Experimental Full Field Investigations of Resonant Vibrations for Piezoceramic Plates by an Optical Interferometry Method

by Chien-Ching Ma and Chi-Hung Huang

ABSTRACT—The experimental measurement of resonant frequencies for piezoelectric material is generally performed by impedance analysis. In this paper we employ an optical interferometry method, called amplitude-fluctuation electronic speckle pattern interferometry (AF-ESPI), to investigate the vibration characteristics of piezoceramic plates. This method demonstrates its advantages of combining noise reduction, like the subtraction method, and high fringe sensitivity, like the time-averaged method. As compared with the film recording and optical reconstruction procedures used for holographic interferometry, the interferometric fringes of AF-ESPI are produced instantly by a video recording system. Based on the fact that clear fringe patterns measured by the AF-ESPI method will be shown only at resonant frequencies, both the resonant frequencies and corresponding mode shapes are obtained experimentally at the same time. Excellent quality for the interferometric fringe patterns of the mode shapes is demonstrated. We find from experimental results that the out-of-plane vibration modes (type A) with lower resonant frequencies cannot be measured by impedance analysis and only the in-plane vibration modes (type B) will be shown. However, both the out-of-plane (bending) and in-plane (extension) vibration modes of piezoceramic plates are obtained by the AF-ESPI method. Finally, numerical finite element calculations are also performed, and the results are compared with the experimental measurements. Excellent agreement for the resonant frequencies and mode shapes are obtained from both results.

KEY WORDS—AFESPI, resonant frequency, mode shape, partially-electrode, piezoceramic plate

Introduction

Electronic speckle pattern interferometry (ESPI), which was first proposed by Butters and Leendertz[1] to investigate the out-of-plane vibration of disks, is a full-field, noncontact, and real-time optical metrology to measure the deformation of structures subjected to various kinds of loadings. This method has opened a new field of activity and been used in many industrial applications involving the investigation of static and dynamic deformation and condition monitoring of machinery. Although different methods of holographic interferometry[2] have been developed for vibration analysis, the slow and cumbersome process of film development limits the application of holographic vibration analysis in industry. ESPI was developed by combining the techniques of holographic and speckle interferometry, employing an image hologram configuration and the method of double-exposure holography. Wang *et al.*[3] proposed the amplitude-fluctuation ESPI method for out-of-plane vibration measurement to increase the visibility of the fringe pattern and to reduce environmental noise simultaneously. In the AF-ESPI method, the reference frame is recorded in a vibrating state and is subtracted from the incoming frames.

Piezoelectric transducers are widely used in electromechanical sensors, actuators, and non-destructive testing devices, as well as in electro-optic modulators, etc. Show[4] used an optical interference technique in which a stroboscopically illuminated multiple beam is applied to measure the surface motion of thick barium titanate disks. Holland[5] used the Rayleigh–Ritz method to study the extensional modes of rectangular piezoelectric plates and classified them into four distinct symmetry types. Chang[6] employed dual-beam speckle interferometry to measure the in-plane vibration amplitude on the PZT surface. Brissaud *et al.*[7] proposed a two-dimensional model to determine the for shear displacement and electrical impedance for the shear modes of a piezoceramic plate. The simulated admittance curve for the fundamental natural frequency was presented and compared with the experimental result. Bisegna and Maceri[8] derived the bending and stretching behavior of a piezoceramic plate from the three-dimensional theory of piezoelectricity, which was based on the initial function method. Ma and Huang[9,10] used the AF-ESPI method to investigate the three-dimensional vibration of piezoelectric rectangular parallelepipeds and cylinders, and both resonant frequencies and mode shapes were presented.

The study of the vibration behavior of piezoelectric materials is a problem of great practical interest. However, there are very few experimental results, especially for the full field measurement of mode shapes, available in the literature. In this paper, both an optical method based on the amplitude-fluctuation ESPI (AF-ESPI) and impedance analysis are employed to study the experimental vibration characteristics of piezoceramic plates. The advantage of using the AF-ESPI method is that both resonant frequencies and the corresponding mode shapes can be obtained simultaneously from the

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experimental investigation. The fringe patterns shown in the experimental results correspond to the vibrating mode shapes. According to the experimental results obtained in this study, the vibration modes of piezoceramic plates can be classified into two types, type A (out-of-plane) and type B (in-plane) modes. It is interesting to note that type A modes with lower resonant frequencies cannot be measured by impedance analysis and the vibration mode shapes are similar to those of isotropic plates. To validate the conclusion, investigation of a piezoelectric plate with partial electrodes is also performed. In addition to the AF-ESPI experimental technique, numerical computations based on a finite element package are presented and good agreement for resonant frequencies and mode shapes are found.

The Theory of the AF-ESPI Method

The basic theory for out-of-plane vibrating measurement by ESPI is briefly discussed first. If the image is taken after the specimen vibrates periodically, the light intensity detected by a charge-coupled device (CCD) camera is indicated as I_1. The AF-ESPI method is employed in this study by taking two images while the specimen vibrates and assuming that the vibration amplitude of the second image has changed from A to $A + \Delta A$ due to the instability of apparatus. The light intensity of the second image is indicated as I_2. When these two images (I_1 and I_2) are subtracted and rectified by the image processing system, the resulting image intensity can be expressed as

$$I = I_2 - I_1 = \frac{\sqrt{I_A I_B}}{2} \left| (\cos\phi)\Gamma^2(\Delta A)^2 J_0(\Gamma A) \right|, \quad (1)$$

where I_A is the object light intensity, I_B is the reference light intensity, ϕ is the phase difference between object and reference light, J_0 is a zeroth-order Bessel function of the first kind, and

$$\Gamma = \frac{2\pi}{\lambda}(1 + \cos\theta), \quad (2)$$

in which λ is the wavelength of the laser and θ is the angle between object light and observation direction.

Similar to the out-of-plane vibration case, the first and second image intensities for in-plane vibration are denoted as I_1 and I_2, respectively. When these two images (I_1 and I_2) are subtracted and rectified by the image processing system, the image intensity is

$$I = I_2 - I_1 = \frac{\sqrt{I_A I_B}}{2} \left| (\cos\phi)\Gamma'^2(\Delta A')^2 J_0(\Gamma' A') \right|, \quad (3)$$

where I_A and I_B are the object light intensities, A' is the amplitude of in-plane vibration, and

$$\Gamma' = \frac{2\pi}{\lambda}(2\sin\theta'), \quad (4)$$

in which θ' is half of the angle between two illumination lights.

From eq (1) and eq (3), it is indicated that both the out-of-plane and in-plane vibration fringe patterns obtained by the AF-ESPI method are dominated by the zeroth-order Bessel function J_0. A detailed discussion of the AF-ESPI method was provided by Ma and Huang.[9] Combining the out-of-plane and in-plane optical setups by the AF-ESPI method, we can construct complete vibration characteristics of the piezoelectric plate, including resonant frequencies and mode shapes at the same time. This is different from impedance analysis, which has been used widely in determining the resonant frequency for piezoelectric materials.

Experimental and Numerical Results

A piezoelectric plate made of $Pb(Zr{\cdot}Ti)O_3$ ceramic is selected for experimental investigations and the modal number is PIC-151 (Germany Physik Instrumente Company). The polarized piezoelectric ceramic has the same symmetry as a hexagonal crystal in class $C_{6v} = 6$ mm, which can be modeled as a transversely isotropic material. The elastic, piezoelectric, dielectric constants and geometric dimensions of the specimens are shown in Table 1 and Fig. 1, respectively. The polarization axis is in the x_3 direction and two opposite faces (x_1-x_2 plane) of the specimen are completely coated with silver electrodes.

TABLE 1—MATERIAL PROPERTIES OF PIC-151

Quantity	PIC-151
c_{11}^E (10^{10} N/m^2)	10.76
c_{33}^E	10.04
c_{12}^E	6.312
c_{13}^E	6.385
c_{44}^E	1.962
$c_{66}^E = \left(c_{11}^E - c_{12}^E\right)/2$	2.224
e_{31} (N/Vm)	−9.6
e_{33}	15.1
e_{15}	12.0
$\varepsilon_{11}^S/\varepsilon_0$	1110
$\varepsilon_{33}^S/\varepsilon_0$	852
ρ (kg/m^3)	7760

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

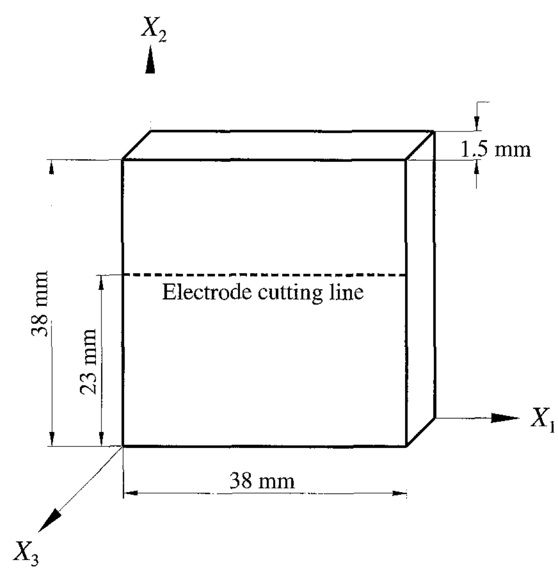

Fig. 1—Geometric dimensions of the piezoceramic plate

The schematic layout of self-arranged time-averaged AF-ESPI optical systems, as shown in Figs. 2 and 3, are used to perform the out-of-plane and in-plane experimental measurements for resonant frequencies and corresponding mode shapes. A He–Ne laser with 30 mW and wavelength λ = 632.8 nm is used as the coherent light source. We use a CCD camera (Pulnix Company) and a P360F (Dipix Technologies, Inc.) frame grabber with a digital signal processor onboard to record and process the images. As shown in Fig. 2 for the out-of-plane measurement, the laser beam is divided into two parts, the object and reference beams, by a beamsplitter. The object beam travels to the specimen and then reflects to the CCD camera. The reference beam is directed to the CCD camera via a mirror and a reference plate. For the in-plane measurement system shown in Fig. 3, two laser beams with the same optical path and light intensity are symmetrically incident to the specimen, and then reflect to the CCD camera. The CCD camera converts the intensity distribution of the interference pattern of the object into a corresponding video signal at 30 frames per second. The signal is electronically processed and finally converted into an image on the video monitor. The interpretation of the fringe image is similar to reading of a contour map. To achieve the sinusoidal output, a function generator HP33120A (Hewlett Packard) connected to a 4005 power amplifier (NF Electronic Instruments) is used.

The experimental procedure of the AF-ESPI technique is performed as follows. First, a reference image is taken after the piezoceramic plate vibrates, then the second image is taken, and the reference image is subtracted by the image processing system. If the vibrating frequency is not the natural frequency, only random distributed speckles are displayed and no fringe patterns will be shown. However, if the vibrating frequency is in the neighborhood of the resonant frequency, stationary distinct fringe patterns will be observed. Then the function generator is carefully and slowly turned; the number of fringes will increase and fringe pattern will become clearer as the resonant frequency is approached. From the aforementioned experimental procedure, the resonant frequencies and the correspondent mode shapes can be determined at the same time.

According to the experimental results from using the AF-ESPI optical system, we find that the natural frequencies and corresponding mode shapes of the piezoceramic plate can be classified into two types, named type A and B modes in this paper. Comparing with type B modes, the natural frequencies of type A modes are much lower and the mode shapes can be obtained only by the out-of-plane measurement. Hence we can conclude that the type A modes are the out-of-plane vibration modes and the type B modes are the in-plane modes. In addition to the experimental measurement, numerical calculation is also investigated by the commercially available software ABAQUS[11] finite element package in which a 20-node three-dimensional brick element (C3D20E) is selected to analyze the problem. Both the experimental measurements obtained by impedance analysis and the numerical calculation will be used in the comparison with the result measured by using the AF-ESPI method. Figs. 4 and 5 show the experimental and numerical results for the first five vibration mode shapes of type A and type B modes, respectively. We indicate the phase of displacement in the finite element results as solid or dashed lines; the solid lines are in the opposite direction to the dashed lines. The transition from solid lines to dashed

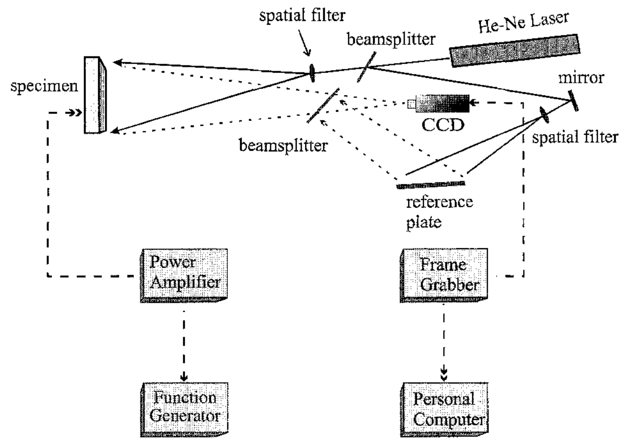

Fig. 2—Schematic diagram of the AF-ESPI setup for out-of-plane measurement

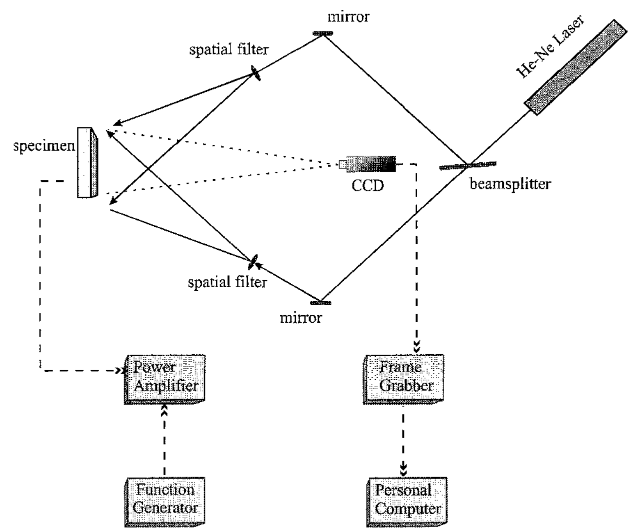

Fig. 3—Schematic diagram of the AF-ESPI setup for in-plane measurement

lines corresponds to a zero displacement line, or a nodal line. The zeroth-order fringe, which is the brightest fringe in the experimental results, represents the nodal lines of the vibrating piezoceramic plate at resonant frequencies. The rest of the fringes are contours of constant amplitudes of displacement, which can be quantitatively calculated by $J_0'(\Gamma A) = 0$ (or $J_0'(\Gamma' A') = 0$) according to eq (1) (or eq (3)) for out-of-plane (or in-plane) measurement. Considering the sensibility of the experimental measurement, we choose $\theta = 10°$ and $\theta' = 60°$ for the experimental setup. The mode shapes obtained by experimental results can be checked by the nodal lines and fringe patterns with the numerical finite element calculations, and excellent agreement is found.

Because the electrical impedance of the piezoceramic material drops to a local minimum when it vibrates at a resonant frequency, the resonant frequency can also be determined by impedance analysis. Here it is carried out by using an HP4194A impedance/gain-phase analyzer (Hewlett Packard) and the impedance curve for the piezoceramic plate measured from HP4194A is shown in Fig. 6. Unexpectedly we find that

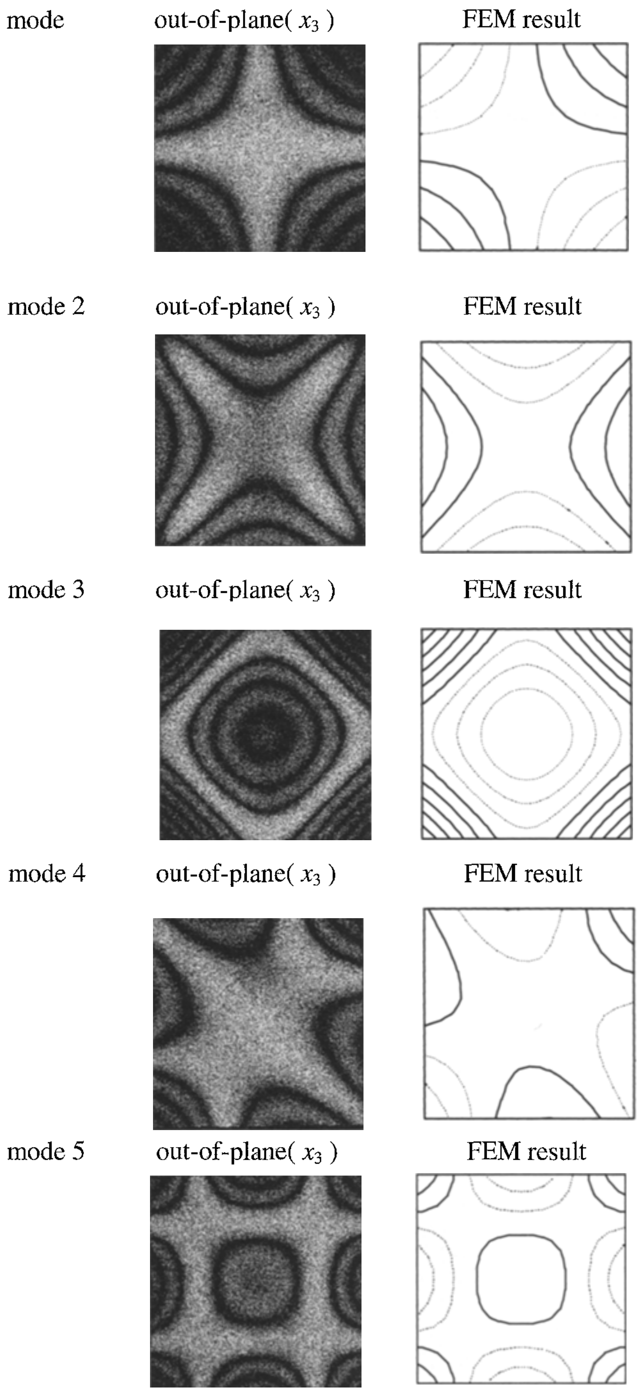

Fig. 4—Mode shapes of type A obtained by AF-ESPI and FEM (full electrodes)

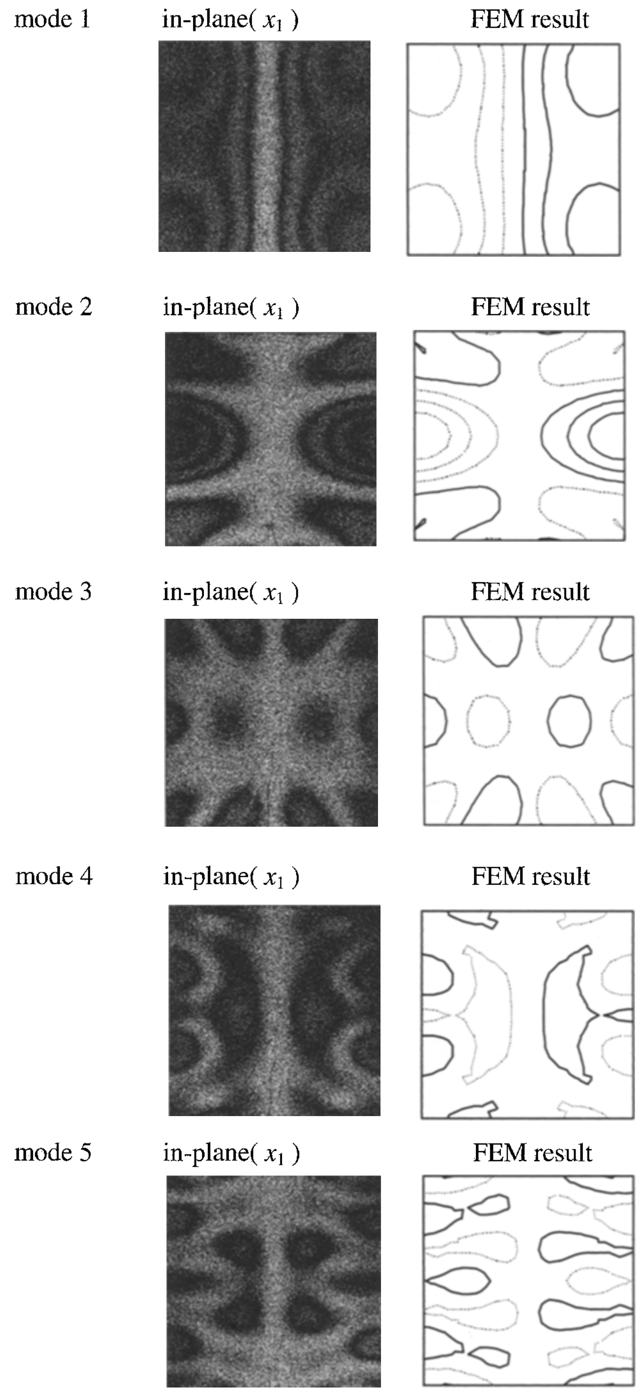

Fig. 5—Mode shapes of type B obtained by AF-ESPI and FEM (full electrodes)

only the resonant frequencies of the type B modes are indicated in Fig. 6, i.e., those of the type A modes cannot be obtained by impedance analysis. This phenomenon can be explained by the characteristics of piezoelectricity. When the piezoelectric plate vibrates at a resonant frequency, the charge will be induced on the electrode surfaces owing to the vibration deformation (the direct piezoelectric effect) and the impedance will drop to a local minimum value. This is why we can measure the resonant frequencies of piezoceramic plates by using the impedance analyzer. If the summation of the induced charge distributed over the electrode surfaces is zero, we will not be able to find a large variation of impedance at the resonant frequency. Type A modes are just the situation mentioned above and we cannot obtain the resonant frequencies from the impedance curve shown in Fig. 6. Table 2 shows the first five resonant frequencies of the piezoceramic plate obtained by using AF-ESPI, impedance analysis, and the FEM method. The discrepancy of resonant frequencies between AF-ESPI and impedance analysis for type B modes is smaller than that between AF-ESPI and FEM. However, the difference between the experimental data and FEM may be a result of the measurement of the material properties and a defect of the piezoelectric material, which is generated by the manufacturing process.

TABLE 2—RESULTS OF RESONANT FREQUENCIES OBTAINED FROM AF-ESPI, IMPEDANCE ANALYSIS AND FEM FOR THE PIEZOCERAMIC PLATE (FULL ELECTRODES)

Type A			
Mode	AF-ESPI (Hz)	Impedance Analysis (Hz)	FEM (Hz)
1	1850	—	1829
2	2820	—	2763
3	4200	—	4286
4	5000	—	4938
5	9210	—	9210
Type B			
Mode	AF-ESPI (Hz)	Impedance Analysis (Hz)	FEM (Hz)
1	43800	43755	42653
2	62600	62753	60772
3	118600	118625	115535
4	122300	122375	119276
5	136700	136625	134083

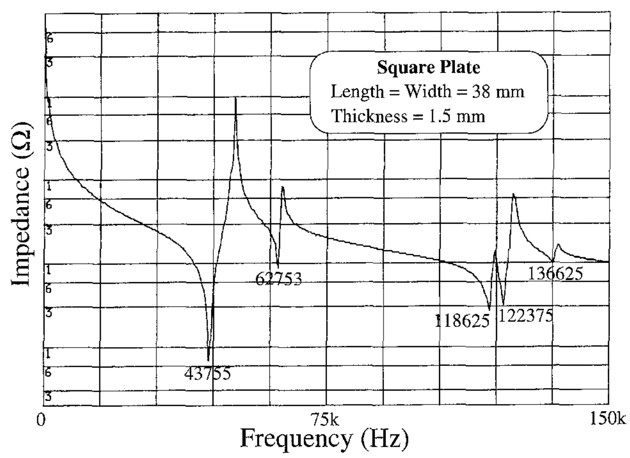

Fig. 6—Impedance variation curve of the piezoceramic plate (full electrodes)

From observing the mode shapes of type A, we find that the vibration characteristics are similar to that of the out-of-plane vibration for an isotropic plate. Next, we slit the silver electrode of one surface of the piezoceramic plate to form the partial electrodes shown in Fig. 1. After performing the experimental procedure and numerical calculation as mentioned above, we obtain results for the resonant frequencies and mode shapes for type A modes that are almost the same as those for the full-electrode specimen. Table 3 shows the resonant frequencies of the partial-electrode specimen obtained by AF-ESPI, impedance analysis, and FEM methods. Since the type A mode shapes for the partial-electrode specimen are the same as that for the full-electrode case, only type B mode shapes will be shown. Because the symmetry of electrodes has been destroyed, we should measure the vibration of in-plane modes in both the x_1 and x_2 directions in order to construct the complete structure of the vibration behavior. Fig. 7 shows the experimental and numerical results for the first eight mode shapes of type B for the partial-electrode piezoceramic plate. It is noted that the mode shapes of type B for the partial-electrode piezoceramic plate are completely different from that of the full-electrode plate shown in Fig. 5.

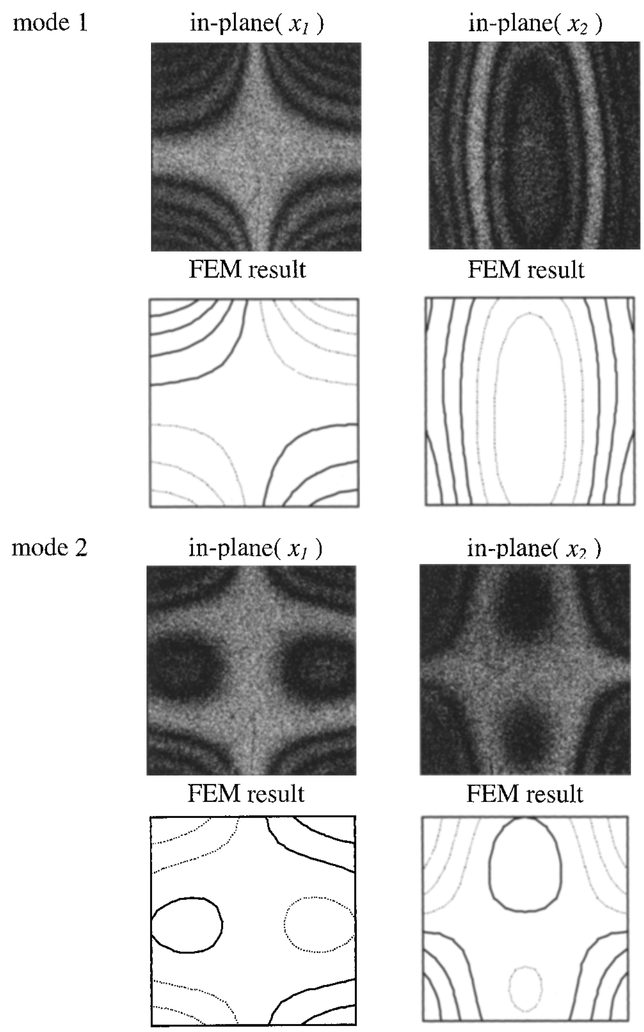

Fig. 7—Mode shapes of type B obtained by AF-ESPI and FEM (partial electrodes) *(continued on next page)*

The impedance variation curve is shown in Fig. 8 and the resonant frequencies of type A modes cannot be found as we have mentioned in the full-electrode case.

TABLE 3—RESULTS OF RESONANT FREQUENCIES OBTAINED FROM AF-ESPI, IMPEDANCE ANALYSIS AND FEM FOR THE PIEZOCERAMIC PLATE (PARTIAL ELECTRODES)

Mode	AF-ESPI (Hz)	Impedance Analysis (Hz)	FEM (Hz)
Type A			
1	1887	—	1829
2	2850	—	2763
3	4220	—	4286
4	5050	—	4939
5	9260	—	9211
Type B			
1	31160	31225	30606
2	37260	37200	37134
3	45570	45675	44479
4	46960	47075	44929
5	63050	63100	61511
6	77240	77225	75014
7	83110	83113	81708
8	89360	89488	88225

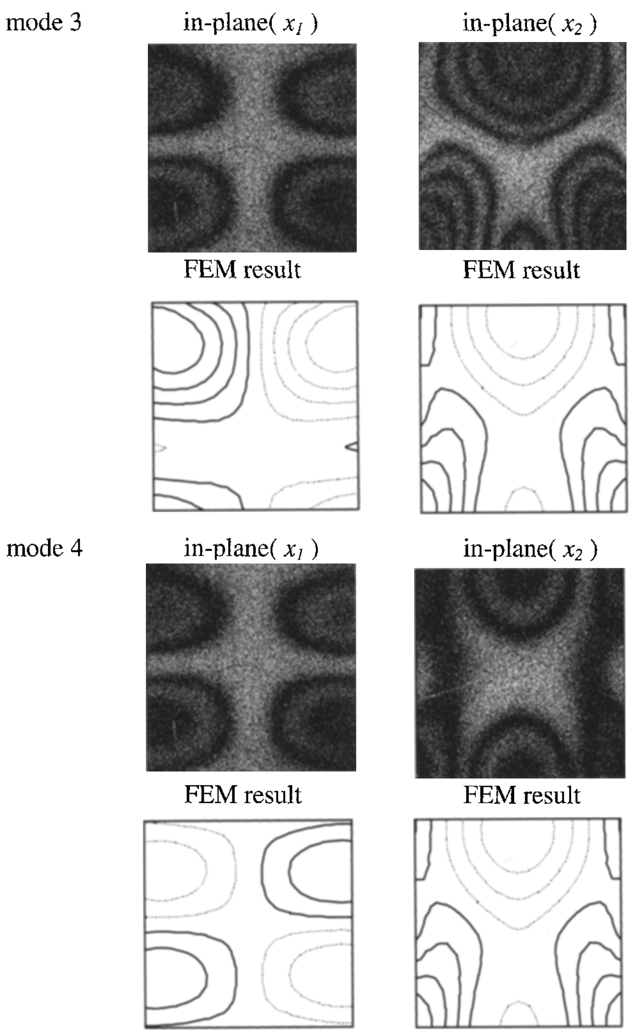

Fig. 7—*(continued from previous page)*

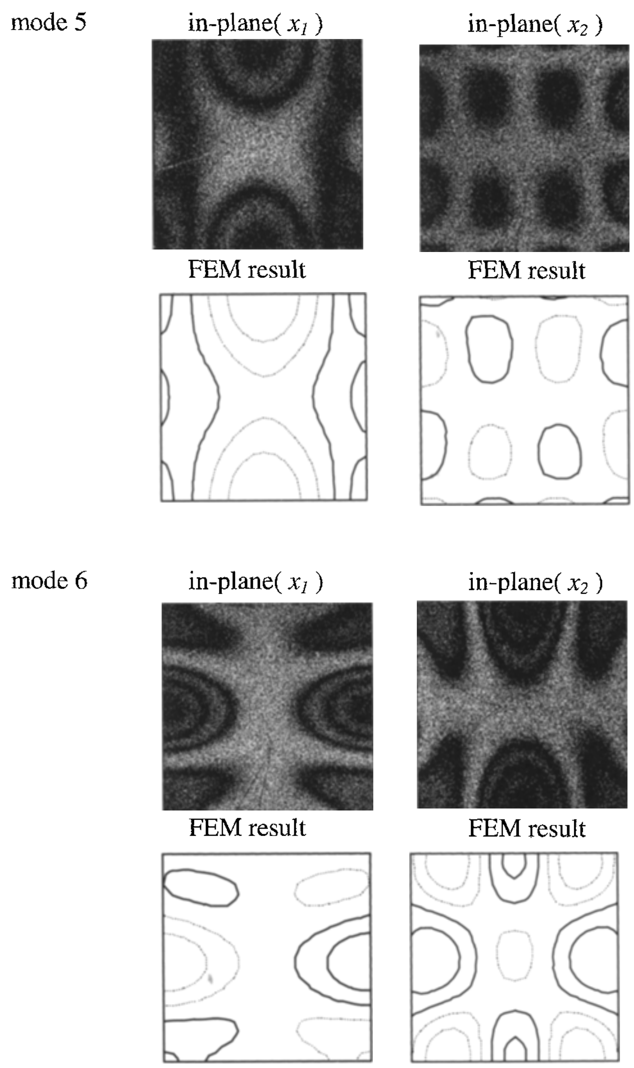

Fig. 7—*(continued)*

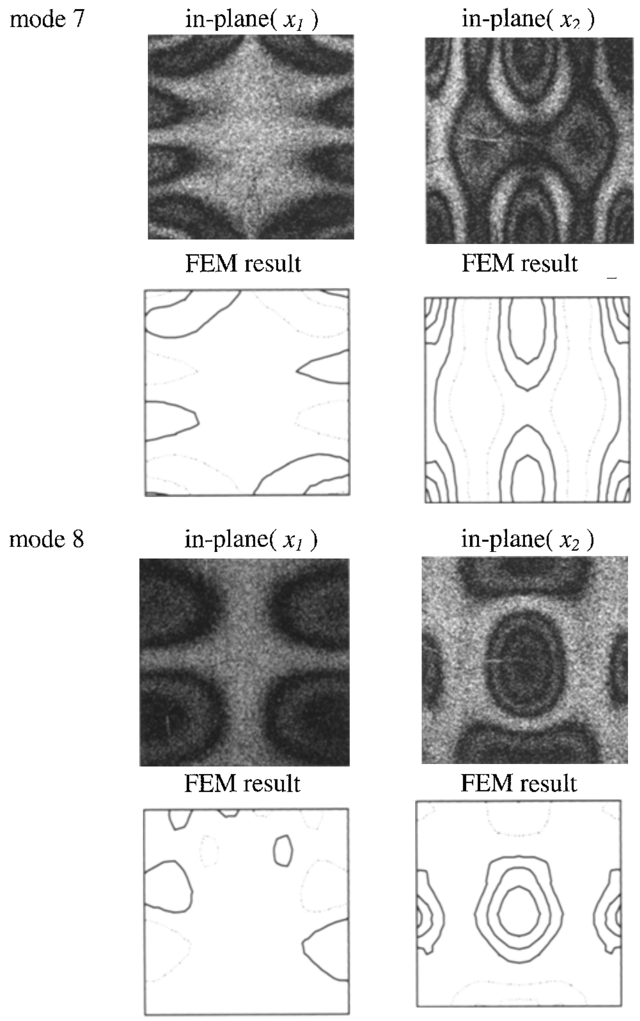

Fig. 7—*(continued)*

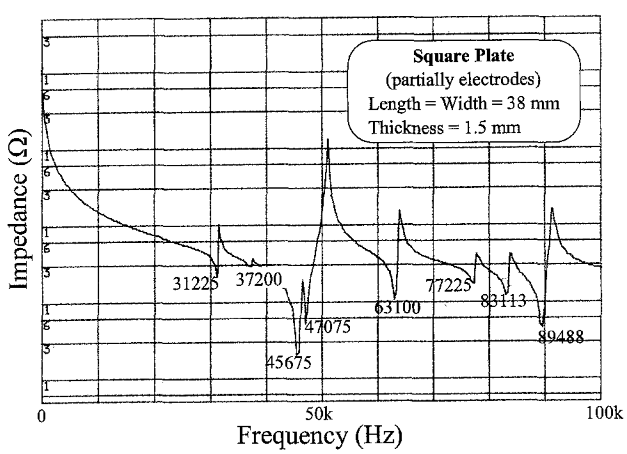

Fig. 8—Impedance variation curve of the piezoceramic plate (partial electrodes)

Conclusions

Optical techniques have been proved to have certain advantages for vibration analysis and ESPI has been applied to many vibration problems. The advantages of the optical ESPI method include noncontact and full-field measurement, real-time observation, submicron sensitivity, and validity of both static deformation and dynamic vibration. As compared with the film recording and optical reconstruction procedures used for holographic interferometry, the interferometric fringes of AF-ESPI are produced instantly by a video recording system.

It is known that the vibration characteristics of piezoelectric materials are important in many engineering applications. Most of the works for vibration analysis of piezoelectric plates published in the literature are analytical and numerical results. There are only few experimental results available for the full-field configuration of mode shapes for vibrating plates. In this study, a self-arranged amplitude-fluctuation ESPI optical setup with good visibility and noise reduction has been established to obtain the resonant frequencies and the corresponding mode shapes of vibrating piezoceramic plates at the same time. The resonant frequencies of piezoceramic plates are also determined by impedance analysis in this study. Both full-electrode and partial-electrode piezoceramic plates are investigated. Based on the experimental results obtained by the AF-ESPI method, we have classified the vibration modes into two types, the out-of-plane (type A) and in-plane (type B) modes. However, the resonant frequencies of type A modes cannot be measured by impedance analysis. Numerical calculations of resonant frequencies and mode shapes based on a finite element package are also performed and excellent agreement for the mode shapes are obtained if compared with results obtained by AF-ESPI. The resonant frequencies of type B modes obtained by AF-ESPI are in good agreement with the impedance analysis but there is a slight difference compared with finite element calculations. The results shown in this study demonstrate that the AF-ESPI method is applicable to many situations in engineering vibration analysis as long as the vibration amplitude reaches the sensitivity of the AF-ESPI method.

Acknowledgments

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