行政院國家科學委員會專題研究計畫成果報告

小波分析在非均質弦的破壞診斷上的應用

The Application of Wavelet Analysis to the Damage Diagonsis of Nonuniform String

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中文摘要

本計劃提出了一個基於小波理論用於偵測結構破壞點的方法。我們利用均勻弦代表原始結構，非均勻弦代表受損結構。由兩者振動訊號的小波係數的差異可以很清楚的判別結構有無受損及受損處。我們同時發現這個方法對早期細小的破壞非常敏感。
關鍵詞：小波、破壞檢測

Abstract

We present a new method based on the wavelet transform for the detection of structural damage. A uniform string is used to represent the original system, while a little amount of density variation distributed over a small range is superimposed to the uniform string to simulated the damaged part. The vibration signals of the damaged system are processed with a discrete wavelet transform and the result is compared with that of the original system. It is found that minor localized faults can induce significant changes in the wavelet coefficients of the vibration signals. Furthermore, the maximum difference between wavelet coefficients of the damaged and the original systems occurs at the location of the damaged part.
Keywords: wavelet, defect dection

1. Introduction

The real-time condition monitoring for structural damage detection is an important research topic. This subject has drawn the attention of many researchers in the past [1-4]. Besides early abnormality detection, it is also desirable to identify the damaged parts of the mechanical structure from the monitored signal for effective maintenance. Most of the methods currently in use for this purpose are based on the processing of structure vibration signals. In these methods, vibration signals measured at convenient locations are processed and compared with the output of the undamaged system. The difference in the response is used for damage detection and identification. Many different techniques for processing vibration signals have been proposed [5, 6]. Among them, a commonly used method is modal analysis [7, 8]. The main idea behind damage detection schemes that use modal data is that a change in the system due to damage will manifest itself as changes of the natural frequencies and the associated mode shapes. However, this method may exhibit low sensitivity to faults in the early stage of development and/or poor diagnostic capability. For example, the change of the fundamental frequency of a cracked shaft is merely 6% of the original value while the length of the crack is one half of the diameter of the shaft [4, 7, 8].

A defect in the early stage, e.g. a crack, is localized in space and the induced vibration signals are not stationary. This may be the main reason why the modal data, which are global and stationary, are not sensitive to the early localized damage. On the other hand, the wavelet analysis, which possesses multi-resolution, provides a promising method to solve this problem. The wavelet transform is still evolving and has been applied with some successes in different areas [9-16].

In this paper we present a method based on wavelet transform to identify the existence and location of structural damage. In order to concentrate on the studying of the effectiveness of this method, a nonuniform string is used to simulate the damaged system while a uniform string represents the original undamaged system. We study several different kinds of defects that are simulated by mass points attached to a uniform string and added mass distributed over a small portion of a uniform string. The system output is processed with wavelet transform and the result is compared with that of the undamaged system for damage detection.

2. Theoretical Background of Wavelet Transform

2.1 Continuous Wavelet Transform

The continuous wavelet transform $(Tf)(a,b)$ of a function $f(t)$ is defined by [17]

$$(Tf)(a,b) = \langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t)\, \psi^{*}_{a,b}(t)\, dt, \tag{1}$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\, \psi\left(\frac{t-a}{b}\right), \tag{2}$$

where $a, b \in \mathbf{R}$, $a \neq 0$ and the asterisk indicates the complex conjugate. The function $\psi(t)$ is termed the mother wavelet and satisfies the admissibility condition:

$$\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\mu)|^2}{|\mu|}\, d\mu < \infty, \tag{3}$$

where $\hat{\psi}(\mu)$ indicates the Fourier transform of $\psi(t)$. The elements of the wavelet basis, $\psi_{a,b}$, are generated by dilating and translating the mother wavelet ψ as defined in equation (2), where a and b are called the dilation parameter and the translation parameter, respectively. The normalization factor $|a|^{-1/2}$ has been chosen so that $\|\psi_{a,b}\| = \|\psi\|$.

2.2 Discrete Wavelet Transform

In order to calculate the WT numerically, discrete methods are required. In this case the dilation parameter a and the translation parameter b both take only discrete values. A commonly used discretization method, the dyadic discretization, is given by:

$$a_j = 2^{-j} a_0, \quad b_k^j = k a_j b_0 \text{, and}$$

$$\psi_k^j(t) = a_j^{-1/2} \psi(x - b_k^j / a_j), \tag{4}$$

where $a_0 > 0$ and ψ is a wavelet. A function $f \in L^2(R)$ can be expressed as the superposition of the wavelets ψ_k^j:

$$f = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_k^j \psi_k^j . \tag{5}$$

For clarity of discussion wavelets corresponding to the same j are called the wavelets of the j level of resolution.

In a closed interval $\Omega \equiv [x_l, x_r]$, if we take

$$a_j = 2^{-j} a_0, \quad b_k^j = (x_r + x_l)/2 + k a_j b_0, \tag{6}$$

where $a_0 = 2^{-L}(x_r - x_l)/b_0$, $L \in Z$, then the set of wavelets $\{\psi_k^j(x): j, k \in Z\}$ constitutes a frame. Note that for any j there are values of k such that the wavelets are centered outside of the domain Ω. For clarity of discussion, all wavelets whose centers are located within the domain will be called internal wavelets; all other wavelets will be called external wavelets. Since all wavelets are either with compact support or decays fast enough away from its center, there are always a finite number of external wavelets whose influence inside of the domain must be accounted. Thus at any level j there is a subset of integers

$$Z_\Omega^j : \{-2^{L+j-1} - N_l, \ldots, 2^{L+j-1} + N_r\} \tag{7}$$

such that for $k \in Z_\Omega^j$ the wavelet ψ_k^j affects significantly the interior of the domain Ω. In equation (7), N_l and N_r are the numbers of external wavelets on each side of the domain. It can be shown that there exist b_0, L, N_l and N_r such that a function $u(x)$ defined on Ω can be approximated as

$$u^J(x) = \sum_{j=0}^{J} \sum_{k \in Z_\Omega^j} c_k^j \psi_k^j(x), \tag{8}$$

where the levels $j = 0$ and $j = J$ correspond respectively to the coarsest and the finest scales. The largest scale present in the approximation is determined by L. The collocation method developed by Vasilyev et al. [11] is adopted in this paper to find the wavelet coefficients c_k^j.

3. Physical Model – Non-uniform String

In order to concentrate on the study of the effectiveness of the proposed method, a relatively simple model, a string with non-uniform density, is employed here. A uniform string is used to represent the original undamaged system, while a little amount of density variation distributed over a small interval is superimposed to the uniform string to simulate the damaged part. We apply the wavelet transform to analyze the vibration signals of the string to find the relation between wavelet coefficients and the density change.

4. Results and Discussion

We first consider the case of a point mass m attached to a uniform string with length L=2. We use the point mass to represent a highly localized damaged part of the original system. The point mass is located at the center. The total mass of the uniform string is 2. Fig. 1 shows the displacement of the string at time $t = 0.2$ of various values of m under the same initial conditions: $u(x,0) = \sin(\pi x/2)$ and $\dot{u}(x,0) = 0$. As can be seen from the figure, the deflection of the uniform string ($m = 0$) at $t = 2$ and that of the non-uniform string of $m = 0.02$ are almost identical. When the added point mass is one percent of the mass of the uniform string, it is difficult to identify the existence of the point mass by merely monitoring the deflection. In

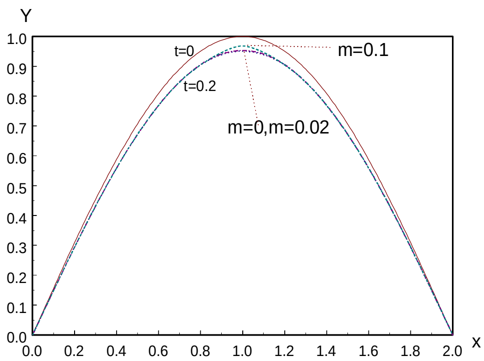

Fig. 1. Deflections at $t = 2$ of a uniform string with a central concentrated mass of various magnitudes under the same initial conditions.

order to demonstrate the effectiveness of the proposed method, we further reduce the point mass to $m = 0.005$ (0.25% of the mass of the uniform string). Then we use the method described in section 2 to determine the wavelet coefficients of these two systems at different levels of resolution. The well-known "Mexican hat" wavelet function is employed as the mother wavelet in this paper. The highest level of resolution is $J = 5$. Fig. 2 shows the results, where solid and dashed lines indicate results of the uniform ($m=0$) and non-uniform ($m=0.005$) strings, respectively. Although the deflections of these two systems are almost identical, their wavelet coefficients are much different from each other. It can be observed that the maximum difference occurs at the center of the string, where the point mass is located. For the convenience of discussion, we denote the region where the difference between the two sets of wavelet coefficients is significant as the "deviation region". As can be seen from the figure, the "deviation region" at each level is centered at the location of the point mass and its width decreases as the level of resolution increases. Therefore the "deviation region" is a good indication regarding the existence and location of the point mass.

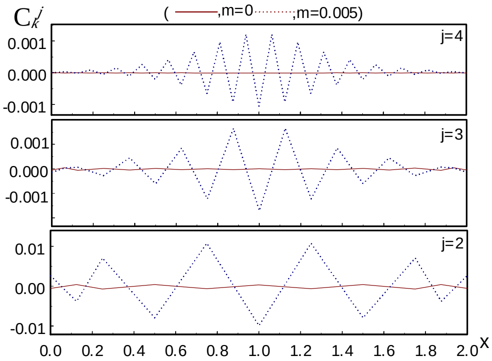

Fig. 2. Wavelet coefficients of a uniform string (solid line) and a string with a central concentrated mass $m=0.005$ (dashed line) at levels j=2, 3, 4.

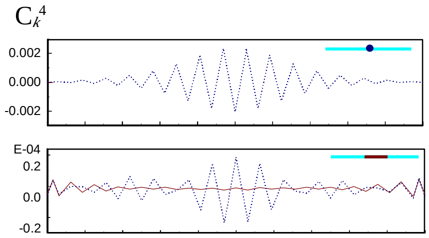

Fig. 3. The fourth level wavelet coefficients of a concentrated mass (top) and a distributed mass (bottom).

Then we study the situation that the superimposed mass is distributed uniformly over a finite interval. Two different cases of superimposed mass are considered here: case 1 is a concentrated mass; in case 2 the superimposed mass is distributed uniformly over 1/4 of the string length. The amount of the superimposed mass is the same, 0.01, in both cases. The wavelet coefficients of the displacement of the non-uniform string at $t=0.2$ are calculated and compared with those of the uniform string, starting from rest with the same initial shape. Fig. 3 shows the results at the 4th level of resolution, where solid and dashed lines indicate uniform and non-uniform strings, respectively. The maximum difference between the wavelet coefficients of the uniform and the non-uniform strings of case 1 is much larger than that of case 2. That is to say, a concentrated mass can cause more significant changes in wavelet coefficients than a distributed mass.

Fig. 4 compares the 4th level wavelet coefficients of the deflections of the uniform string and a string with two concentrated masses, each of magnitude 0.005, located at 1/4 and 7/8 of the string length. It is seen that there are two "deviation regions". Furthermore, the maximum differences between the two sets of wavelet coefficients in these two regions occur at the locations of the concentrated masses.

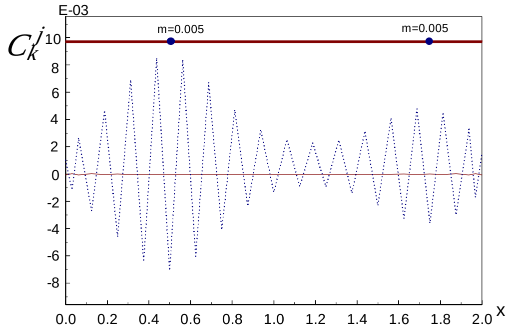

Fig. 4. Wavelet coefficients of a uniform string (solid line) and a string with two concentrated masses located at $x=0.5$ and $x=1.75$ (dashed line).

Mode shapes are often monitored for machine diagnosis. It is obsvered that the first mode shape of the uniform string and that with a point mass of $m=0.005$ are almost the same. The 5th level wavelet coefficients of the first mode shape are calculated and the results are shown in Fig. 5. It is obvious that light point mass added to the uniform string has significant effects on the wavelet coefficients of the mode shapes.

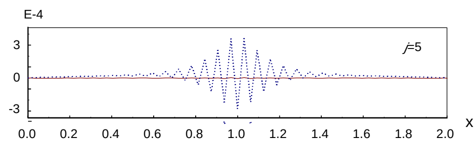

Fig. 5. Comparison of the 5th level wavelet coefficients of the first mode of a uniform string (solid line) and a point mass $m=0.005$ added to the center of a uniform string (dashed line).

5. Summary

In this paper, we presented a method based on wavelet transform for structural damage detection and identification. A collocation wavelet transform method was employed to analyze the vibration signals of the system. We used a uniform string to represent the original system, while some added mass distributed over small intervals to represent initial localized damaged parts. It is found that minor localized damage, which generates vibration signals almost undistinguishable from those of the undamaged system, can induce significant changes in the wavelet coefficients. Furthermore the maximum difference between the wavelet coefficients of the original and damaged systems occurs close to the location of the damaged part. The precision in locating the damaged parts increases with the level of resolution employed in wavelet transform. This method provides structural engineers an alternative and possibly a more reliable way to locate damaged parts in structures.

Acknowledgement

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