行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※

※　　　多指式機械手之構造合成與幾何關係之研究　　　※

※　　　※

※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：■個別型計畫　　□整合型計畫

計畫編號：NSC　89-2212-E-002-015

執行期間：　88 **年**　8 **月** 1 **日至**　89 **年**　7 **月**　31 **日**

計畫主持人：李志中

共同主持人：

本成果報告包括以下應繳交之附件：

- □赴國外出差或研習心得報告一份
- □赴大陸地區出差或研習心得報告一份
- □出席國際學術會議心得報告及發表之論文各一份
- □國際合作研究計畫國外研究報告書一份

執行單位：台灣大學　機械系

中　華　民　國　89 **年**　9 **月**　18 **日**

行政院國家科學委員會專題研究計畫成果報告

題目：多指式機械手之構造合成與幾何關係之研究

Structural Synthesis and Geometry Analysis for Multi-Finger Hands

計劃編號：NSC 89-2212-E-002-015

執行期間：八十八年八月一日至八十九七月三十一日

計劃主持人：李志中　台灣大學 機械系;　Email: jjlee@ccms.ntu.edu.tw

計劃參與人員：陳鈞泊、黃長耀　台灣大學 機械系

中文摘要

本文旨在合成非對稱多指式機械手抓取物體時之拓樸構造。利用自由度判別式和力量包圍的理論，本文首先探討多指式機械手抓取物體的拓樸構造，其次找出藏於中之限制條件，接著，藉由限制條件發展合成多指式機械手抓取物體之拓樸構造。最後，以舉例之方式說明機構的合成步驟。

本文並根據所合成出之機構，分析其工作空間與幾何尺寸之關係，此一結果可協助設計者設計機械手指在抓取物體時之幾何尺寸限制參考用・

ABSTRACT

This work presents a new approach for the structural synthesis of multi-fingered hands. It takes into account both the total mobility and the force closure criterion of the system. Based upon the Grübler's mobility equation, relations regarding the numbers of fingers, contact geometry, and object freedoms are established. Subsequently, by applying the force closure criterion, the total number of possible multi-fingered hands with given mobility are synthesized.

This work also performs the workspace analysis of planar hand mechanisms. The result of analysis provides necessary conditions for the geometry constraints between the object and finger geometry in order to have minimum non-empty workspace.

INTRODUCTION

The structure of robotic manipulator generally employs a serial type of kinematic chain attached with an end-effector, or the gripper, capable of simple gripping. The use of such structure has shown limited functions, such as unable to provide fine motion or force controls. Therefore, anthropomorphic hands with multiple fingers have been developed to meet the condition where the dexterous manipulation or fine motion controls are required. The mechanical hands are designed to have a plurality of fingers, each with a number of joints, such that they can emulate human hand's functions. Various multi-finger hands could be found in the literature [3,5,6,7,11,13]. Generally, these designs have been made to develop improved hands for different applications, for example, prosthetic devices or remote manipulation. The structures of the mechanical hands basically resemble the skeleton of human hands. Salisbury ever performed the number synthesis of hand mechanisms up to three fingers. Subsequently, a three-finger hand with three joints on each finger was designed [10]. Except this, most other mechanical hands were developed from the emulating work of grasping patterns observed from human hands. Not much literature addressed the multi-finger hands from the structural synthesis viewpoint.

The objective of this study is to enumerate total number of admissible hand mechanisms where the object and the hand base are kinematically linked together through fingers. The contact geometry between the object and the hand will be firstly reviewed. Then, several design constraints for the structural synthesis will be discussed and developed. Finally, admissible hand structures with mobility up to six and finger numbers up to six will be enumerated.

CONTACT GEOMETRY

A rigid body without any constraint has six degree-of-freedom (DOF) in the three-dimensional space. If the body is brought into contact with another rigid body, the motion of the rigid body will be restricted. The degree to which the motion of the body is restricted depends upon the nature of the contact, that is, the pairing geometry as well as the frictional restraint in act [2,11]. Thus, the relative DOF between two contacting bodies with or without frictional restraint can be summarized as shown in Table 1. It can be seen that there is every possible physical pairing to generate the contact DOF from zero to five between two contacting bodies. This result can facilitate our structural synthesis procedure since the contact freedom can be assigned from zero to five.

Table 1.　Contact geometry and degree-of-freedom

Type of contact	Point	Line	Plane	Soft finger[1]
Without friction	5(2)[2]	4(1)	3	2
With friction	3(1)	1(0)	0	-

1. See ref.[11]. 2. Numbers in ( ) represents the 2-dimensional case

CONSTRAINT CONDITIONS

When a multi-finger hand grasps an object, it becomes a mechanism with multiple loops, as shown in Fig. 1. In this work, we assumed that the contact between the hand and the object only occurs at the distal portion of the finger, i.e. the fingertip position. Further, all the contact is of the same type of contact geometry. In addition, each joint on the finger is considered to have single degree of freedom. Hence, the total mobility of the hand-object mechanism from Grübler's formula can be written as

$$F = 6(2 + \sum_{i=1}^{n} N_i - 1) - [5\sum_{i=1}^{n} N_i + n(6 - f)] \qquad (1)$$

where

F : total mobility of the hand-object mechanism,

n : number of the fingers,

N_i : number of the links of the ith finger, also number of the joints of the ith finger,

f: contact DOF between the finger and the object,

After some simplification, Equation (1) becomes

$$F = 6 + \sum_{i=1}^{n} N_i - n(6 - f) \qquad (2)$$

Equation (2) can be used to calculate the mobility of the mechanism formed by the hand and object. This mobility number serves a necessary condition index for dexterous manipulation. The more dexterously the object can be manipulated, the greater the mobility number is. Yet, the reverse statement does not always hold as will be discussed later.

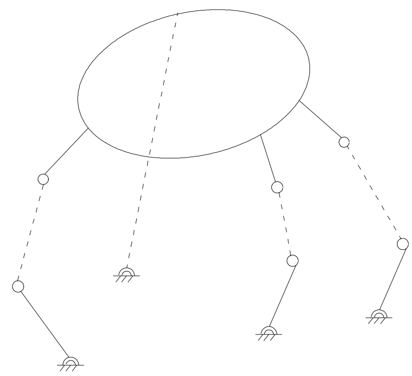

Fig. 1 A multi-fingered hand with n fingers

To carry out the structural synthesis by Eq. (2), one can specify the mobility F, finger number n and the contact DOF f, and then find the joints of each finger. It is possible to enumerate the answers through the exhaustive combination and permutation. However, it can be more efficient to impose the embedded constraints prior to the enumeration. Such procedure can reduce the number of mechanisms so that we can obtain more useful cases only. Following are the establishment of the constraints.

Constraint Condition 1

From the practical point of view, it is desirable for the grasped object to have a full mobility six. On the other hand, it is reasonable to specify three as the minimum mobility of the grasped object since the mobility less than three will be of little use. Similar analogy can be applied to the limitation of the finger number. We specify three for the finger number as the lower limit since it is trivial to discuss one or dual fingers. We then specify six as the upper limit since it is reasonable to consider the number greater than human fingers by 1 and too complex to be greater than six. Thus, the limit for the total mobility F and the finger number n are specified as

$$3 \le F \le 6 \qquad (3)$$

$$3 \le n \le 6 \qquad (4)$$

Constraint Condition 2

The mobility equation Eq. (2) provides only an index of moveable condition for the mechanism. It is not sufficient to indicate whether the grasped object can be held securely. To determine if the hand imposes sufficient constraint to immobilize the object completely, one must further count the constraints applied on object. Reuleaux [8] found that at least 4 frictionless point contacts are required to immobilize two-dimensional objects. Somoff [14] and later Lakshminarayana [4] showed that at least 7 frictionless contact points are required to immobilize three-dimensional objects. Much later, Rimon and Burdick[9] and Yashikawa[15] showed that objects with special geometry can be immobilized with number of contacts less than that derived by Reuleaux. In this work, we shall apply the general force closure criteria by Smoff and Lakshminarayana to provide further constraint conditions. Therefore, when the multi-finger hand grasps the object, the constraints on the object should be greater than or equal to 7. This results in

$$n(6 - f) \ge 7 \qquad (5.a)$$

$$\text{or,}\quad f \le (6n - 7)/n \qquad (5.b)$$

where f assumes to the nearest integer number.

Constraint Condition 3

Davies [1] ever studied the mobility relationship between a kinematic chain and its kinematic subchain. It was pointed that kinematic chains of mobility F in which all kinematic subchains have mobility ≥ F form mechanisms that always have total mobility. We wish to synthesize mechanisms as having total mobility. Therefore, the kinematic subchains formed by any two or more fingers contacting with the object should have the mobility greater than or equal to the total mobility. Consider a multi-finger hand of n fingers, the kinematic subchains formed by l fingers should have mobility greater than or equal to the total mobility F. Thus, the mobility of the kinematic subchains formed by 2 fingers contacting with the object is

$$N_i + N_j - 2(6 - f) + 6 \ge F \qquad (6.a)$$

Mobility of 3 fingers contacting with the object,

$$N_i + N_j + N_k - 3(6 - f) + 6 \ge F \qquad (6.b),$$

Or, in a concise form, the mobility of l fingers where ($2 \le l < n$),

$$\overbrace{N_i + N_j + ... + N_x}^{l\ \text{terms}} - l(6 - f) + 6 \ge F \qquad (6.c)$$

The theory developed by Davies did not include the condition of the open kinematic chain. In what follows, we shall deduce the joint number constraint for one finger condition from Eq.(6.a). First we write (n-1) mobility inequalities for the kinematic subchains when finger i and another finger ($\neq i$) contacting with the object

$N_1 \geq (F+6-2f)-N_i$ (7.a)

$N_2 \geq (F+6-2f)-N_i$ (7.b)

……

$N_{i-1} \geq (F+6-2f)-N_i$ (7.c)

$N_{i+1} \geq (F+6-2f)-N_i$ (7.d)

…

$N_n \geq (F+6-2f)-N_i$ (7.e)

Adding the terms on each individual side and an N_i to both sides of the inequalities, one yields:

$$\sum_{m=1}^{n} N_m \geq (n-1)(F+6-2f)-(n-2)N_i \quad (8)$$

Recall from the Eq.(2) that the total joint number is

$$\sum_{m=1}^{n} N_m = F + n(6-f) - 6$$

Substituting the above term into the left term of Inequality (8) yields

$$N_i \geq F - f \quad (9)$$

Since the subscript i is a dummy variable, Equation (9) can be a general constraint for the joint number of a single finger. We note that Eq.(9) can be obtained from Eq.(6.c) if the limit of variable l is extended to ($1 \leq l < n$).

In summary, the constraints for the structural synthesis of multi-finger hands are rewritten as

$$\begin{cases} F = 6 + \sum_{i=1}^{n} N_i - n(6-f) & (2) \\ 3 \leq F \leq 6 & (3) \\ 3 \leq n \leq 6 & (4) \\ f \leq \frac{1}{n}(6n-7) & (5.b) \\ \overbrace{N_i + N_j + \dots + N_x}^{l \text{ terms}} - l(6-f) + 6 \geq F \quad (1 \leq l < n) & (6.c) \end{cases}$$

STRUCTURAL SYNTHESIS

In view of the constraints mentioned above, it can be seen that the determination of the joint numbers of each finger will be the main part of the synthesis procedure. In what follows, we shall demonstrate the procedure by two examples.

Example 1 Synthesize the structures of multi-finger hand with mobility 3 and three fingers.

Step 1. Since F=3 and n=3, find the limit for contact DOF f from Eq.(5.b)

$f \leq 3$ (10)

Equation (10) means the contact DOF between the finger and the object must be less than or equal to 3; otherwise, the hand is unable to restrain the object.

Step 2. Set the values of f. Substituting f=0 into Eq.(2),and (6.c), yields

$N_1 + N_2 + N_3 = 15$ (11)

$N_1 \geq 3$ (12.a)

$N_2 \geq 3$ (12.b)

$N_3 \geq 3$ (12.c)

$N_1 + N_2 \geq 9$ (13.a)

$N_1 + N_3 \geq 9$ (13.b)

$N_2 + N_3 \geq 9$ (13.c)

Step 3. Although there are 6 inequalities in Eqs.(12) and (13), it is not necessary to check all the inequalities for Ni's. If we let $N_1 \leq N_2 \leq N_3$, the inequalities (12.b), (12.c), (13.b), and (13.c) are satisfied accordingly. Adding this constraint can avoid the repetition during the distribution of Ni's. Thus, the admissible solutions for (N_1, N_2, N_3) are (3,6,6), (4,5,6), and (5,5,5).

Step 4. The structures for various f's can be obtained via the same procedure from Step 2 to Step 3. The results are listed in Table 2.

Table. 2 Structures for F=3, n=3

f	(N_1, N_2, N_3)
0	(3,6,6), (4,5,6), (5,5,5)
1	(2,5,5), (3,4,5), (4,4,4)
2	(1,4,4), (2,3,4), (3,3,3)
3	(1,2,3), (2,2,2)

Example 2 Find the structures of multi-finger hand with mobility 6 and four fingers.

Step 1. Find the limit for the contact DOF. We obtain

$f \leq 4$ (14)

Step 2. Let f=0 and $N_1 \leq N_2 \leq N_3 \leq N_4$, we have the joint number equation and inequalities as

$N_1 + N_2 + N_3 + N_4 = 24$ (15)

$N_1 \geq 6$ (16)

$N_1 + N_2 \geq 12$ (17)

$N_1 + N_2 + N_3 \geq 18$ (18)

Solving the above constrains yields (N_1, N_2, N_3, N_4)=(6,6,6,6). Similarly, for various contact DOF f's, the admissible combinations for Ni's are listed in Table 3. It can be noted that the structures of this kind of multi-finger hand have a symmetric form of joint number.

From the above illustration, a systematic procedure for the enumeration of structures subject to the range 3 $\leq F \leq$6 and 3$\leq n \leq$6 can be developed and the admissible structures of multi-finger hands can be obtained. The total number of admissible multi-finger hands with given mobility and finger numbers are listed in the Table 4.

Table. 3 Structures for F=6, n=4

f	(N_1, N_2, N_3, N_4)
0	(6,6,6,6)
1	(5,5,5,5)
2	(4,4,4,4)
3	(3,3,3,3)
4	(2,2,2,2)

WORKSPACE ANALYSIS OF THE PLANAR-HANDS

The workspace of a hand-object system can be defined as the area swept out by one particular point fixed in the object as the object is moved through all possible positions. The workspace is generally determined by kinematic properties of the hand-object system, such as the contact geometry at the contacts, shape of the object, and structure of the hand mechanism (Kerr and Roth, 1986). In what follows, we will assume that the contact geometry between the fingertips and the object is point contact with friction, and the contact points do not change during the manipulation. Under this assumption, the revolute-pairing can be used to replace the contact geometry between the fingertip and the object. In addition, we will only consider the planar hand for our purpose.

A three-fingered planar hand

The workspace for a three-fingered planar hand can be analyzed in a way similar to that for a two-fingered hand. Figure 2(a) shows a three-fingered hand. For the sake of symmetry, the proximal joint of each finger is located on the vertex of an equilateral triangle with edge length L. The links length will also be symmetric for each finger. Points B_1, B_2, B_3 are referred to the contact points and form a grasp triangle. Let P be the centroid of the grasp triangle. Then, for a specified orientation of the grasp triangle, f, the workspace of the point P is to be analyzed. By using the kinematic inversion method, the trace of P generated by $E_1A_1B_1P$ is an annular region centered at C where E_1C_1 is parallel to B_1P. Likewise, the trace of P generated by the other fingers can be obtained in a similar means. Note the annular regions generated by three fingers will be of equal size. In order for the object to have non-empty workspace, the three annuluses must have common intersecting area, i.e., the radii of the outer circles must be greater than that of the circumscribed circle of $C_1C_2C_3$ and radii of the inner circles smaller than that of the circle of $C_1C_2C_3$. This becomes:

$$(l_1 + l_2) \le d/[2\sin(D)] \qquad (19a)$$

$$(l_1 + l_2) \le d/[2\sin(D)] \qquad (19b)$$

where d denotes an edge of the triangle $\Delta C_1C_2C_3$ formed by centers of annuluses and D is the corresponding opposite angle as shown in Fig. 2(b).

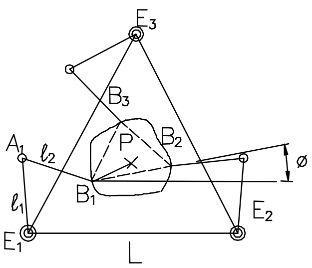

Fig. 2(a)

In case the grasp triangle $\Delta B_1B_2B_3$ is equilateral, the triangle formed by points C_1, C_2, and C_3 will also be equilateral. Let the coordinate system be defined as X-axis pointing along the edge E_1E_2. Then, the coordinates

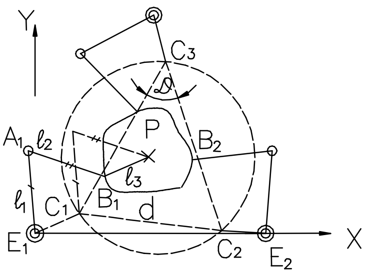

Fig. 2(b)

for the triangle vertices C_1, C_2, and C_3 are:

$$C_1 : [l_3 \cdot \cos(\phi + \pi/6),\ l_3 \cdot \sin(\phi + \pi/6)]$$

$$C_2 : [L - l_3 \cdot \cos(\phi - \pi/6), -l_3 \cdot \sin(\phi - \pi/6)]$$

$$C_3 : [L/2 + l_3 \cdot \cos(\pi/2 - \phi), \sqrt{3}/2 \cdot L - l_3 \cdot \sin(\pi/2 - \phi)]$$

where l_3 is the radius of the circumscribed circle of grasp triangle $\Delta B_1B_2B_3$. Substituting the calculation of d into Eq.(11a&b) and squaring both sides of (11a&b), yields:

$$(l_1 + l_2)^2 \ge l_3^{\,2} + L^2/3 - 2/\sqrt{3} \cdot l_3 \cdot L\cos(\phi) \qquad (20a)$$

$$(l_1 - l_2)^2 \le l_3^2 + L^2/3 - 2/\sqrt{3} \cdot l_3 \cdot L\cos(\phi) \qquad (20b)$$

While equality holds, Eqs.(20a) and (20b) denote a family of hyperbolas in l_3-(l_1+l_2) and l_3-$|l_1-l_2|$ space, respectively. It can also be noted that both equations degenerate into two straight lines at $\phi = 0$ and $\phi = \pi$. In order for the object to have least non-empty workspace, i.e., occurred at $\phi = 0$, Eq. (20a) can be deduced to:

$$Max\{0, -(l_1 + l_2) + \frac{L}{\sqrt{3}}\} \le l_3 \le (l_1 + l_2) + \frac{L}{\sqrt{3}} \qquad (21)$$

On the other hand, Eq. (20b) may be rearranged as:

$$(l_1 - l_2)^2 \le (l_3 - L/\sqrt{3})^2 \qquad (22)$$

Solving the admissible range for l_3 results in

If $l_3 \ne L/\sqrt{3}$, then $l_3 \le L/\sqrt{3} - |\ l_1 - l_2\ |^2$ or

$$l_3 \ge |\ l_1 - l_2\ |^2 + L/\sqrt{3} \qquad (23a)$$

Else $l_1 = l_2$ (23b)

Solving the intersecting ranges of Eq. (21) and (23a, b), one can obtain the feasible range of l_3:

If $l_3 \ne L/\sqrt{3}$, then l_3 must fall within the following two inequalities:

$$L/\sqrt{3} + |\ l_1 - l_2\ | \le l_3 \le L/\sqrt{3} + |\ l_1 + l_2\ | \qquad (24a)$$

or

$$Max\{0, L/\sqrt{3} - (l_1 + l_2)\} \le l_3 \le L/\sqrt{3} - |\ l_1 - l_2\ | \qquad (24b)$$

If $l_3 = L/\sqrt{3}$, then l_1 must be equal to l_2 and the workspace always exists. (24c)

Equation (24) gives a necessary condition for the range of the object size with respect to the hand mechanism geometry in order for the object to have least non-empty workspace. Likewise, in order for the object to have maximum orientation range and non-empty workspace, i.e., occurred at $\phi = \pi$, Eqs. (20a) and (20b) yield to:

$$Max\{0, | l_1 - l_2 | - L/\sqrt{3}\} \le l_3 \le (l_1 + l_2) - L/\sqrt{3} \quad (25)$$

The object satisfying the above condition will have a non-empty workspace for an orientation.

CONCLUSIONS

In this work, the structural synthesis of multi-finger hands has been studied via the mobility equation and force closure theory. Certain constraints for the synthesis procedure have been established and the admissible multi-finger hand structures within the limits $3 \le F \le 6$ and $3 \le n \le 6$ have been enumerated. It is hoped that the result will help designers in the initial stage of designing multi-finger hands.

The workspace of planar hand-object mechanisms was investigated via the kinematic inversion method. Criteria for obtaining non-empty workspace were also established. In order for the object to have least non-empty workspace and maximum orientation range, geometry constraints between the object and the hand mechanism must fulfill Eq. (24-25) for the three-fingered one. It is hoped that this work will help designers realize the topological as well as geometry characteristics of hand-object mechanisms in the initial stage of designing mechanical hands.

REFERENCE

Ý Davies T., 1968, “An Extension of Manolescu’s Classification of Planar Kinematic Chains and Mechanisms of Mobility M ≥ 1, Using Graph theory,” J. of Mechanisms, Vol. 3, pp.87-100.

Ý Hunt, K. H., 1978, Kinematic Geometry of Mechanisms, Oxford University Press.

Ý Jacobsen, S.C., Wood, J.E., Knutti, D.F. and Biggers, K.B., 1984, “The Utah/MIT Dexterous Hand,” Int’l J. of Robotics Research, Vol. 3, No. 4, pp.21-50.

Ý Lakshminarayana, K., 1978, "Mechanics of Form Closure," ASME Paper No. 78-DET-32.

Ý Lin, L. R. and Huang, H. P., 1996, "Mechanism Design of a New Multifingered Robot Hand," IEEE Intl. Conference on Robotics and Automation, April 21-27, Minneapolis, U.S.A., pp.1471-1476.

Ý Okada, T., 1979, “Computer Control of Multi-Jointed Finger System,” Sixth Int’l Joint Conference on Artificial Intelligence, Tokyo, Japan.

Ý Pham, D. T. and Heginbotham, W. B., 1986, *Robot Grippers*, IFS (Publications) Ltd, UK.

Ý Reuleaux, F., 1963, The Kinematics of Machinery, Dover Publications.

Ý Rimon, E. and Burdick, J., 1996, “On Force and Form Closure for Multiple Finger Grasps,” IEEE Intl. Conference on Robotics and Automation, pp.1795-1800.

Ý Rouff, C. F. and Salisbury, J. K., 1990, “Multi-Fingered Robotic Hand,” U.S. Patent 4,921,293.

Ý Salisbury, J.K., 1982, "Kinematic and Force Analysis of Articulated Hands," Ph.D. Thesis, Dept. of Mechanical Engineering, Stanford University.

Ý Salisbury, J. K. and Roth, B., 1983, “Kinematic and Force Analysis of Articulated Mechanical Hands, ” ASME J. of Mechanisms, Transmissions and Automation in Design, Vol. 105, March.

Ý Skinner, F., 1975, “Designing a Multiple Prehension Manipulator,” Mechanical Engineering, September, pp.30-37.

Ý Somov, P., 1897, “Über Schraubengeschwindigkeiten eines festen Körpers beiverschiedener Zahl von Stützflachen,” *Zeitschrift für Mathematik und Physik*, V. 42, pp. 133-153, 161-182.

Ý Yashikawa, T., 1996, “Passive and Active Closures by Constraining Mechanisms,” IEEE Intl. Conference on Robotics and Automation, pp.1477-1484.

Table. 4 The total number of admissible structures of multi-finger hands with given mobility and finger numbers.

Mobility	Finger number	Contact freedom	Admissible structures of multi-finger hands
3	3	0	(3,6,6), (4,5,6), (5,5,5)
		1	(2,5,5), (3,4,5), (4,4,4)
		2	(1,4,4), (2,3,4), (3,3,3)
		3	(1,2,3), (2,2,2)
	4	0	(3,6,6,6), (4,5,6,6), (5,5,5,6)
		1	(2,5,5,5), (3,4,5,5), (4,4,4,5)
		2	(1,4,4,4), (2,3,4,4), (3,3,3,4)
		3	(1,2,3,3), (2,2,2,3), (1,1,1,2)
	5	0	(3,6,6,6,6), (4,5,6,6,6), (5,5,5,6,6)
		1	(2,5,5,5,5), (3,4,5,5,5), (4,4,4,5,5)
		2	(1,4,4,4,4), (2,3,4,4,4), (3,3,3,4,4)
		3	(1,2,3,3,3), (2,2,2,3,3)
		4	(1,1,1,2,2)
	6	0	(3,6,6,6,6,6), (4,5,6,6,6,6), (5,5,5,6,6,6)
		1	(2,5,5,5,5,5), (3,4,5,5,5,5), (4,4,4,5,5,5)
		2	(1,4,4,4,4,4), (2,3,4,4,4,4), (3,3,3,4,4,4)
		3	(1,2,3,3,3,3), (2,2,2,3,3,3)
		4	(1,1,1,2,2,2,)
4	3	0	(4,6,6), (5,5,6)
		1	(3,5,5), (4,4,5)
		2	(2,4,4), (3,3,4)
		3	(1,3,3), (2,2,3)
	4	0	(4,6,6,6), (5,5,6,6)
		1	(3,5,5,5), (4,4,5,5)
		2	(2,4,4,4), (3,3,4,4)
		3	(1,3,3,3), (2,2,3,3)
		4	(1,1,2,2)
	5	0	(4,6,6,6,6), (5,5,6,6,6)
		1	(3,5,5,5,5), (4,4,5,5,5)
		2	(2,4,4,4,4), (3,3,4,4,4)
		3	(1,3,3,3,3), (2,2,3,3,3)
		4	(1,1,2,2,2)
	6	0	(4,6,6,6,6,6), (5,5,6,6,6,6)
		1	(3,5,5,5,5,5), (4,4,5,5,5,5)
		2	(2,4,4,4,4,4), (3,3,4,4,4,4)
		3	(1,3,3,3,3,3), (2,2,3,3,3,3)
		4	(1,1,2,2,2,2,)
5	3	0	(5,6,6)
		1	(4,5,5)
		2	(3,4,4)
		3	(2,3,3)
	4	0	(5,6,6,6)
		1	(4,5,5,5)
		2	(3,4,4,4)
		3	(2,3,3,3)
		4	(1,2,2,2)
	5	0	(5,6,6,6,6)
		1	(4,5,5,5,5)
		2	(3,4,4,4,4)
		3	(2,3,3,3,3)
		4	(1,2,2,2,2)
	6	0	(5,6,6,6,6,6)
		1	(4,5,5,5,5,5)
		2	(3,4,4,4,4,4)
		3	(2,3,3,3,3,3)
		4	(1,2,2,2,2,2,)
6	3	0	(6,6,6)
		1	(5,5,5)
		2	(4,4,4)
		3	(3,3,3)
	4	0	(6,6,6,6)
		1	(5,5,5,5)
		2	(4,4,4,4)
		3	(3,3,3,3)
		4	(2,2,2,2)
	5	0	(6,6,6,6,6)
		1	(5,5,5,5,5)
		2	(4,4,4,4,4)
		3	(3,3,3,3,3)
		4	(2,2,2,2,2)
	6	0	(6,6,6,6,6,6)
		1	(5,5,5,5,5,5)
		2	(4,4,4,4,4,4)
		3	(3,3,3,3,3,3)
		4	(2,2,2,2,2,2,)