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拱形元件受移動負載作用時之動態反應與穩定性分析

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計畫主持人：陳振山

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Dynamics and Stability of Shallow Arches under Moving Loads

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計畫主持人：陳振山 jschen@ccms.ntu.edu.tw

執行機關：國立台灣大學機械系

摘要

本計畫探討一個拱形元件受移動集中力作用時的動態反應，我們想知道當集中力移動速度夠大時，拱形元件是否會發生折斷式挫曲。我們首先推導出拱形元件受移動力作用時的運動方程式，再將之離散化成一組常微分方程式，其中與外力有關的項是時間的函數。我們接著先探討靜態的情況以決定拱形元件平衡位置及穩定性與外力大小及位置的關係。然後再考慮移動速度不可忽略時，以數值及解析的方法，來決定臨界負載，在此臨界負載的定義是當外力低於此時就不會有折斷式挫曲發生。

關鍵詞：拱形元件，動態折斷式挫曲，移動負載

Abstract

In this report we study the dynamic behavior of a shallow arch under a point load Q traveling at a constant speed. Emphasis is placed on finding whether snap-through buckling will occur. In the quasi-static case when the moving speed is almost zero, there exists a critical load Q_{cr} in the sense that no static snap-through will occur as long as Q is smaller than Q_{cr}. In the dynamic case when the point load travels with a nonzero speed, the critical load Q_{cr}^{d} is in general smaller than the static one.

Keywords: shallow arch, dynamic snap-through, moving loads

Introduction

In previous researches, the lateral loading on a shallow arch, either distributed or concentrated, is assumed to be fixed in space. In this project we study the dynamic stability of a shallow arch under a moving point force. The proposed problem is potentially important because shallow arches have been crucial elements in numerous structures for public transportation. For better understanding of the response and safety of these structures, it is necessary to study the behavior of an arch under high-speed moving loads. In the first part of this paper we study the quasi-static case when the moving speed of the point force is very small. Our analysis shows that there exists a static critical load in the sense that no static snap-through will occur as long as

the point load is smaller than this critical load. The second part of the paper considers the case when the point load travels with a nonzero speed. Similar to the quasi-static case there exists a dynamic critical load when the point load travels with a nonzero speed. The dynamic critical load is in general smaller than the static one. When the point load is greater than the dynamic critical load there exists a finite speed zone within which the arch runs the risk of dynamic snap-through. A simpler but more conservative criterion is then proposed to determine the boundary of this dangerous speed zone.In general

Equations of Motion

The dimensionless equation of motion of an arch under a moving point load can be written as,

$$u_{,\tau\tau} = -(u - u_0)_{,\xi\xi\xi\xi} + pu_{,\xi\xi} - \frac{\pi}{2} Q\delta(\xi - c\tau) \qquad \textbf{(1)}$$

$$p = \frac{1}{2\pi}\int_0^{\pi}\left(u_{,\xi}^2 - u_{0,\xi}^2\right)d\xi\,, \qquad \textbf{(2)}$$

The initial shape and the deformed shape of the arch are $u_0(x)$ **and** $u(\xi,\tau)$**.** ξ **is the dimensionless coordinate in the axial direction of the arch.** τ **is time.** ***c*** **is the constant speed of the traveling point load** ***Q*****. The initial shape of the arch is assumed to be in the form**

$$u_0 = h\sin\xi \qquad \textbf{(3)}$$

h **is the rise parameter of the arch. It is assumed that the shape of the arch after stretching can be expanded as**

$$u(\xi,\tau) = \lim_{N\to\infty}\sum_{n=1}^{N}\alpha_n(\tau)\sin n\xi \qquad \textbf{(4)}$$

After substituting Eqs. (3) and (4) into (1) and (2) we obtain the equations governing α_n**,**

$$\ddot{\alpha}_n = -n^4\alpha_n - n^2 p\alpha_n - q_n\,, \quad \boldsymbol{n} = 1,2,3, \qquad \textbf{(5)}$$

where

$$p = \frac{1}{4}\sum_{k=1}^{\infty}k^2\alpha_k^2 - \frac{h^2}{4}\,, \qquad \textbf{(6)}$$

$$q_1 = Q\sin e - h\,, \qquad \textbf{(7)}$$

$$q_n = Q\sin ne\,, \quad \boldsymbol{n} = 2,3,\ldots \qquad \textbf{(8)}$$

$$e(\tau) = c\tau\,. \qquad \textbf{(9)}$$

The parameter $0 < e(\tau) < \pi$ **represents the position of the point load on the arch. The overhead dot in Eq. (5) represents differentiation with respect to** τ**. The initial**

conditions for Eq. (5) are

$$\alpha_1(0) = h \qquad \textbf{(10)}$$

$$\alpha_n(0) = 0\,, \qquad n=2,3,\ldots \qquad \textbf{(11)}$$

$$\dot{\alpha}_n(0) = 0\,, \qquad n=1,2,3,\ldots \qquad \textbf{(12)}$$

Equilibrium Configurations

We first consider the case when the moving speed *c* of the point load is small and the acceleration terms in Eq. (5) can be neglected. The equilibrium equations governing α_n can then be written as

$$n^4\alpha_n + n^2 p\alpha_n + q_n = 0\,, \qquad n=1,2,3,\ldots \quad \textbf{(13)}$$

Equation (13) represents an infinite number of coupled nonlinear equations for the infinite number of coordinates α_n. While it is in general impossible to solve for the infinite number of α_n simultaneously, it is possible to use a deduction method to derive the equation for α_1. The resulted equation is,

$$f_k(\alpha_1) = f_1(\alpha_1)\prod_{j=2}^{k}\left[\alpha_1 - \alpha_1\left(\overline{P}_{1j}^{\pm}\right)\right]^2 + \sum_{i=2}^{k}\left\{\frac{q_i^2\alpha_1^3}{4i^2(i+1)^2(i-1)^2}\prod_{\substack{j=2\\ j\neq i}}^{k}\left[\alpha_1 - \alpha_1\left(\overline{P}_{1j}^{\pm}\right)\right]^2\right\} = 0 \qquad (14)$$

$$f_1(\alpha_1) = \alpha_1 + \frac{\alpha_1}{4}\left(\alpha_1^2 - h^2\right) + q_1 \qquad (15)$$

$\overline{P}_{1k}^{+}$ and $\overline{P}_{1k}^{-}$ are the *k*-mode solutions, whose coordinates can be calculated exactly as

$$\alpha_n(\overline{P}_{1k}^{\pm}) = \frac{q_n}{n^2(k^2 - n^2)}\,, n=1,2, k-1 \qquad (16)$$

$$\alpha_k(\overline{P}_{1k}^{\pm}) = \pm\frac{2}{k}\sqrt{\frac{h^2}{4} - k^2 - \sum_{n=1}^{k-1}\frac{n^2\alpha_n^2(\overline{P}_{1k}^{\pm})}{4}} \qquad (17)$$

In Fig. 1 we show the α_1 as a function of load position *e* for the case when *h*=8. Figures 1(a), (b), and (c) are for *Q*=18, 20, and 18.16, respectively. The number of modes *N* used in the expansion is 4. While there are at most 9 equilibrium positions for *N*=4, only 7 of them are real. In the case when *e*=0, i.e., the arch is free from the point load, there are three one-mode solutions (P_0, P_1^+, P_1^-) involving only α_1, and two pairs of two-mode solutions ($P_{12}^{\pm}$, $P_{13}^{\pm}$). It can be shown that for a free arch with *h*>4 only positions P_0 and P_1^- are stable while all others are unstable. As the point load moves across the arch, these one-mode and two-mode solutions will involve all the harmonic modes in expansion (8). However, we retain the names of the equilibrium positions when the point load moves across the arch. In

Fig. 1(a) we observe that all the solution curves experience no bifurcation as the point load moves across the arch. Therefore both P_0 and P_1^- remain stable while all others remain unstable. We use solid and dashed lines to denote stable and unstable solutions. The stability properties of these equilibrium positions for a loaded arch can also be determined by the conventional energy method. As a consequence for *Q*=18 no snap-through will occur as the point load moves across the arch quasi-statically. However, it remains unknown yet whether snap-through will occur if the moving speed of the point load is no longer negligible. This will be a subject for later discussion.

In Fig. 1(b) for *Q*=20 we notice that as the point load moves across the arch the stable P_0 solution merges with the unstable P_{12}^- solution via a saddle-node bifurcation at *e*=0.76. As a consequence the arch snaps from the P_0 position to the stable P_1^- position at this bifurcation point. Three additional saddle-node bifurcation points are at *e*=1.35, 1.79, and 2.38. As *Q* decreases from 20 the saddle-node bifurcation points in Fig. 1(b) approach each other and eventually merge into transcritical bifurcation points when *Q*=18.16, as shown in Fig. 1(c). Therefore, *Q*=18.16 is a critical load for *h*=8 in the sense that no snap-through will occur when a point load smaller than this value moves across the arch quasi-statically.

Dynamic Snap-Through

The response of the arch will be different when the point load travels with a nonzero speed. The response history can be calculated by integrating Eq. (4) numerically with the initial conditions (10), (11) and (12). We use only the first two equations with *n*=1 and 2 in Eq. (4) for simplicity. In the numerical simulation we also add damping terms $\mu\dot{\alpha}_1$ and $\mu\dot{\alpha}_2$ in these two equations of motion. It is noted that there is no technical difficulty in using more than two modes in expansion (3) for numerical simulation except that the calculation time will increase.

The quasi-static analysis in Fig. 1 shows that the arch with *h*=8 will remain in the P_0 position as long as *Q* is smaller than Q_{cr} =18.16. However, when the speed *c* is increased from zero, dynamic snap-through may occur even when the load *Q* is smaller than Q_{cr}. One of the thick lines in Fig. 2 shows the dynamic response of the arch with *h*=8, *Q*=18, $\mu = 0.001$, and *c*=0.1. The arch snaps to the P_1^- position when the point load moves to the position *e*=2.2 and settles to P_1^- position thereafter. The equilibrium positions from quasi-static analysis (*c*=0) are also shown in Fig.2 with thin lines for comparison. On the other hand, when the speed *c* is further increased to 1.5, the arch does not have sufficient time to snap before the load reaches the other end and leaves the arch, as demonstrated by another thick line in Fig. 2. However, it must be emphasized that there still exists a possibility that the arch may continue to deform and snap to P_1^- position after the point load leaves the arch. Therefore, to determine whether the arch will snap or not, we have to continue the numerical simulation until the arch settles to one of the stable equilibrium positions. For the case with *c*=1.5 in Fig.2 our numerical simulation shows that the arch will settle to P_0 position eventually. Furthermore, the arch never snaps to P_1^- at any instant before it settles to P_0. Therefore, in the case when *c*=1.5 no dynamic snap-through occurs when the point load is either still on the arch or after the point load leaves the arch.

It is noted in Fig.2 that the static analysis does not predict the dynamic behavior for a small speed *c*=0.1. To demonstrate that the dynamic behavior indeed approaches the quasi-static result, we show the response for a smaller speed *c*=0.05. It can be seen that the curve for *c*=0.05 follows, with small oscillation, the quasi-static position P_0 between the two ends of the arch. The dynamic result for an even smaller speed *c*=0.01 is indistinguishable from the quasi-static curve.

Figure 3 shows another case with *h*=8 and *Q*=25. The equilibrium positions from the quasi-static analysis are similar to those in Fig. 1(b) and are shown with thin lines. Apparently, for this case the critical speed c_{cr}^- is zero. On the other hand, when the moving speed is large enough, say, *c*=2.6, then the arch will not have enough time to snap before the load reaches the other end, as demonstrated by the thick line. Further calculation confirms that the arch settles to P_0 position after the point load leaves the arch. To demonstrate the situation that the arch may continue to deform and snap after the point load leaves the arch, we show the response history for a different speed *c*=2.4 in Fig.3. Again the arch does not have enough time to snap before the load reaches the other end. However, further calculation shows that the arch will snap to P_1^- after the point load leaves the arch and settles to P_1^- eventually.

Conclusions

In this report we study the dynamic behavior of a shallow arch under a point load *Q* traveling at a constant speed *c*. Emphasis is placed on finding whether snap-through will occur if the arch possesses two stable equilibrium positions P_0 and P_1^- when it is free of lateral loading. The first part of the paper considers the quasi-static case when the moving speed is almost zero. Several conclusions can be summarized.

(1) There exists a critical load Q_{cr} in the sense that no static snap-through will occur as long as the load *Q* is smaller than Q_{cr}. Q_{cr} increases with larger *h*.

(2) In the case when *Q* is greater than Q_{cr}, the solution curves of P_0 and P_{12}^-

configurations will merge into a saddle-node bifurcation point. At the bifurcation point the arch will snap to the position P_1^-.

The second part of the paper considers the case when the point load travels with a nonzero speed. Some more conclusions can be summarized in this regard.

(3) Similar to the quasi-static case there exists a dynamic critical load Q_{cr}^d when the point load travels with a nonzero speed. No dynamic snap-through will occur as long as the point load Q is smaller than Q_{cr}^d. The dynamic critical load Q_{cr}^d is in general smaller than the static critical load Q_{cr}.

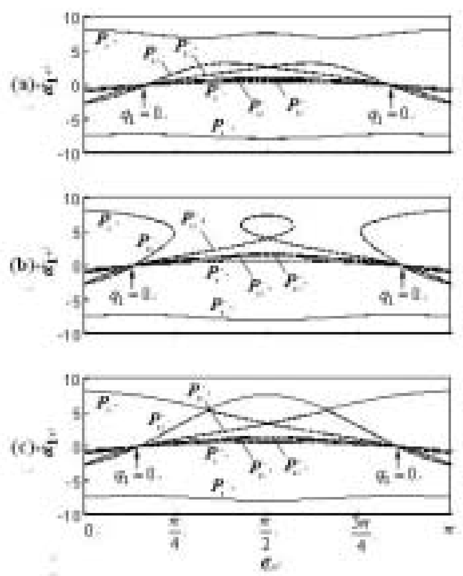

Figure 1 Coordinate α_1 as a function of load position *e* for an arch with *h*=8. (a) *Q*=18. (b) *Q*=20. (c) *Q*=18.16.

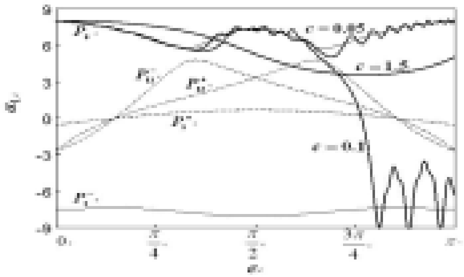

Figure 2 The thick lines are the dynamic responses for an arch with *h*=8, μ =0.001, and *Q*=18.

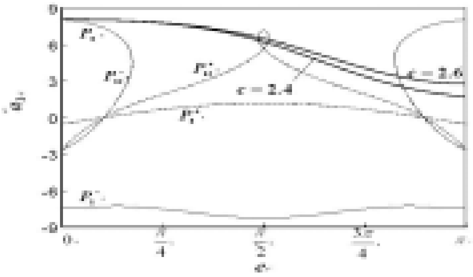

Figure 3 The thick lines are the dynamic responses for an arch with *h*=8, *Q*=25, and μ =0.001.