行政院國家科學委員會專題研究計畫　成果報告

# 無漂移系統之片段閉迴路控制

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行政院國家科學委員會專題研究計劃成果報告

# 無飄移系統之片段閉迴路控制

# Piecewise Closed-Loop Control of Driftless Systems

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**摘要**

在本計劃中，我們提出了一個無飄移系統的運動規劃演算方法，這個控制器的設計是使用片段平滑的控制輸入信號的方法。為了改進系統的性能，我們提出閉迴路控制的設計，這個閉迴路設計的提出可以將一個非線性無飄移系統的狀態帶到任何我們要求的位置並可以呈現漸進穩定。

**關鍵字**：運動規劃、無飄移系統、閉迴路控制

### Abstract:

In this project, we propose a motion planning algorithm for driftless nonlinear systems. This control design use piecewise smooth control signals to steer the state. To improve the system performance, we develop a closed loop control design for the motion planning. The proposed closed-loop control design can drive the state of a driftless nonlinear system to any desired position asymptotically.

**Keyword**: motion planning, driftless system, closed-loop control

## 1. Introduction

In this project, we consider the closed-loop control design using piecewise smooth control signals. Consider a drift-free system, which is nonholonomic, described as:

$$\dot{x} = \sum_{i=1}^{m} g_i(x)u_i,\ x(0) = x_0 \qquad (1)$$

where $x$, $g_i(x)$, $u_i$ are the system state, the input vector, and the control respectively. The objective is to find a piecewise continuous closed-loop control

$$u(t) = U(x(k\Delta t)), \quad t \in [k\Delta t,\ (k+1)\Delta t),$$

where $\Delta t$ is the sampling time, that drives the state of Eq.(1) to the origin of the coordinates asymptotically.

## 2. Control Design

The closed-loop control algorithm we use to solve this motion planning problem can be separated into three steps:

(1) Design the fictitious control input $v(t)$, $t \in [k\Delta t, (k+1)\Delta t)$, based on $x(k\Delta t)$.
(2) Use $v(t)$ to calculate *Philip Hall coordinates* $h(t)$, $t \in [k\Delta t, (k+1)\Delta t)$.
(3) Use $h(t)$ to compute the true control input $u(t)$, $t \in [k\Delta t, (k+1)\Delta t)$

*Step1. Design the fictitious control* $v(t)$

In the first step of control design, we choose the fictitious inputs, $v_i$ 's, in the extended system of Eq.(1)

$$\dot{x} = \sum_{i=1}^{s} B_i(x)v_i,\ x(0) = x_0 \qquad (2)$$

that can drive the state, $x$, to the origin exponentially. Note that in Eq.(2) the number of the fictitious control inputs, $s(\geq n)$, is chosen such that the following assumption holds :

$rank\left[B_1(x), B_2(x), \cdots, B_s(x)\right] = n$, for all $x$.

Choose a Lyapunov function $V = \| x \|^2$. The time derivative of $V$ along the controlled extended system Eq.(2) is found to be

$$\dot{V} = 2x^T \dot{x}$$

$$= 2x^T \left[B_1(x) \quad B_2(x) \quad \cdots \quad B_s(x)\right] \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_s(t) \end{bmatrix}$$

The design for the fictitious control inputs is given by

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_s(t) \end{bmatrix} = -\gamma \left[B_1(x) \quad B_2(x) \quad \cdots \quad B_s(x)\right]^T x(t) \tag{3}$$

Substituting the fictitious inputs into the derivative of V, we obtain

$$\dot{V} = -2\gamma \left\| \left[B_1(x) \quad B_2(x) \quad \cdots \quad B_s(x)\right]^T x(t) \right\|^2 \tag{4}$$

**Lemma 1** *Lasalle's Principle* **:** Let $V(x): R^n \to R$ be a p.d.f and $\dot{V}(x) \le 0$ for all $x \in R^n$. Define the set

$$S = \left\{ x \in R^n : \dot{V}(x) = 0 \right\}$$

Then

$$\lim_{t \to \infty} x(t) \in S$$

According to the Lasalle's principle, Eq.(4) yields $\lim_{t \to \infty} \left[B_1(x) \quad B_2(x) \quad \cdots \quad B_s(x)\right]^T x(t) = 0$ Since $\left[B_1(x) \quad B_2(x) \quad \cdots \quad B_s(x)\right]$ is full rank, one concludes that

$$\lim_{t \to \infty} x(t) = 0$$

*Step2. Calculate the desired Philip Hall coordinate* $h^d(t)$

The relationship between Philip Hall coordinate and the fictitious input is as follows

$$\begin{aligned}
\dot{h}_1^d &= v_1 & t &\in (t_k^+, t_{k+1}^-) \\
\dot{h}_2^d &= v_2 & t &\in (t_k^+, t_{k+1}^-) \\
\dot{h}_3^d &= h_1 v_2 & t &\in (t_k^+, t_{k+1}^-) \\
\dot{h}_4^d &= \tfrac{1}{2} h_1^{d\,2} v_2 + h_1^d v_3 + v_4 & t &\in (t_k^+, t_{k+1}^-) \\
\dot{h}_5^d &= h_2^d v_3 + h_1^d h_2^d v_2 + v_5 & t &\in (t_k^+, t_{k+1}^-) \\
&\vdots
\end{aligned} \tag{5}$$

(4-4)

with the initial conditions $h_i^d(k\Delta t) = 0,\ i = 1,\ 2,\ \cdots,\ s$, and $t_k = k\Delta t$.

Unfortunately, since $v(t)$ depends on $x(t)$ according to the control law in Eq.(3), at the time instant $t_k = k\Delta t$, we have no prediction of the future value of $x(t), t \in (t_k, t_{k+1})$ and hence $v(t), t \in (t_k, t_{k+1})$ .Therefore, the differential equation, Eq.(5), can not be solved for $h^d(t), t \in (t_k, t_{k+1})$ .However, $h^d(t)$ can be approximately solved under the assumption:

(4-5)

$$v(t) \cong v(t_k), \ t \in [t_k, t_{k+1})$$
$$= -\gamma\left[B_1(x(t_k)) \quad B_2(x(t_k)) \quad \cdots \quad B_s(x(t_k))\right]^T x(t(t_k)) \tag{6}$$

when $\Delta t$ is small. Substituting Eq.(6) into Eq.(5), we can now solve the approximate solution $h^d(t), t \in [t_k, t_{k+1})$, without the causality problem. The solution obtained is defined as $h^d((k+1)\Delta t) = h^d$.

*Step3. Design the continuous control* $u(t)$

The last step is to find a continuous control $u(t) = U(x(k\Delta t)),\ t \in [t_k, t_{k+1})$ of the system in Eq.(1) that does the same work as $v(t)$. Consider the two-input case ($m = 2$) in Eq.(1), which is a nilpotent system of order 3. That is, $B_5 = B_6 = \cdots = B_\infty = 0$. The real *Philip Hall coordinate* reduces to

$$
\begin{aligned}
\dot{h}_1(t) &= u_1(t) & t \in (t_k^+, t_{k+1}^-) \\
\dot{h}_2(t) &= u_2(t) & t \in (t_k^+, t_{k+1}^-) \\
\dot{h}_3(t) &= h_1(t)u_2(t) & t \in (t_k^+, t_{k+1}^-) \\
\dot{h}_4(t) &= \tfrac{1}{2}h_1^2(t)u_2(t) & t \in (t_k^+, t_{k+1}^-)
\end{aligned} \tag{7}
$$

Given $h_i(k\Delta t) = 0$ in Eq.(7), and the desired end point $h_i((k+1)\Delta t) = h_i^d$, the control inputs $u_1(t),\ u_2(t)$ in Eq.(7) can be calculated as, according to the open-loop control formula in linear system theory,

$$
u_1(t) = \sum_{n=0,1,2,\Lambda} \frac{a_n(2n+1)\pi}{\Delta t}\cos\left(\frac{(2n+1)\pi}{\Delta t}t\right) + \frac{h_1^d}{\Delta t}
$$

$$
u_2(t) = -\begin{bmatrix} 1 & h_1(t) & \tfrac{1}{2}h_1^2(t) \end{bmatrix} W_c^{-1}(\Delta t) \left[ \begin{bmatrix} h_2(k\Delta t) \\ h_3(k\Delta t) \\ h_4(k\Delta t) \end{bmatrix} - \begin{bmatrix} h_2^d \\ h_3^d \\ h_4^d \end{bmatrix} \right]
$$

where $a_n$ 's are constants, $t \in [t_k, t_{k+1})$, and

$$
W_c(t) = \int_0^t \begin{bmatrix} 1 \\ h_1(\tau) \\ \tfrac{1}{2}h_1^2(\tau) \end{bmatrix} \begin{bmatrix} 1 & h_1(\tau) & \tfrac{1}{2}h_1^2(\tau) \end{bmatrix} d\tau .
$$

The above control input steers $h_i(t)$ 's to the desired end point $h_i^d$ 's, and hence steers the system state to follow the flow

$$
x(t_{k+1}) = e^{h_4^d B_4} e^{h_3^d B_3} e^{h_2^d B_2} e^{h_1^d B_1}(x(t_k))
$$

where $h_i^d$ 's are obtained based on $x(t_k)$.

In conclusion, the above proposed design steps uses a piecewise smooth control :

$$
u(t) = U(h(t_{k+1}^-)) = U(x(t_k)), \quad t \in [t_k, t_{k+1})
$$

to achieve asymptotic stability, that is,

$$
\lim_{k \to \infty} x(k\Delta t) = 0
$$

## 3. Simulation Example

Now we illustrate the above closed-loop control with an example. Consider the nilpotent system of order 2

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \tan(x_3) \\ 0 \end{bmatrix} w_1(t) + \begin{bmatrix} 0 \\ 0 \\ \cos^2(x_3) \end{bmatrix} w_2(t) \quad (4\text{-}9)
$$

$$
x_0 = x(0) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T
$$

where $w_i(t) \in R^1$. In this simulation we choose a sampling time $\Delta t = 0.5$ sec. The simulation is stopped if $\| x(k\Delta t) \|^2 \le 10^{-3}$. Figure 1 shows the desired *Philip Hall coordinate* $h_i^d(t)$ with time. Figure 2 shows the control input $w_1(t)$, $w_2(t)$ with time. Figure 3 shows the state trajectory $x(t)$ with time. Figure 4 shows the motion of the cart. (4-10a)

(4-10b)

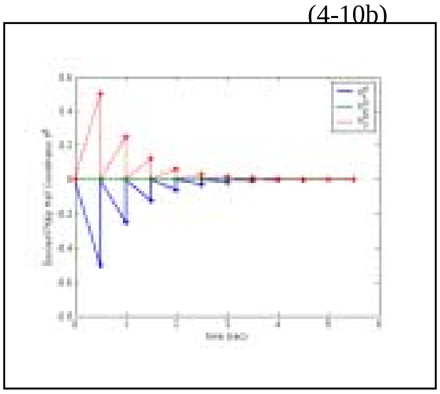

**Fig1. Desired Philip Hall coordinates with time**

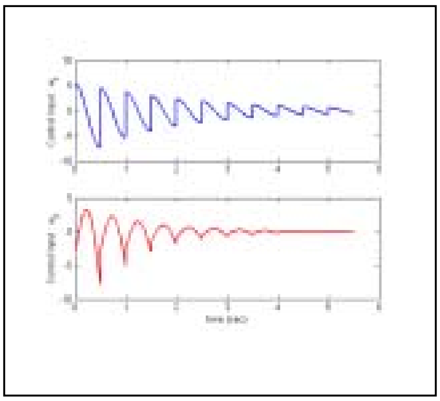

**Fig2 . Control input with time**

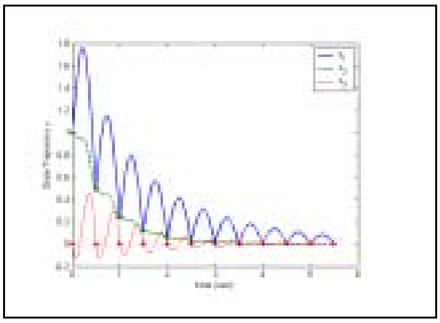

**Fig3.  State trajectory with time**

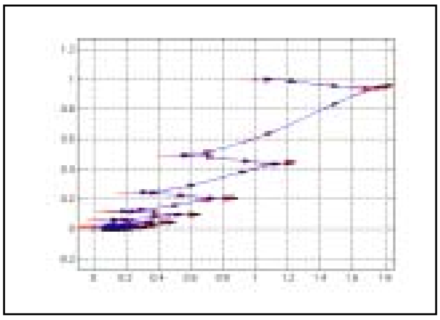

**Fig4.  The motion of the unicycle**