Transient Analysis for Antiplane Crack Subjected to Dynamic Loadings

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The problem considered here is the antiplane response of an elastic solid containing a half-plane crack subjected to suddenly applied concentrated point forces acting at a finite distance from the crack tip. A fundamental solution for the dynamic dislocation is obtained to construct the dynamic fracture problem containing a characteristic length. Attention is focused on the time-dependent full-field solutions of stresses and stress intensity factor. It is found that at the instant that the first shear wave reaches the crack tip, the stress intensity factor jumps from zero to the appropriate static value. The stresses will take on the appropriate static *value instantaneously upon arrival of the shear wave diffracted from the crack tip, and this static value is thereafter maintained. The dynamic stress intensity factor of a kinked crack from this stationary semi-infinite crack after the arrival of shear wave is obtained in an explicit form as a function of the kinked crack velocity, the kink angle, and time. A perturbation method, using the kink angle as the perturbation parameter, is used. If the maximum energy release rate is accepted as the crack propagation criterion, then the crack will propagate straight ahead of the original crack when applying point load at the crack face.*

1 Introduction

Most of the analysis done regarding cracked bodies are quasi-static. Because of loading conditions and material characteristics, there are numerous problems for which the assumption that the deformation is quasi-static is invalid and the inertia of the material must be taken into account. The inherent time dependence of the dynamic fracture problems makes them more complex than equivalent quasi-static models. Both the case of a stationary crack in a body subjected to dynamic loading and the case of a rapidly propagating crack in a stressed body are considered as dynamic fracture problems.

When dynamic loading is applied to a body with an internal crack, the resulting stress waves may initiate crack growth. Few solutions for a cracked elastic solid subjected to dynamic loading are available. The most notable of these are the analysis of diffraction of a plane pulse for a semi-infinite crack by de Hoop (1958) and the equivalent problem for a finite length crack by Thau and Lu (1971). The study of propagation crack in a brittle solid began with the pioneering analysis of Yoffe (1951), and considerable progress has been made in the area of dynamic brittle fracture. Transient problems for constant crack propagation velocity along the fracture plane have been studied by Baker (1962), Broberg (1960), and Achenbach and Nuismer (1971). In a series of papers, Freund (1972a, 1972b, 1973, 1974a) developed important analytical methods for evaluation of the transient stress field of a propagating crack in a two-dimensional geometric configuration. In Freund's papers, a fundamental solution is obtained and is used to develop the solution for negation the stress distribution on the prospective fracture plane by superposition. A generalization of this idea also led to solutions of crack kinking problems under dynamic loading analyzed by Ma and Burgers (1986, 1987, 1988) and Ma (1988).

The difficulty in determining the transient stress field in a cracked body subjected to dynamic loading is well known. The complete solution of a spatially uniform traction distribution acting on the crack faces can be obtained by integral transformation methods. If the problem is modified by replacing a nonuniform distribution having a characteristic length, then the same solution procedure using integral transformation methods does not apply. Freund (1974b) developed a technique which makes it possible to solve this modified problem. Freund solved the problem of an elastic solid containing a half-plane crack subjected to concentrated impact loading on the faces of the crack. An exact expression for the dynamic stress intensity factor was derived by superposition over a one-parameter family of continuously distributed moving dislocations. The complete elastic solution can also be determined by this scheme, but only the stress intensity factor was studied in detail by Freund (1974b). Freund found that if the applied point loading on the crack faces is a step function of time dependence, the dynamic stress intensity factor is zero until the longitudinal wave, which was generated at the loading point, arrives at the crack tip. At the instant the Rayleigh wave arrives,

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the stress intensity factor takes on its appropriate static value, and this value in maintained thereafter. Dynamic stress wave interaction with cracks was analyzed by Brock (1982, 1984, 1986) and Brock et al. (1985). The analysis of the interaction of dynamic dislocations with stationary semi-infinite cracks was studied by Brock (1983a,b). Brock also focused his attention mainly on the investigation of the dynamic stress intensity factor.

In some classes of dynamic problems of impact loading, the ability to find a static field may hinge on waiting for the wave front to pass and the transient effect to die away. For point dynamic loading with the step function suddenly applied on the surface of a half plane, the stress field becomes a static value as the time tends to infinity. When a propagating antiplane crack subjected to dynamic loading suddenly stops, Eshelby (1969) and Ma and Burgers (1988) found that the stationary crack solution is radiated out behind the shear wave centered at the stopped crack tip. In this study, the problem to be considered is the antiplane response of an elastic solid containing a half-plane crack subjected to impact loading on the crack faces with finite distance to the crack tip as shown in Fig. 1. The techniques used in this study were first described by Freund (1974b) who investigated the same problem in the plane case but focused only on the dynamic stress intensity factor. In this study, we analyze not only the dynamic stress intensity factor but also the transient full-field solution. The main results are that the stress intensity factor is zero before the shear wave arrives at the crack tip, and then it jumps from zero to its appropriate static value at the instant of the wave arrival. The full-field solution of stresses will take on the appropriate static value instantaneously upon arrival of the secondary shear wave (SS wave) diffracted from the shear wave (S wave) which is generated by the suddenly applied load. A kinked crack which suddenly propagates out of the original semi-infinite crack with constant velocity is also considered. The direction of propagation, as well as the velocity of crack propagation will depend on the local stress field around the crack tip. To understand the observed bifurcation events, the dynamic stress intensity factor for cracks which suddenly kink is obtained in closed form by a perturbation method. The energy flux into the propagating kinked crack tip is derived and these results are discussed in terms of an assumed fracture criterion.

2 Statements of the Problem

Consider a stress-free elastic homogeneous isotropic infinite medium that contains a semi-infinite crack, a Cartesian coordinate system is defined in the body in such a way that the antiplane deformation is in the y-direction. The planar crack lies in the plane $z=0$, $x<0$. At time $t=0$, a concentrated force of magnitude σ_0 (per unit length in the y-direction) acts at $x=-l$ on each face of the crack as shown in Fig. 1. The relevant stress components are denoted by σ_{yz} and σ_{xy}, and the nonzero out-of-plane displacement is denoted by w. In a stationary coordination systems of x and z, two-dimensional antiplane wave motions are governed by

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}=b^2\frac{\partial^2 w}{\partial t^2}, \tag{1}$$

where b is the slowness of the transverse wave given by

$$b=\frac{1}{v_s}=\sqrt{\frac{\rho}{\mu}}.$$

Here, μ and ρ are the shear modulus and the mass density of the material, respectively. The nonvanishing shear stresses are

$$\sigma_{xy}=\mu\frac{\partial w}{\partial x},\quad \sigma_{yz}=\mu\frac{\partial w}{\partial z}.$$

The crack faces are traction-free, except for the point of

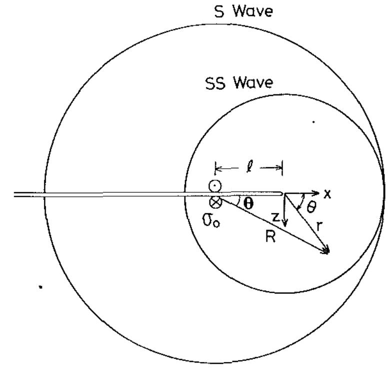

Fig. 1 Wavefronts for a stationary crack subject to dynamic point loading at the crack faces

application of the concentrated forces. Because of symmetry with respect to the plane $z=0$, the boundary conditions can be written as

$$\sigma_{yz}(x,\,0,\,t)=\sigma_0\delta(x+l)H(t),\ x<0 \tag{2}$$

$$w(x,\,0,\,t)=0,\ x>0, \tag{3}$$

where H is the Heaviside step function and δ is the Dirac delta function. The formulation is completed by specifying zero initial condition. Because of the presence of the characteristic length l in the formulation, the standard Wiener-Hopt technique cannot be used. Therefore, some other approach must be followed. If the boundary condition (2) is extended to the entire boundary, then the problem is reduced to the antiplane analog of Lamb's problem in the plane case. Hence, the problem described in (2) and (3) is Lamb's problem with a concentrated loading at $x=-l$, but with surface displacement negated for $x>0$. By solving for the fundamental solution of a distribution dislocation, this problem can be solved by superposition. This methodology was first discussed and used to solve the correspondence problem in plane strain by Freund (1974b).

3 Transient Solutions for Impact Loading on the Crack Faces

Now let us consider the same unbounded body containing a semi-infinite crack. At time $\tau=0$, a screw dislocation of strength 2Δ begins to move from the crack tip at constant speed v in the positive x-direction. This problem is also symmetric with respect to the plane $z=0$, the boundary conditions can be written as

$$\sigma^F_{yz}(x,\,0,\,\tau)=0,\ x<0 \tag{4}$$

$$w^F(x,\,0,\,\tau)=\Delta H(v\tau-x),\ x>0. \tag{5}$$

The solution of this fundamental problem can be obtained by using integral transformation and the standard Wiener-Hopf technique. The exact full-field solutions can be expressed as follows:

$$\sigma^F_{yz}(x,\,z,\,\tau)=-\frac{\Delta\mu(b+h)^{1/2}}{\pi}\mathrm{Im}\left[\frac{(b+\lambda)^{1/2}}{\lambda+h}\frac{\partial\lambda}{\partial\tau}\right]H(\tau-br), \tag{6}$$

$$\sigma^F_{xy}(x,\,z,\,\tau)=\frac{\Delta\mu(b+h)^{1/2}}{\pi}\mathrm{Im}\left[\frac{\lambda}{(b-\lambda)^{1/2}(\lambda+h)}\frac{\partial\lambda}{\partial\tau}\right]H(\tau-br), \tag{7}$$

$$w^F(x,\,z,\,\tau)=\frac{\Delta(b+h)^{1/2}}{\pi}\int_{br}^{\tau}\mathrm{Im}\left[\frac{1}{(b-\lambda)^{1/2}(\lambda+h)}\frac{\partial\lambda}{\partial\tau}\right]d\tau\, H(\tau-br), \tag{8}$$

and

$$\lim_{x\to 0^+}[x^{1/2}\sigma_{yz}^F(x,\,0,\,\tau)]=-\frac{\mu\Delta(b+h)^{1/2}}{\pi\tau^{1/2}},\tag{9}$$

where

$$h=1/v,$$

$$\lambda=-\frac{\tau}{r}\cos\theta+i\sin\theta\left(\frac{\tau^2}{r^2}-b^2\right)^{1/2},$$

$$r=\sqrt{x^2+z^2}.$$

Now, consider the transient wave in a half-space generated by a impact point loading σ_0 at $x=0$, $z=0$, which can be viewed as the Lamb's problem in antiplane analogy. The solutions are

$$\sigma_{yz}^L(x,\,z,\,t)=\frac{\sigma_0 t\sin\theta}{\pi br^2[(t/br)^2-1]^{1/2}}H(t-br),\tag{10}$$

$$\sigma_{xy}^L(x,\,z,\,t)=\frac{\sigma_0 t\cos\theta}{\pi br^2[(t/br)^2-1]^{1/2}}H(t-br),\tag{11}$$

$$w^L(x,\,z,\,t)=-\frac{\sigma_0}{\mu\pi}\ln\left[\frac{t}{br}+\sqrt{\left(\frac{t}{br}\right)^2-1}\right]H(t-br).\tag{12}$$

With the fundamental solution and Lamb's solution at hand, it is now possible to construct the field solutions for the problem of impact loading on the crack faces at $x=-l$. As described in the previous section, this solution can be superimposed by two solutions, one is the Lamb's problem with a concentrated force at $x=-l$, $z=0$, the other problem is that which cancels out the surface displacement for $z=0$, $x>0$ of Lamb's problem. The surface displacement for Lamb's problem by applying concentrated loading σ_0 at $x=-l$, $z=0$ and at time $t=0$ is obtained from (12)

$$w^L(x,\,0,\,t)=-\frac{\sigma_0}{\mu\pi}\mathrm{Im}\left[\frac{t}{b(x+l)}+\sqrt{\left(\frac{t}{b(x+l)}\right)^2-1}\right]H(t-b(x+l)).\tag{13}$$

It is observed from (13) that w^L depends only on the ratio $t/(x+l)$, which means any given displacement level radiates out at a constant speed $(x+l)/t$ along the x-axis for $t>0$. The speed varies between zero and the shear wave speed. For a particular speed v_τ arriving at $x=0$, at any time t will be $v_\tau=l/t$. Then, the full-field transient solution of stress $\sigma_{yz}(x,\,z,\,t)$ can be constructed by superposition over a one-parameter family of dislocation velocity. The result is

$$\sigma_{yz}(x,\,z,\,t)=\sigma_{yz}^L(x,\,z,\,t)-\int_{v_s}^{v_\tau}\sigma_{yz}^F(x,\,z,\,\tau-\tau_0,\,v)\frac{dw^L(v)}{dv}dv.\tag{14}$$

The coordinate systems and the wavefronts are shown in Fig. 1. The wavefront for Lamb's problem consists of only the shear wavefront, denoted by S, propagating away from the loading point. When this shear wave reaches the crack tip, an additional shear wave indicated by SS diffracted from the crack tip is generated. The first term in (14) represents the contribution from the S wave while the second term is from the SS wave. It is convenient to change the integration variable from v to $h=1/v$. After the substitution of the explicit expressions for σ_{yz}^L, σ_{yz}^F and dw^L/dv into (14), the expression becomes

$$\sigma_{yz}(r,\,\theta,\,t)=\frac{\sigma_0 t\sin\Theta}{\pi bR^2[(t/bR)^2-1]^{1/2}}H(t-bR)-\frac{\sigma_0}{\pi^2}\int_b^{(t-br)/l}\mathrm{Im}\left[\frac{(b+\eta_1)^{1/2}}{\eta_1+h}\frac{\partial\eta_1}{\partial\tau}\right]\frac{dh}{(h-b)^{1/2}}H(t-b(r+l)),\tag{15}$$

where

$$\eta_1=-\frac{t-lh}{r}\cos\theta+i\sin\theta\left[\left(\frac{t-lh}{r}\right)^2-b^2\right]^{1/2},\tag{16}$$

$$\frac{\partial\eta_1}{\partial\tau}=\frac{1}{r}\left[-\cos\theta+i\sin\theta\frac{t-lh}{((t-lh)^2-b^2r^2)^{1/2}}\right],$$

$$R=[(l+r\cos\theta)^2+r^2\sin^2\theta]^{1/2}.\tag{17}$$

By a similar procedure, we also get the result for σ_{xy}

$$\sigma_{xy}(r,\,\theta,\,t)=\frac{\sigma_0 t\cos\Theta}{\pi bR^2[(t/bR)^2-1]^{1/2}}H(t-bR)+\frac{\sigma_0}{\pi^2}\int_b^{(t-br)/l}\mathrm{Im}\left[\frac{\eta_1}{(\eta_1+\mathrm{h})(\mathrm{b}-\eta_1)^{1/2}}\frac{\partial\eta_1}{\partial\tau}\right]\frac{dh}{(h-b)^{1/2}}H(t-b(r+l)).\tag{18}$$

The stress intensity factor is defined by the following limit

$$K=\lim_{x\to 0^+}(2\pi x)^{1/2}\sigma_{yz}(x,\,0,\,t).\tag{19}$$

It is clear that the stress in Lamb's problem is not singular at $x=0$, so that the stress intensity factor is determined by the second term in (15). From the result of (9) and in the same manner as the construction of the full-field solution, it is found that the stress intensity factor is given by

$$K(t)=-\sigma_0\sqrt{\frac{2}{\pi l}}\frac{1}{\pi}\int_b^{h*}\frac{dh}{(h^*-h)^{1/2}(h-b)^{1/2}}H(t-bl)=-\sigma_0\sqrt{\frac{2}{\pi l}}H(t-bl),\tag{20}$$

where $h^*=t/l$. The interesting result of (20) is that the stress intensity factor jumps from zero to the static value after the shear wave generated from the loading point arrives at the crack tip. In the plane-strain case, Freund (1974b) found that the stress intensity factor takes on its static value instantaneously upon arrival of the Rayleigh wave generated by the suddenly applied load.

Now, we focus our attention on the stress along the crack-tip line. After making the indicated change of variable, the result is

$$\sigma_{yz}(x,\,0,\,t)=\frac{\sigma_0}{\pi x^{1/2}}\int_b^{1/v_\tau}\frac{(t-lh-xb)^{1/2}}{(hx+hl-t)\sqrt{h-b}}dh\,H(t-b(l+x)).\tag{21}$$

After some suitable change of variables and working out the details, it is found that

$$\sigma_{yz}(x,\,0,\,t)=-\frac{\sigma_0}{\pi}\sqrt{\frac{l}{x}}\frac{1}{x+l}H(t-b(x+l)).\tag{22}$$

The remarkable result shown in (22) is that the stress at any point along the crack line takes on its static value instantaneously after the shear wave has passed this point.

The static full-field solutions of stresses for applied point loading σ_0 at the crack faces of the same problem are

$$\sigma_{yz}^s(r,\,\theta)=-\frac{\sigma_0}{\pi}\sqrt{\frac{l}{r}}\frac{r\cos(3\theta/2)+l\cos(\theta/2)}{r^2+2rl\cos\theta+l^2},\tag{23}$$

$$\sigma_{xy}^s(r,\,\theta)=\frac{\sigma_0}{\pi}\sqrt{\frac{l}{r}}\frac{r\sin(3\theta/2)+l\sin(\theta/2)}{r^2+2rl\cos\theta+l^2}.\tag{24}$$

The numerical calculations of the full-field solutions of stress σ_{yz} in (15) are shown in Fig. 2. The most interesting feature as shown in this figure is that the full-field solutions of stresses jump from the dynamic transient solution to the appropriate static value expressed in (23) instantaneously upon arrival of

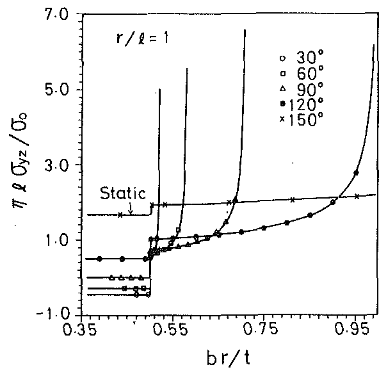

Fig. 2 The transient shear stresses σ_{yz} for applying dynamic point loading on the crack faces

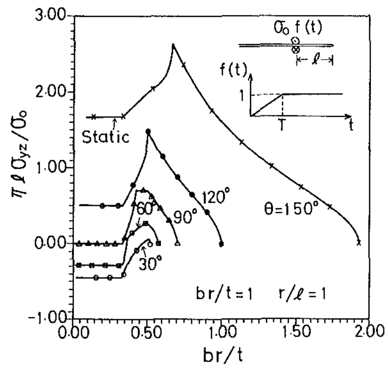

Fig. 3 Nondimensionalized transient shear stress σ_{yz} for applied point loading with a finite rise time

the secondary shear wave (SS) diffracted from the shear wave (S) which is generated by the suddenly applied load. Along the crack-tip line, the SS and S wave coincide. From the general features of the numerical results of full-field solutions indicated above, it is then very easy to draw the conclusion just made regarding the stress along the crack-tip line. Hence, the transient full-field solution of stress σ_{yz} can be expressed as follows:

$$\begin{cases} \sigma_{yz}(r,\theta,t)=0 & \text{for } t<bR, \\ \sigma_{yz}(r,\theta,t)=\sigma_{yz}^{L} & \text{for } t>bR \text{ and } b(r+l)>t, \\ \sigma_{yz}(r,\theta,t)=\sigma_{yz}^{s} & \text{for } b(r+l)<t, \end{cases} \tag{25}$$

where

$$R=[(x+l)^2+z^2]^{1/2}.$$

In order to indicate the correctness of the solutions for point loading shown previously, these solutions for point loading are regarded as the Green function and are used to construct the solutions of uniform loading σ_0 applied to the crack faces. The method used to obtain solutions for stresses is very straightforward. The results are

$$\sigma_{yz}(r,\theta,t)=-\frac{\sigma_0}{\pi}\left[2\cos(\theta/2)\sqrt{\frac{t}{br}-1}\right. \\ \left.-\tan^{-1}\sqrt{\frac{t/br-1}{1-\sin\theta}}-\tan^{-1}\sqrt{\frac{t/br-1}{1+\sin\theta}}\right], \tag{26}$$

$$\sigma_{xy}(r,\theta,t)=\frac{2\sigma_0}{\pi}\sin(\theta/2)\sqrt{\frac{t}{br}-1}. \tag{27}$$

The result of the numerical integration of stress σ_{yz} in (15) over the crack faces is compared with the solution shown in (26). The two results are in excellent agreement as indicated in Table 1.

Table 1 Comparison of the numerical results for σ_{yz} from integration of the solution of point loading and the solutions of applying uniform loading on the crack faces

br/t	Green function	Uniform loading
	$\theta = 0^0$	
0.9	$-7.385E-3$	$-7.385E-3$
0.8	$-2.316E-2$	$-2.316E-2$
0.7	$-4.778E-2$	$-4.778E-2$
0.6	$-8.393E-2$	$-8.393E-2$
0.5	$-1.366E-1$	$-1.366E-1$
0.4	$-2.157E-1$	$-2.157E-1$
0.3	$-3.416E-1$	$-3.416E-1$
0.2	$-5.678E-1$	$-5.678E-1$
0.1	$-1.115E-0$	$-1.115E-0$
	$\theta = 30^0$	
0.9	$1.982E-2$	$1.982E-2$
0.8	$1.180E-2$	$1.181E-2$
0.7	$-8.609E-3$	$-8.593E-3$
0.6	$-4.217E-2$	$-4.214E-2$
0.5	$-9.302E-2$	$-9.294E-2$
0.4	$-1.700E-1$	$-1.698E-1$
0.3	$-2.927E-1$	$-2.925E-1$
0.2	$-5.132E-1$	$-5.132E-1$
0.1	$-1.046E-0$	$-1.042E-0$
	$\theta = 60^0$	
0.9	$1.275E-1$	$1.275E-1$
0.8	$1.348E-1$	$1.348E-1$
0.7	$1.190E-1$	$1.190E-1$
0.6	$8.713E-2$	$8.714E-2$
0.5	$3.807E-2$	$3.806E-2$
0.4	$-3.510E-2$	$-3.510E-2$
0.3	$-1.494E-1$	$-1.494E-1$
0.2	$-3.512E-1$	$-3.512E-1$
0.1	$-8.293E-1$	$-8.294E-1$

Up to this point, the time dependency of the point loading profile is a simple step function in time. The stress for spatially distributed traction on crack faces or for more general time dependence can be obtained by superposition. Suppose that the rate of increase in magnitude of the point loading from zero is taken to be linear, after some finite rise time, say T, the magnitude of the loading is held constant. In this case, the stress σ_{yz} can be obtained from superposition of (15) over time. The numerical results are shown in Fig. 3. The transient solutions will become static at time $t=b(l+r)+T$ as expected. Now, let us consider uniform loading of step function applied on parts of the crack faces from $x=-l_1$ to $x=-(l_1+l_2)$ at $t=0$. The transient solution of σ_{yz} can also be obtained from superposition of (15), the numerical results are shown in Fig. 4. The solutions become a static value after time $t=b(l_1+l_2+r)$ as expected.

4 Dynamic Crack Kinking

In this section, we will analyze the dynamic crack growth

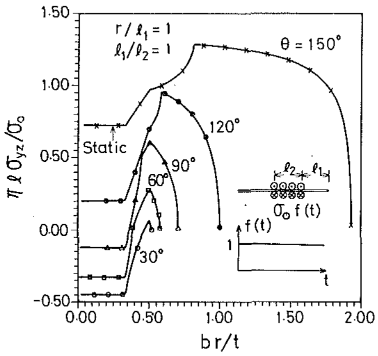

Fig. 4 Nondimensionalized transient shear stress σ_{yz} for suddenly applied uniform loading over part of the crack faces

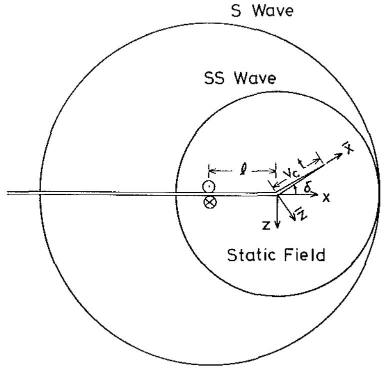

Fig. 5 Geometry of wavefronts for a kinking crack subject to point loading on the crack faces

out of the original semi-infinite crack at an angle to the original crack after applying dynamic point loadings on the original crack faces. From the results indicated in the previous section, we know that the dynamic stress intensity factor jumps from zero to the static value after the S wave arrives at the crack tip. Hence, if the critical stress intensity factor criteria for crack growth is adopted, the crack will be expected to grow immediately after the S wave reaches the crack tip.

Now, let us consider a stationary, semi-infinite crack in an initially stress-free isotropic elastic full space. A sharp crack, which will be referred to as the original crack, is subjected to a point dynamic loading at the crack faces with a distance l to the crack tip. A short time later, at $t=0$, the S wave generated from the point loading arrives at the crack tip. A crack referred to as the new crack, propagates out from the tip of the semi-infinite crack. The velocity of propagation v_c is constant and less than the shear wave speed v_s. The line of propagation is straight, making an angle δ with the original crack, thus producing a kinked crack. The pattern of wavefronts and the position of the crack tip for $t>0$ are shown in Fig. 5. The field solution for a kinked crack geometry can be considered as the superposition of the field for the S and SS waves of the stationary crack and the field from the new crack faces subjected to crack-face tractions which are opposite in sign to the stresses computed from the stationary crack. This computation involves coupled integral equations which must be solved numerically.

The method used to solve the problem in this study relies on an asymptotic approach. The perturbation procedure indicated by Kuo and Achenbach (1985), by using the kinking angle δ as the perturbation parameter, is adopted to construct the solution. The first-order approximation of the dynamic stress intensity factor for a kinked crack can be expressed by the stress intensity factor for a straight crack, propagating in its own plane, subjected to the negative of the traction computed from the stationary crack problem along the line of the kinked crack.

$$\sigma^k_{yz}=0 \quad \text{for} \quad \bar{x}<0,$$
$$\sigma^k_{yz}=-\sigma_{\theta y}(\theta=\delta) \quad \text{for} \quad 0<\bar{x}<v_c t. \tag{28}$$

A fundamental solution needed to construct the problem indicated in (28) is similar to that proposed by Freund (1972a) in the plane-strain case. Consider a crack extending straight at a constant speed v_c in the $\bar{x}$-direction. For $t<0$, there are no body forces or tractions acting on the body. At time $t=0$, the position of the crack tip is $\bar{x}=0$ and concentrated forces of unit magnitude appear at the crack tip. For $t>0$, the crack tip continues to move in the positive $\bar{x}$-direction, but the concentrated forces continue to act at $\bar{x}=0$. The shear stress on the plane $\bar{z}=0$ can be obtained from results analyzed by Ma and Burgers (1986) as follows:

$$\sigma^f_{yz}(\xi,\,0,\,t)=-\frac{\sqrt{d}[\lambda^2-b^2-b^2\lambda^2/d^2-2b^2\lambda/d]^{1/2}}{\xi\pi(d+\lambda)[b+\lambda(1+b/d)]^{1/2}}\;t>b_2\xi, \tag{29}$$

where

$$\lambda=t/\xi,\; \xi=\bar{x}-v_c t,$$
$$b_2=b/(1-b/d),\; d=1/v_c.$$

As indicated in the previous section, stresses for material points behind the SS wave are essentially a static value. With this special feature in mind, the kinked crack problem can be greatly simplified. Hence, if the kinked crack velocity is less than the shear wave speed, the dynamic stress intensity factor for the kinked crack tip subjected to dynamic point loading at the crack faces is the same as that for the static point loading case. For this reason the dynamic stress $\sigma_{\theta y}$ in (28) can be replaced by the equivalent static value $\sigma^s_{\theta y}$ which can be obtained from (23) and (24) as follows:

$$\sigma^s_{\theta y}(r,\,\theta)=-\frac{\sigma_0}{\pi}\sqrt{\frac{l}{r}}\,\frac{(r+l)\cos(\theta/2)}{r^2+2rl\cos\theta+l^2}.$$

The dynamic problem as indicated in (28) can be solved in a similar manner to that considered above, with the exception that a traction which gives rise to $\sigma^s_{\theta y}(\bar{x})$ on $0<\bar{x}<v_c t$, instead of concentrated forces appears through the moving crack tip. The solution for the case of a distributed traction $\sigma^s_{\theta y}(\bar{x})$ appearing through the crack tip on $0<\bar{x}<v_c t$ is given by the following superposition integral

$$\sigma^k_{yz}(\bar{x},\,0,\,t)=\int_0^{v_c t}\sigma^f_{yz}(\bar{x}-x_0,\,0,\,t-x_0/v_c)\sigma^s_{\theta y}(x_0)dx_0. \tag{30}$$

The first-order approximation of the dynamic stress intensity factor K^d is obtained by considering the limiting behavior $\xi\to 0^+$ of (30) at the moving crack tip

$$K^d(t,\,v_c,\,\delta)=\lim_{\xi\to 0^+}\sqrt{2\pi\xi}\,\sigma^k_{yz}(\bar{x},\,0,\,t). \tag{31}$$

Taking the limit $\xi\to 0^+$ in (29) first and working out the detail of (31) yields

$$K^d(t,\,v_c,\,\delta)=-\frac{\sigma_0}{\pi}\sqrt{\frac{2l}{\pi}}\,(1-b/d)^{1/2}\int_0^{v_c t}\frac{(v_c t+l-\eta)\cos(\delta/2)}{\sqrt{\eta}\sqrt{v_c t-\eta}[(v_c t-\eta)^2+2(v_c t-\eta)l\cos\delta+l^2]}d\eta. \tag{32}$$

Equation (32) can be worked out and expressed in explicit form as follows:

$$K^d(t,\,v,\,\delta)=-\sigma_0\frac{\sqrt{1-V}}{\sqrt{\pi l}}\left\{\frac{\sqrt{(VT)^2+2VT\cos\delta+1}+VT+\cos\delta}{(VT)^2+2VT\cos\delta+1}\right\}, \tag{33}$$

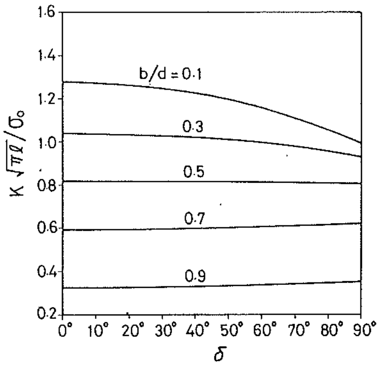

Fig. 6 Dynamic stress intensity factor for various values of the crack kinking angle and crack speed

where $V=b/d$ and $T=t/bl$ are nondimensional quantities. Figure 6 shows the dimensionless dynamic stress intensity factor for various values of the crack kinking angle δ and the normalized crack speed b/d. The error by using this approximation method is less than 10 percent for the any kinking angle less than 90 deg; if the kinking angle is less than 45 deg, the error is less than 2 percent, as indicated by Ma and Burgers (1986) for kinking crack under stress wave loading. We believe that the result in (33) will also be comparably accurate. For the kinking angle $\delta=0$, the crack propagates straight out of the original crack, the solution in (33) is an exact result without any approximation.

$$K^d(t,\ v,\ 0)=-\sigma_0\sqrt{\frac{2}{\pi(vt+l)}}(1-b/d)^{1/2}=K^s\kappa(d). \quad (34)$$

The expression for K^d in (34) has the interesting form of the product of a function of the crack velocity $\kappa(d)$ and the corresponding static stress intensity factor K^s for applying concentrated loading σ_0 at the crack face with a distance $l+vt$ from the crack tip. The value $\kappa(d)=(1-b/d)^{1/2}$ is an universal function which depends only on crack speed and material properties.

At the instant that the kinking has just occurred, we have

$$\lim_{t\to 0^+} K^d(t,\ v,\ \delta)=-\sqrt{\frac{2}{\pi l}}(1-b/d)^{1/2}\cos(\delta/2)$$
$$=K^s\kappa(d)\cos(\delta/2). \quad (35)$$

That is, the stress intensity factor just after the initiation of the kinked crack has the form of the universal function of the crack-tip speed $\kappa(d)$ times the stress intensity factor appropriate for static value K^s times the spatial angular dependence of the stationary crack field.

The energy flux into the propagating crack tip can be written in terms of the corresponding dynamic stress intensity factor by

$$E=\frac{K^2}{2\mu d(1-b^2/d^2)^{1/2}}=\frac{\sigma_0^2}{2\mu l b}E^*, \quad (36)$$

where

$$E^*=\frac{V(1-V)^{1/2}}{\pi(1+V)^{1/2}}\frac{\sqrt{(VT)^2+2VT\cos\delta+1}+VT+\cos\delta}{(VT)^2+2VT\cos\delta+1}.$$

If the maximum energy release rate criterion is accepted as the kinking condition, then the combination of the kinking angle and the crack speed can be determined at which the energy flux into the propagating crack tip achieves a maximum value. The conditions for this to occur are

$$\frac{\partial E^*}{\partial V}=0,\ \ \frac{\partial^2 E^*}{\partial V^2}<0,$$

and

$$\frac{\partial E^*}{\partial \delta}=0,\ \ \frac{\partial^2 E^*}{\partial \delta^2}<0.$$

If one wants to study the criterion for a crack kinking event, it is clear that the most significant time involved will be when the crack kinking has just occurred, i.e., $T\to 0$. From the maximum energy release rate criterion, it is found that the crack will tend to propagate straight ahead of the original crack with a constant crack speed $v_c=0.618v_s$ which makes $E^*_{\max}=0.191$.

5 Conclusions

The difficulty in determining the transient stress field in a cracked body subjected to dynamic loading is well known. The problem considered in this study is the antiplane response of an elastic solid containing a half-plane crack subjected to impact loading on the crack faces. Attention is focused on the transient stress fields for an applied load with step-function time dependence. The remarkable results are that the full stress takes on its static value a very short time after the *SS* wave diffracted from the crack tip has passed, while the stress intensity factor takes on the appropriate static value after the shear wave generated from the point loading reaches the crack tip. Generalizations are discussed for spatially distributed and time-varying impact loads.

Because of the interesting feature that the static field propagates out behind the *SS* wave front, the evaluation of the dynamic stress intensity factor of the kinking crack propagating with constant crack speed is the same for the dynamic point loading and for the static point loading on the crack faces. A perturbation method is used to obtain the first-order analytic closed-form solution of the dynamic stress intensity factor for the kinking crack. The elastodynamic stress intensity factors of the kinking crack tip are used to compute the corresponding fluxes of energy flux into the propagating crack tip. With these theoretical results for the stress intensity factor at hand, an attempt can be made to determine the kink angle and the new kinked crack speed using different fracture criteria and to compare them with the experimental results available. An energy based fracture criterion is used to look at the initiation of the crack-tip motion. The energy criterion suggests that the crack will choose to propagate in the direction and at the velocity for which the energy flux into the crack tip has a maximum value. Based on the maximum energy release rate criterion, it is found that the crack will tend to propagate straight ahead with crack speed of 0.618 v_s.

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