha_2}{\alpha_1} > \left|\frac{(\mu_{mn})_r}{n}\right| \qquad (22)$$

where subscripts f and r denote forward and reflected waves, respectively.

Conclusions

Several conclusions can be summarized in the following.

(1) In the sub-critical speed range, a controller measuring the displacement information alone works well in suppressing the disk vibration.

(2) A controller using only the pitching slope information is unable to suppress the disk vibration either in the sub- or the super-critical speed range.

(3) In order to control the disk vibration in the super-critical speed range, we need to use a controller which measures both the displacement and the pitching slope simultaneously. The range of the control gain parameter which renders the spinning disk asymptotically stable can be determined by a stability analysis.

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