行政院國家科學委員會專題研究計畫　成果報告

運動分解於齒輪機構之功能導向概念設計的應用

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計畫主持人：陳達仁

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計畫類別：■個別型計畫　　□整合型計畫
計畫編號：NSC　91-2212-E-002-031
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執行單位：國立台灣大學機械系

中　華　民　國　91 年　10 月　27 日

運動分解於齒輪機構之功能導向概念設計的應用
Functional-Guided Conceptual Design of Geared Mechanisms Using the Concept of Kinematic Fractionation

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主持人：陳達仁，國立台灣大學機械系教授
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摘要

本計劃將藉由運動分解之概念，將齒輪機構視為運動單元之組合，而闡明齒輪機構中其運動特性與拓樸構造間之依存關係，進而引導出以運動特性需求為導向之齒輪機構設計方法。在本計劃中，經由探討運動單元之特性，齒輪機構中機構自由度及運動單元個數間依存之關係將因此揭露，而齒輪機構之拓樸構造特性也將以運動單元之方式加以呈現。本計劃將探討出構造上不可分解之齒輪機構之構造特徵。在獲得齒輪機構之構造特徵後，再在其構造上安排可行的輸出入桿桿件位置，則可得出其可能的對應之運動傳遞路線。藉由齒輪機構之拓樸構造及運動傳遞路線之揭露，則亦即建立出其拓樸構造及運動特性之依存關係。相較於傳統以拓樸特徵為拓樸合成考量之方法，相信本計劃之研究成果，將可使齒輪機構之拓樸合成流程，更具運動之觀點，並可使拓樸合成之程序更有效率。

ABSTRACT

The design procedure of geared mechanism from both topological and kinematic viewpoints is introduced in this paper by exploring the correspondences between structural and kinematic characteristics of geared mechanism. The structure of geared mechanism will be considered as the combination of kinematic units (KUs). The characteristics of KUs are applied to reveal the relations between the number of degree of freedom (DOF) and the number of KUs. It leads to admissible connections of KUs in structurally non-fractionated geared kinematic chains (GKCs). By designating permissible locations of input and output in the configuration of the connection of KUs, the corresponding kinematic propagation paths of 1-DOF, up to 3-KU and 2-DOF, up to 4-KU geared mechanisms can be obtained. Hence, the corresponding kinematic behavior of the GKCs can be exposed. It is believed that the correspondences between kinematic and topological characteristics provide a more kinematic perspective to synthesize the topological structure of geared mechanism comparing to traditionally topological-centered approach, and make the design procedure more efficiently.

1. INTRODUCTION

For decades, many researches for topological analysis of geared mechanisms had been devoted to facilitate the design procedure. The purpose of topological analysis is determining the locations of ground, input, and output links; avoiding the redundancy to reduce the power attrition. Olson, et al. [1] established the concept of coincident-joint graph to characterize the adjacency between links in planetary kinematic geared chains (PKGCs). Due to the exposure of the characteristics of adjacency, an approach is developed to determine the ground, input, and output links in 1-DOF, single output, and 5-link PKGCs with both input and output links are adjacent to the fixed links. However, the majority of their results are excluded after additional redundancy check, since they contain redundant links. Liu and Chen [2] developed the concept of kinematic fractionation to clarify the motion transmission in geared mechanism. By listing the topological requirements of the ground, input, and output links, an efficient method of topological analysis avoiding the occurrence of redundant links is exposed thereafter.

There have also many researches for establishing efficient methodologies to kinematic analysis of geared mechanism. In virtue of the application of graph theory [3], the concept of fundamental circuit was applied to the kinematic analysis of geared mechanism [4, 5]. By solving a segment of linear equations, the kinematic relations are derived; however, mathematical manipulation cannot provide enough kinematic information of the mechanism. Chatterjee and Tsai [6] established the concept of fundamental geared entity (FGE) for automatic transmission mechanisms; an eplicyclic gear mechanism (EGM) can hence be divided into several FGE.

Chen and Shiue [7] showed that a geared robotic mechanism can be fractionated into input units and transmission units. Chen [8] further verified the forward and backward gains of each unit and proposed a unit-by-unit evaluation procedure for the kinematic analysis of geared robotic mechanisms. The approach can only be applied to geared robotic mechanisms. The kinematic analysis method via the concept of kinematic unit (KU), exposed by Liu et al. [9], can penetrate the propagation paths by trace-back of the motion transmission flow from output to input. This approach allows a wider scope of application, and provides a more kinematic insight.

Traditionally, the design procedure of geared mechanism is based on topological perspective, whereas kinematic analysis serves as a step of final check. That is, the topological structure of GKC is synthesized due to the number of DOF, links, and joints. Then the configuration of geared mechanism is generated via topological analysis including the assignment of the ground, input, and output links in GKC. The fulfillment of functional requirements of geared mechanism is examined by kinematic analysis afterward. Such design procedure seems as a result from trial and error, which is less than a systematic approach, and time-consuming.

In this paper, geared mechanism is regarded as the combination of KUs connected by the common linkages. Based on the characteristics of KUs and common linkages, admissible connections of KUs are derived; thereafter the structural characteristics of geared mechanism are attained. By designating the locations of input and output in the configurations of connections, all possible kinematic propagation paths of 1-DOF, up to 3-KU and 2-DOF, up to 4-KU geared mechanisms can be acquired accordingly. Via analyzing these paths, the 2-DOF path can be separated into 1-DOF paths, and three types of path patterns, serial, branch, and feedback types, can be obtained thereafter. The corresponding kinematic behavior of the GKCs can then be revealed. In virtue of the exposure of the correspondences, the creation of GKCs will direct to the required kinematic relation and have more kinematic insight. It can make the topological synthesis of geared mechanism systematic and present a concept for functional-orientated design procedure.

2. CHARACTERISTICS OF KU

2.1 Kinematic unit

In the graph representation of geared mechanisms, links are represented as vertices, turning pairs as thin edges, geared pairs as heavy edges, and each thin edge is labeled as the associating axis orientation. Liu et al. [10] exposed that a KU is composed of a carrier and all gears on it; and the fundamental circuits in a KU are in series connection, as shown in Fig. 1. In which g_i is the gear pair in the *i*th fundamental circuit, and l_j is the axis label of the *j*th thin edge. Since the gear pairs consist an independent power transmission path, the kinematic relation between the two ends of the heavy-edge path can be described by an augmented fundamental circuit equation. It means that the motion of links in a KU can be determined by a single input. Since the links have such coupled kinematic relation, they are considered as a unit.

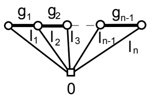


Figure 1: The connected heavy-edge path in a typical KU.

Each KU can be considered as a 1-DOF sub-mechanism; the motion of each KU is initiated by the local input and then transmitted by the geared pairs to the local output. Both local input and output can be expressed as the angular displacement between two links connected by a thin edge. The thin edges in a KU can be classified into two types:

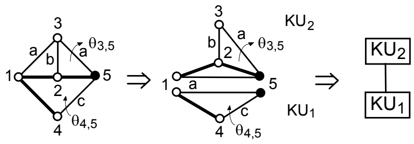
(1) gear-carrier (g-c) type: one end of the thin edge is connected to a gear vertex, and the other is connected to carrier vertex.

(2) gear-gear (g-g) type: both the ends are connected to gear vertex.

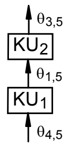
Hence, there are three forms of local gains of KUs, such as g-c vs. g-c, g-c vs. g-g, and g-g vs. g-g types. In which local gain is the gear ratio from local input to local output in a KU [9].

The geared mechanism can be regarded as the combination of KUs, and the common linkages are the adjacencies between KUs. Liu et al. [9] exposed that there are two types of common linkages in the GKCs with up to 7-link and up to 2-DOF: 2-link chain type and coaxial triangle type. The motion transmission in the common linkages can be classified as follows.

2-link chain type: Figure 2(a) shows a graph of 1-DOF 5-link geared mechanism [3], which is composed of two KUs and has a 2-link chain type common linkage, containing link 1, 5 and thin edge between link 1 and 5. The ground is link 5, denoted as a rigid circle, input is link 4, and output is link 3. As shown in the middle graph of Fig. 2(a), the motion is transmitted from the prior KU, KU_1, to the succeed KU, KU_2, and the motion between the two KUs is communicated by the 2-link chain type common linkage. Hence, the kinematic relations in the geared mechanism can be expressed as what follows.



(a) The connection of KUs.



(b) The motion transmission path.

Figure 2: A 1-DOF 5-link geared mechanism.

In KU_1, $\theta_{4,5}$ is the local input while $\theta_{1,5}$ is the local output, thus we have:

$$\theta_{1,5} = e_{4,1}\theta_{4,5} \tag{1}$$

where $e_{4,1}$ is the local gain of KU_1.

In KU_2, $\theta_{1,5}$ is the local input while $\theta_{3,5}$ is the local output, thus we have:

$$\theta_{3,5} = (1 - e_{5,1})\theta_{1,5} \tag{2}$$

where $(1 - e_{5,1})$is the local gain of KU_2.

Substitute Eq. (1) into Eq. (2), the kinematic relation can be expressed as:

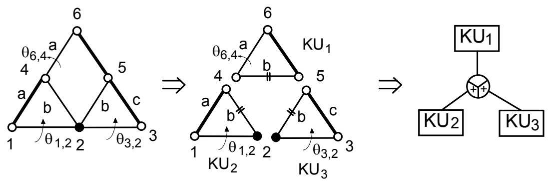
$$\theta_{3,5} = e_{4,1}(1 - e_{5,1})\theta_{4,5} \tag{3}$$

The 2-link chain type common linkage plays as a bridge communicating the two KUs.

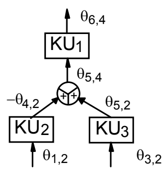
In the representation of the connection of KUs, the KU is represented as a block and the 2-link chain common linkage is represented as a thin edge as shown in the right graph of Fig. 2(a). Due to the representation and the derivation of kinematic relation, the motion transmission is clearly exposed, as shown in

Figure 2(b), in which the arrowhead indicates the direction of the motion signal.

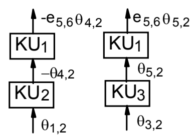
Coaxial triangle type: Figure 3(a) shows a graph of 2-DOF 6-link geared mechanism [11], which is composed of 3 KUs, and has a coaxial triangle type common linkage, containing link 2, 4, and 5 and three thin edges between link 2 and 4, link 2 and 5, and link 4 and 5. The coaxial triangle type common linkage appears only when the number of KUs is equal to or greater than three. The coaxial triangle is denoted by the short thin edges as shown in Fig. 3(a). The ground link is link 2, denoted as a rigid circle, the input links are link 1 and 3, and the output link is link 6, as shown in the middle graph of Fig. 3(a). The kinematic relation can hence be expressed as:



(a) The connection of KUs.



(b) The motion transmission path.



(c) The fractionation of motion transmission path.

Figure 3: A 2-DOF 6-link geared mechanism.

In KU_2, $\theta_{1,2}$ is the local input while $\theta_{4,2}$ is the local output, thus we have:

$$\theta_{4,2} = e_{1,4}\theta_{1,2} \tag{4}$$

where $e_{1,4}$ is the local gain of KU_2.

In KU_3, $\theta_{3,2}$ is the local input while ${}_{5,2}$ is the local output, thus we have:

$$\theta_{5,2} = e_{3,5}\theta_{3,2} \tag{5}$$

where $e_{3,5}$ is the local gain of KU_3.

Among the coaxial triangle type common linkage, the kinematic relation between the three links on the coaxial triangle can be expressed as:

$$\theta_{5,4} = \theta_{5,2} + (-\theta_{4,2}) \tag{6}$$

From Eq. (6), it needs two local inputs to maintain the mobility between the three links on the coaxial triangle [9].

In KU_1, $\theta_{5,4}$ is the local input while $\theta_{6,4}$ is the local output, thus we have:

$$\theta_{6,4} = e_{5,6}\theta_{5,4} \tag{7}$$

where $e_{5,6}$ is the local gain of KU_1.

Substitute Eq. (6) into Eq. (7), we have:

$$\begin{aligned}\theta_{6,4} &= e_{5,6}[\theta_{5,2} + (-\theta_{4,2})] \\ &= e_{5,6}\theta_{5,2} + (-e_{5,6}\theta_{4,2})\end{aligned} \tag{8}$$

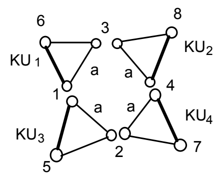
Substitute Eq. (4) and (5) into Eq. (8), the kinematic relation can be expressed as:

$$\theta_{6,4} = e_{5,6}e_{3,5}\theta_{3,2} + (-e_{5,6}e_{1,4}\theta_{1,2}) \tag{9}$$

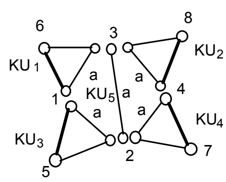
From Eq. (6), the coaxial triangle type common linkage, as an addition operator, adds up two motion signals into one. Hence, an addition operator is used to represent the coaxial triangle, as shown in the right graph in Fig. 3(a).

According to the representation and the derivation of kinematic relation, the motion transmission is then shown in Fig. 3(b). It can be seen that the output can be separated into two groups, each containing one input. Hence, the motion transmission path can be fractionated into two sub-paths, as shown in Fig. 3(c).

2.2 Degenerated KU



(a) The coaxial quadrangle type common linkage.



(b) The degenerated KU.

Figure 4: Degenerated KU.

However, as the number of links and the number of DOF of GKCs increase, the types of common linkages may extend to coaxial quadrangle type, coaxial pentagon type, and so on. In order to simplify the types of common linkages, the coaxial polygon can be considered as the combination of coaxial triangles. For instance, Fig. 4(a) shows a mechanism having coaxial quadrangle composed of four KUs. The quadrangle is also a combination of two coaxial triangles as shown in Fig. 4(b). The fifth KU, KU_5, is formed by connecting the two opposite coaxial links on the coaxial quadrangle with a thin edge. KU_5 is composed of two links and a thin edge without a heavy edge, so it is called a degenerated KU, existing only in the connections of KUs with coaxial polygon.

3. TOPOLOGICAL CHARACTERISTICS OF GEARED MECHANISMS

3.1 Rules of connections of KUs

By clarifying the relations between the numbers of DOF, common linkages, and KUs in geared mechanisms, the rules of the connections of KUs are therefore sustained. The structural characteristics of geared mechanisms can be exposed thereafter.

The formula for computing the number of DOF of geared mechanisms can be expressed as:

$$F = E - H \tag{10}$$

where E is the number of thin edges in the geared mechanism, and H the number of heavy edges in the geared mechanism.

However, as shown in Fig. 2(a) and 3(a), since there are common linkages between KUs after kinematic fractionation, a thin edge may appear in several KUs. It takes 2 KUs to form a 2-link chain type common linkage, and 3 KUs to form a coaxial triangle type common linkage. Both two types of common linkages have a repeat thin edge. Thereafter, it will increase the summation of the number of thin edges that fails to meet the authentic number. Therefore, to fit the number of thin edges in geared mechanism, the number of thin edges after kinematic fractionation must subtract the number of 2-link chain and coaxial triangle type common linkages. Hence, Eq. (10) can be modified as:

$$F = \sum (e_i - h_i) - t - c \tag{11}$$

where e_i is the number of thin edges in *i*th KU, and h_i the number of heavy edges in *i*th KU, t the number of 2-link chain type common linkages, and c the number of coaxial triangle type common linkages.

Since the number of DOF of a KU is equal to one, the difference between the number of thin and heavy edges in *i*th KU can be expressed as:

$$e_i - h_i = 1 \tag{12}$$

From Eq. (12), since the difference of the number of thin and heavy edges in a KU is equal to one, the difference between thin and heavy edges in geared mechanism must be equal to the number of KUs. Thus, we have:

$$\sum_{i=1}^{u} e_i - h_i = u \tag{13}$$

where u is the number of KUs in the geared mechanism.

Substitute Eq. (13) into Eq. (11), we have:

$$F = \sum_{i=1}^{u} (e_i - h_i) - t - c = u - (t + c) \tag{14}$$

Equation (14) can also be expressed as:

$$u = F + (t + c) \tag{15}$$

where (t + c) is the number of common linkages in a mechanism.

According to Eq. (15), we can have

Axiom 1: The number of KUs is equal to the number of DOF adding the number of common linkages in the geared mechanism; or in other words, the number of DOF is equal to the number of KUs subtracting the number of common linkages in the geared mechanism.

3.2 Connections of KUs for structurally non-fractionated GKCs

Since a structurally fractionated GKC can be decomposed into structurally non-fractionated GKCs, the following will discuss the connections of non-fractionated GKCs.

Based on the characteristics of common linkages and the mobility of geared mechanisms, the basic rules of admissible connections of KUs can be listed as follows:

The occurrence of cut links leads to the structurally fractionated GKC. Therefore, the GKC with cut links can be separated into several structurally non-fractionated GKCs, which have independent kinematic relation to each other. In the KU representation, the appearance of disconnected KUs indicates cut links, since the disconnected KUs have no communication with others. In order to obtain structurally non-fractionated GKCs, we have:

Rule 1: There should have no disconnected KUs in the connection.

Since it takes two KUs to form a 2-link type common linkage, and three KUs to form a coaxial triangle type common linkage, we have:

Rule 2: The number of 2-link chain type common linkages is greater than or equal to zero when the number of KUs is greater than or equal to two.

Rule 3: The number of coaxial triangle type common linkages is greater than or equal to zero when the number of KUs is greater than or equal to three.

From Eq. (15), the relation between the number of KUs and the number of common linkages can be expressed as below:

For 1-DOF geared mechanisms,

$$u = 1 + t + c \tag{16}$$

For 2-DOF geared mechanisms,

$$u = 2 + t + c \tag{17}$$

From Eq. (17), if the number of KUs is equal to two, we have:

$$2 = 2 + t + c \tag{18}$$

Hence,

$$t + c = 0 \tag{19}$$

It will leads to the appearance of disconnected KUs. Thus, we have:

Rule 4: For the 2-DOF geared mechanism, the number of KUs must be greater than two.

If the number of KUs is equal to three, we have:

$$3 = 2 + t + c \tag{20}$$

Thus,

$$t + c = 1 \tag{21}$$

If t = 1, it will lead to the appearance of disconnected KUs. Hence, c should be equal to one. Thus, we have:

Rule 5: There must exist coaxial triangle type common linkage(s) in the 2-DOF structurally non-fractionated geared mechanism.

Based on Eq. (16), (17), **R 1**, **R 2**, **R 3**, **R 4**, and **R 5**, admissible connections in structurally non-fractionated GKCs can be selected, as shown in Table 1. In which, the admissible connections are labeled by letters alphabetically. There are five 1-DOF, up to 3-KU connections and three 2-DOF, up to 4-KU connections, and their configurations are shown in Table 2 and 3.

Table 1: Selection of admissible connections.

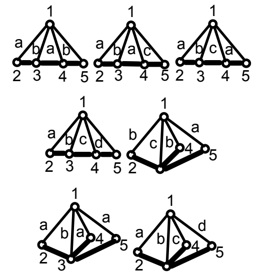
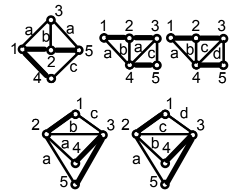
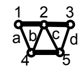
F	u	t	c	Notes
1	1	0	0	Connection (a)
1	2	1	0	Connection (b)
1	2	0	1	Against R3
1	3	2	0	Connection (c)
1	3	1	1	Connection (d)
1	3	0	2	Connection (e)
2	2	0	0	Against R1, R4, R5
2	3	1	0	Against R1, R5
2	3	0	1	Connection (f)
2	4	2	0	Against R1, R5
2	4	1	1	Connection (g)
2	4	0	2	Connection (h)

The corresponding 1-DOF 5-link GKCs can be found, as shown in Table 2; while the corresponding 2-DOF, up to 7-link GKCs are found, as shown in Table 3. It is noticed that the configuration of connection (d) has no corresponding GKCs in 1-DOF 5-link graphs; but the corresponding ones can be found in 1-DOF 7-link GKCs; the configuration of connection (e) will not be discussed, because GKCs with such configuration has not been seen in the atlas of 1-DOF, up to 7-link GKCs.

3.3 Admissible locations of the ports in geared mechanisms

Liu et al. [2] exposed the topological requirements of the locations of ports, which contain ground, input, and output links. In order to maintain the mobility of the geared mechanism, the number and the location of input in KUs should be constrained. Thus, we have:

Table 2: Configurations of 1-DOF, up to 3-KU connections (connection a to e).

	Configurations	Possible paths	Graphs
a	KU$_1$	→ KU$_1$ →	
b	KU$_1$ — KU$_2$	→ KU$_1$ → KU$_2$ →	
c	KU$_1$ — KU$_2$ — KU$_3$	→ KU$_1$ → KU$_2$ → KU$_3$ → → KU$_2$ → KU$_3$ →, KU$_1$ →	
d	KU$_3$, KU$_2$ — KU$_1$	KU$_3$, KU$_2$, KU$_1$	N/A in 1-DOF 5-link graphs
		KU$_3$, KU$_2$ → KU$_1$	N/A in 1-DOF 5-link graphs
e	KU$_3$, KU$_2$, KU$_1$		N/A in 1-DOF, up to 7-link graphs

Constraint 1: The number of inputs is equal to the number of DOF of the geared mechanism.

Constraint 2: A KU should have only one local input, since the number of DOF of a KU is equal to one.

2-link chain type common linkage needs one local input to determine the motion while coaxial triangle type needs two [9]. Thus, we have:

Constraint 3: The number of local inputs for the 2-link chain common linkage is equal to one; and that for coaxial triangle type is equal to two.

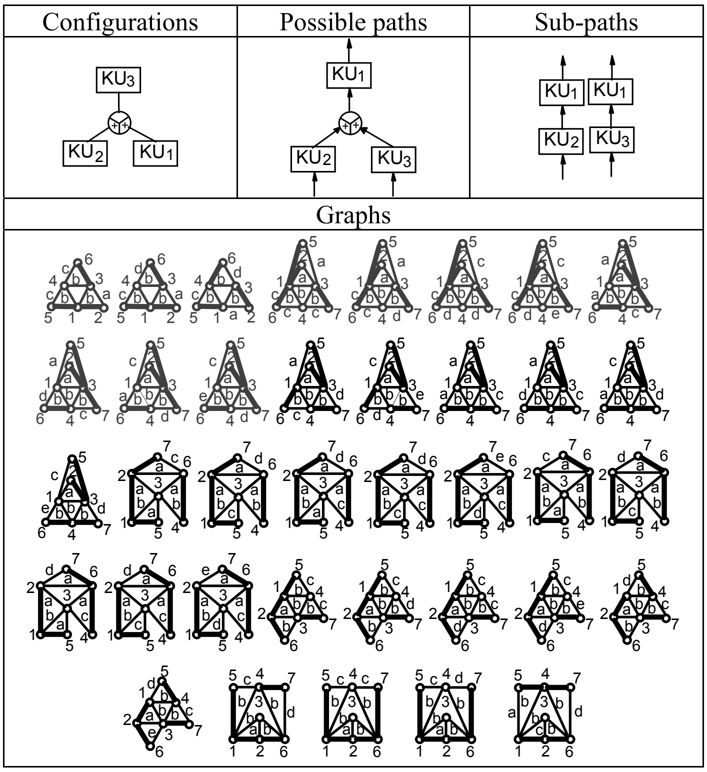
To reduce the attrition of power transmission, the redundant KU(s), which does not transmit motion, should be deducted. Thus, we have:

Constraint 4: The output link(s) should be located at the redundant end KU(s) which no ground or input assigned on it. In which the end KU is the terminal of motion transmission.

By determining the locations of input and output in the configurations of connections, redundant KUs can be avoided, and possible motion transmission paths in 1-DOF, up to 3-KU and 2-DOF, up to 4-KU geared mechanisms hence are generated, as shown in Table 2 and 3.

Table 3: Configurations of 2-DOF, up to 4-KU connections.

(a) Connection f.



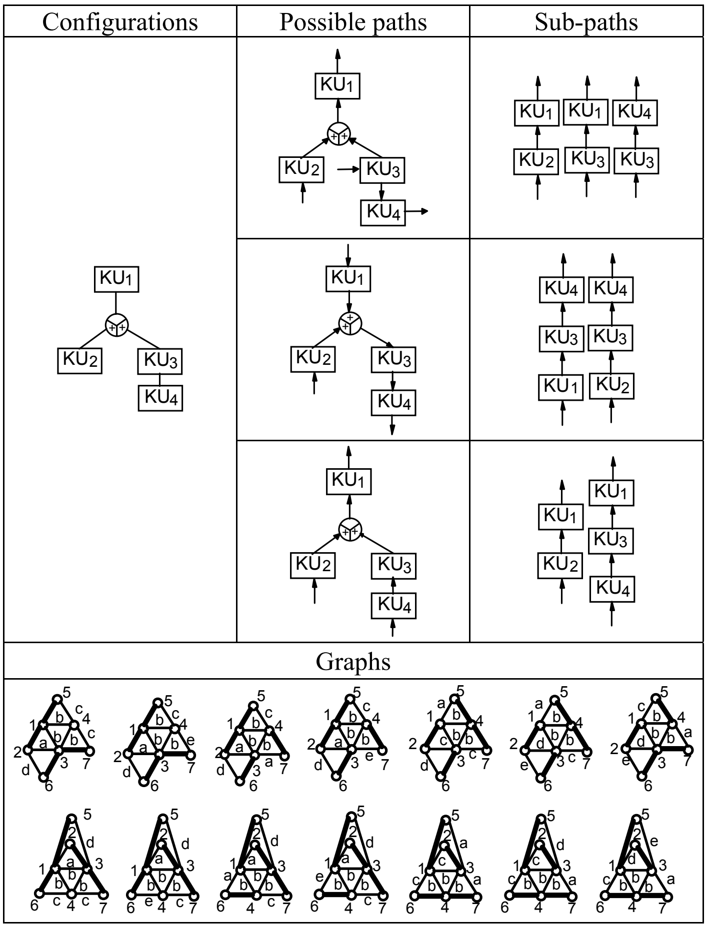
4. KINEMATIC PROPAGATION PATHS

4.1 Path patterns

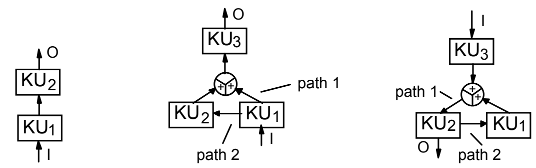
The kinematic propagation paths are the motion transmission paths from input to output, which clearly expose the kinematic relations between KUs. By observation, it can be seen that the configuration of connection of KUs is similar to the block diagram of electric circuit. Hence, the kinematic propagation path can be considered as the electric circuit, and the KUs can be regarded as the amplifiers. The analysis procedure of kinematic propagation path can be also simulated as that of electric circuit [12]. The paths can be separated into sub-paths, each containing single input and ending at the output, according to the direction of the transmission of input signal. The number of the sub-paths of a geared mechanism is equal to the number of DOF. The sub-path is composed of the KUs, which propagate the kinematic signal from the input to the output. For instance, the kinematic propagation path of the geared mechanism in Fig. 3(b) can be separated into two sub-paths, as shown in Fig. 3(c). It should be noticed that the output in Fig. 3(b) is equal to the sum of the two outputs in Fig. 3(c). The number of DOF of each sub-path is equal to one, and the kinematic analysis of complicated geared mechanisms can then be processed more efficiently.

Table 3 (continued)

(b) Connection g.



Due to this point of view, the kinematic propagation path can be separated into 1-DOF sub-paths according to the locations of input and output. That is, start from the KU, an input located, and along the direction of input signal to the output, then the path is a sub-path of the kinematic propagation path.



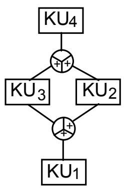
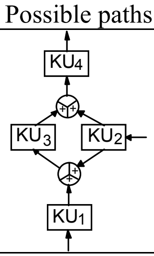
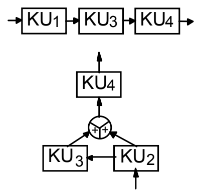
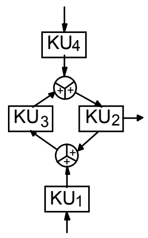
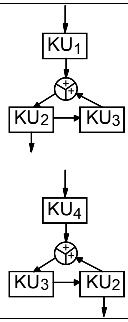
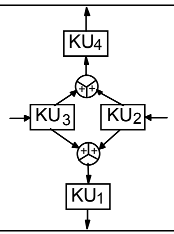
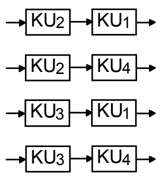
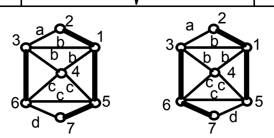
(a) Serial type. (b) Branch type. (c) Feedback type.

Figure 5: Path patterns.

The 1-DOF paths can be considered as the modules of paths, since the sub-paths of 2-DOF paths can be found in the 1-DOF paths, as shown in Table 2 and 3. And the 1-DOF, 3-KU path can be considered as the combination of two 1-DOF, 2-KU paths. Thus, the connections of module paths are connection (b) and (d), and their possible paths are module paths. Those modules are denoted as path patterns, and are shown in Fig. 5. Due to the characteristics of the kinematic propagation, the three path patterns are denominated. Path pattern in Fig. 5(a) is denoted as serial type path pattern, and path pattern in Fig. 5(b) is denoted as branch type path pattern, and path pattern in Fig. 5(c) is denoted as feedback type path pattern.

Table 3 (continued)

(c) Connection h.

Configurations	Possible paths	Sub-paths
		
		
		
		

4.2 The derivation of path gain

Due to the kinematic propagation path, the derivation of the total gear ratio of geared mechanism is facile. Since the kinematic signal flow can be simulated as the electric flow in the electric circuit, the formula for deriving the total gain can be used to derive the path gain. Hence, the gear ratio can be efficiently derived in virtue of Mason's gain formula [12]:

$$P = \frac{1}{\Delta} P_k \Delta_k \tag{22}$$

where P is the gain of an input to an output, P_k the path gain of *k*th forward path, Δ the determinant of graph = 1 – (sum of all different loop gains) + (sum of gain products of all possible combination of two nontouching loops) – …, and Δ_k the cofactor of the *k*th forward path determinant of the graph with the loops touching the *k*th forward path removed.

The total gain of geared mechanism can be derived by connection the total gains of the path patterns since the kinematic propagation path can be regarded as the combination of path patterns. Therefore, only the gains of path patterns are derived in this paper.

From Eq. (22), since there is no loop in the serial type path pattern, as shown in Fig. 5(a), the loop gain is equal to zero:

$$\Delta = 1, \Delta_k = 1 \tag{23}$$

and there is one forward path in the path pattern:

$$P_1 = p_1 \times p_2 \tag{24}$$

where P_1 is the forward path gain, p_1 the local gain of KU_1, and p_2 the local gain of KU_2.

Substitute Eq. (23), (24) into Eq. (22), the gain of the serial type path pattern can be derived as:

$$P = p_1 \times p_2 \tag{25}$$

The gain of the branch type path pattern, as shown in Fig. 5(b), can be expressed as what follows:

Since there is no loop in the path:

$$\Delta = 1, \Delta_k = 1 \tag{26}$$

There are two forward paths in the path pattern, and the two path gains can be expressed as:

$$P_1 = p_1 p_3 \tag{27}$$

$$P_2 = p'_1 p_2 p_3 \tag{28}$$

where P_1 is the first forward path gain, P_2 is the second forward path gain, p_1 is the local gain path 1 in KU_1, and p'_1 the local gain of path 2 in KU_1.

Substitute Eq. (26), (27), and (28) into Eq. (22), we have:

$$P = P_1 + P_2 = p_1 p_3 + p'_1 p_2 p_3 \tag{29}$$

Similarly, the gain of the feedback type path pattern, as shown in Fig. 5(c), can be derived as follows:

There is one forward path and one loop in the path pattern. The gain of the forward path is:

$$P_1 = p_2 p_3 \tag{30}$$

where p_2 is the local gain path 1 in KU_2.

The loop gain is:

$$L_1 = p_1 p_2 p'_2 \quad (31)$$

where p'_2 the local gain of path 2 in KU_2.

From Eq. (22), since the forward path touches the loop, we have:

$$\Delta = 1 - L_1 = 1 - p_1 p_2 p'_2 \quad (32)$$

and Δ_1 is equal to Δ removes the loop gain, hence we have:

$$\Delta_k = 1 \quad (33)$$

Substitute Eq. (30), (31), (32), (33) into Eq. (22), the gain of the feedback type path pattern can be expressed as:

$$P = p_2 p_3 / (1 - p_1 p_2 p'_2) \quad (34)$$

Equations (25), (29), and (34) show forms of the gains of the three path patterns. Since kinematic propagation paths are the compositions of the three path patterns, the forms of the total gains of the paths can be derived by combining gains of the path patterns.

4.3 Application of kinematic propagation path

It is seen that both the configurations and the locations of input and output determine the kinematic characteristics of geared mechanisms, as shown in Table 2 and 3. The configuration may have different kinematic propagation paths, when the locations of input and output are different.

Due to the exposure of admissible configurations of the connections and their corresponding kinematic propagation paths, it can be seen that the geared mechanism with the same number of KUs and DOF may have quite different kinematic characteristics. As shown in Table 2, the kinematic propagation paths of configuration (c) and (d) are considerably different, thus configuration (c) and (d) are suitable for different functional requirements. However, the traditional design procedure cannot deal with this problem but only find the right one from trail and error.

Due to the correspondences exposed in this paper, the enumeration of geared mechanisms can be focusing on both topological and kinematic points of view. The topological structure of geared mechanism can be synthesized directing to the required kinematic relation due to the exposure of kinematic propagation path.

5. CONCLUSION

The concept of KU is used to expose the correspondences between topological and kinematic characteristics in 1-DOF, up to 3-KU and 2-DOF, up to 4-KU geared mechanisms. It is shown that kinematic propagation paths can be considered as the combinations of path patterns to simplify the kinematic analysis procedure. All the possible kinematic paths and the forms of total gains are obtained thereafter. The corresponding kinematic characteristics of GKCs can then be efficient obtained. It is believed that the correspondences exposed in this paper will lead to a systematic creation of GKCs, and a functional-orientated design procedure of geared mechanisms.

Acknowledgement

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