行政院國家科學委員會補助專題研究計畫成果報告

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計畫類別：■個別型計畫　　□整合型計畫
計畫編號：NSC 90-2218-E-002-024
執行期間：　90 年 1 月 1 日至 90 年 10 月 31 日

計畫主持人：楊燿州
共同主持人：

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中　華　民　國　90 年　10　月　31　日

行政院國家科學委員會專題研究計畫成果報告

受擠薄膜式阻尼效應在多孔式微機電系統之研究
Squeezed-film damping for Perforated MEMS Devices

計畫編號：NSC 90-2218-E-002-024
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主持人：楊燿州　台大機械系
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一、中文摘要

本計劃使用模型降階演算法（Arnoldi model order reduction algorithm），來計算受擠薄膜式阻尼在多孔式微機電系統的效應。此方法所產生之精簡模型，不但可與電路模擬軟體相結合來做系統整合之暫態計算及頻率分析，更可將計算效率提高百倍以上。由於多孔式平行板電容式的元件結構非常複雜，電腦的計算資源往往無法有效支援。本研究的之加速計算的技巧，可將以往無法計算的系統，在極短的時間內求出其結果。

關鍵詞： 模型降階演算法、受擠薄膜式阻尼、多孔式平行板

Abstract

In this work, we present an application of an Arnoldi-based model order reduction (MOR) technique on squeezed-film damping (SQFD) effects for arbitrarily -shaped perforated geometry. The compact model generated by this approach not only can be easily inserted into a system-level modeling package for transient and frequency analysis, but also accounts for the effect of flow impedance of each perforation. We also demonstrated that this approach is at least 100 times faster than previous works.

Keywords: damping, model order reduction, squeeze film damping, perforation

INTRODUCTION

Perforations are often used in MEMS devices either to reduce release-etch time or to control SQFD. Modeling of arbitrarily perforated MEMS devices can be extremely complicated. Most work in damping has focused on getting more accurate simulations of the small or large signal gas damping and spring effects [1-6]. However, these studies are not capable of effectively simulating perforated surfaces because of neglecting the flow impedance of perforation holes. Veijola [7] introduced an special term into the SQFD governing equation (the Reynolds equation) to account for the *total* acoustic impedance effect due to perforation, and derived compact analytically solutions. This approach significantly reduces the complexity of the perforated model, but the solution is limited to the cases of rectangular-shaped surfaces with uniformly -distributed perforations. Yang [8] has developed a flow impedance model with the Reynolds equation, and solved the system using finite-difference method. Although this approach is applicable for cases of arbitrarily chosen plate-shapes and perforation configurations, the computational cost is very expensive. In this work, we employ an Arnoldi-based MOR algorithm [9] to generate low-order models from a Finite element or Finite difference approximation of the *modified Reynolds equation* that is coupled with pressure leakage due to perforations. The procedure of this approach is shown in Figure 1. Because the model size of the low order system is much small than the original system, the transient calculation of is improved by about two order of magnitude. Also, the frequency analysis can be easily performed using eigen expansion technique.

THEORY

The Arnoldi-based model reduction algorithm known as PRIMA [9], which is commonly used for model reduction of electrical interconnects, is used for model extraction. Our approach is similar to the model reduction approach used in [10]. In [10], model reduction was applied to a linearized form of the fully coupled electro-mechanical-fluid damped system. Here, we treat the squeeze film damping separately and build models in the mechanical mode shape basis. Such an approach would allow these models to be readily combined with the mode shape based models in the low order model of the entire system [3]. Additionally, we show that the models can be used for large-signal motion with certain restrictions.

To begin, the linearized Reynolds equation for squeeze film damping from [1] is

$$\frac{h_0}{P_0} \cdot \frac{\partial p}{\partial t} = \frac{h_0^3}{12\mu} \nabla^2 p - \frac{\partial e}{\partial t} \qquad (1)$$

where the variation in plate spacing h is assumed to be small compared to the mean spacing, h_0, given by

$$h = h_0 + e(x,t)$$

with $x \in \Re^2$ and $e << h_0$. The variation in pressure, P, will thus be small compared to ambient,

$$P = P_0 + p(x,t)$$

Equation (1) can be solved by finite element analysis for a given $e(x,t)$. Let $f(x)$ be the shape (perhaps mode shape) of the displacement so that $e(x,t) = f(x) \cdot u(t)$. The dynamic system from discretizing (1) by finite elements can be written in state space form as

$$\begin{aligned} \frac{h_0}{P_0} B \frac{d\mathbf{p}}{dt} &= \frac{h_0^3}{12\mu} A\mathbf{p} + B\mathbf{f}u(t) \\ y &= (B\mathbf{f})^T \mathbf{p} \end{aligned} \qquad (2)$$

where $A, B \in \Re^{n \times n}$, where n is the number of nodal degrees of freedom, $\mathbf{p}$ is the pressure at the nodes, and $\mathbf{f}$ is $f(x)$ evaluated at the node points. y is then the net force projected into the shape defined by $f(x)$ in a finite element sense.

The dynamic system of (2) is too large to insert directly into a system simulator such as SPICE or SABER. We thus apply PRIMA to generate a low order representation of (2) which still accurately captures the dynamic behavior. To apply the PRIMA algorithm to generate a k-th order model, k orthogonal vectors $\{v_i\} \in \Re^n$ are computed which span the vector space known as Krylov subspace:

$$K_k = \left\{ (B\mathbf{f}), A^{-1}B(B\mathbf{f}), ..., (A^{-1}B)^{k-1}(B\mathbf{f}) \right\}$$

These vectors can be stably computed via the Arnoldi algorithm [8]. Given the matrix V whose columns are $\{v_i\}$, the reduced order model is

$$\begin{aligned} \frac{h_0}{P_0} \tilde{B} \frac{d\tilde{\mathbf{p}}}{dt} &= \frac{h_0^3}{12\mu} \tilde{A}\tilde{\mathbf{p}} + \tilde{\mathbf{f}}u(t) \\ y &= \tilde{\mathbf{f}}^T \tilde{\mathbf{p}} \end{aligned} \qquad (3)$$

where $\tilde{B} = V^T BV$, $\tilde{A} = V^T AV$, and $\tilde{\mathbf{f}} = V^T B\mathbf{f}$. The attractive properties of such an approach are that the first k Taylor series coefficients of the transfer function of (3) match those of the original model in (1). In addition, the model is guaranteed to be passive. Finally, the method easily extends to a single model with multiple inputs $\{u_1(x), u_2(x), ...\}$ corresponding to multiple mode shapes, $\{\mathbf{f}_1(x), \mathbf{f}_2(x), ...\}$.

As will be seen in the next section, k = 5 is generally adequate for an accurate damping model. This low order model can be inserted directly into a system simulator such as SPICE or SABER. Note that since the vector space spanned by $\{v_i\}$ does not depend on the mean gap, ambient pressure or viscosity, the above model is valid for any choice of those parameters. Going one step further, we can model large signal behavior by letting the mean gap vary with time, $h_0 = h_0(t)$. Such an approach would be valid if $h_0(t)$ varies slowly compared to $u(t)$. In fact, from numerical experiment, we find that replacing h_0 in (2) with $h_0(t) = h_0 + u(t)$ for even large $u(t)$ gives good results.

In order to account for the perforation effect, we employ an the modified Reynolds equation that is coupled with pressure leakage due to perforations:

$$\frac{\partial P}{\partial t} = \frac{P_a h^2}{12\mu}\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) - \frac{P_a}{h}\cdot\frac{dh}{dt} - \frac{P_a}{h}\cdot\frac{P}{Z} \qquad (4)$$

where P is pressure variation, h is gap thickness, Pa is ambient pressure and is viscosity. Note that the derivation of this equation is based on the condition of flow continuity [8]. The last term on the right-hand side of Equation (4) accounts for the acoustic pressure leakage due to perforation.

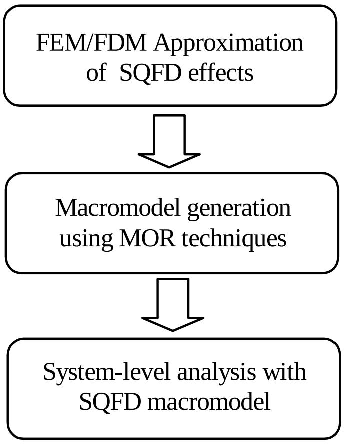

Figure 1 *Procedure of the efficient and accurate air damping modeling for perforated MEMS devices described in this paper*

The impedance Z in the equation represents flow impedance for each hole. The value of Z, which can be found in [8], depends on the perforation size and location. Z is infinite at the location where a hole does not exist (i.e., the pressure leakage term is eliminated).

RESULTS

The modeling procedure is shown in Figure 1. The results of damping and spring components for a 515x515 m2 perforated accelerometer are shown on Figure 2. The original system has 66564 nodes (i.e., 66564x66564 system matrix), which is too large to be compatible with system-level simulation. The result indicates that a reduced macromodel of order 20 (i.e., 20x20 system matrix) will give accurate and converged results.

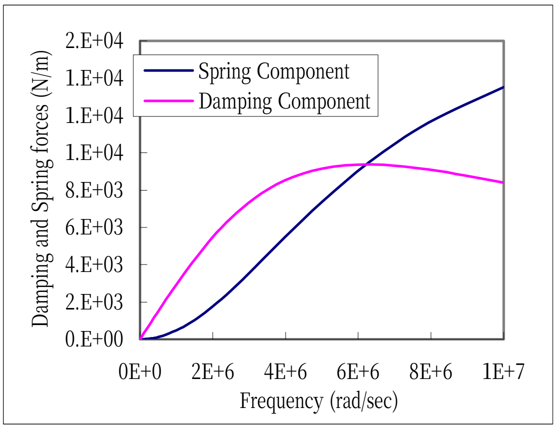

Figure 2 *Spring and damping components of an order-20 macromodel for a 515x515μm^2 perforated accelerometer. There are a total of 36x36 holes (2μm in diameter) evenly distributed on the proof mass. The gap is 1μm.*

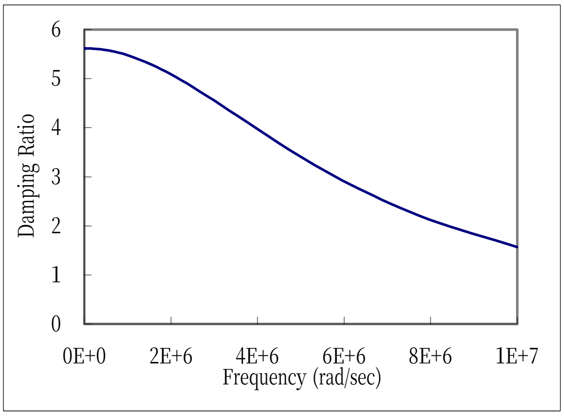

Figure 3 *Damping ratio of the accelerometer. The mechanical spring constant is 4.75 N/m, and the mass is 1.52e-8 kg. The experimental damping ratio at 10 kHz is about 5.*

The simulated damping ratio of 5.6 (at 10 KHz) is very close to the experimental result of about 5 [8,10], as shown in Figure 3. The extracted macromodels can be easily inserted into circuit simulators [9] or system-level simulators such as Saber or Simulink [12]. Figure 4 shows the schematic of a Mass-Spring-SQFD model under Simulink environment. The extracted SQFD macromodel is described as a state-space model of the Simulink. The

Simulink transient simulation of the system under a step input voltage is shown in Figure 5.

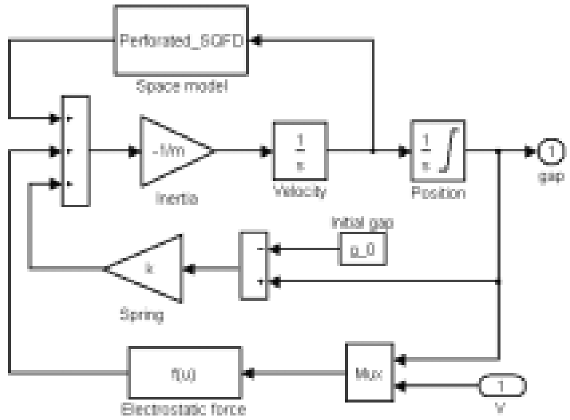

Figure 4 *Schematic of mass-spring-SQFD model of an accelerometer with electrostatic actuator. The extracted SQFD macromodel is described as a state-space model of the Simulink.*

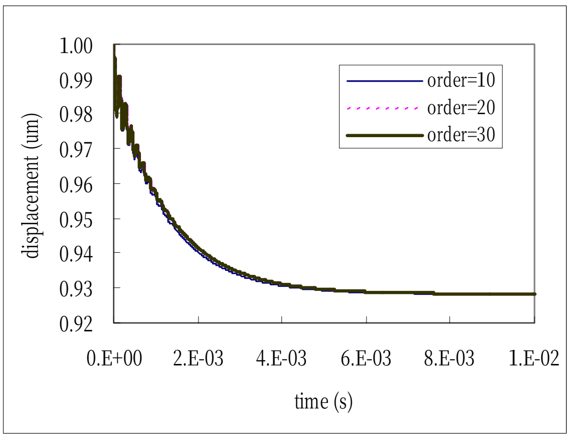

Figure 5 *Transient simulation result of the accelerometer system shown in Figure 4. The applied voltage is 0.5 volt. The ambient pressure is set to 0.001atm so that oscillation will be observed. The curves of order 20 and 30 systems are identical*

CONCLUSION

A new approach to extract frequency-dependent gas damping models for perforated arbitrary geometries of MEMS devices is demonstrated. The Arnoldi-based algorithm is applied for creating a low-order model from the transient FE/FD system matrices. The frequency-dependant gas damping and spring effects can be obtained using the low-order models without any computationally intensive transient simulation for wide frequency range. After constructing 3D solid models, more than two order of magnitude reduction in computational time has been demonstrated. Examples of transient analysis and comparison to experimental results are also provided.

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