依據改進的 Riccati 方程式之線性時變系統控制
A new Riccati equation design for LTV system control

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一、中文摘要

本計畫提出一種新的，對於線性時變系統的控制方法。本方法是針對轉換過的狀態變數作控制器設計。新的狀態變數具有與轉換前的狀態變數大小成反比的性質，因此新的控制方法是希望能找到一種狀態回授增益，使得轉換過的系統呈全域性指數形式發散。本文所提出的控制方法與傳統控制時變系統的方法比較，有兩個優點：一是計算狀態回授增益時僅需要過去及現在的系統資訊，不需預測系統未來的變化；另一是不需對系統參數作微分，因此新的控制方法也可以運用在具有片段連續性時變參數的系統上。

關鍵詞：線性時變系統，控制設計，Riccati 方程式，LQ 控制。

Abstract

This project proposes a new control design for linear time-varying systems. The design is based on an inversion state transfromation, which converts the problem of stabilizing the system state into that of de-stabilizing the transformed state. The new design utilizes a forward Riccati equation while conventional control ustilizes backward Riccati equation. The forward Riccati equation design has the advantage that only present and past information of the system matrices is required in calculating the control. The other advantage is that no differentiation of the time-varying system matrices is required so that the new control can be applied to systems whose time-varying parameters are piecewise continuous only.

Keywords：linear time-varying systems, control design, Riccati equation, LQ control.

二、Introduction

There are essentially two approaches to the state feedback control design of Linear Time-Varying (LTV) systems. In the first approach, one assumes knowledge of future information on the system matrices, and uses such information to synthesize the state feedback gain. A well-known example is the LQ control (Kalman, 1964; Kwakernaak and Sivan, 1972), in which the control Riccati equation is solved backwards starting from some future time to the present time. However, their assumption is rarely met since prediction of how system matrices vary in the future can be very difficult in most practical applications. The second approach to the state feedback control design of LTV systems is the pole-placement-like control (Wolovich, 1968; Valasek and Olgac, 1993). In this approach, one must differentiate thesystem matrices with respect to time consecutively up to the order of the system dimension. In practice, information of the system matrices is often obtained through sensor measurement. Such measurement inevitably introduces noise that can easily destroy the fidelity of the required information in a time differentiation process. Hence, it is generally not advisable to calculate the state feedback gain by differentiating a noise-corrupted system matrix.

The goal of this project is to devise a

control design that avoids the shortcomings of the above approaches. In other words, the control design should require neither time differentiation of the system matrices nor prediction of future information of the system matrices. The calculation of the control law should be based soly on past and present information of the system matrices.
In this project, it will first be shown that by the introduction of an inversion state transformation, the problem of stabilizing the system state is converted into that of destabilizing the transformed system. Second, it is shown that the differential Riccati equation for destabilizing the transformed state becomes a forward Riccati equation with the boundary condition set at the initial time of operation instead of at some future time, similar to the differential Riccati equation used in the Kalman filter design. The requirement of future information on the system matrices can thus be avoided.

三、Preliminaries

Consider a multivariable linear time-varying system:

$$\begin{aligned} x &= A(t)x + B(t)u(t), \quad x(0) = x_0 \\ y &= C(t)x \end{aligned} \tag{1}$$

where $x(t)$ is the system state, $u(t)$ is the control input, $y(t)$ is the system output, the system matrix $A(t)$, the input matrix $B(t)$, and the output matrix $C(t)$ are time-varying matrices with piecewise continuous and bounded elements.

It is assumed that the system (1) is uniformly controllable as defined below.

Definition 1 : The pair $(A(t), B(t))$ is uniformly controllable if there exist a constant Δ and another constant α depending on Δ such that the controllability grammian $I(t-\Delta, t)$ satisfies

$$I(t-\Delta,t) = \int_{t-\Delta}^{t} \Psi(t,\tau)B(\tau)B^T(\tau)\Psi^T(t,\tau)d\tau \geq \alpha I > 0,$$

where $\Psi(t,\tau)$ is the state transition matrix of the free system in Eq.(1).

The goal is to find a state feedback gain $K(t,x)$ such that the control input

$$u(t) = K(t,x)x(t)$$

stabilizes the closed-loop system exponentially.

四、Control Design

The proposed control design is based on the construction of an inversion state transformation

$$z(t) = \frac{x(t)}{\|x(t)\|^2}, \quad \forall x(t) \neq 0. \tag{2}$$

It follows from this definition that

$$z(t) = \frac{1}{\|x(t)\|} \tag{3}$$

By differentiating Eq.(2) along the trajectory (1), one obtains

$$\dot{z}(t) = T(x)A(t)z(t) + T(x)B(t)v(t), \tag{4}$$

where $v(t)$ is the transformed input defined by

$$v(t) = \frac{u(t)}{\|x(t)\|^2}, \tag{5}$$

and $T(x)$ is a Householder transformation (Chen, 1984)

$$T(x) = I - 2\frac{x(t)x^T(t)}{x^T(t)x(t)}.$$

It is important to note that $T(x)$ is symmetric and remains uniformly bounded for any $x(t) \neq 0$ since

$$\|T(x)\| = 1. \tag{6}$$

Equation (6) can be verified by checking that

$$T^2(x) = I.$$

Another consequence of the above equation is that $T(x)$ is invertible for any nonzero $x(t)$:

$$T^{-1}(x) = T(x).$$

One can also verify that $T^{-1}(x)z(t) = -z(t)$;hence, $z(t) = -T^{-1}(x)z(t)$. Substituting this relationship into the right-hand side of Eq.(4) yields

$$\begin{aligned} \dot z(t) &= \overline{A}(t)z(t) + \overline{B}(t)v(t), \\ \overline{A}(t) &= -T(x)A(t)T^{-1}(x), \\ \overline{B}(t) &= T(x)B(t). \end{aligned} \qquad (7)$$

Here, both $\overline{A}(t)$ and $\overline{B}(t)$ are functions of $x(t)$; however, the argument x is omitted for the sake of brevity.

Equation (3) suggests that stabilization of the original system state $x(t)$ is achieved if one can find a transformed input $v(t)$ in Eq.(7) to destabilize the transformed system (7). Such a destabilizing control input is given by

$$v(t) = R_2^{-1}(t)\overline{B}^T(t)P(t)z(t), \qquad (8)$$

where $P(t)$ satisfies the forward differential Riccati Equation

$$\begin{aligned} P(t) = -\overline{A}^T(t)P(t) - P(t)\overline{A}(t) + R_1(t) \\ - P(t)\overline{B}(t)R_2^{-1}(t)\overline{B}^T(t)P(t), \\ P(0) = P_0 > 0, \end{aligned} \qquad (9)$$

in which $\overline{A}(t)$, $\overline{B}(t)$ are defined in Eqs.(7), $R_1(t) > 0$, $R_2(t) > 0$, and $R_1(t)$, $R_2(t)$, $R_1^{-1}(t)$, $R_2^{-1}(t)$ are all uniformly bounded. It follows from Eqs.(8), (2), and (5) that the actual control input $u(t)$ is given by

$$u(t) = R_2^{-1}(t)\overline{B}^T(t)P(t)x(t). \qquad (10)$$

Observe that $P(t)$, and hence the state feedback gain in the above control depend only on past information of the system matrices $A(\tau)$ and $B(\tau)$, and the past state $x(\tau)$, where $\tau \in [0, t]$. In other words, the proposed control (10) is in fact a nonlinear dynamic state feedback control that does not require future information of system matrices.

Under the assumption of existence of solutions, the computation of $P(t)$ is directly based on an *on-line* integration of the forward differential Riccati equation (9) starting at $t = 0$. This is quite different from the computation of the backward differential Riccati equation appearing in the time-varying LQ control, where the integration has to be done *off-line*, starting from some future time.

The following lemmas state some properties of the transformed system matrices $\overline{A}(t)$ and $\overline{B}(t)$.

Lemma 1: If the pair $(A(t), B(t))$ in Eq.(1) is uniformly controllable, the pair $(\overline{A}(t), \overline{B}(t))$ in Eq.(7) is also uniformly controllable.

The second lemma, whose proof is omitted, is an immediate consequence of Lemma 1 and Duality (Callier and Desoer, 1992).

Lemma 2: Under the Hypothesis of Lemma 1, the pair $(\overline{A}(t), \overline{B}(t))$ is uniformly observable in the sense of Definition 1.

Observe that the differential Riccati Equation (9) can be re-written as

$$\begin{aligned} P(t) = (-\overline{A}^T(t))P(t) + P(t)(-\overline{A}^T(t))^T \\ + R_1(t) - P(t)(\overline{B}(t))^T R_2^{-1}(t)(\overline{B}^T(t))P(t), \end{aligned}$$

which has exactly the same form as the observer Riccati Equation below:

$$\begin{aligned} \dot Q(t) = A(t)Q(t) + Q(t)A^T(t) + V_1(t) \\ - Q(t)C^T(t)V_2^{-1}(t)C(t)Q(t), \\ Q(0) = Q_0 > 0 \end{aligned}$$

Therefore, based on Lemma 2, one can follow the procedure in Wonham, 1968 and Anderson and Moore, 1968, to show that $P(t)$ in Eq.(9) is uniformly bounded above and below.

Lemma 3: Under the Hypothesis of Lemma 1, there exist positive constants $¥_1$ and $¥_2$ such that the solution $P(t)$ of the Riccati Eq.(9) satisfies

$$¥_1 I \leq P(t) \leq ¥_2 I \qquad \forall t > 0.$$

One can now state the main theorem of this paper, which shows that the proposed control (10) is globally stabilizing for the system (1).

Theorem: Consider the system (1) and the state feedback control (10) and (9). If the system (1) is uniformly controllable, the closed-loop system is globally exponentially stable.

五、Simulation Example

Example: Consider a multivariable time-varying system (1) with

$$A(t) = \begin{bmatrix} -1+1.5\cos^2 t & 1-1.5\sin t\cos t & 1 \\ -1-1.5\sin t\cos t & -1+1.5\sin^2 t & 1 \\ 0 & -\sin t & -5+\sin t \end{bmatrix},$$

$$B(t) = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{bmatrix}.$$

The initial condition of the system is $x^T(0) = [3,3,3]$. From simulation studies, it is found that the open-loop system is unstable. The proposed control (10) is then applied to the system with the following design parameters $P(0) = I$, $R_1(t) = I$ and $R_2(t) = I$ in Eq. (9).
Figure 1 shows the time history of the System State, which converges to zero as guaranteed by Theorem 2, and Figure 2 shows the time history of the control inputs.

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