行政院國家科學委員會補助專題研究計畫成果報告

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計畫編號：NSC　90-2212-E-002-212
執行期間：90 年 8 月 1 日　至　91 年 7 月 31 日

計畫主持人：陳達仁

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
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□國際合作研究計畫國外研究報告書一份

執行單位：國立台灣大學機械系

中　華　民　國　91 年　9 月　1 日

人工導引卸載裝置的設計與實作：彈簧平衡機構的構想設計與實作

Design of Personal Guided Vehicle: Conceptual Design of Spring Counter-balanced Mechanisms

計劃編號：NSC 90-2212-E-002-212
執行期間：九十年八月一日至九十一年七月三十一日
主持人：陳達仁，國立台灣大學機械系教授
計畫參與人員：林宏舜、陳志龍

摘要

本研究利用原有之靜平衡機構，在限定彈簧之彈性係數為常數下找出基本之二桿靜平衡機構的構型特徵及彈簧參數化設計公式；藉由此構型特徵而由基本之二桿平衡機構找出構型之條件去推導單自由度四至六桿平衡機構之構型特徵，再以此特徵推得四桿及六桿之單自由度平衡機構之可行構型，最後根據彈簧設計公式設計得出之四、六桿構型之彈簧彈性係數，即可得到一個簡單的單自由度六桿彈簧平衡機構；最後，可將此單自由度之平衡機構視為一個單元，以兩種不同之方法－串連與並聯－予以合成，將各個單元結合起來，即可得到一多自由度之平衡機構，或可將原來平面中桿件的關係延伸至空間中面的關係，而合成為一空間中的多自由度機構。因此，可利用本研究所提出之設計方法方便設計出一平衡機構，即可符合工作搬運的各項需求，如所需構型設計、可靠的搬運、、複雜的動作、簡單的動力設計等，亦可使設計的過程正確而快速。

ABSTRACT

A systematic procedure for the design in one-dof static balancing mechanisms is developed in this paper. Structural characteristics for the two-link static balancing mechanism are investigated, from which general structural characteristics for up-to-six link static balancing mechanisms are generated. According to these structural characteristics, it is shown that the configuration of a static balancing mechanism is a parallel mechanism and the admissible graphs for parallel mechanisms can be obtained. From the admissible graph, an atlas of valid configurations for six-link static balancing mechanism is obtained. It will be shown that a static balancing mechanism can be integrated by connecting multiple one-dof static balancing mechanisms to adapt to versatile design requirements. It is believed that the proposed approach provides better physical comprehension in the design process and is beneficial to extent the application of static balancing mechanisms.

1. INTRODUCTION

For pick-and-place objects securely, an exterior balancing moment is required to be exerted to cope with the moment due to the weight of links and payload in a mechanism. A static balancing mechanism can be balanced at arbitrary position without exterior moments. Based on concept of the self-balance, the static balancing mechanism is developed by Nathan (1985). With a constant force acting on a rotating link, Nathan (1985) configured a zero-free-length spring to automatically equilibrate weight of links and payload at arbitrary position. The zero-free-length spring is also referred to as the ideal spring, and has been extensively discussed by Soper et al. (1997).

The concept of static balancing mechanism has been applied to various kinds of mechanisms. Streit and Gilmore (1989) studied the static balancing of a rigid body fixed at one point with multiple normal springs. Fisher (1992) made a surgical light statically balanced by the friction moment and a normal spring. Shin and Streit (1991) statically balanced pantographs. Idlani, et al. (1993) designed an equilibrator to statically balance an arbitrary link of various planar linkages. French and Widden (2000) poised a 5-link, 2-dof mechanism statically with two ideal springs. By arranging parallelogram linkages in series, Rahman, et al. (1995) constructed an n-link static balancing mechanism to statically balance the self-weight. Wang and Gosselin (1998) and Ebert-Uphoff, et al. (2000) applied the concept of static balancing mechanisms to statically balance a spatial parallel mechanism. Above studies are stressed on the derivation of spring constants for a given configuration. However, no complete design procedure of a static balancing mechanism is provided in these studies.

The objective of the study is to propose a systematic design procedure for up-to-six link static balancing mechanisms. Through the design procedure, a new static balancing mechanism with a heavy payload can be constructed with the required configuration. A multi-dof static balancing mechanism can be obtained by connecting many one-dof modules.

2. TWO-LINK STATIC BALANCING MECHANISM

As shown in Fig. 1, the 2-link pick-and-place mechanism picks heavy payloads, m_p, with its end, and m_1 is the mass of link 1, which is called the primary link, and β is the angle from the perpendicular to the ground. Since the primary link is statically balanced for arbitrary configuration at rest, there is no change in potential energy (Ebert-Uphoff, et al., 2000). However, gravitational potential energy, e_g, is dependent on the configuration of the mechanism and e_g can be written as

$$e_g = M \cdot g \cdot r_1 \cdot \cos\theta_1 \tag{1}$$

where

$$M = m_p + m_1 / 2 \tag{2}$$

It can be observed from Eq. (1) that e_g varies with the change of rotating angle, θ. Hence, potential energy can not keep constant as only gravitational potential energy is taken into account.

To keep potential energy conservative for arbitrary rest configuration, the energy change in e_g can be compensated by installing springs into the mechanism. The spring provides elastic potential energy, e_s, which can be designed to fulfill the following condition:

$$e_p = e_g + e_s = \text{const.} \tag{3}$$

where e_p is the total potential energy.

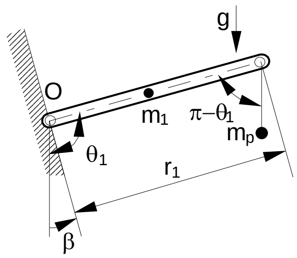


Figure 1: A two-link pick-and-place mechanism

Spring installation

From Eq. (1), it can be seen that eg is a function of cos 1. Hence, es is also required as a function of cos 1 such that Eq. (3) can be satisfied. Elastic potential energy of the helical spring can be made as a function of cos 1 with a zero-free-length spring with a constant stiffness (Nathan, 1985), and such a spring is referred to as an ideal helical spring. Hence, only ideal helical springs are considered in the following.

For a two-link pick-and-place mechanism with only revolute joints, the spring installation is discussed in the following.

For compensating the change in gravitational potential energy, the spring should be stretched as primary link rotates downward. In order to achieve the requirement, the spring is installed as shown in Fig. 2, where spring R is the ideal helical spring with spring constant K_R, and a, b are the installation lengths. From Fig. 2, it can be seen that the extension of spring R, δL, can be written as

$$\delta L = \sqrt{(a^2 + b^2 - 2a \cdot b \cdot \cos(\pi - (\theta_1 - \beta)))} \tag{4}$$

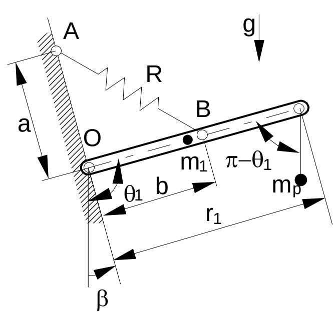


Figure 2: A 2-link static balancing mechanism with a R-joint

For the 2-link model, total potential energy, e_p, can be written as

$$e_p = M \cdot g \cdot r_1 \cdot \cos\theta_1 + K_R (a^2 + b^2 - 2a \cdot b \cdot \cos(\theta_1 - \beta))/2 \tag{5}$$

The partial derivative of e_p to θ_1 can be written as

$$\frac{\partial e_p}{\partial \theta_1} = -M \cdot g \cdot r_1 \cdot \sin\theta_1 + K_R \cdot a \cdot b \cdot \sin(\theta_1 - \beta) \tag{6}$$

In order to make total potential energy conservative for arbitrary θ_1, Eq. (6) is set as zero, from which K_R can be rearranged as

$$K_R = M \cdot g \cdot r_1 /(a \cdot b) \cdot (\sin\theta_1 / \sin(\theta_1 - \beta)) \tag{7}$$

Since spring R is an ideal helical spring, K_R in Eq. (7) should be a constant, from which it can be seen that β should be zero and thus line OA is parallel to the gravity. From Fig. 2, it can be seen that line OA should be the ground link since all links are assumed as straight links.

By setting β as zero in Eq. (7), K_R can be determined as

$$K_R = M \cdot g \cdot r_1 /(a \cdot b) \tag{8}$$

It is shown that the structural characteristic for a 2-link static balancing mechanism with revolute joint is that the ground link should be parallel to the payload.

Admissible graphs for up-to-six link mechanisms

In order to expand the workspace without reducing the stiffness, up-to-six link static balancing mechanisms are considered in this paper. By specializing the payload in a 2-link static balancing mechanism as a rigid link, which is called the distal link, the 2-link model can be expanded to 1-dof up-to-six link static balancing mechanism in virtue of forming parallel mechanisms with the distal link parallel to the ground and letting lighter links other than the primary link. Note that lighter links are referred to as secondary links. In an up-to-six link parallel mechanism as shown in Fig. 3, the distal link, AB, is parallel to the ground link and the angle, θ, measured from the perpendicular to the distal link is called the distal angle, which should be equal to π for arbitrary input. The generalized structural characteristic of a static balancing mechanism can be rewritten as

C1. The ground link is parallel to the distal link, and they re connected via the primary link

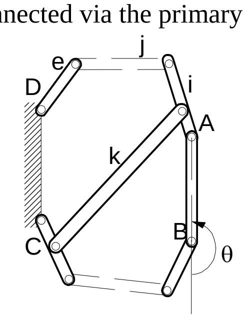


Figure 3: An up-to-six link static balancing mechanism

From C1, for 4-link case, admissible graphs can be identified from 4-link kinematic chains as shown in Fig. 4 by identifying two non-connecting links as the ground link and the distal link. As shown in Fig. 5, one admissible 4-link graph can be obtained by identifying link 3 as the distal link

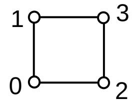


Figure 4: A four-link kinematic chain

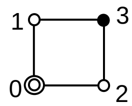


Figure 5: An admissible four-link graph

There are two non-isomorphic 6-link kinematic chains, i.e. Watt's type and Stephenson's type as shown in Fig. 6. In the Watt's mechanism as shown in Fig. 6, links 0, 2, 4 and 5 are topologically symmetric and links 1 and 3 are topologically symmetric. In the Stephenson's mechanism as shown, links 0, 3, links 2, 4, and links 1,5 are topologically symmetric respectively. Hence, the ground link in Watt's mechanism can be assigned on non-topological symmetric links, 0 and 1 and the ground link in Stephenson's mechanism can be assigned on non-topological symmetric links, 0, 1 and 4. For 1-dof 6-link mechanisms, as the parallel links are in the same 4-link loop, the other loop is redundant in, and thus a generalized characteristic can be obtained as

C2. The distal link and the ground link cannot be in the same 4-link loop.

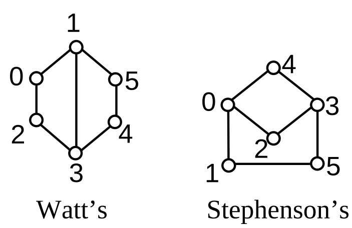


Figure 6: Six-link kinematic chains

As shown in Fig. 7, admissible graphs for up-to-six link parallel mechanisms are generated from the existing atlas via C1 and C2. Generally, it can be said that the distal link and the ground link are connected via primary linkage, which is composed of one link or two links. In w-2, it is noted that the ground link and the distal link are connected via primary linkage with two links

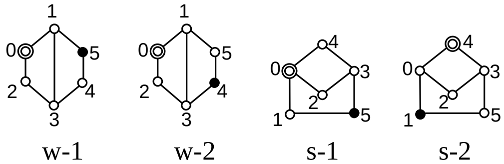


Figure 7: Admissible six-link graphs

<u>The properties of links</u>

For binary link i, $\vec{r}_i$ is the vector from the pre-connecting joint to the post-connecting joint and θ_i is the angle measured from the perpendicular to link i. Joint ij are the joint between link i and link j. In a one-dof, 6-link mechanism, the links, which are shared by both loops, are called the common links. In a 1-dof, 6-link mechanism, a ternary link must be a common link. In order to define a ternary link in a 6-link mechanism, the reference point is referred to be the joint connecting with common links. For ternary link i, $\vec{v}_{ij}$ is the vector from the reference point to joint ij. In a ternary link, the two joint described by the two vectors may be in the same side or in the opposite side of the reference point. Hence, for the both vectors in a ternary link, both angles are the same as the reference is an end joint and the difference between both angles is π as the reference point is in the middle. The positions of the reference points on ternary links differ from the configuration of the distal loop. From Fig. 3, it can be seen that $\vec{r}_j$ is the vector from joint ej to joint ij. For ternary link i in Fig. 3, the reference point is joint ik and $\vec{v}_{ij}$ is the vector from joint ik to joint ij.

3. CONFIGURATION SYNTHESIS FOR FOUR-LINK PARALLEL MECHANISM

As shown in Fig. 8, the admissible 4-link mechanism as shown in Fig. 5 is represented schematically. With only lower pairs in this mechanism, the distal angle can be represented as a function of the input angle. Corresponding to Fig. 8, the loop closure equation defining the constraint between the angles can be written as

$$\vec{r}_0 + \vec{r}_2 - \vec{r}_1 - \vec{r}_3 = 0 \tag{9}$$

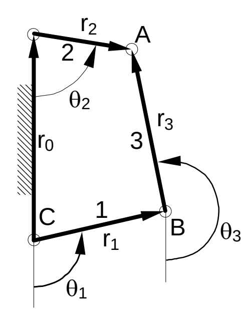


Figure 8: Schematic representation of admissible 4-link graph

From Eq. (9), the scalar components of obtained loop closure equation can be rewritten as

$$r_0\cos\theta_0 + r_2\cos\theta_2 - r_1\cos\theta_1 - r_3\cos\theta_3 = 0 \tag{10a}$$

and

$$r_0\sin\theta_0 + r_2\sin\theta_2 - r_1\sin\theta_1 - r_3\sin\theta_3 = 0 \tag{10b}$$

From Eqs. (10), the representation of θ_3 can be written as

$$\tan(\theta_3/2) = (a \pm \sqrt{a^2 + b^2 - c^2})/(b + c) \tag{11}$$

where

$$a = \sin\theta_1 \quad (12a)$$
$$b = \cos\theta_1 + r_0 / r_1 \quad (12b)$$

and

$$c = \frac{r_0}{r_3}\cdot\cos\theta_1 + \frac{r_0^2 + r_1^2 + r_3^2 - r_2^2}{2r_1\cdot r_3} \quad (12c)$$

By introducing the semi-angle trigonometric function, θ_3 can be rewritten as an implicit function of θ_1, which can be expressed as

$$r_0^2 + r_1^2 + r_3^2 - r_2^2 + 2r_0 r_1 \cos\theta_1 + 2r_0 r_3 \cos\theta_3 + 2r_1 r_3 \cos(\theta_1 - \theta_3) = 0 \quad (13)$$

In this mechanism, the distal angle, θ_3, is required as π for arbitrary input. By setting θ_3 as π, Eq. (13) can be modified as

$$C_1\cdot\cos^2\theta_1 + C_2\cdot\cos\theta_1 + C_3 = 0 \quad (14)$$

Since Eq. (14) is valid for arbitary θ_3, C_i's should be zero, which leads to

$$r_0 = r_3 \quad (15a)$$

and

$$r_1 = r_2 \quad (15b)$$

By substituting Eqs. (15a) into (15b) into C_3, C_3 is zero. As shown in Fig. 9, the 4-link mechanism, which is generated from Eqs. (15a) and (15b), constitutes a parallelogram. In this mechanism, links 0 and 3 are parallel for arbitrary configuration.

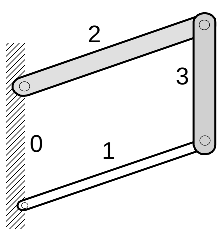


Figure 9: Functional representation of 4-link parallel mechanism

4. CONFIGURATION SYNTHESIS FOR SINGLE-DOF SIX-LINK PARALLEL MECHANISM

In a 6-link parallel mechanism, the reference link is referred to be the ground link in the distal loop. In this loop, angles of the other links are measured from the line parallel to the reference link. In order to derive of the representation of the distal angle for a 1-dof 6-link parallel mechanism, steps are describes as follows.

Step 1: Assign the reference link and the distal link for each loop

For the base loop, the common link in the contraposition to the ground link is taken as the distal link. For the distal loop, the common link in the contraposition to the distal link is taken as the reference link.

Step 2: Determine the position of reference points on ternary links

The position of the reference points on both ternary links should be discussed for two cases (i) the middle joint (ii) an end joint.

Step 3: Represent the distal angle for each loop in terms of the input link

With loop closure equations, the distal angle can be represented as a function of an input angle for each loop.

Step 4: Rewrite the distal angle as a function of the input angle

By substituting the relations for the base loop into the relations for the distal loop, the distal angle can be rewritten as an function of the input angle.

<u>Graph w-1</u>

For graph w-1, ternary links 1 and 3 the common links. In the distal loop, link 3 is taken as the distal link in the base loop and the reference link. For ternary links, four cases about the position of the reference points on both links are discussed as follows.

(i) Reference points are the middle joints of link 1 and an end joint of link 3.

As shown in Fig. 10, graph w-1 is represented schematically and link 1 and 3 are the common links. Based on derivation process of the 4-link parallel mechanism, the distal angle can be represented as a function of the input angle for each loop.

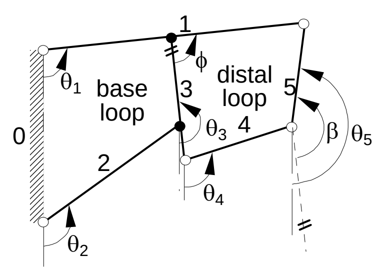


Figure 10: Schematic representation of graph w-1

For the distal loop, the distal angle, β, is represented as a function of ϕ as

$$\tan(\frac{\beta}{2}) = \frac{a_3 \pm \sqrt{a_3^2 + b_3^2 - c_3^2}}{b_3 + c_3} \quad (16)$$

where

$$a_3 = \sin\phi \quad (17a)$$
$$b_3 = \cos\phi - v_{34} / v_{15} \quad (17b)$$

and

$$c_3 = -\frac{v_{34}}{r_5}\cdot\cos\phi + \frac{v_{15}^2 + v_{34}^2 + r_5^2 - r_4^2}{2v_{15}\cdot r_5} \quad (17c)$$

Similarly, for the base loop, θ_3 can be represented as a function of θ_1 as

$$\tan(\frac{\theta_3}{2}) = \frac{a_1 \pm \sqrt{a_1^2 + b_1^2 - c_1^2}}{b_1 + c_1} \quad (18)$$

where

$$a_1 = \sin\theta_1 \tag{19a}$$

$$b_1 = \cos\theta_1 + r_0 / v_{10} \tag{19b}$$

and

$$c_1 = \frac{r_0}{v_{32}} \cdot \cos\theta_1 - \frac{v_{32}^2 + v_{10}^2 + r_0^2 - r_2^2}{2v_{32} \cdot v_{10}} \tag{19c}$$

Since link 3 is the reference link, relations between the angles in the distal loop can be written as

$$\beta = \theta_5 - \theta_3 \,;\, \phi = \theta_1 - \theta_3 \tag{20}$$

By introducing the semi-angle trigonometric function, Eq. (18) can be rewritten as

$$\frac{2(a_3 \pm \sqrt{a_3^2 + b_3^2 - c_3^2})}{b_3 + c_3} = (1 + (\frac{2(a_3 \pm \sqrt{a_3^2 + b_3^2 - c_3^2})}{b_3 + c_3})^2)\sin\beta \tag{21}$$

With the same derivation procedure in 4-link parallel mechanism, the distal angle, θ_5, can be represented as an implicit function of the input angle, θ_1, by substituting Eqs. (17), (18), (19) and (20) into Eq. (21). In this mechanism, θ_5 should be π for arbitrary θ_1. Based on the 4-link parallel mechanism, the representation of θ_5 can be rewritten as the constraint polynomial by setting θ_5 as π, which can be written as

$$(C_1 \cdot \cos\theta_1 + C_2)^2 (C_3 \cdot \cos^3\theta_1 + C_4 \cdot \cos^2\theta_1 + C_5 \cdot \cos\theta_1 + C_6) = 0 \tag{22}$$

where C_i is the coefficient formed by link lengths.

By setting C_i's as zero, a set of relations between the link-lengths are obtained as

$$v_{34} / v_{32} = r_5 / r_0 = r_4 / r_2 = v_{15} / v_{10} \tag{23}$$

Thus, Eq. (23) can be used to design a 1-dof 6-link parallel mechanism from w-1. In Eq. (23), the design variable is the ratio between the link-lengths, such as r_5/r_0, and it is called the parallel factor. For case (i), the admissible configurations are shown in Table 1(a).

(ii) Reference points are the middle joints of both links.

As shown in Fig.11, the difference in angles of vectors of link 3 is π since the reference point of link 3 is the middle joint. Hence, for the distal loop, the distal angle, β, is represented as a function of ϕ as Eq. (16) by changing a_3, b_3 and c_3 as

$$a_3 = \sin\phi \tag{24a}$$

$$b_3 = \cos\phi + v_{34} / v_{15} \tag{24b}$$

and

$$c_3 = -\frac{v_{34}}{r_5} \cdot \cos\phi - \frac{v_{15}^2 + v_{34}^2 + r_5^2 - r_4^2}{2v_{15} \cdot r_5} \tag{24c}$$

With the same approach, the constraint polynomial is the same as Eq. (22). By setting C_i's as zero, available relations are the same as Eq. (23). For case (ii), the admissible configurations are shown in Table 1(b).

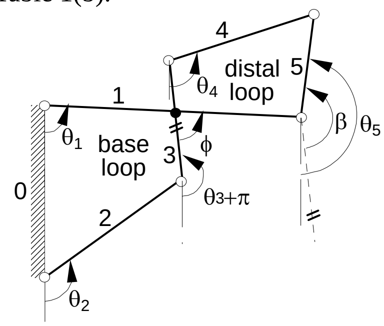


Figure 11: Schematic representation of graph w-1 for case (ii)

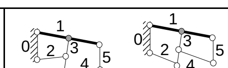
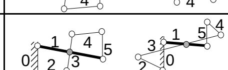
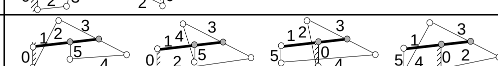
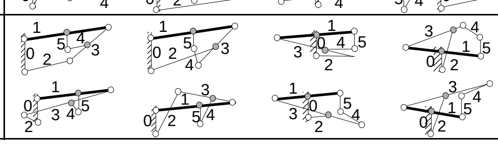
(iii) Reference points are an end joint of link 1 and the middle joint of link 3.

Since the reference point of link 3 is the middle joint, the difference in angles of vectors of link 3 is π. For link 1, the difference in angles of vectors on it is π. For the distal loop, the distal angle, β, is represented as a function of ϕ as Eq. (16) and Eqs. (17), and thus the constraint polynomial is the same as Eq. (22). Hence, valid relations between link-lengths obtained are the same as Eq. (23). For this case, the admissible configurations are shown in Table 1(c).

(iv) Reference points are end joints of link 1 and 3

Since the reference point of link 1 is an end joint, the difference in angles of vectors of link 1 is π. For the distal loop, the distal angle, β, is represented as a function of ϕ as Eq. (16) and Eqs. (24) in case (ii). Hence, the constraint polynomial is the same as that obtained in case (ii) and relations between link-lengths obtained are the same as Eq. (23). For this case, the admissible configurations are shown in Table 1(d).

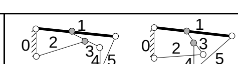
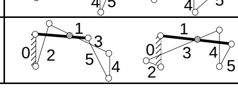
Table 1 Admissible configurations for graph w-1

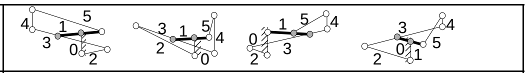
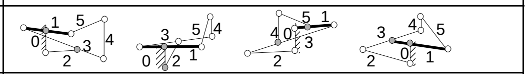
(a)	
(b)	
(c)	
(d)	

Graph w-2

Following the same procedure, admissible configurations for graphs w-2 can be obtained as shown in Table 2.

Table 2 Admissible configurations for graph w-2

(a)	
(b)	

(c)	
(d)	

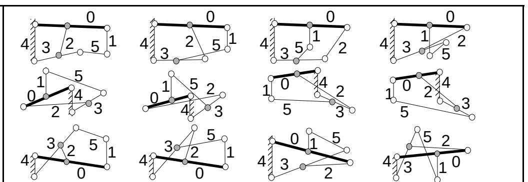
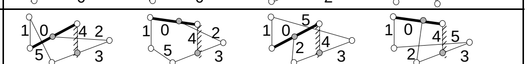
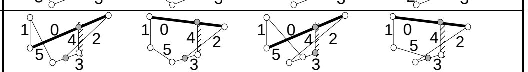
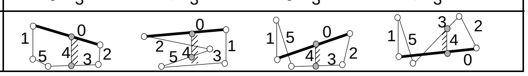
Graph s-1

No configuration without redundant loop can be generated from graph s-1.

Graph s-2

Following the same procedure, admissible configurations for graphs s-2 can be obtained as shown in Table 2.

Table 3 Admissible configurations for graph s-2

(a)	
(b)	
(c)	
(d)	

5. ONE-DOF 6-LINK STATIC BALANCING MECHANISM

In a parallel mechanism, there are two types of connecting primary linkage (i) a primary link only, (ii) two primary links. In order to apply the spring arrangement of two-link static balancing mechanism, the two conditions will be discussed as follows.

(i) Only a primary link

For the 4-link parallel mechanism and w-1, s-1 and s-2 in Fig. 7, there is only a primary link between the distal link and the ground link. In these parallel mechanisms, m_L is the mass of the primary link, m_p is the mass of the payload, m_d is the mass of the distal link, r_L is link-length of the primary link, r_d is link-length of the distal link, and θ_L is angle of the primary link. Since the distal link and the ground link is connected via only a primary link, gravitational potential energy of these mechanism can be written as

$$e_g = \left(\frac{m_L}{2} + m_d + m_p\right)\cdot g \cdot r_L \cdot \cos\theta_L + m_d \cdot g \cdot \frac{r_d}{2} + m_p \cdot g \cdot r_d \tag{25}$$

The partial derivative of gravitational potential energy to θ_L can be written as

$$\frac{\partial e_g}{\partial \theta_L} = \left(\frac{m_L}{2} + m_d + m_p\right)\cdot g \cdot r_L \cdot \sin\theta_L \tag{26}$$

From Eq. (26), it can be seen that the partial derivative of gravitational potential energy can be made to be the same as the partial derivative of gravitational potential energy in Eq. (6) in two-link static balancing mechanism by rewriting the system mass as

$$M = m_L / 2 + m_d + m_p \tag{27}$$

Accordingly, the equation of spring constant of the two-link static balancing mechanism can be applied to the up-to-six link static balancing mechanism. Based on the 2-link static balancing mechanism, the spring is installed to the 6-link static balancing mechanism with installation lengths, a and b, and the spring constant, k, can be rewritten as

$$k = M \cdot g \cdot r_L / (a \cdot b) \tag{28}$$

where a and b are the installation lengths.

(ii) Two primary links

In the parallel mechanism w-2, the ground link and the distal link are connected via two primary links. In graph w-2, link 1 and link 5 are primary links, link 4 is the distal link, m_i is mass of link i, m_p is the mass of the payload, r_i is the link-length of link i, and θ_i is angle of link i. For this mechanism, gravitational potential energy can be written as

$$\begin{aligned} e_g = [\, & m_1 \frac{r_1}{2}\cos\theta_1 + m_5 (r_1 \cdot \cos\theta_1 + \frac{r_5}{2}\cdot\cos\theta_5) \\ & + m_4 (r_1 \cdot \cos\theta_1 + r_5 \cdot \cos\theta_5 + \frac{r_4}{2}) \\ & + m_p (r_1 \cdot \cos\theta_1 + r_5 \cdot \cos\theta_5 + r_4)] \cdot g \end{aligned} \tag{29}$$

Since this mechanism has one dof, θ_5 can be written as a function of θ_1 and thus Eq. (29) can be rewritten as

$$\begin{aligned} e_g = [\, & (\frac{m_1}{2} + m_5 + m_4 + m_p) r_1 \cos\theta_1 \\ & + (\frac{m_5}{2} + m_4 + m_p) r_5 \cdot \frac{f_1 + \sqrt{-f_1^2 + f_2^2}}{f_3} \\ & + (\frac{m_4}{2} + m_p) r_4] \cdot g \end{aligned} \tag{30}$$

where f_i's are composed of functions of $\cos\theta_1$ and $\sin\theta_1$.

Since elastic potential energy of an ideal helical spring can only be made as a function of $\cos\theta_1$, the change of gravitational potential energy of this mechanism cannot be compensated by arranging an ideal helical spring. Thus, the parallel mechanism, w-2, cannot be used to construct a one-dof six-link static balancing mechanism.

By attaching the spring with required spring constant to the configuration, a 1-dof 6-link static balancing mechanism can be constructed to pick-and-place objects securely. For a parallel mechanism from graph w-1, the ideal helical spring is attached to the ground link, 0, and to the primary link, 1, as shown in Fig. 12, by applying the spring arrangement in two-link static balancing mechanism. In Fig. 12, link 5 is the distal link, m_i is the mass of link i, m_p is the mass of the payload, r_i is link-length of link i, and a, b are the installation lengths. In this six-link static balancing mechanism, total potential energy can be written as

$$e_p = (\frac{m_1}{2} + m_5 + m_p) \cdot g \cdot r_1 \cdot \cos\theta_1$$
$$+ m_5 \cdot g \cdot \frac{r_5}{2} + m_p \cdot g \cdot r_5$$
$$+ \frac{k}{2}(a^2 + b^2 - 2a \cdot b \cdot \cos\theta_1) \quad (31)$$

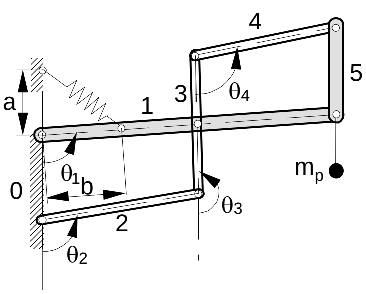


Figure 12: One-dof 6-link static balancing mechanism of w-1

In order to make e_p is conservative for arbitrary θ_1, the partial derivative of e_p with respect to θ_1 should be equal to zero. By setting the partial derivative of Eq. (31) with respect to θ_1 as zero, the spring constant, k, can be written as

$$k = (m_1 / 2 + m_5 + m_p) \cdot g \cdot r_1 /(a \cdot b) \quad (32)$$

For another parallel mechanism from graph s-2, the ideal helical spring is attached to the ground link, 4, and to the primary link, 0, as shown in Fig. 13, by applying the spring arrangement in two-link static balancing mechanism. In Fig. 13, link 5 is the distal link, m_i is the mass of link i, m_p is the mass of the payload, r_0 is link-length of link 0, and a, b are the installation lengths. In this six-link static balancing mechanism, total potential energy can be written as

$$e_p = (\frac{m_0}{2} + m_1 + m_p) \cdot g \cdot r_0 \cdot \cos\theta_0$$
$$+ m_1 \cdot g \cdot \frac{r_1}{2} + m_p \cdot g \cdot r_1$$
$$+ \frac{k}{2}(a^2 + b^2 - 2a \cdot b \cdot \cos\theta_0) \quad (33)$$

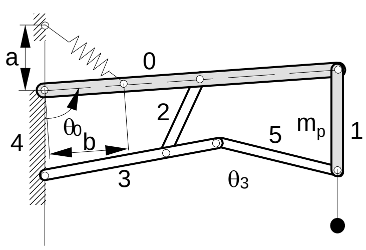


Figure 13: One-dof 6-link static balancing mechanism of s-2

In order to make e_p is conservative for arbitrary θ_1, the partial derivative of e_p with respect to θ_1 should be equal to zero. By setting the partial derivative of Eq. (33) with respect to θ_1 as zero, the spring constant, k, can be written as

$$k = (m_0 / 2 + m_1 + m_p) \cdot g \cdot r_0 /(a \cdot b) \quad (34)$$

6. SUMMARY

A systematic design procedure for a static balancing mechanism is proposed in this article. In the proposed procedure, the configuration design and the spring arrangement have been developed. In the configuration design, it is shown that the configuration of a static balancing mechanism can be designed to meet versatile requirements. For the new configuration, a spring can be arranged to construct a single-dof static balancing mechanism. It is believed that the new static balancing mechanism can be used to replace the ordinary types of pick-and-place mechanism to pick and place objects stably without exterior balancing moment in modern manufacture process.

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