

# Dynamic interfacial crack propagation in elastic–piezoelectric bi-materials subjected to uniformly distributed loading

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## Abstract

In transversely isotropic elastic solids, there is no surface wave for anti-plane deformation. However, for certain orientations of piezoelectric materials, a surface wave propagating along the free surface (interface) will occur and is called the Bleustein–Gulyaev (Maerfeld–Tournois) wave. The existence of the surface wave strongly influences the crack propagation event. The nature of anti-plane dynamic fracture in piezoelectric materials is fundamentally different from that in purely elastic solids. Piezoelectric surface wave phenomena are clearly seen to be critical to the behavior of the moving crack. In this paper, the problem of dynamic interfacial crack propagation in elastic–piezoelectric bi-materials subjected to uniformly distributed dynamic anti-plane loadings on crack faces is studied. Four situations for different combination of shear wave velocity and the existence of MT surface wave are discussed to completely analyze this problem. The mixed boundary value problem is solved by transform methods together with the Wiener–Hopf and Cagniard–de Hoop techniques. The analytical results of the transient full-field solutions and the dynamic stress intensity factor for the interfacial crack propagation problem are obtained in explicit forms. The numerical results based on analytical solutions are evaluated and are discussed in detail.

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## 1. Introduction

More than a century has passed since the Curie brothers discovered the piezoelectric effect in 1880. Today, over a hundred piezoelectric materials or composites are used in many engineering applications (Pohanka and Smith, 1988). Due to their intrinsic electromechanical coupling behaviors, piezoelectric materials, particularly piezoelectric ceramics, have been widely used for applications such as sensors, filters, ultrasonic generators and actuators. Because of the brittle properties of most piezoelectric materials, the failure analysis of piezoelectric devices has attracted more attention from researchers. A closed-form solution of the anti-plane fracture prob-

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lem related to quasi-static condition was obtained for an unbounded piezoelectric medium. For a fixed value of the mechanical load, it was shown that the crack growth can be either enhanced or retarded depending on the magnitude, the direction and the type of the applied electrical load (Pak, 1990). Because of the mathematical complications, less attention has been paid to the study of dynamic fracture mechanics of piezoelectric materials. Shindo and Ozawa (1990) first investigated the steady-state response of a cracked piezoelectric material subjected to plane harmonic waves. Recently, the dynamic fracture analysis of piezoelectric materials was developed rapidly. The anti-plane dynamic fracture problem of a homogeneous infinite piezoelectric strip with a crack was studied using integral transform method by Chen (1998). By solving a Fedholm integral equation of second kind, all the relevant quantities such as the anti-plane mechanical displacement, electric potential, stress and electric displacement were obtained. When the width of the strip tends to infinite, these results were reduced to the solving of two pairs of dual integral equations. The problem of multiple cracks had been studied by Wang et al. (2000). He proposed a method to analyze the dynamic response of a multi-layered piezoelectric material containing non-collinear cracks. Based on Fourier and Laplace transforms, the boundary value problem was reduced to a system of generalized singularity integral equations in the Laplace transform domain. By utilized numerical Laplace inversion, the time-dependent full field solutions were obtained in the time domain. However, due to the mathematical difficulties, most of the researchers represented their solutions by numerical methods. The exact analytical solutions for transient crack propagation in piezoelectric materials are rare in the literature.

In the study of crack propagation, Yoffe (1951) was the first one to investigate a steady-state crack growth problem of a crack of fixed length propagating in an infinite and purely elastic body subjected to a uniform remote tensile stress. Subsequently, many researchers were devoted to the study of crack propagation for purely elastic solids. For example, it was shown that a horizontally polarized shear wave can incite transient extension of a crack by Achenbach (1970). The stress intensity factors of a semi-infinite crack extending non-uniformly in an isotropic elastic solid subjected to stress wave loading were considered by Freund (1972). It was found that the mode I and mode II stress intensity factors each have the form of the product of a universal function of instantaneous crack tip speed with the stress intensity factor for an equivalent stationary crack. Ma and Hou (1990, 1991) analyzed a series of problems of an unbounded medium containing a semi-infinite crack subjected to impact loadings. For piezoelectric crack problem, Li and Mataga (1996a,b) first obtained transient closed-form solutions for dynamic stress and electric displacement intensities and dynamic energy release rate of a propagating crack in homogeneous hexagonal piezoelectric materials. They assumed that the crack surfaces are electrode- or vacuum-type boundary conditions and the dynamic anti-plane point loading was initially applied at the stationary crack tip. Hence there is no characteristic length presented in their problems. Ing and Ma (1996, 1997) solved the problem of a finite crack subjected to a dynamic anti-plane point loading and a horizontally polarized shear wave in isotropic solids, but only dynamic stress intensity factors were obtained in closed forms. Chen and Yu (1997) studied the problem of anti-plane Yoffe's crack in an unbounded piezoelectric medium. Later, Chen et al. (1998) investigated the response of a finite Griffith crack propagating along the interface of two dissimilar piezoelectric half-planes. Ma and Liao (1999) studied the transient full-field response of a semi-infinite interface crack lying between dissimilar anisotropic elastic media subjected to a dynamic body force. Recently, Kwon and Lee (2000) investigated the crack problem of an infinitely long piezoelectric ceramic strip containing a Griffith crack moving with constant velocity.

In the literature, there are a number of experimental results concerning the fracture behavior of piezoelectric ceramics under purely mechanical loading. The effects of an alternating electric field on crack initiation and growth have long been a focus of interest in the study of the reliability of piezoelectric ceramics. However, experimental studies on the subject of combined electrical and mechanical loadings are limited. McHenry and Koepke (1983) observed the piezoelectric ceramics subjected to deadweight loading in a double torsion mode, crack propagation was enhanced by the application of an alternating electric field perpendicular to existing cracks. Furuta and Uchino (1993) studied crack initiation and propagation near the internal electrode tip in multilayer piezoelectric actuators during cyclic loading. Aburatani et al. (1994) investigated the failure mechanisms in multilayer actuators and found that cracks initiate from the edges of internal electrodes and propagate to other electrodes in piezoelectric samples. With pre-cracks introduced by indentation, Tajima et al. (2000) observed steady crack growth perpendicular to the applied field. Jiang and Sun (1999) investigated

the fatigue behavior of PZT-4 ceramics using pre-notched compact tension specimens and indicated that the magnitude and direction of the electric field influence the crack growth rate significantly.

In engineering applications, piezoelectric materials often take the form of multi-layered structures that use the accumulative results of stacks to enhance the efficiency and sensitivity. The integrity of interfaces between the stacks is of great concern because interfacial fracture is one of the major failure modes in multi-layered structures. The problem of interfacial crack of piezoelectric materials is of paramount importance and has drawn much attention in recent years. Because the analysis of an interfacial crack in piezoelectric materials is complex and difficult, most of the current research efforts have focused on static fracture behaviors. Only very few of them consider dynamic effects (e.g., To et al., 2005; Li et al., 2005) for stationary interface cracks. Recently, To et al. (2006) analyzed the transient response of an interfacial crack propagating between two dissimilar piezoelectric half spaces. They assumed that a pair of concentrated shear forces is applied at the crack tip of a pre-existing semi-infinite interfacial crack, and the crack propagated at a constant speed. The transient solutions for dynamic stress and electric displacement intensity factors and dynamic energy release rate are obtained and discussed in detail.

In this paper, the dynamic fracture problem of a purely elastic and piezoelectric bi-material containing a semi-infinite crack along the interface is considered. The transient stress fields and the dynamic stress intensity factor of the semi-infinite crack propagating along the interface between two different media are analyzed. The crack is initially at rest and, at a certain instant, is subjected to an anti-plane uniformly loading on the stationary crack faces and the crack begins to propagate along the interface with a constant velocity, which is less than the smaller of the shear wave speed of these two materials and the surface wave of the piezoelectric material. The complete solutions of this problem can be obtained by integral transform methods in conjunction with the direct application of the Wiener–Hopf technique (Noble, 1958) and the Cagniard–de Hoop method (Cagnard, 1939; de Hoop, 1960) of Laplace inversion. The crack surfaces are assumed to be the electrode-type boundary condition. The same assumption had been proposed by Bleustein (1968). Under this assumption, exact analytical transient solutions are obtained and numerical calculations have been carried out. The existence of the Maerfeld–Tournois surface wave (Maerfeld and Tournois, 1971) may play a significant role in determining the transient fields. Hence, the existence condition of a propagating MT surface wave along the interface is analyzed and discussed in this study. The propagation velocity of the surface wave is explicitly expressed if this surface wave does exist. Base on the existence of the MT surface wave, four different cases are analyzed in order to construct the complete solutions for this problem. The analytical results for the transient full-field solutions obtained in this study are exact and are expressed in explicit closed forms, each term representing a physical transient wave. The dynamic stress intensity and dynamic electric displacement intensity factors for the interfacial crack propagation problem are also presented in explicit forms. The results obtained in this investigation provide important analytical solutions for dynamic fracture problem especially on the crack propagation event.

## 2. Problem statement and formulation

Consider a purely elastic and piezoelectric bi-material containing a semi-infinite crack along the interface. The interface crack faces are subjected to uniformly distributed dynamic anti-plane loadings with magnitude  $\tau_0$  applied at time  $t = 0$ , as shown in Fig. 1. The interface crack lies along the negative  $\xi$ -axis and propagates with a constant velocity  $v$  along the crack tip line after a dynamic loading is applied on crack faces. It is

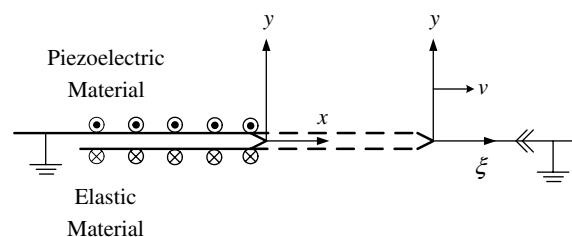


Fig. 1. Configuration and coordinate systems of a propagating crack in elastic–piezoelectric bi-materials.

assumed that the propagating crack surfaces are perfectly covered with an infinitesimally thin conducting electrode that is grounded, such that the electrostatic potential vanishes over the entire crack surfaces.

If we consider only the out-of-plane displacement and the in-plane electric fields, the governing equations for a hexagonal piezoelectric material (6 mm) can be described in the fixed coordinate system  $x$ – $y$  by

$$c_{44}\nabla^2 w + e_{15}\nabla^2 \phi = \rho \ddot{w}, \quad (1)$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2 \phi = 0, \quad (2)$$

where  $w = w(x, y, t)$  is the anti-plane displacement in the  $z$ -direction (which is assumed to be aligned with the hexagonal symmetry axis),  $\phi = \phi(x, y, t)$  is the electric potential,  $c_{44}$  is the elastic modulus measured in a constant electric field,  $\varepsilon_{11}$  is the dielectric permittivity measured at a constant strain,  $e_{15}$  is the piezoelectric constant and  $\rho$  is the material density.  $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$  is the in-plane Laplacian and a dot denotes material time derivative. The dynamic anti-plane governing equation for a purely elastic material is

$$\nabla^2 w^{(e)} = b^{(e)2} \ddot{w}^{(e)}, \quad (3)$$

where  $b^{(e)}$  is the slowness of the shear wave in the purely elastic material given by

$$b^{(e)} = 1/c^{(e)} = \sqrt{\rho^{(e)}/\mu^{(e)}}, \quad (4)$$

where  $c^{(e)}$  is the velocity of the shear wave in the purely elastic material and  $\mu^{(e)}$  is the respective shear modulus. Note that, the superscript (e) indicates the purely elastic material. The constitutive equations for the piezoelectric material can be expressed as

$$\tau_{yz} = c_{44} \frac{\partial w}{\partial y} + e_{15} \frac{\partial \phi}{\partial y}, \quad (5)$$

$$\tau_{xz} = c_{44} \frac{\partial w}{\partial x} + e_{15} \frac{\partial \phi}{\partial x}, \quad (6)$$

$$D_y = e_{15} \frac{\partial w}{\partial y} - \varepsilon_{11} \frac{\partial \phi}{\partial y}, \quad (7)$$

$$D_x = e_{15} \frac{\partial w}{\partial x} - \varepsilon_{11} \frac{\partial \phi}{\partial x}, \quad (8)$$

where  $\tau_{yz}$  and  $\tau_{xz}$  are the shear stress components and  $D_y$  and  $D_x$  are the electric displacements. The constitutive equations for the purely elastic material can be written as

$$\tau_{yz}^{(e)} = \mu^{(e)} \frac{\partial w^{(e)}}{\partial y}, \quad (9)$$

$$\tau_{xz}^{(e)} = \mu^{(e)} \frac{\partial w^{(e)}}{\partial x}. \quad (10)$$

In analyzing the problem of an interface crack propagating with a constant velocity, it is convenient to express the relevant equations in the moving  $\xi$ – $y$  coordinates. The coordinate  $\xi$  defined by  $\xi = x - vt$  is fixed with respect to the moving crack tip. Making use of the transformation, the governing and constitutive equations for a hexagonal piezoelectric material may be rewritten as follows:

$$(1 - b^2 v^2) \frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial y^2} + 2b^2 v \frac{\partial^2 w}{\partial \xi \partial t} - b^2 \frac{\partial^2 w}{\partial t^2} = 0, \quad (11)$$

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial y^2} = 0, \quad (12)$$

$$\tau_{yz} = \bar{c}_{44} \frac{\partial w}{\partial y} + e_{15} \frac{\partial \psi}{\partial y}, \quad (13)$$

$$\tau_{\xi z} = \bar{c}_{44} \frac{\partial w}{\partial \xi} + e_{15} \frac{\partial \psi}{\partial \xi}, \quad (14)$$

$$D_y = -\varepsilon_{11} \frac{\partial \psi}{\partial y}, \tag{15}$$

$$D_\xi = -\varepsilon_{11} \frac{\partial \psi}{\partial \xi}, \tag{16}$$

where

$$\psi = \phi - \frac{e_{15}}{\varepsilon_{11}} w \tag{17}$$

and

$$\bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}} \tag{18}$$

is the piezoelectrically stiffened elastic constant. The governing and constitutive equations for the purely elastic material in the moving coordinates  $\xi - y$  are expressed as follows:

$$(1 - b^{(e)2} v^2) \frac{\partial^2 w^{(e)}}{\partial \xi^2} + \frac{\partial^2 w^{(e)}}{\partial y^2} + 2b^{(e)2} v \frac{\partial^2 w^{(e)}}{\partial \xi \partial t} - b^{(e)2} \frac{\partial^2 w^{(e)}}{\partial t^2} = 0, \tag{19}$$

$$\tau_{yz}^{(e)} = \mu^{(e)} \frac{\partial w^{(e)}}{\partial y}, \tag{20}$$

$$\tau_{\xi z}^{(e)} = \mu^{(e)} \frac{\partial w^{(e)}}{\partial \xi}. \tag{21}$$

For the problem as indicated in Fig. 1, the mixed boundary conditions can be written as

$$\tau_{yz}(x, 0^+, t) = \tau_{yz}^{(e)}(x, 0^-, t) = -\tau_0 H(-x)H(t) \quad \text{for } -\infty < x < 0, \tag{22}$$

$$w(\xi, 0^+, t) = w^{(e)}(\xi, 0^-, t) \quad \text{for } 0 < \xi < \infty, \tag{23}$$

$$\phi(\xi, 0^+, t) = \phi(\xi, 0^-, t) = 0 \quad \text{for } -\infty < \xi < \infty, \tag{24}$$

where  $H(t)$  is the Heaviside function of  $t$ . To solve the dynamic interfacial crack propagation problem with the governing Eqs. (11), (12) and (19) and the mixed-type boundary conditions (22)–(24), the integral transform method and the Wiener–Hopf technique will be used. The one-sided Laplace transform over time  $t$  and the bilateral Laplace transform on the spatial variable  $\xi$  are defined by

$$\bar{w}(\xi, y, s) = \int_0^\infty w(\xi, y, t) e^{-st} dt, \tag{25}$$

$$\bar{w}^*(\lambda, y, s) = \int_{-\infty}^\infty \bar{w}(\xi, y, s) e^{-s\lambda\xi} d\xi, \tag{26}$$

where  $s$  which is the Laplace transform parameter is a positive real number, large enough to ensure the convergence of the integral and  $\lambda$  is a complex variable. From (11), (12) and (19), the general solutions in the double transformed domain can be obtained as follows:

$$\bar{w}^*(\lambda, y, s) = A(\lambda, s) e^{-s\alpha^*(\lambda)y}, \tag{27}$$

$$\bar{\psi}^*(\lambda, y, s) = B(\lambda, s) e^{-s\beta^*(\lambda)y}, \tag{28}$$

$$\bar{w}^{(e)*}(\lambda, y, s) = A^{(e)}(\lambda, s) e^{s\alpha^{(e)*}(\lambda)y}, \tag{29}$$

where

$$\alpha^*(\lambda) = \sqrt{b^2 - \lambda^2 + b^2 v^2 \lambda^2 - 2b^2 v \lambda} = \sqrt{b + \lambda(1 - bv)} \sqrt{b - \lambda(1 + bv)} = \alpha_+^*(\lambda) \alpha_-^*(\lambda), \tag{30}$$

$$\beta^*(\lambda) = \lim_{\varepsilon \rightarrow 0} \sqrt{\varepsilon^2 - \lambda^2} = \lim_{\varepsilon \rightarrow 0} \sqrt{\varepsilon + \lambda} \sqrt{\varepsilon - \lambda} = \lim_{\varepsilon \rightarrow 0} \beta_+^*(\lambda) \beta_-^*(\lambda), \tag{31}$$

$$\alpha^{(e)*}(\lambda) = \sqrt{b^{(e)2} - \lambda^2 + b^{(e)2} v^2 \lambda^2 - 2b^{(e)2} v \lambda} = \sqrt{b^{(e)} + \lambda(1 - b^{(e)}v)} \sqrt{b^{(e)} - \lambda(1 + b^{(e)}v)} = \alpha_+^{(e)*}(\lambda) \alpha_-^{(e)*}(\lambda) \tag{32}$$

and

$$b = 1/c = \sqrt{\rho/\bar{c}_{44}} \tag{33}$$

is the slowness of the shear wave in the piezoelectric material and  $c$  is the velocity of the shear wave in the piezoelectric material. Here  $\varepsilon$  is introduced as an auxiliary positive real perturbation parameter. It is noted that the final expressions involved are always evaluated at  $\varepsilon = 0$  in the end of manipulation. This technique has been widely used in applying transform methods to solve partial differential equations. The boundary conditions (22)–(24) expressed in the two-sided Laplace transform domain are

$$\bar{\tau}_{yz}^*(\lambda, 0^+, s) = \bar{\tau}_{yz}^{(e)*}(\lambda, 0^-, s) = \frac{-\tau_0 d}{s^2 \lambda(\lambda - d)} + \bar{\tau}_+^*(\lambda, s) \quad \text{for } -\infty < \xi < \infty, \tag{34}$$

$$\bar{w}^*(\lambda, 0^+, s) = \bar{w}^{(e)*}(\lambda, 0^-, s) = A_+(\lambda, s) \quad \text{for } 0 < \xi < \infty, \tag{35}$$

$$\bar{\phi}^*(\lambda, 0^+, s) = \bar{\phi}^*(\lambda, 0^-, s) = 0 \quad \text{for } -\infty < \xi < \infty, \tag{36}$$

where  $d = 1/v$  and  $\text{Re}(\lambda) < 0$ . The unknown function  $\bar{\tau}_+^*$  is defined to be the shear stress  $\tau_{yz}$  on the plane  $y = 0$  for  $0 < \xi < \infty$  in the transform domain. Likewise,  $A_+$  is defined to be the displacement in the  $z$ -direction of the interface  $y = 0$  for  $0 < \xi < \infty$ . Substitution of (27) and (28) into (17) and (36), we will have  $B(\lambda, s) = -e_{15}A(\lambda, s)/\varepsilon_{11}$ . Substitution of (27) and (29) into (34) and (35) leads to

$$-\bar{c}_{44}A(\lambda, s)[\alpha^*(\lambda) - k_e^2\beta^*(\lambda)] = \mu^{(e)}[\alpha^{(e)*}(\lambda)A^{(e)}(\lambda, s)] = \frac{-\tau_0 d}{s^3 \lambda(\lambda - d)} + \frac{1}{s}\bar{\tau}_+^*(\lambda, s), \tag{37}$$

$$\bar{w}^*(\lambda, 0^+, s) = A(\lambda, s) = A_+(\lambda, s) + A_-(\lambda, s), \tag{38}$$

$$\bar{w}^{(e)*}(\lambda, 0^-, s) = A^{(e)}(\lambda, s) = A_+(\lambda, s) + A_-^{(e)}(\lambda, s), \tag{39}$$

where  $k_e^2 = e_{15}^2/\bar{c}_{44}\varepsilon_{11}$  is the electro-mechanical coupling coefficient for the electrode boundary condition.  $A_-$  is defined to be the displacement in the  $z$ -direction of the crack face  $y = 0^+$  for  $-\infty < \xi < 0$ . Likewise,  $A_-^{(e)}$  is defined to be the displacement in the  $z$ -direction of the crack face  $y = 0^-$  for  $-\infty < \xi < 0$ . From (37), we have

$$A^{(e)}(\lambda, s) = -\bar{c}_{44} \left[ \frac{\alpha^*(\lambda) - k_e^2\beta^*(\lambda)}{\mu^{(e)}\alpha^{(e)*}(\lambda)} \right] A(\lambda, s). \tag{40}$$

From (37)–(39), we cancel out  $A_+$  to have

$$\tilde{A}_-(\lambda, s) = \frac{\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)}{[\alpha^*(\lambda) - k_e^2\beta^*(\lambda)]\mu^{(e)}\alpha^{(e)*}(\lambda)} \left[ \frac{-\tau_0 d}{s^3 \lambda(\lambda - d)} + \frac{1}{s}\bar{\tau}_+^*(\lambda, s) \right], \tag{41}$$

where  $\tilde{A}_-(\lambda, s) \equiv A^{(e)}(\lambda, s) - A(\lambda, s) = A_-^{(e)}(\lambda, s) - A_-(\lambda, s)$ . Then rewriting (41) and we have the following Wiener–Hopf equation

$$[\alpha^*(\lambda) - k_e^2\beta^*(\lambda)]\mu^{(e)}\alpha^{(e)*}(\lambda)\tilde{A}_-(\lambda, s) = [\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)] \left[ \frac{-\tau_0 d}{s^3 \lambda(\lambda - d)} + \frac{1}{s}\bar{\tau}_+^*(\lambda, s) \right], \tag{42}$$

where  $k = \mu^{(e)}/\bar{c}_{44}$ . It is noted that the bracketed term  $\alpha^*(\lambda) - k_e^2\beta^*(\lambda)$  in the left-hand side of (42) corresponds to Bleustein–Gulyaev wave function (Bleustein, 1968; Gulyaev, 1969). The Bleustein–Gulyaev wave function will be decomposed and we introduce a new function  $S_1^*(\lambda)$  by defining

$$S_1^*(\lambda) = \frac{\alpha^*(\lambda) - k_e^2\beta^*(\lambda)}{\sqrt{1 + (c_{bg} - v)\lambda}\sqrt{1 - (c_{bg} + v)\lambda}} \frac{\sqrt{c_{bg}^2 - v^2}}{\sqrt{1 - b^2v^2 - k_e^2}}, \tag{43}$$

where

$$c_{bg} = \frac{\sqrt{1 - k_e^4}}{b} \tag{44}$$

is the Bleustein–Gulyaev wave speed in classical piezoelectric theory with electrode. It is assumed in this study that the crack speed  $v$  does not exceed  $c_{bg}$ . Under this assumption, the function  $S_1^*(\lambda)$  has the properties that

$S_1^*(\lambda) \rightarrow 1$  as  $|\lambda| \rightarrow \infty$ , and  $S_1^*(\lambda)$  has neither poles nor zeros in the  $\lambda$ -plane by cuts along  $-1/(c_{bg} - v) < \lambda < -\varepsilon$  and  $\varepsilon < \lambda < 1/(c_{bg} + v)$ . By using the general product factorization method,  $S_1^*(\lambda)$  can be further decomposed as the product of two regular functions  $S_{1+}^*(\lambda)$  and  $S_{1-}^*(\lambda)$ , where

$$S_{1+}^*(\lambda) = \sqrt{\frac{1/(c_{bg} - v) + \lambda}{1/(c - v) + \lambda}} Q_{1+}^*(\lambda) \tag{45}$$

and

$$S_{1-}^*(\lambda) = \sqrt{\frac{1/(c_{bg} + v) - \lambda}{1/(c + v) - \lambda}} Q_{1-}^*(\lambda), \tag{46}$$

in which

$$Q_{1+}^*(\lambda) = \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c-v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2}}{\alpha^*(-z)} \right\} \frac{dz}{z + \lambda} \right\}, \tag{47}$$

$$Q_{1-}^*(\lambda) = \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c+v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2}}{\alpha^*(z)} \right\} \frac{dz}{z - \lambda} \right\}. \tag{48}$$

The bracketed term  $\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)$  in the right-hand side of (42) will be decomposed in the next step. It is noted that  $\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)$  may have two roots (the Maerfeld–Tournois surface wave) or not depends only on the material combinations of elastic–piezoelectric bi-materials. A convenient method to determine the number of roots for the equation  $\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda) = 0$  is by means of the principle of argument (Achenbach, 1976). Let  $G_\lambda(z)$  be analytic everywhere inside and on a simple closed curve  $C_\lambda$ , except for a finite number of zeros inside  $C_\lambda$  and let  $G_\lambda(z)$  have no zeros on  $C_\lambda$ . Then

$$\frac{1}{2\pi i} \int_{C_\lambda} \frac{dG_\lambda}{dz} \frac{dz}{G_\lambda(z)} = Z_\lambda, \tag{49}$$

where  $z$  is a complex variable.  $Z_\lambda$  is the number of zeros inside  $C_\lambda$ . The numbers  $Z_\lambda$  include the orders of zeros. The formula can be rewritten by replacing  $C_\lambda$  by a sum of contours surrounding each zero of  $G_\lambda(z)$ , since expansions these individual integrals are easily evaluated and their sum yields the number  $Z_\lambda$ . To apply the principle of the argument to  $\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda) = 0$ , we express it in the form

$$\begin{aligned} R(\lambda) &= \alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda) \\ &= \sqrt{b^2 - \lambda^2 + b^2 v^2 \lambda^2 - 2b^2 v \lambda} + k \sqrt{b^{(e)2} - \lambda^2 + b^{(e)2} v^2 \lambda^2 - 2b^{(e)2} v \lambda} - k_e^2 \sqrt{\varepsilon^2 - \lambda^2} = 0. \end{aligned} \tag{50}$$

In the complex  $\lambda$ -plane the function  $R(\lambda)$  is rendered single-valued by introducing branch cuts. Now consider the contour  $C_\lambda$  consisting of  $\Gamma_t$ ,  $\Gamma_l$  and  $\Gamma_r$  as indicated in Fig. 2. Within the contour  $C_\lambda = \Gamma_t + \Gamma_l + \Gamma_r$ , the number of zeros is given by

$$Z_\lambda = \frac{1}{2\pi i} \int_{C_\lambda} \frac{dR}{d\lambda} \frac{d\lambda}{R(\lambda)}. \tag{51}$$

The counting of the number of zeros is carried out by mapping the  $\lambda$ -plane on the  $\omega$ -plane through the relation

$$\omega = R(\lambda), \quad d\omega = \frac{dR(\lambda)}{d\lambda} d\lambda. \tag{52}$$

If  $C_\omega$  is the mapping of  $C_\lambda$  in the  $\omega$ -plane, the integral (51) in the  $\omega$ -plane becomes

$$\frac{1}{2\pi i} \int_{C_\omega} \frac{d\omega}{\omega} = Z_\lambda. \tag{53}$$

The latter integral has a simple pole at  $\omega = 0$  and thus  $Z_\lambda$  is simply the number of times the image contour  $C_\omega$  encircles the origin in the  $\omega$ -plane in the counter-clockwise direction. To determine the number of zeros in

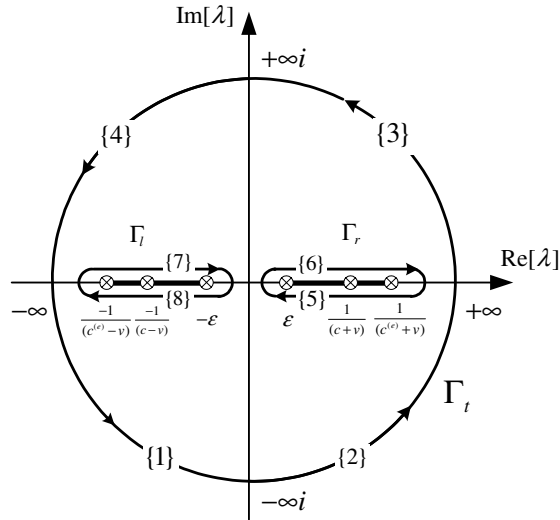


Fig. 2. The  $\lambda$ -plane for  $b^{(e)} > b$ .

the  $\lambda$ -plane, we thus carefully trace the mapping of the contour  $C_\lambda$  into the  $\omega$ -plane. There are two cases to be considered, that are  $b^{(e)} > b$  and  $b > b^{(e)}$ , as follows:

Case (a):  $b^{(e)} > b$  (see Fig. 2).

we find that

$$\text{Im}[R(b^{(e)}/(1 + b^{(e)}v))] > 0 \tag{54}$$

and

$$\text{Im}[R(-b^{(e)}/(1 - b^{(e)}v))] > 0, \tag{55}$$

where the mapping into the  $\omega$ -plane is thus qualitatively indicated in Fig. 3 that the contours  $\Gamma_t, \Gamma_r$  and  $\Gamma_l$  in the  $\lambda$ -plane are mapped to the contours  $\Gamma'_t, \Gamma'_r$  and  $\Gamma'_l$  in the  $\omega$ -plane. The contour  $\Gamma'_t$  encircles the origin (pole) counterclockwise in the  $\omega$ -plane and  $\Gamma'_l$  and  $\Gamma'_r$  encircle the origin clockwise in the  $\omega$ -plane.

The contour of  $\Gamma'_t$  is  $\{1\} \rightarrow \{2\}$  ( $\{3\} \rightarrow \{4\}$ ), and we have  $Z_\lambda = 2 \times (1/2) = 1$ .

The contour of  $\Gamma'_r$  is  $\{5\} \rightarrow \{6\}$ , and we have  $Z_\lambda = -1/2$ .

The contour of  $\Gamma'_l$  is  $\{7\} \rightarrow \{8\}$ , and we have  $Z_\lambda = -1/2$ .

Hence the number of zeros is zero, i.e.,  $Z_\lambda = 2 \times (1/2) - 2 \times (1/2) = 0$ . For the case that

$$\text{Im}[R(b^{(e)}/(1 + b^{(e)}v))] < 0 \tag{56}$$

and

$$\text{Im}[R(-b^{(e)}/(1 - b^{(e)}v))] < 0, \tag{57}$$

where the mapping into the  $\omega$ -plane is qualitatively indicated in Fig. 4 that the contour  $\Gamma'_t$  encircles the origin counterclockwise in the  $\omega$ -plane and  $\Gamma'_l$  and  $\Gamma'_r$  also encircle the origin counterclockwise in the  $\omega$ -plane.

The contour of  $\Gamma'_t$  is  $\{1\} \rightarrow \{2\}$  ( $\{3\} \rightarrow \{4\}$ ), and we have  $Z_\lambda = 2 \times (1/2) = 1$ .

The contour of  $\Gamma'_r$  is  $\{5\} \rightarrow \{6\}$ , and we have  $Z_\lambda = 1/2$ .

The contour of  $\Gamma'_l$  is  $\{7\} \rightarrow \{8\}$ , and we have  $Z_\lambda = 1/2$ .

Hence the number of zeros for the function  $R(\lambda)$  is two, i.e.  $Z_\lambda = 2 \times (1/2) + 2 \times (1/2) = 2$ . Note that, if the crack velocity  $v$  is zero, (56) and (57) are the same as the MT condition which had been obtained by Maerfeld and Tournois (1971).



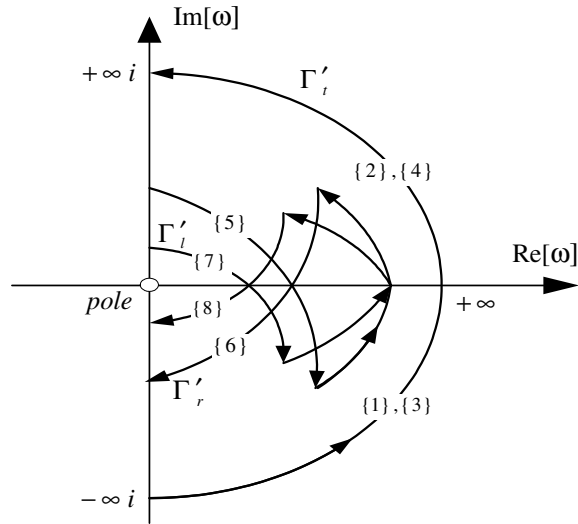


Fig. 3. The  $\omega$ -plane for  $\text{Im}[R(b^{(e)}/(1 + b^{(e)}v))] > 0$  and  $\text{Im}[R(-b^{(e)}/(1 - b^{(e)}v))] > 0$ .

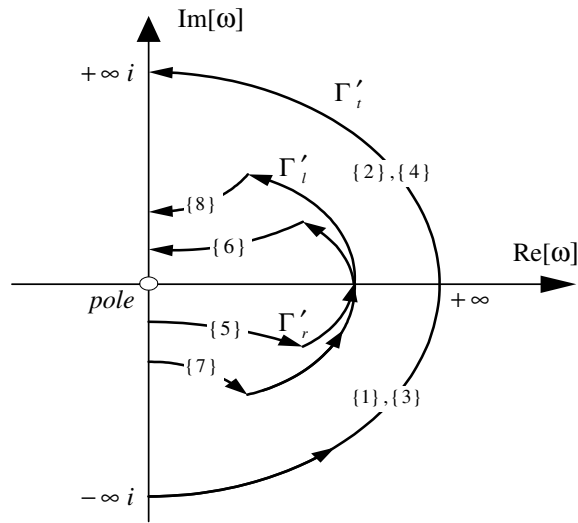


Fig. 4. The  $\omega$ -plane for  $\text{Im}[R(b^{(e)}/(1 + b^{(e)}v))] < 0$  and  $\text{Im}[R(-b^{(e)}/(1 - b^{(e)}v))] < 0$ .

Case (b):  $b > b^{(e)}$ .

From the similar analyzed procedure, we find that for

$$\text{Im}[R(b/(1 + bv))] > 0 \tag{58}$$

and

$$\text{Im}[R(-b/(1 - bv))] > 0, \tag{59}$$

$Z_\lambda = 2(1/2) - 2(1/2) = 0$ . For

$$\text{Im}[R(b/(1 + bv))] < 0 \tag{60}$$

and

$$\text{Im}[R(-b/(1 - bv))] < 0, \tag{61}$$

the number of zeros is two, i.e.,  $Z_\lambda = 2(1/2) + 2(1/2) = 2$ . If (56), (57) (or (60), (61)) are satisfied and  $\text{Im}[R(\lambda)]|_{\lambda \rightarrow \infty} > 0$ , then (50) has two real roots along the real axis for  $b^{(e)}/(1 + b^{(e)}v) < \lambda < \infty$  (or  $b/(1 + bv) < \lambda < \infty$ ) and  $-\infty < \lambda < -b^{(e)}/(1 - b^{(e)}v)$  (or  $-\infty < \lambda < -b/(1 - bv)$ ). When (50) possesses two real roots, the function  $R(\lambda)$  has two distinct real roots,  $-1/c_{mt1}^*$  and  $1/c_{mt2}^*$ , such that  $c_{mt1}^*$  and  $c_{mt2}^*$  are defined as Maerfeld–Tournois surface wave (MT surface wave) velocities, i.e.,

$$c_{mt1}^* = \sqrt{2A_1 / \left( -B_1 - \sqrt{B_1^2 - 4A_1C_1} \right)} - v, \quad c_{mt2}^* = \sqrt{2A_1 / \left( -B_1 - \sqrt{B_1^2 - 4A_1C_1} \right)} + v,$$

where

$$\begin{aligned} A_1 &= [1 + (\mu^{(e)}/\bar{c}_{44})^2 - k_e^4]^2 - 4(\mu^{(e)}/\bar{c}_{44})^2, \\ B_1 &= -2[b^2 + (\mu^{(e)}/\bar{c}_{44})^2 b^{(e)^2}][1 + (\mu^{(e)}/\bar{c}_{44})^2 - k_e^4] + 4(\mu^{(e)}/\bar{c}_{44})^2(b^2 + b^{(e)^2}), \\ C_1 &= b^4 - 2(\mu^{(e)}/\bar{c}_{44})^2 b^2 b^{(e)^2} + (\mu^{(e)}/\bar{c}_{44})^4 b^{(e)^4}. \end{aligned}$$

If  $R(\lambda)$  has two distinct real roots, i.e.,  $-1/c_{mt1}^*$  and  $1/c_{mt2}^*$ , there are two cases, i.e.,  $b < b^{(e)}$  and  $b > b^{(e)}$ , to be discussed in detail.

Case (1):  $b < b^{(e)}$  (The existence of MT surface wave)

We introduce a new function  $S_2^*(\lambda)$  as follows

$$S_2^*(\lambda) = \frac{\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)}{\sqrt{1/c_{mt1}^* + \lambda}\sqrt{1/c_{mt2}^* - \lambda}} \frac{1}{\sqrt{1 - b^2v^2 + k}\sqrt{1 - b^{(e)2}v^2 - k_e^2}}. \tag{62}$$

The function  $S_2^*(\lambda)$  has the property that  $S_2^*(\lambda) \rightarrow 1$  as  $|\lambda| \rightarrow \infty$ , and  $S_2^*(\lambda)$  has neither poles nor zeros in the  $\lambda$ -plane by cuts along  $-1/c_{mt1}^* < \lambda < -\varepsilon$  and  $\varepsilon < \lambda < 1/c_{mt2}^*$ . Similarly, by using the general product factorization method,  $S_2^*(\lambda)$  can be further decomposed as the product of two regular functions  $S_{2+}^*(\lambda)$  and  $S_{2-}^*(\lambda)$ , where

$$S_{2+}^*(\lambda) = \sqrt{\frac{1/c_{mt1}^* + \lambda}{1/(c^{(e)} - v) + \lambda}} \mathcal{Q}_{2+}^*(\lambda) \tag{63}$$

and

$$S_{2-}^*(\lambda) = \sqrt{\frac{1/c_{mt2}^* - \lambda}{1/(c^{(e)} + v) - \lambda}} \mathcal{Q}_{2-}^*(\lambda), \tag{64}$$

in which

$$\begin{aligned} \mathcal{Q}_{2+}^*(\lambda) &= \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c-v}}^{\frac{1}{c^{(e)}-v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2 - \varepsilon^2} - \sqrt{(1 - b^2v^2)z^2 - 2vb^2z - b^2}}{k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z + \lambda} \right\} \\ &\cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c-v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2 - \varepsilon^2}}{\alpha^*(-z) + k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z + \lambda} \right\} \end{aligned} \tag{65}$$

and

$$\begin{aligned}
 Q_{2-}^*(\lambda) = & \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c+v}}^{\frac{1}{c^{(e)+v}}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2} - \sqrt{(1 - b^2 v^2) z^2 + 2vbz - b^2}}{k\alpha^{(e)*}(z)} \right\} \frac{dz}{z - \lambda} \right\} \\
 & \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c+v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2}}{\alpha^*(z) + k\alpha^{(e)*}(z)} \right\} \frac{dz}{z - \lambda} \right\}.
 \end{aligned} \tag{66}$$

In view of the previous derivation and discussion, the Wiener–Hopf equation (42) can be rewritten as

$$\begin{aligned}
 \alpha_{-}^{(e)*}(\lambda) \frac{1/(c_{bg} + v) - \lambda}{1/c_{m2}^* - \lambda} \frac{\sqrt{1/(c^{(e)} + v) - \lambda}}{\sqrt{1/(c + v) - \lambda}} \frac{Q_{1-}^*(\lambda)}{Q_{2-}^*(\lambda)} \tilde{A}_{-}^*(\lambda, s) \\
 = \tilde{k} \frac{M_{2+}^*(\lambda)}{\alpha_{+}^{(e)*}(\lambda) M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \bar{\tau}_{+}^*(\lambda, s) \right],
 \end{aligned} \tag{67}$$

where

$$\begin{aligned}
 \tilde{k} &= \frac{\sqrt{1 - b^2 v^2} + k \sqrt{1 - b^{(e)2} v^2 - k_e^2}}{\mu^{(e)} (\sqrt{1 - b^2 v^2} - k_e^2)}, \quad M_{1+}^*(\lambda) = \frac{1/(c_{bg} - v) + \lambda}{\sqrt{1/(c - v) + \lambda}} Q_{1+}^*(\lambda), \\
 M_{2+}^*(\lambda) &= \frac{1/c_{m1}^* + \lambda}{\sqrt{1/(c^{(e)} - v) + \lambda}} Q_{2+}^*(\lambda).
 \end{aligned}$$

The first term on the right-hand side of (67) is regular for  $\text{Re}(\lambda) > -\varepsilon$ , except for the poles at  $\lambda = 0$  and  $d$ . These poles can be removed and (67) will be rearranged into the following form

$$\begin{aligned}
 \alpha_{-}^{(e)*}(\lambda) \frac{1/(c_{bg} + v) - \lambda}{1/c_{m2}^* - \lambda} \frac{\sqrt{1/(c^{(e)} + v) - \lambda}}{\sqrt{1/(c + v) - \lambda}} \frac{Q_{1-}^*(\lambda)}{Q_{2-}^*(\lambda)} \tilde{A}_{-}^*(\lambda, s) - \tilde{k} \frac{M_{2+}^*(0)}{\alpha_{+}^{(e)*}(0) M_{1+}^*(0)} \frac{\tau_0}{s^3 \lambda} - \tilde{k} \frac{M_{2+}^*(d)}{\alpha_{+}^{(e)*}(d) M_{1+}^*(d)} \frac{-\tau_0}{s^3 (\lambda - d)} \\
 = \tilde{k} \frac{M_{2+}^*(\lambda)}{\alpha_{+}^{(e)*}(\lambda) M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \bar{\tau}_{+}^*(\lambda, s) \right] - \tilde{k} \frac{M_{2+}^*(0)}{\alpha_{+}^{(e)*}(0) M_{1+}^*(0)} \frac{\tau_0}{s^3 \lambda} - \tilde{k} \frac{M_{2+}^*(d)}{\alpha_{+}^{(e)*}(d) M_{1+}^*(d)} \frac{-\tau_0}{s^3 (\lambda - d)}.
 \end{aligned} \tag{68}$$

The left-hand side of this equation is regular for  $\text{Re}(\lambda) < \varepsilon$ , while the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ . Applying the analytic continuation argument, therefore, each side of (68) represents a single entire function, say  $E_1^*(\lambda)$ . By Liouville’s theorem, the bounded entire function  $E_1^*(\lambda)$  is a constant. The magnitude of the constant can be obtained from order conditions on  $E_1^*(\lambda)$  as  $|\lambda| \rightarrow \infty$ , which in turn are obtained from order conditions on the dependent field variables in the vicinity of  $\xi = 0$ . Consequently,  $\bar{\tau}_{+}(\xi, 0, s)$  is expected to be square root singular near  $\xi = 0$ , i.e.,  $\bar{\tau}_{+}(\xi, 0, s) = O(|\xi|^{-1/2})$  as  $\xi \rightarrow 0^+$ . By using of the Abelian theorem,  $E_1^*(\lambda)$  vanishes identically, and we can solve for  $A_{-}^*$  from the left-hand side of (68). From  $A_{-}^*(\lambda, s) \equiv A^{(e)}(\lambda, s) - A(\lambda, s)$  and (37), we find

$$\begin{aligned}
 A(\lambda, s) = & \frac{-\tau_0 M_{2+}^*(0) \sqrt{1/(c^{(e)} - v) + \lambda} \sqrt{1/(c + v) - \lambda}}{\bar{c}_{44} (\sqrt{1 - b^2 v^2} - k_e^2) \sqrt{1/(c^{(e)} - v)} M_{1+}^*(0) s^3 \lambda M_{2+}^*(\lambda) [1/(c_{bg} + v) - \lambda] Q_{1-}^*(\lambda)} \\
 & + \frac{\tau_0 M_{2+}^*(d) \sqrt{1/(c^{(e)} - v) + \lambda} \sqrt{1/(c + v) - \lambda}}{\bar{c}_{44} (\sqrt{1 - b^2 v^2} - k_e^2) \sqrt{1/(c^{(e)} - v) + d} M_{1+}^*(d) s^3 (\lambda - d) M_{2+}^*(\lambda) [1/(c_{bg} + v) - \lambda] Q_{1-}^*(\lambda)}
 \end{aligned} \tag{69}$$

and

$$\begin{aligned}
 A^{(e)}(\lambda, s) = & \frac{\tau_0 M_{2+}^*(0) M_{1+}^*(\lambda) \sqrt{1/(c^{(e)} - v) + \lambda}}{\mu^{(e)} \sqrt{1/(c^{(e)} - v)} M_{1+}^*(0) s^3 \lambda (1/c_{m1}^* + \lambda) Q_{2+}^*(\lambda) \alpha_{-}^{(e)*}(\lambda)} \\
 & - \frac{\tau_0 M_{2+}^*(d) M_{1+}^*(\lambda) \sqrt{1/(c^{(e)} - v) + \lambda}}{\mu^{(e)} \sqrt{1/(c^{(e)} - v) + d} M_{1+}^*(d) s^3 (\lambda - d) (1/c_{m1}^* + \lambda) Q_{2+}^*(\lambda) \alpha_{-}^{(e)*}(\lambda)}.
 \end{aligned} \tag{70}$$

The amplitude of  $\bar{\psi}^*$  can be obtained as follows:

$$B(\lambda, s) = \frac{\tau_0 k_e^2 M_{2+}^*(0) \sqrt{1/(c^{(e)} - v)} + \lambda \sqrt{1/(c + v)} - \lambda}{e_{15}(\sqrt{1 - b^2 v^2 - k_e^2}) \sqrt{1/(c^{(e)} - v)} M_{1+}^*(0) s^3 \lambda M_{2+}^*(\lambda) [1/(c_{bg} + v) - \lambda] Q_{1-}^*(\lambda)} - \frac{\tau_0 k_e^2 M_{2+}^*(d) \sqrt{1/(c^{(e)} - v)} + \lambda \sqrt{1/(c + v)} - \lambda}{e_{15}(\sqrt{1 - b^2 v^2 - k_e^2}) \sqrt{1/(c^{(e)} - v)} + d M_{1+}^*(d) s^3 (\lambda - d) M_{2+}^*(\lambda) [1/(c_{bg} + v) - \lambda] Q_{1-}^*(\lambda)}. \quad (71)$$

Substituting (69)–(71) into (27)–(29) and making use of (17) and (5)–(10), the complete field solutions in the transform domain are obtained. Finally, the Cagniard–de Hoop scheme (de Hoop, 1960) is employed to invert the transform domain into the time domain. The Cagniard–de Hoop inversion method is used to perform the two integrations in one single operation leaving only the convolution to be done. We have to include the integral around the branch cut whatever different slowness combines. This additional integral path represents the head wave. There are two situations, i.e.,  $[1/(c^{(e)} - v)]\cos\theta > 1/(c - v)$  and  $[1/(c^{(e)} - v)]\cos\theta < 1/(c - v)$ , and the corresponding  $\lambda$ -contours are shown in Figs. 5 and 6, respectively. The transient explicit full-field solutions for shear stresses and electric displacements in the time domain are presented as follows:

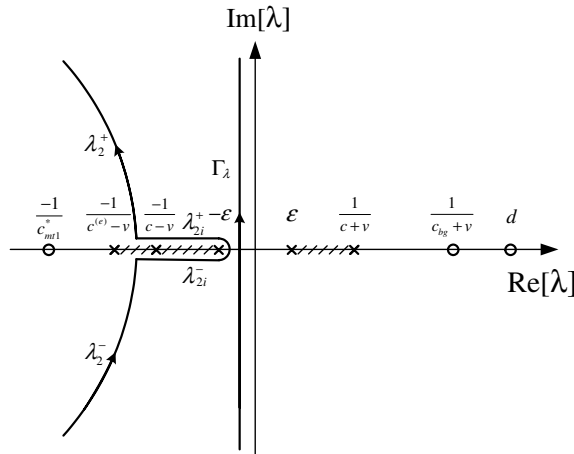


Fig. 5. The Cagniard–de Hoop contour for  $[1/(c^{(e)} - v)]\cos\theta > 1/(c - v)$ .

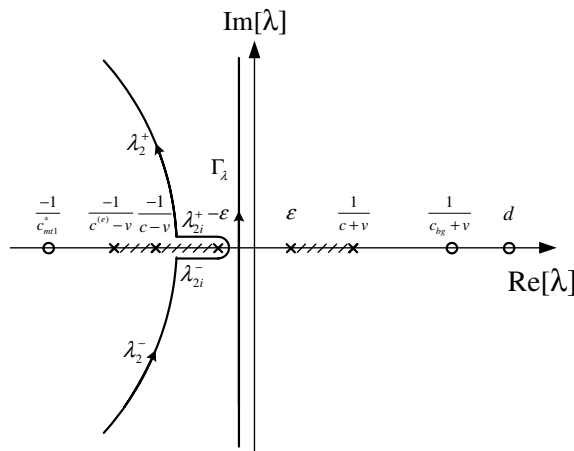


Fig. 6. The Cagniard–de Hoop contour for  $[1/(c^{(e)} - v)]\cos\theta < 1/(c - v)$ .

$$\begin{aligned}
 \tau_{\xi z}(\xi, y, t) = & \frac{-\tau_0 \sqrt{1/(c-v)}(1/c_{m1}^*)Q_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v)][1/(c_{bg}-v)]Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c^{(e)}-v) + \lambda_2^{*+}] \sqrt{1/(c+v) - \lambda_2^{*+}}}{Q_{2+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg}+v) - \lambda_2^{*+}]Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_s^*) \\
 & + \frac{\tau_0 \sqrt{1/(c-v) + d}(1/c_{m1}^* + d)Q_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v) + d][1/(c_{bg}-v) + d]Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_2^{*+} [1/(c^{(e)}-v) + \lambda_2^{*+}] \sqrt{1/(c+v) - \lambda_2^{*+}}}{(\lambda_2^{*+} - d)Q_{2+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg}+v) - \lambda_2^{*+}]Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_s^*) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v)}(1/c_{m1}^*)Q_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v)][1/(c_{bg}-v)]Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c^{(e)}-v) + \lambda_{2i}^{*+}] \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{Q_{2+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg}+v) - \lambda_{2i}^{*+}]Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t-t_{hms}^*) - H(t-t_s^*)] \\
 & + \frac{\tau_0 \sqrt{1/(c-v) + d}(1/c_{m1}^* + d)Q_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v) + d][1/(c_{bg}-v) + d]Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{2i}^{*+} [1/(c^{(e)}-v) + \lambda_{2i}^{*+}] \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{(\lambda_{2i}^{*+} - d)Q_{2+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg}+v) - \lambda_{2i}^{*+}]Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t-t_{hms}^*) - H(t-t_s^*)] \\
 & + \frac{\tau_0 k_e^2 \sqrt{1/(c-v)}(1/c_{m1}^*)Q_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v)][1/(c_{bg}-v)]Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c^{(e)}-v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{Q_{2+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg}+v) - \lambda_3^{*+}]Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_m^*) \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1/(c-v) + d}(1/c_{m1}^* + d)Q_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v) + d][1/(c_{bg}-v) + d]Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} [1/(c^{(e)}-v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d)Q_{2+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg}+v) - \lambda_3^{*+}]Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t-t_m^*), \tag{72}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}(\xi, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)}(1/c_{m1}^*)Q_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c^{(e)}-v)][1/(c_{bg}-v)]Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_2^{*+})[1/(c^{(e)}-v) + \lambda_2^{*+}] \sqrt{1/(c+v) - \lambda_2^{*+}}}{\lambda_2^{*+} (1/c_{m1}^* + \lambda_2^{*+})Q_{2+}^*(\lambda_2^{*+})[1/(c_{bg}+v) - \lambda_2^{*+}]Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_s^*)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) \mathcal{Q}_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_2^{*+}) [1/(c^{(e)} - v) + \lambda_2^{*+}] \sqrt{1/(c+v) - \lambda_2^{*+}}}{(\lambda_2^{*+} - d) (1/c_{m1}^* + \lambda_2^{*+}) \mathcal{Q}_{2+}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}] \mathcal{Q}_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^*) \\
 & + \frac{\tau_0 \sqrt{1/(c-v)} (1/c_{m1}^*) \mathcal{Q}_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v)] [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+}) [1/(c^{(e)} - v) + \lambda_{2i}^{*+}] \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{\lambda_{2i}^{*+} (1/c_{m1}^* + \lambda_{2i}^{*+}) \mathcal{Q}_{2+}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) \mathcal{Q}_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+}) [1/(c^{(e)} - v) + \lambda_{2i}^{*+}] \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{(\lambda_{2i}^{*+} - d) (1/c_{m1}^* + \lambda_{2i}^{*+}) \mathcal{Q}_{2+}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1/(c-v)} (1/c_{m1}^*) \mathcal{Q}_{2+}^*(0)}{\pi(\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v)] [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) [1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{\lambda_3^{*+} (1/c_{m1}^* + \lambda_3^{*+}) \mathcal{Q}_{2+}^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 k_e^2 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) \mathcal{Q}_{2+}^*(d)}{\pi(\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) [1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) (1/c_{m1}^* + \lambda_3^{*+}) \mathcal{Q}_{2+}^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) - \tau_0 H(t - by) H(-\cos \theta), \tag{73}
 \end{aligned}$$

$$\begin{aligned}
 D_\xi(\xi, y, t) & = \frac{-\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} (1/c_{m1}^*) \mathcal{Q}_{2+}^*(0)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v)] [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{\mathcal{Q}_{2+}^*(\lambda_3^{*+}) (1/c_{m1}^* + \lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) \mathcal{Q}_{2+}^*(d)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2 - k_e^2}) [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} [1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) \mathcal{Q}_{2+}^*(\lambda_3^{*+}) (1/c_{m1}^* + \lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*), \tag{74}
 \end{aligned}$$

$$\begin{aligned}
 D_y(\zeta, y, t) = & \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} (1/c_{m1}^*) Q_{2+}^*(0)}{\pi e_{15} (\sqrt{1-b^2 v^2 - k_e^2}) [1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) [1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{\lambda_3^{*+} Q_{2+}^*(\lambda_3^{*+}) (1/c_{m1}^* + \lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{-\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{\pi e_{15} (\sqrt{1-b^2 v^2 - k_e^2}) [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) [1/(c^{(e)} - v) + \lambda_3^{*+}] \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) Q_{2+}^*(\lambda_3^{*+}) (1/c_{m1}^* + \lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*), \tag{75}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{\zeta z}^{(e)}(\zeta, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)} (1/c_{m1}^*) Q_{2+}^*(0)}{\pi [1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+}) \sqrt{1/(c^{(e)} - v) + \lambda_1^{*+}}}{(1/c_{m1}^* + \lambda_1^{*+}) Q_{2+}^*(\lambda_1^{*+}) \sqrt{1/(c-v) + \lambda_1^{*+}} \alpha_{-}^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{\pi [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+}) \sqrt{1/(c^{(e)} - v) + \lambda_1^{*+}}}{(\lambda_1^{*+} - d) (1/c_{m1}^* + \lambda_1^{*+}) Q_{2+}^*(\lambda_1^{*+}) \sqrt{1/(c-v) + \lambda_1^{*+}} \alpha_{-}^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1/(c-v)} (1/c_{m1}^*) Q_{2+}^*(0)}{\pi [1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c^{(e)} - v) + \lambda_{1i}^{*+}}}{(1/c_{m1}^* + \lambda_{1i}^{*+}) Q_{2+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}} \alpha_{-}^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{\pi [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c^{(e)} - v) + \lambda_{1i}^{*+}}}{(\lambda_{1i}^{*+} - d) (1/c_{m1}^* + \lambda_{1i}^{*+}) Q_{2+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}} \alpha_{-}^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta), \tag{76}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}^{(e)}(\zeta, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)} (1/c_{m1}^*) Q_{2+}^*(0)}{\pi [1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+}) [1/(c^{(e)} - v) + \lambda_1^{*+}]}{\lambda_1^{*+} (1/c_{m1}^* + \lambda_1^{*+}) Q_{2+}^*(\lambda_1^{*+}) \sqrt{1/(c-v) + \lambda_1^{*+}}} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{\pi [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)}
 \end{aligned}$$

$$\begin{aligned}
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_{1+}^{*+}] Q_{1+}^*(\lambda_{1+}^{*+}) [1/(c^{(e)} - v) + \lambda_{1+}^{*+}]}{(\lambda_{1+}^{*+} - d)(1/c_{m1}^* + \lambda_{1+}^{*+}) Q_{2+}^*(\lambda_{1+}^{*+}) \sqrt{1/(c - v) + \lambda_{1+}^{*+}}} \frac{\partial \lambda_{1+}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1/(c - v)} (1/c_{m1}^*) Q_{2+}^*(0)}{\pi [1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+}) [1/(c^{(e)} - v) + \lambda_{1i}^{*+}]}{\lambda_{1i}^{*+} (1/c_{m1}^* + \lambda_{1i}^{*+}) Q_{2+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c - v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta) \\
 & + \frac{-\tau_0 \sqrt{1/(c - v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{\pi [1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{[1/(c_{bg} - v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+}) [1/(c^{(e)} - v) + \lambda_{1i}^{*+}]}{(\lambda_{1i}^{*+} - d)(1/c_{m1}^* + \lambda_{1i}^{*+}) Q_{2+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c - v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta) \\
 & - \tau_0 H(t + b^{(e)}y) H(-\cos \theta),
 \end{aligned} \tag{77}$$

where

$$\begin{aligned}
 \lambda_{1+}^{*+} &= \frac{-(\xi t + b^{(e)2}vy^2) + i|y|\sqrt{t^2 - b^{(e)2}[y^2 + (\xi + tv)^2]}}{\xi^2 + (1 - b^{(e)2}v^2)y^2}, \quad \lambda_{1i}^{*+} = \frac{-(\xi t + b^{(e)2}vy^2) + |y|\sqrt{b^{(e)2}[y^2 + (\xi + tv)^2] - t^2}}{\xi^2 + (1 - b^{(e)2}v^2)y^2} + i\epsilon, \\
 \lambda_{2+}^{*+} &= \frac{-(\xi t + b^2vy^2) + i|y|\sqrt{t^2 - b^2[y^2 + (\xi + tv)^2]}}{\xi^2 + (1 - b^2v^2)y^2}, \quad \lambda_{2i}^{*+} = \frac{-(\xi t + b^2vy^2) + |y|\sqrt{b^2[y^2 + (\xi + tv)^2] - t^2}}{\xi^2 + (1 - b^2v^2)y^2} + i\epsilon, \\
 \lambda_{3+}^{*+} &= \frac{-[\xi t + \epsilon^2vy^2] + i|y|\sqrt{t^2 - \epsilon^2[y^2 + (\xi + tv)^2]}}{\xi^2 + [1 - \epsilon^2v^2]y^2}, \quad t_s^* = \frac{b[bv\xi + \sqrt{\xi^2 + (1 - b^2v^2)y^2}]}{1 - b^2v^2}, \quad t_m^* \rightarrow 0^+, \quad t_{hms}^* = by, \\
 t_s^{(e)*} &= \frac{b^{(e)}[b^{(e)}v\xi + \sqrt{\xi^2 + (1 - b^{(e)2}v^2)y^2}]}{1 - b^{(e)2}v^2}, \quad t_{hms}^{(e)*} = -b^{(e)}y, \quad r = \sqrt{\xi^2 + y^2}, \quad \cos \theta = \xi/r, \quad \sin \theta = y/r.
 \end{aligned}$$

The corresponding transient results of the dynamic stress intensity factor and the dynamic electric displacement intensity factor expressed in the time domain are

$$\begin{aligned}
 K_{III}^{(e)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} \tau_{yz}(\xi, 0, t) \\
 &= \frac{2\tau_0 \sqrt{1/(c - v)} (1/c_{m1}^*) Q_{2+}^*(0)}{[1/(c^{(e)} - v)] [1/(c_{bg} - v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} - \frac{2\tau_0 \sqrt{1/(c - v) + d} (1/c_{m1}^* + d) Q_{2+}^*(d)}{[1/(c^{(e)} - v) + d] [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}}
 \end{aligned} \tag{78}$$

and



$$\begin{aligned}
 K_{III}^{(D)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} D_y(\xi, 0, t) \\
 &= \frac{2\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} (1/c_{m1}^*) Q_{2+}^*(0)}{e_{15} (\sqrt{1-b^2v^2-k_e^2} [1/(c^{(e)}-v)] [1/(c_{bg}-v)] Q_{1+}^*(0))} \sqrt{\frac{2t}{\pi}} \\
 &\quad - \frac{2\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)+d} (1/c_{m1}^*+d) Q_{2+}^*(d)}{e_{15} (\sqrt{1-b^2v^2-k_e^2} [1/(c^{(e)}-v)+d] [1/(c_{bg}-v)+d] Q_{1+}^*(d))} \sqrt{\frac{2t}{\pi}},
 \end{aligned} \tag{79}$$

respectively.

Case (2):  $b > b^{(e)}$  (The existence of MT surface wave)

If the function  $R(\lambda)$  has two distinct real roots  $-1/c_{m1}^*$  and  $1/c_{m2}^*$ , then a new function  $S_3^*(\lambda)$  is introduced as

$$S_3^*(\lambda) = \frac{\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)}{\sqrt{1/c_{m1}^* + \lambda} \sqrt{1/c_{m2}^* - \lambda}} \frac{1}{\sqrt{1-b^2v^2+k} \sqrt{1-b^{(e)2}v^2-k_e^2}}. \tag{80}$$

The function  $S_3^*(\lambda)$  has similar properties as  $S_2^*(\lambda)$ , but the product factorization will be different. Similarly,  $S_3^*(\lambda)$  can be further decomposed as the product of two regular functions  $S_{3+}^*(\lambda)$  and  $S_{3-}^*(\lambda)$ , where

$$S_{3+}^*(\lambda) = \sqrt{\frac{1/c_{m1}^* + \lambda}{1/(c-v) + \lambda}} Q_{3+}^*(\lambda) \tag{81}$$

and

$$S_{3-}^*(\lambda) = \sqrt{\frac{1/c_{m2}^* - \lambda}{1/(c+v) - \lambda}} Q_{3-}^*(\lambda), \tag{82}$$

in which

$$\begin{aligned}
 Q_{3+}^*(\lambda) &= \exp \left\{ -\frac{1}{\pi} \int_{\frac{1}{c^{(e)}-v}}^{\frac{1}{c-v}} \tan^{-1} \left\{ \frac{k\sqrt{(1-v^2b^{(e)2})z^2 - 2vb^{(e)2}z - b^{(e)2} - k_e^2\sqrt{z^2 - \varepsilon^2}}}{\alpha^*(-z)} \right\} \frac{dz}{z + \lambda} \right\} \\
 &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c^{(e)}-v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2 - \varepsilon^2}}{\alpha^*(-z) + k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z + \lambda} \right\}
 \end{aligned} \tag{83}$$

and

$$\begin{aligned}
 Q_{3-}^*(\lambda) &= \exp \left\{ -\frac{1}{\pi} \int_{\frac{1}{c^{(e)}+v}}^{\frac{1}{c+v}} \tan^{-1} \left\{ \frac{k\sqrt{(1-v^2b^{(e)2})z^2 + 2vb^{(e)2}z - b^{(e)2} - k_e^2\sqrt{z^2 - \varepsilon^2}}}{\alpha^*(z)} \right\} \frac{dz}{z - \lambda} \right\} \\
 &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c^{(e)}+v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2 - \varepsilon^2}}{\alpha^*(z) + k\alpha^{(e)*}(z)} \right\} \frac{dz}{z - \lambda} \right\}.
 \end{aligned} \tag{84}$$

In view of the previous discussion, (42) can be rewritten as

$$\alpha_-^{(e)*}(\lambda) \frac{1/(c_{bg}+v) - \lambda}{1/c_{m2}^* - \lambda} \frac{Q_{1-}^*(\lambda)}{Q_{3-}^*(\lambda)} \tilde{A}_-(\lambda, s) = \tilde{k} \frac{M_{3+}^*(\lambda)}{\alpha_+^{(e)*}(\lambda) M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda-d} \right) + \frac{1}{s} \tilde{\tau}_+^*(\lambda, s) \right], \tag{85}$$

where

$$M_{3+}^*(\lambda) = \frac{1/c_{m1}^* + \lambda}{\sqrt{1/(c-v) + \lambda}} Q_{3+}^*(\lambda).$$

The first term on the right-hand side of (85) is regular for  $\text{Re}(\lambda) > -\varepsilon$ , except for the poles at  $\lambda = 0$  and  $d$ . However, these poles can be removed and (85) is rearranged into the following form

$$\begin{aligned} \alpha_-^{(e)*}(\lambda) & \frac{1/(c_{bg} + v) - \lambda}{1/c_{m1}^* - \lambda} \frac{Q_{1-}^*(\lambda)}{Q_{3-}^*(\lambda)} \tilde{A}_-^*(\lambda, s) - \tilde{k} \frac{M_{3+}^*(0)}{\alpha_+^{(e)*}(0)M_{1+}^*(0)} \frac{\tau_0}{s^3\lambda} - \tilde{k} \frac{M_{3+}^*(d)}{\alpha_+^{(e)*}(d)M_{1+}^*(d)} \frac{-\tau_0}{s^3(\lambda - d)} \\ & = \tilde{k} \frac{M_{3+}^*(\lambda)}{\alpha_+^{(e)*}(\lambda)M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \bar{\tau}_+^*(\lambda, s) \right] - \tilde{k} \frac{M_{3+}^*(0)}{\alpha_+^{(e)*}(0)M_{1+}^*(0)} \frac{\tau_0}{s^3\lambda} - \tilde{k} \frac{M_{3+}^*(d)}{\alpha_+^{(e)*}(d)M_{1+}^*(d)} \\ & \quad \times \frac{-\tau_0}{s^3(\lambda - d)}. \end{aligned} \tag{86}$$

The left-hand side of this equation is regular for  $\text{Re}(\lambda) < \varepsilon$ , while the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ . Applying the analytic continuation argument, therefore, each side of (86) represents a single entire function, say  $E_2^*(\lambda)$ . By Liouville’s theorem, the bounded entire function  $E_2^*(\lambda)$  is a constant. By using of the Abelian theorem,  $E_2^*(\lambda)$  vanishes identically. The unknown  $\tilde{A}_-^*$  can be obtained from (86). From  $\tilde{A}_-^*(\lambda, s) \equiv A^{(e)}(\lambda, s) - A_-(\lambda, s)$  and (37), we find

$$\begin{aligned} A(\lambda, s) & = \frac{-\tau_0}{s^3\lambda} \frac{M_{3+}^*(0)\sqrt{1/(c+v) - \lambda}\alpha_+^{(e)*}(\lambda)}{\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(0)M_{1+}^*(0)M_{3+}^*(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^*(\lambda)} + \frac{\tau_0}{s^3(\lambda - d)} \\ & \quad \times \frac{M_{3+}^*(d)\sqrt{1/(c+v) - \lambda}\alpha_+^{(e)*}(\lambda)}{\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(d)M_{1+}^*(d)M_{3+}^*(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^*(\lambda)} \end{aligned} \tag{87}$$

and

$$A^{(e)}(\lambda, s) = \frac{\tau_0}{s^3\lambda} \frac{M_{3+}^*(0)M_{1+}^*(\lambda)}{\mu^{(e)}\alpha_+^{(e)*}(0)M_{1+}^*(0)M_{3+}^*(\lambda)\alpha_-^{(e)*}(\lambda)} - \frac{\tau_0}{s^3(\lambda - d)} \frac{M_{3+}^*(d)M_{1+}^*(\lambda)}{\mu^{(e)}\alpha_+^{(e)*}(d)M_{1+}^*(d)M_{3+}^*(\lambda)\alpha_-^{(e)*}(\lambda)}. \tag{88}$$

The amplitude of  $\bar{\psi}^*$  can be obtained as follows:

$$\begin{aligned} B(\lambda, s) & = \frac{\tau_0}{s^3\lambda} \frac{k_e^2 M_{3+}^*(0)\sqrt{1/(c+v) - \lambda}\alpha_+^{(e)*}(\lambda)}{e_{15}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(0)M_{1+}^*(0)M_{3+}^*(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^*(\lambda)} - \frac{\tau_0}{s^3(\lambda - d)} \\ & \quad \times \frac{k_e^2 M_{3+}^*(d)\sqrt{1/(c+v) - \lambda}\alpha_+^{(e)*}(\lambda)}{e_{15}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(d)M_{1+}^*(d)M_{3+}^*(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^*(\lambda)}. \end{aligned} \tag{89}$$

We can obtain transient full-field solutions for shear stresses and electric displacements in the time domain from the Cagniard–de Hoop scheme and the results are

$$\begin{aligned} \tau_{\xi z}(\xi, y, t) & = \frac{-\tau_0(1/c_{m1}^*)Q_{3+}^*(0)}{\pi(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(0)[1/(c_{bg} - v)]Q_{1+}^*(0)} \\ & \quad \cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c-v) + \lambda_2^{*+}} \sqrt{1/(c+v) - \lambda_2^{*+}} \alpha_+^{(e)*}(\lambda_2^{*+})}{Q_{3+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg} + v) - \lambda_2^{*+}]Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} dt H(t - t_s^*) \\ & \quad + \frac{\tau_0(1/c_{m1}^* + d)Q_{3+}^*(d)}{\pi(\sqrt{1 - b^2v^2 - k_e^2})\alpha_+^{(e)*}(d)[1/(c_{bg} - v) + d]Q_{1+}^*(d)} \\ & \quad \cdot \int_0^t \text{Im} \left[ \frac{\lambda_2^{*+} \sqrt{1/(c-v) + \lambda_2^{*+}} \sqrt{1/(c+v) - \lambda_2^{*+}} \alpha_+^{(e)*}(\lambda_2^{*+})}{(\lambda_2^{*+} - d)Q_{3+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg} + v) - \lambda_2^{*+}]Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} dt H(t - t_s^*) \end{aligned}$$

$$\begin{aligned}
 & + \frac{-\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c-v)} + \lambda_{2i}^{*+} \sqrt{1/(c+v)} - \lambda_{2i}^{*+} \alpha_+^{(e)*}(\lambda_{2i}^{*+})}{\mathcal{Q}_{3+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg}+v) - \lambda_{2i}^{*+}]\mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau[H(t-t_{hms}^*) - H(t-t_s^*)] \\
 & + \frac{\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{2i}^{*+} \sqrt{1/(c-v)} + \lambda_{2i}^{*+} \sqrt{1/(c+v)} - \lambda_{2i}^{*+} \alpha_+^{(e)*}(\lambda_{2i}^{*+})}{(\lambda_{2i}^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg}+v) - \lambda_{2i}^{*+}]\mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau[H(t-t_{hms}^*) - H(t-t_s^*)] \\
 & + \frac{\tau_0 k_e^2(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c-v)} + \lambda_3^{*+} \sqrt{1/(c+v)} - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}{\mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg}+v) - \lambda_3^{*+}]\mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_m^*) \\
 & + \frac{-\tau_0 k_e^2(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} \sqrt{1/(c-v)} + \lambda_3^{*+} \sqrt{1/(c+v)} - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}{(\lambda_3^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg}+v) - \lambda_3^{*+}]\mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t-t_m^*), \tag{90}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}(\xi, y, t) = & \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_2^{*+})\sqrt{1/(c-v)} + \lambda_2^{*+} \sqrt{1/(c+v)} - \lambda_2^{*+} \alpha_+^{(e)*}(\lambda_2^{*+})}{\lambda_2^{*+} \mathcal{Q}_{3+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg}+v) - \lambda_2^{*+}]\mathcal{Q}_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t-t_s^*) \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_2^{*+})\sqrt{1/(c-v)} + \lambda_2^{*+} \sqrt{1/(c+v)} - \lambda_2^{*+} \alpha_+^{(e)*}(\lambda_2^{*+})}{(\lambda_2^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_2^{*+})(1/c_{m1}^* + \lambda_2^{*+})[1/(c_{bg}+v) - \lambda_2^{*+}]\mathcal{Q}_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t-t_s^*) \\
 & + \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+})\sqrt{1/(c-v)} + \lambda_{2i}^{*+} \sqrt{1/(c+v)} - \lambda_{2i}^{*+} \alpha_+^{(e)*}(\lambda_{2i}^{*+})}{\lambda_{2i}^{*+} \mathcal{Q}_{3+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg}+v) - \lambda_{2i}^{*+}]\mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau[H(t-t_{hms}^*) - H(t-t_s^*)] \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)}
 \end{aligned}$$

$$\begin{aligned}
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+}) \sqrt{1/(c-v) + \lambda_{2i}^{*+} \sqrt{1/(c+v) - \lambda_{2i}^{*+} \alpha_+^{(e)*}(\lambda_{2i}^{*+})}}{(\lambda_{2i}^{*+} - d) \mathcal{Q}_{3+}^*(\lambda_{2i}^{*+})(1/c_{m1}^* + \lambda_{2i}^{*+})[1/(c_{bg} + v) - \lambda_{2i}^{*+}] \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 k_e^2 (1/c_{m1}^*) \mathcal{Q}_{3+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{\lambda_3^{*+} \mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 k_e^2 (1/c_{m1}^* + d) \mathcal{Q}_{3+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{(\lambda_3^{*+} - d) \mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & - \tau_0 H(t - by) H(-\cos \theta), \tag{91}
 \end{aligned}$$

$$\begin{aligned}
 D_\xi(\xi, y, t) &= \frac{-\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^*) \mathcal{Q}_{3+}^*(0)}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{\mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^* + d) \mathcal{Q}_{3+}^*(d)}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} \sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{(\lambda_3^{*+} - d) \mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*), \tag{92}
 \end{aligned}$$

$$\begin{aligned}
 D_y(\xi, y, t) &= \frac{\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^*) \mathcal{Q}_{3+}^*(0)}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{\lambda_3^{*+} \mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{-\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^* + d) \mathcal{Q}_{3+}^*(d)}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2 - k_e^2}) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c-v) + \lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+})}}{(\lambda_3^{*+} - d) \mathcal{Q}_{3+}^*(\lambda_3^{*+})(1/c_{m1}^* + \lambda_3^{*+})[1/(c_{bg} + v) - \lambda_3^{*+}] \mathcal{Q}_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*), \tag{93}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{\xi z}^{(e)}(\xi, y, t) = & \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_1^{*+}]\mathcal{Q}_{1+}^*(\lambda_1^{*+})}{\mathcal{Q}_{3+}^*(\lambda_1^{*+})(1/c_{m1}^* + \lambda_1^{*+})\alpha_-^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_1^{*+}]\mathcal{Q}_{1+}^*(\lambda_1^{*+})}{(\lambda_1^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_1^{*+})(1/c_{m1}^* + \lambda_1^{*+})\alpha_-^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}]\mathcal{Q}_{1+}^*(\lambda_{1i}^{*+})}{\mathcal{Q}_{3+}^*(\lambda_{1i}^{*+})(1/c_{m1}^* + \lambda_{1i}^{*+})\alpha_-^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta) \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}]\mathcal{Q}_{1+}^*(\lambda_{1i}^{*+})}{(\lambda_{1i}^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_{1i}^{*+})(1/c_{m1}^* + \lambda_{1i}^{*+})\alpha_-^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta), \tag{94}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}^{(e)}(\xi, y, t) = & \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_1^{*+})[1/(c_{bg}-v) + \lambda_1^{*+}]\mathcal{Q}_{1+}^*(\lambda_1^{*+})}{\lambda_1^{*+}\mathcal{Q}_{3+}^*(\lambda_1^{*+})(1/c_{m1}^* + \lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_1^{*+})[1/(c_{bg}-v) + \lambda_1^{*+}]\mathcal{Q}_{1+}^*(\lambda_1^{*+})}{(\lambda_1^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_1^{*+})(1/c_{m1}^* + \lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\pi\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{1i}^{*+})[1/(c_{bg}-v) + \lambda_{1i}^{*+}]\mathcal{Q}_{1+}^*(\lambda_{1i}^{*+})}{\lambda_{1i}^{*+}\mathcal{Q}_{3+}^*(\lambda_{1i}^{*+})(1/c_{m1}^* + \lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta) \\
 & + \frac{-\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\pi\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{1i}^{*+})[1/(c_{bg}-v) + \lambda_{1i}^{*+}]\mathcal{Q}_{1+}^*(\lambda_{1i}^{*+})}{(\lambda_{1i}^{*+} - d)\mathcal{Q}_{3+}^*(\lambda_{1i}^{*+})(1/c_{m1}^* + \lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta) \\
 & - \tau_0 H(t + b^{(e)}y)H(-\cos \theta), \tag{95}
 \end{aligned}$$

The corresponding results of the dynamic stress intensity factor and the dynamic electric displacement intensity factor are

$$\begin{aligned}
 K_{III}^{(\tau)}(t) = & \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} \tau_{yz}(\xi, 0, t) \\
 = & \frac{2\tau_0(1/c_{m1}^*)\mathcal{Q}_{3+}^*(0)}{\alpha_+^{(e)*}(0)[1/(c_{bg}-v)]\mathcal{Q}_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} - \frac{2\tau_0(1/c_{m1}^* + d)\mathcal{Q}_{3+}^*(d)}{\alpha_+^{(e)*}(d)[1/(c_{bg}-v) + d]\mathcal{Q}_{1+}^*(d)} \sqrt{\frac{2t}{\pi}} \tag{96}
 \end{aligned}$$

and

$$\begin{aligned}
 K_{\text{III}}^{(D)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} D_y(\xi, 0, t) \\
 &= \frac{2\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^*) Q_{3+}^*(0)}{e_{15}(\sqrt{1-b^2v^2-k_e^2}) \alpha_+^{(e)*}(0) [1/(c_{bg}-v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} \\
 &\quad - \frac{2\tau_0 \varepsilon_{11} k_e^2 (1/c_{m1}^* + d) Q_{3+}^*(d)}{e_{15}(\sqrt{1-b^2v^2-k_e^2}) \alpha_+^{(e)*}(d) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}},
 \end{aligned} \tag{97}$$

respectively.

If the function  $R(\lambda)$  has no root, there are two cases, i.e.,  $b < b^{(e)}$  and  $b > b^{(e)}$ , will be discussed in detail. The full-field solutions can be obtained in a similar way as indicated in cases (1) and (2). In order to save the space of this manuscript, we only present the results for intensity factors.

Case (3):  $b < b^{(e)}$  (No MT surface wave)

We introduced a new function  $Q_4^*(\lambda)$  as

$$Q_4^*(\lambda) = \frac{\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)}{\alpha^{(e)*}(\lambda)} \frac{\sqrt{1-b^{(e)2}v^2}}{\sqrt{1-b^2v^2} + k\sqrt{1-b^{(e)2}v^2 - k_e^2}}. \tag{98}$$

The function  $Q_4^*(\lambda)$  has the property that  $Q_4^*(\lambda) \rightarrow 1$  as  $|\lambda| \rightarrow \infty$ , and  $Q_4^*(\lambda)$  has neither poles nor zeros in the  $\lambda$ -plane by cuts along  $-1/(c^{(e)} - v) < \lambda < -\varepsilon$  and  $\varepsilon < \lambda < 1/(c^{(e)} + v)$ . Similarly, by using the general product factorization method,  $Q_4^*(\lambda)$  can be further decomposed as the product of two regular functions  $Q_{4+}^*(\lambda)$  and  $Q_{4-}^*(\lambda)$ , where

$$\begin{aligned}
 Q_{4+}^*(\lambda) &= \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c-v}}^{\frac{1}{c^{(e)}-v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2-\varepsilon^2} - \sqrt{(1-b^2v^2)z^2 - 2vb^2z - b^2}}{k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z+\lambda} \right\} \\
 &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c-v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2-\varepsilon^2}}{\alpha^*(-z) + k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z+\lambda} \right\}
 \end{aligned} \tag{99}$$

and

$$\begin{aligned}
 Q_{4-}^*(\lambda) &= \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c+v}}^{\frac{1}{c^{(e)}+v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2-\varepsilon^2} - \sqrt{(1-b^2v^2)z^2 + 2vb^2z - b^2}}{k\alpha^{(e)*}(z)} \right\} \frac{dz}{z-\lambda} \right\} \\
 &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c+v}} \tan^{-1} \left\{ \frac{k_e^2\sqrt{z^2-\varepsilon^2}}{\alpha^*(z) + k\alpha^{(e)*}(z)} \right\} \frac{dz}{z-\lambda} \right\}.
 \end{aligned} \tag{100}$$

Eq. (42) can be rewritten as

$$\frac{[1/(c_{bg} + v) - \lambda]}{\sqrt{1/(c + v) - \lambda}} \frac{Q_{1-}^*(\lambda)}{Q_{4-}^*(\lambda)} \tilde{A}_-(\lambda, s) = \hat{k} \frac{Q_{4+}^*(\lambda)}{M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \bar{\tau}_+^*(\lambda, s) \right], \tag{101}$$

where

$$\hat{k} = \frac{\sqrt{1-b^2v^2} + k\sqrt{1-b^{(e)2}v^2 - k_e^2}}{\mu^{(e)}\sqrt{1-b^{(e)2}v^2}(\sqrt{1-b^2v^2 - k_e^2})}.$$

The first term on the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ , except for the poles at  $\lambda = 0$  and  $d$ . However, these poles can be removed and (101) is rearranged in the following form

$$\begin{aligned} & \frac{[1/(c_{bg} + v) - \lambda] Q_{1-}^*(\lambda)}{\sqrt{1/(c + v) - \lambda} Q_{4-}^*(\lambda)} \tilde{A}_-^*(\lambda, s) - \hat{k} \frac{Q_{4+}^*(0)}{M_{1+}^*(0)} \frac{\tau_0}{s^3 \lambda} - \hat{k} \frac{Q_{4+}^*(d)}{M_{1+}^*(d)} \frac{-\tau_0}{s^3(\lambda - d)} \\ &= \hat{k} \frac{Q_{4+}^*(\lambda)}{M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \tau_+^*(\lambda, s) \right] - \hat{k} \frac{Q_{4+}^*(0)}{M_{1+}^*(0)} \frac{\tau_0}{s^3 \lambda} - \hat{k} \frac{Q_{4+}^*(d)}{M_{1+}^*(d)} \frac{-\tau_0}{s^3(\lambda - d)}, \end{aligned} \tag{102}$$

The left-hand side of this equation is regular for  $\text{Re}(\lambda) < \varepsilon$ , while the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ . Applying the analytic continuation argument, therefore, each side of (102) represents a single entire function, say  $E_3^*(\lambda)$ . From Liouville’s theorem, the bounded entire function  $E_3^*(\lambda)$  is a constant. By using of the Abelian theorem,  $E_3^*(\lambda)$  vanishes identically, and we can solve for  $\tilde{A}_-^*$  from the left-hand side of (102). From  $\tilde{A}_-^*(\lambda, s) \equiv A^{(e)}(\lambda, s) - A(\lambda, s)$  and (37), we find

$$\begin{aligned} A(\lambda, s) &= \frac{-\tau_0}{s^3 \lambda} \frac{\sqrt{1/(c - v) + d} Q_{4+}^*(0) \sqrt{1/(c + v) - \lambda}}{\bar{c}_{44}(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] Q_{1+}^*(0) Q_{4+}^*(\lambda) Q_{1-}^*(\lambda) [1/(c_{bg} + v) - \lambda]} + \frac{\tau_0}{s^3(\lambda - d)} \\ &\times \frac{\sqrt{1/(c - v) + d} Q_{4+}^*(d) \sqrt{1/(c + v) - \lambda}}{\bar{c}_{44}(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] Q_{1+}^*(d) Q_{4+}^*(\lambda) Q_{1-}^*(\lambda) [1/(c_{bg} + v) - \lambda]}, \end{aligned} \tag{103}$$

and

$$\begin{aligned} A^{(e)}(\lambda, s) &= \frac{\tau_0}{s^3 \lambda} \frac{\sqrt{1/(c - v)} Q_{4+}^*(0) [1/(c_{bg} - v) + \lambda] Q_{1+}^*(\lambda)}{\mu^{(e)} [1/(c_{bg} - v)] Q_{1+}^*(0) Q_{4+}^*(\lambda) \alpha^{(e)*}(\lambda) \sqrt{1/(c - v) + \lambda}} \\ &- \frac{\tau_0}{s^3(\lambda - d)} \frac{\sqrt{1/(c - v) + d} Q_{4+}^*(d) [1/(c_{bg} - v) + \lambda] Q_{1+}^*(\lambda)}{\mu^{(e)} [1/(c_{bg} - v) + d] Q_{1+}^*(d) Q_{4+}^*(\lambda) \alpha^{(e)*}(\lambda) \sqrt{1/(c - v) + \lambda}}. \end{aligned} \tag{104}$$

$$\begin{aligned} B(\lambda, s) &= \frac{\tau_0}{s^3 \lambda} \frac{e_{15} \sqrt{1/(c - v)} Q_{4+}^*(0) \sqrt{1/(c + v) - \lambda}}{\varepsilon_{11} \bar{c}_{44}(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] Q_{1+}^*(0) Q_{4+}^*(\lambda) Q_{1-}^*(\lambda) [1/(c_{bg} + v) - \lambda]} \\ &+ \frac{-\tau_0}{s^3(\lambda - d)} \frac{e_{15} \sqrt{1/(c - v) + d} Q_{4+}^*(d) \sqrt{1/(c + v) - \lambda}}{\varepsilon_{11} \bar{c}_{44}(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] Q_{1+}^*(d) Q_{4+}^*(\lambda) Q_{1-}^*(\lambda) [1/(c_{bg} + v) - \lambda]}. \end{aligned} \tag{105}$$

The explicit transient solutions in the time domain are

$$\begin{aligned} \tau_{\xi z}(\xi, y, t) &= \frac{-\tau_0 \sqrt{1/(c - v)} Q_{4+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c + v) - \lambda_2^{*+}}}{Q_{4+}^*(\lambda_2^{*+}) Q_{1-}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}]} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\ &\cdot d\tau H(t - t_s^*) \\ &+ \frac{\tau_0 \sqrt{1/(c - v) + d} Q_{4+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\ &\cdot \int_0^t \text{Im} \left[ \frac{\lambda_2^{*+} \sqrt{1/(c + v) - \lambda_2^{*+}}}{(\lambda_2^{*+} - d) Q_{4+}^*(\lambda_2^{*+}) Q_{1-}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}]} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^*) \\ &+ \frac{-\tau_0 \sqrt{1/(c - v)} Q_{4+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\ &\cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c + v) - \lambda_{2i}^{*+}}}{Q_{4+}^*(\lambda_{2i}^{*+}) Q_{1-}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}]} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\ &+ \frac{\tau_0 \sqrt{1/(c - v) + d} Q_{4+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \end{aligned}$$

$$\begin{aligned}
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\lambda_{2i}^{*+} \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{(\lambda_{2i}^{*+} - d) \mathcal{Q}_{4+}^*(\lambda_{2i}^{*+}) \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}]} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{\tau_0 k_e^2 \sqrt{1/(c-v)} \mathcal{Q}_{4+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\sqrt{1/(c+v) - \lambda_3^{*+}}}{\mathcal{Q}_{4+}^*(\lambda_3^{*+}) \mathcal{Q}_{1-}^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1/(c-v) + d} \mathcal{Q}_{4+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) \mathcal{Q}_{4+}^*(\lambda_3^{*+}) \mathcal{Q}_{1-}^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*), \tag{106}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}(\xi, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)} \mathcal{Q}_{4+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\alpha^*(\lambda_2^{*+}) \sqrt{1/(c+v) - \lambda_2^{*+}}}{\lambda_2^{*+} \mathcal{Q}_{4+}^*(\lambda_2^{*+}) \mathcal{Q}_{1-}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}]} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^*) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} \mathcal{Q}_{4+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\alpha^*(\lambda_2^{*+}) \sqrt{1/(c+v) - \lambda_2^{*+}}}{(\lambda_2^{*+} - d) \mathcal{Q}_{4+}^*(\lambda_2^{*+}) \mathcal{Q}_{1-}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}]} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^*) \\
 & + \frac{\tau_0 \sqrt{1/(c-v)} \mathcal{Q}_{4+}^*(0)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v)] \mathcal{Q}_{1+}^*(0)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+}) \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{\lambda_{2i}^{*+} \mathcal{Q}_{4+}^*(\lambda_{2i}^{*+}) \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}]} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} \mathcal{Q}_{4+}^*(d)}{\pi(\sqrt{1 - b^2 v^2 - k_e^2}) [1/(c_{bg} - v) + d] \mathcal{Q}_{1+}^*(d)} \\
 & \cdot \int_0^t \operatorname{Im} \left[ \frac{\alpha^*(\lambda_{2i}^{*+}) \sqrt{1/(c+v) - \lambda_{2i}^{*+}}}{(\lambda_{2i}^{*+} - d) \mathcal{Q}_{4+}^*(\lambda_{2i}^{*+}) \mathcal{Q}_{1-}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}]} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau}
 \end{aligned}$$



$$\begin{aligned}
 & \cdot d\tau[H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c_{bg}-v)]Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c+v) - \lambda_3^{*+}}}{\lambda_3^{*+} Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 k_e^2 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi(\sqrt{1-b^2v^2-k_e^2})[1/(c_{bg}-v) + d]Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & - \tau_0 H(t - by) H(-\cos \theta),
 \end{aligned} \tag{107}$$

$$\begin{aligned}
 D_\xi(\xi, y, t) &= \frac{-\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2-k_e^2}) [1/(c_{bg}-v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\sqrt{1/(c+v) - \lambda_3^{*+}}}{Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2-k_e^2}) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*),
 \end{aligned} \tag{108}$$

$$\begin{aligned}
 D_y(\xi, y, t) &= \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2-k_e^2}) [1/(c_{bg}-v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c+v) - \lambda_3^{*+}}}{\lambda_3^{*+} Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{-\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi \varepsilon_{15} (\sqrt{1-b^2v^2-k_e^2}) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \sqrt{1/(c+v) - \lambda_3^{*+}}}{(\lambda_3^{*+} - d) Q_{4+}^*(\lambda_3^{*+}) Q_{1-}^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}]} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*),
 \end{aligned} \tag{109}$$

$$\begin{aligned}
 \tau_{\xi z}^{(e)}(\xi, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1+}^{*+}] Q_{1+}^*(\lambda_{1+}^{*+})}{Q_{4+}^*(\lambda_{1+}^{*+}) \alpha^{(e)*}(\lambda_{1+}^{*+}) \sqrt{1/(c-v) + \lambda_{1+}^{*+}}} \frac{\partial \lambda_{1+}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{1+}^{*+} [1/(c_{bg}-v) + \lambda_{1+}^{*+}] Q_{1+}^*(\lambda_{1+}^{*+})}{(\lambda_{1+}^{*+} - d) Q_{4+}^*(\lambda_{1+}^{*+}) \alpha^{(e)*}(\lambda_{1+}^{*+}) \sqrt{1/(c-v) + \lambda_{1+}^{*+}}} \frac{\partial \lambda_{1+}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{Q_{4+}^*(\lambda_{1i}^{*+}) \alpha^{(e)*}(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{1i}^{*+} [1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{(\lambda_{1i}^{*+} - d) Q_{4+}^*(\lambda_{1i}^{*+}) \alpha^{(e)*}(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta), \tag{110}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{yz}^{(e)}(\xi, y, t) = & \frac{\tau_0 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1+}^{*+}] Q_{1+}^*(\lambda_{1+}^{*+})}{\lambda_{1+}^{*+} Q_{4+}^*(\lambda_{1+}^{*+}) \sqrt{1/(c-v) + \lambda_{1+}^{*+}}} \frac{\partial \lambda_{1+}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1+}^{*+}] Q_{1+}^*(\lambda_{1+}^{*+})}{(\lambda_{1+}^{*+} - d) Q_{4+}^*(\lambda_{1+}^{*+}) \sqrt{1/(c-v) + \lambda_{1+}^{*+}}} \frac{\partial \lambda_{1+}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1/(c-v)} Q_{4+}^*(0)}{\pi [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{\lambda_{1i}^{*+} Q_{4+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] \\
 & + \frac{-\tau_0 \sqrt{1/(c-v) + d} Q_{4+}^*(d)}{\pi [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{(\lambda_{1i}^{*+} - d) Q_{4+}^*(\lambda_{1i}^{*+}) \sqrt{1/(c-v) + \lambda_{1i}^{*+}}} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] \\
 & - \tau_0 H(t + b^{(e)}y) H(-\cos \theta), \tag{111}
 \end{aligned}$$

The corresponding results of the dynamic stress intensity factor and the dynamic electric displacement intensity factor are

$$K_{III}^{(\tau)}(t) = \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} \tau_{yz}(\xi, 0, t) = \frac{2\tau_0 \sqrt{1/(c-v)} Q_{4+}^*(0)}{[1/(c_{bg}-v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} - \frac{2\tau_0 \sqrt{1/(c-v)+d} Q_{4+}^*(d)}{[1/(c_{bg}-v)+d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}} \tag{112}$$

and

$$\begin{aligned} K_{III}^{(D)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} D_y(\xi, 0, t) \\ &= \frac{2\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)} Q_{4+}^*(0)}{e_{15}(\sqrt{1-b^2v^2-k_e^2})[1/(c_{bg}-v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} \\ &\quad - \frac{2\tau_0 \varepsilon_{11} k_e^2 \sqrt{1/(c-v)+d} Q_{4+}^*(d)}{e_{15}(\sqrt{1-b^2v^2-k_e^2})[1/(c_{bg}-v)+d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}}, \end{aligned} \tag{113}$$

respectively.

Case (4):  $b > b^{(e)}$  (No MT surface wave)

If  $R(\lambda)$  has no root, a new function  $Q_5^*(\lambda)$  is defined as

$$Q_5^*(\lambda) = \frac{\alpha^*(\lambda) + k\alpha^{(e)*}(\lambda) - k_e^2\beta^*(\lambda)}{\alpha^*(\lambda)} \frac{\sqrt{1-b^2v^2}}{\sqrt{1-b^2v^2} + k\sqrt{1-b^{(e)2}v^2 - k_e^2}}. \tag{114}$$

The function  $Q_5^*(\lambda)$  has the property that  $Q_5^*(\lambda) \rightarrow 1$  as  $|\lambda| \rightarrow \infty$ , and  $Q_5^*(\lambda)$  has neither poles nor zeros in the  $\lambda$ -plane by cuts along  $-1/(c-v) < \lambda < -\varepsilon$  and  $\varepsilon < \lambda < 1/(c+v)$ . The function  $Q_5^*(\lambda)$  can be decomposed as the product of two regular functions  $Q_{5+}^*(\lambda)$  and  $Q_{5-}^*(\lambda)$ , where

$$\begin{aligned} Q_{5+}^*(\lambda) &= \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c^{(e)}-v}}^{\frac{1}{c-v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2} - k \sqrt{(1-v^2b^{(e)2})z^2 - 2vb^{(e)2}z - b^{(e)2}}}{\alpha^*(-z)} \right\} \frac{dz}{z + \lambda} \right\} \\ &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c^{(e)}-v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2}}{\alpha^*(-z) + k\alpha^{(e)*}(-z)} \right\} \frac{dz}{z + \lambda} \right\} \end{aligned} \tag{115}$$

and

$$\begin{aligned} Q_{5-}^*(\lambda) &= \exp \left\{ \frac{1}{\pi} \int_{\frac{1}{c^{(e)}+v}}^{\frac{1}{c+v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2} - k \sqrt{(1-v^2b^{(e)2})z^2 + 2vb^{(e)2}z - b^{(e)2}}}{\alpha^*(z)} \right\} \frac{dz}{z - \lambda} \right\} \\ &\quad \cdot \exp \left\{ \frac{1}{\pi} \int_{\varepsilon}^{\frac{1}{c^{(e)}+v}} \tan^{-1} \left\{ \frac{k_e^2 \sqrt{z^2 - \varepsilon^2}}{\alpha^*(z) + k\alpha^{(e)*}(z)} \right\} \frac{dz}{z - \lambda} \right\}. \end{aligned} \tag{116}$$

Eq. (42) is rewritten as

$$\begin{aligned} \frac{\alpha^{(e)*}(\lambda)[1/(c_{bg}+v) - \lambda]}{[1/(c+v) - \lambda]} \frac{Q_{1-}^*(\lambda)}{Q_{5-}^*(\lambda)} \tilde{A}_-^*(\lambda, s) &= \tilde{k} \frac{\sqrt{1/(c-v) + \lambda}}{\alpha_+^{(e)*}(\lambda)} \\ &\quad \times \frac{Q_{5+}^*(\lambda)}{M_{1+}^*(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda-d} \right) + \frac{1}{s} \bar{\tau}_+^*(\lambda, s) \right], \end{aligned} \tag{117}$$

where

$$\tilde{k} = \frac{\sqrt{1-b^2v^2} + k\sqrt{1-b^{(e)2}v^2 - k_e^2}}{\mu^{(e)}(\sqrt{1-b^2v^2 - k_e^2})}.$$

The first term on the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ , except for the poles at  $\lambda = 0$  and  $d$ . However, these poles can be removed and (117) is rearranged in the following form

$$\begin{aligned} & \frac{\alpha_{-}^{(e)*}(\lambda)[1/(c_{bg} + v) - \lambda]}{[1/(c + v) - \lambda]} \frac{Q_{1-}^{*}(\lambda)}{Q_{5-}^{*}(\lambda)} \tilde{A}_{-}^{*}(\lambda, s) - \tilde{k} \frac{Q_{5+}^{*}(0)\sqrt{1/(c - v)}}{\alpha_{+}^{(e)*}(0)M_{1+}^{*}(0)} \frac{\tau_0}{s^3\lambda} - \tilde{k} \frac{Q_{5+}^{*}(d)\sqrt{1/(c - v) + d}}{\alpha_{+}^{(e)*}(d)M_{1+}^{*}(d)} \\ & \times \frac{-\tau_0}{s^3(\lambda - d)} \\ & = \tilde{k} \frac{Q_{5+}^{*}(\lambda)\sqrt{1/(c - v) + \lambda}}{\alpha_{+}^{(e)*}(\lambda)M_{1+}^{*}(\lambda)} \left[ \frac{-\tau_0}{s^3} \left( \frac{-1}{\lambda} + \frac{1}{\lambda - d} \right) + \frac{1}{s} \bar{\tau}_{+}^{*}(\lambda, s) \right] - \tilde{k} \frac{Q_{5+}^{*}(0)\sqrt{1/(c - v)}}{\alpha_{+}^{(e)*}(0)M_{1+}^{*}(0)} \frac{\tau_0}{s^3\lambda} \\ & - \tilde{k} \frac{Q_{5+}^{*}(d)\sqrt{1/(c - v) + d}}{\alpha_{+}^{(e)*}(d)M_{1+}^{*}(d)} \frac{-\tau_0}{s^3(\lambda - d)}, \end{aligned} \tag{118}$$

The left-hand side of this equation is regular for  $\text{Re}(\lambda) < \varepsilon$ , while the right-hand side is regular for  $\text{Re}(\lambda) > -\varepsilon$ . Applying the analytic continuation argument, therefore, each side of (118) represents a single entire function, say  $E_4^*(\lambda)$ . From Liouville’s theorem, the bounded entire function  $E_4^*(\lambda)$  is a constant and equals to zero. The unknown  $\tilde{A}_{-}^{*}$  can be obtained from the left-hand side of (118). From  $\tilde{A}_{-}^{*}(\lambda, s) \equiv A^{(e)}(\lambda, s) - A(\lambda, s)$  and (37), we find

$$\begin{aligned} A(\lambda, s) &= \frac{-\tau_0}{s^3\lambda} \frac{\sqrt{1 - b^2v^2}Q_{5+}^{*}(0)[1/(c - v)]\alpha_{+}^{(e)*}(\lambda)[1/(c + v) - \lambda]}{\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_{+}^{(e)*}(0)[1/(c_{bg} - v)]Q_{1+}^{*}(0)Q_{5+}^{*}(\lambda)\alpha^{*}(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^{*}(\lambda)} \\ & + \frac{\tau_0}{s^3(\lambda - d)} \\ & \times \frac{\sqrt{1 - b^2v^2}Q_{5+}^{*}(d)[1/(c - v) + d]\alpha_{+}^{(e)*}(\lambda)[1/(c + v) - \lambda]}{\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})\alpha_{+}^{(e)*}(d)[1/(c_{bg} - v) + d]Q_{1+}^{*}(d)Q_{5+}^{*}(\lambda)\alpha^{*}(\lambda)[1/(c_{bg} + v) - \lambda]Q_{1-}^{*}(\lambda)} \end{aligned} \tag{119}$$

and

$$\begin{aligned} A^{(e)}(\lambda, s) &= \frac{\tau_0}{s^3\lambda} \frac{\sqrt{1 - b^2v^2}Q_{5+}^{*}(0)[1/(c - v)][1/(c_{bg} - v) + \lambda]Q_{1+}^{*}(\lambda)}{\mu^{(e)}\alpha_{+}^{(e)*}(0)[1/(c_{bg} - v)]Q_{1+}^{*}(0)Q_{5+}^{*}(\lambda)[1/(c - v) + \lambda]\alpha_{-}^{(e)*}(\lambda)} \\ & - \frac{\tau_0}{s^3(\lambda - d)} \frac{\sqrt{1 - b^2v^2}Q_{5+}^{*}(d)[1/(c - v) + d][1/(c_{bg} - v) + \lambda]Q_{1+}^{*}(\lambda)}{\mu^{(e)}\alpha_{+}^{(e)*}(d)[1/(c_{bg} - v) + d]Q_{1+}^{*}(d)Q_{5+}^{*}(\lambda)[1/(c - v) + \lambda]\alpha_{-}^{(e)*}(\lambda)}. \end{aligned} \tag{120}$$

$$\begin{aligned} B(\lambda, s) &= \frac{\tau_0}{s^3\lambda} \frac{e_{15}\sqrt{1 - b^2v^2}\alpha_{+}^{(e)*}(\lambda)Q_{5+}^{*}(0)[1/(c - v)][1/(c + v) - \lambda]}{\varepsilon_{11}\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})Q_{5+}^{*}(\lambda)\alpha^{*}(\lambda)\alpha_{+}^{(e)*}(0)[1/(c_{bg} - v)]Q_{1+}^{*}(0)[1/(c_{bg} + v) - \lambda]Q_{1-}^{*}(\lambda)} \\ & + \frac{-\tau_0}{s^3(\lambda - d)} \frac{e_{15}\sqrt{1 - b^2v^2}\alpha_{+}^{(e)*}(\lambda)Q_{5+}^{*}(d)[1/(c - v) + d][1/(c + v) - \lambda]}{\varepsilon_{11}\bar{c}_{44}(\sqrt{1 - b^2v^2 - k_e^2})Q_{5+}^{*}(\lambda)\alpha^{*}(\lambda)\alpha_{+}^{(e)*}(d)[1/(c_{bg} - v) + d]Q_{1+}^{*}(d)[1/(c_{bg} + v) - \lambda]Q_{1-}^{*}(\lambda)}. \end{aligned} \tag{121}$$

The transient solutions of the field quantities are

$$\begin{aligned}
 \tau_{\xi z}(\xi, y, t) = & \frac{-\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_2^{*+}) [1/(c + v) - \lambda_2^{*+}]}{Q_{5+}^*(\lambda_2^{*+}) \alpha^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}] Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^*) \\
 & + \frac{\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_2^{*+} \alpha_+^{(e)*}(\lambda_2^{*+}) [1/(c + v) - \lambda_2^{*+}]}{(\lambda_2^{*+} - d) Q_{5+}^*(\lambda_2^{*+}) \alpha^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}] Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^*) \\
 & + \frac{-\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{2i}^{*+}) [1/(c + v) - \lambda_{2i}^{*+}]}{Q_{5+}^*(\lambda_{2i}^{*+}) \alpha^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{2i}^{*+} \alpha_+^{(e)*}(\lambda_{2i}^{*+}) [1/(c + v) - \lambda_{2i}^{*+}]}{(\lambda_{2i}^{*+} - d) Q_{5+}^*(\lambda_{2i}^{*+}) \alpha^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{\tau_0 k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*) \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{(\lambda_3^{*+} - d) Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*), \tag{122}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{\eta z}(\xi, y, t) = & \frac{\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_2^{*+}) [1/(c + v) - \lambda_2^{*+}]}{\lambda_2^{*+} Q_{5+}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}] Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^*) \\
 & + \frac{-\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_2^{*+}) [1/(c + v) - \lambda_2^{*+}]}{(\lambda_2^{*+} - d) Q_{5+}^*(\lambda_2^{*+}) [1/(c_{bg} + v) - \lambda_2^{*+}] Q_{1-}^*(\lambda_2^{*+})} \frac{\partial \lambda_2^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_s^*)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{2i}^{*+}) [1/(c + v) - \lambda_{2i}^{*+}]}{\lambda_{2i}^{*+} Q_{5+}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{2i}^{*+}) [1/(c + v) - \lambda_{2i}^{*+}]}{(\lambda_{2i}^{*+} - d) Q_{5+}^*(\lambda_{2i}^{*+}) [1/(c_{bg} + v) - \lambda_{2i}^{*+}] Q_{1-}^*(\lambda_{2i}^{*+})} \frac{\partial \lambda_{2i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^*) - H(t - t_s^*)] \\
 & + \frac{-\tau_0 k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{\lambda_3^{*+} Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{(\lambda_3^{*+} - d) Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & - \tau_0 H(t - by) H(-\cos \theta),
 \end{aligned} \tag{123}$$

$$\begin{aligned}
 D_\xi(\xi, y, t) & = \frac{-\tau_0 \varepsilon_{11} k_e^2 Q_{5+}^*(0) [1/(c - v)]}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*) \\
 & + \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_3^{*+} \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{(\lambda_3^{*+} - d) Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*),
 \end{aligned} \tag{124}$$

$$\begin{aligned}
 D_y(\xi, y, t) & = \frac{\tau_0 \varepsilon_{11} k_e^2 \sqrt{1 - b^2 v^2} Q_{5+}^*(0) [1/(c - v)]}{\pi \varepsilon_{15} (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c + v) - \lambda_3^{*+}]}{\lambda_3^{*+} Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg} + v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} d\tau H(t - t_m^*)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-\tau_0 \varepsilon_{11} k_c^2 \sqrt{1-b^2 v^2} Q_{5+}^*(d) [1/(c-v) + d]}{\pi \varepsilon_{15} (\sqrt{1-b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\beta^*(\lambda_3^{*+}) \alpha_+^{(e)*}(\lambda_3^{*+}) [1/(c+v) - \lambda_3^{*+}]}{(\lambda_3^{*+} - d) Q_{5+}^*(\lambda_3^{*+}) \alpha^*(\lambda_3^{*+}) [1/(c_{bg}+v) - \lambda_3^{*+}] Q_{1-}^*(\lambda_3^{*+})} \frac{\partial \lambda_3^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_m^*),
 \end{aligned} \tag{125}$$

$$\begin{aligned}
 \tau_{\zeta z}^{(e)}(\zeta, y, t) = & \frac{\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(0) [1/(c-v)]}{\pi \alpha_+^{(e)*}(0) [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+})}{Q_{5+}^*(\lambda_1^{*+}) [1/(c-v) + \lambda_1^{*+}] \alpha_-^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(d) [1/(c-v) + d]}{\pi \alpha_+^{(e)*}(d) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_1^{*+} [1/(c_{bg}-v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+})}{(\lambda_1^{*+} - d) Q_{5+}^*(\lambda_1^{*+}) [1/(c-v) + \lambda_1^{*+}] \alpha_-^{(e)*}(\lambda_1^{*+})} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(0) [1/(c-v)]}{\pi \alpha_+^{(e)*}(0) [1/(c_{bg}-v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{[1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{Q_{5+}^*(\lambda_{1i}^{*+}) [1/(c-v) + \lambda_{1i}^{*+}] \alpha_-^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta) \\
 & + \frac{-\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(d) [1/(c-v) + d]}{\pi \alpha_+^{(e)*}(d) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\lambda_{1i}^{*+} [1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{(\lambda_{1i}^{*+} - d) Q_{5+}^*(\lambda_{1i}^{*+}) [1/(c-v) + \lambda_{1i}^{*+}] \alpha_-^{(e)*}(\lambda_{1i}^{*+})} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})] H(\cos \theta),
 \end{aligned} \tag{126}$$

$$\begin{aligned}
 \tau_{yz}^{(e)}(\zeta, y, t) = & \frac{\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(0) [1/(c-v)]}{\pi \alpha_+^{(e)*}(0) [1/(c_{bg}-v)] Q_{1+}^*(0)} \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_1^{*+}) [1/(c_{bg}-v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+})}{\lambda_1^{*+} Q_{5+}^*(\lambda_1^{*+}) [1/(c-v) + \lambda_1^{*+}]} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{-\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(d) [1/(c-v) + d]}{\pi \alpha_+^{(e)*}(d) [1/(c_{bg}-v) + d] Q_{1+}^*(d)} \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_1^{*+}) [1/(c_{bg}-v) + \lambda_1^{*+}] Q_{1+}^*(\lambda_1^{*+})}{(\lambda_1^{*+} - d) Q_{5+}^*(\lambda_1^{*+}) [1/(c-v) + \lambda_1^{*+}]} \frac{\partial \lambda_1^{*+}}{\partial t} \right]_{t=\tau} \\
 & \cdot d\tau H(t - t_s^{(e)*}) \\
 & + \frac{\tau_0 \sqrt{1-b^2 v^2} Q_{5+}^*(0) [1/(c-v)]}{\pi \alpha_+^{(e)*}(0) [1/(c_{bg}-v)] Q_{1+}^*(0)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{1i}^{*+}) [1/(c_{bg}-v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+})}{\lambda_{1i}^{*+} Q_{5+}^*(\lambda_{1i}^{*+}) [1/(c-v) + \lambda_{1i}^{*+}]} \frac{\partial \lambda_{1i}^{*+}}{\partial t} \right]_{t=\tau}
 \end{aligned}$$

$$\begin{aligned}
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta) \\
 & + \frac{-\tau_0 \sqrt{1 - b^2 v^2} Q_{5+}^*(d) [1/(c - v) + d]}{\pi \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \\
 & \cdot \int_0^t \text{Im} \left[ \frac{\alpha_+^{(e)*}(\lambda_{1i}^{*+}) [1/(c_{bg} - v) + \lambda_{1i}^{*+}] Q_{1+}^*(\lambda_{1i}^{*+}) \partial \lambda_{1i}^{*+}}{(\lambda_{1i}^{*+} - d) Q_{5+}^*(\lambda_{1i}^{*+}) [1/(c - v) + \lambda_{1i}^{*+}] \partial t} \right]_{t=\tau} \\
 & \cdot d\tau [H(t - t_{hms}^{(e)*}) - H(t - t_s^{(e)*})]H(\cos \theta) \\
 & - \tau_0 H(t + b^{(e)}y)H(-\cos \theta),
 \end{aligned} \tag{127}$$

The corresponding results of the dynamic stress intensity factor and the dynamic electric displacement intensity are

$$\begin{aligned}
 K_{III}^{(\tau)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} \tau_{yz}(\xi, 0, t) \\
 &= \frac{2\tau_0 Q_{5+}^*(0) [1/(c - v)]}{\alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} - \frac{2\tau_0 Q_{5+}^*(d) [1/(c - v) + d]}{\alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}}
 \end{aligned} \tag{128}$$

and

$$\begin{aligned}
 K_{III}^{(D)}(t) &= \lim_{\xi \rightarrow 0} \sqrt{2\pi\xi} D_y(\xi, 0, t) \\
 &= \frac{2\tau_0 \varepsilon_{11} k_e^2 Q_{5+}^*(0) [1/(c - v)]}{\varepsilon_{15} (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(0) [1/(c_{bg} - v)] Q_{1+}^*(0)} \sqrt{\frac{2t}{\pi}} \\
 &\quad - \frac{2\tau_0 \varepsilon_{11} k_e^2 Q_{5+}^*(d) [1/(c - v) + d]}{\varepsilon_{15} (\sqrt{1 - b^2 v^2} - k_e^2) \alpha_+^{(e)*}(d) [1/(c_{bg} - v) + d] Q_{1+}^*(d)} \sqrt{\frac{2t}{\pi}},
 \end{aligned} \tag{129}$$

respectively.

### 3. Numerical results

With the explicit transient solution obtained in the previous section, the transient full-field response will be investigated and discussed in detail in this section. The piezoelectric material to be considered in numerical calculations is PZT4. The material constants of PZT4 and a number of elastic materials are listed in Tables 1 and 2, respectively. Table 2 also lists the MT surface wave velocities for PZT4-Aluminum alloy and PZT4-Copper bi-materials. It is noted that the material combinations for PZT4 and elastic materials indicated in Table 2 correspond to four cases analyzed in the previous section.

Table 1  
The properties of PZT4 piezoelectric material

Piezoelectric material	$c_{44}$ (N m <sup>-2</sup> )	$e_{15}$ (C m <sup>-1</sup> )	$\varepsilon_{11}$ (F m <sup>-1</sup> )	$\rho$ (kg m <sup>-3</sup> )	$c$ (m s <sup>-1</sup> )	$c_{bg}$ (m s <sup>-1</sup> )
PZT4	$2.56 \times 10^{10}$	12.70	$6.46 \times 10^{-9}$	7500	2596.26	2257.92

Table 2  
The properties of elastic materials and the MT surface wave velocities

Elastic materials	$\mu$ (N m <sup>-2</sup> )	$\rho^{(e)}$ (kg m <sup>-3</sup> )	$c^{(e)}$ (m s <sup>-1</sup> )	$c_{mt}$ (m s <sup>-1</sup> )
Aluminum alloy	$2.8 \times 10^{10}$	2800	3162.28	2559.25
Brass	$4.1 \times 10^{10}$	8400	2209.29	–
Copper	$4.7 \times 10^{10}$	8900	2298.02	2297.00
Steel	$7.5 \times 10^{10}$	7850	3090.98	–



We consider a piezoelectric–elastic bi-material containing a semi-infinite crack that lies along the negative  $\xi$ -axis and propagates with a constant velocity  $v$  along the crack tip line after the uniformly distributed loading is applied at time  $t = 0$ . It is assumed that the propagating crack surfaces are perfectly covered with an infinitesimally thin conducting electrode that is grounded, such that the electrostatic potential vanishes over the entire crack surfaces. The geometrical configuration of the problem is depicted in Fig. 1. For different slowness combinations, one has faster shear wave velocity in piezoelectric material and the pattern of wave-fronts for the problem is shown in Fig. 7. The other has faster shear wave velocity in elastic material and the pattern of wave-fronts is shown in Fig. 8. The waves shown in Figs. 7 and 8 are composed of incident wave, diffracted wave, head wave and plane wave. There are four cases for different slowness combinations and MT surface wave generation will be considered in numerical calculations. Figs. 9 and 10 represent the shear stress versus normalized time for different values of crack speed  $v$  ( $v = 0$  correspond to the stationary crack) for PZT4-Copper bi-materials (i.e., case (1)). Figs. 11 and 12 represent the result of case (2) for PZT4-Aluminum alloy bi-

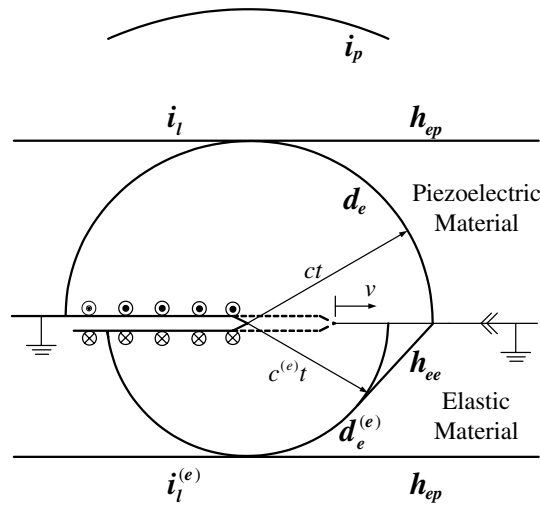


Fig. 7. The pattern of wave-fronts for  $b^{(e)} > b$  subjected to uniformly distributed dynamic anti-plane loadings on the crack faces.

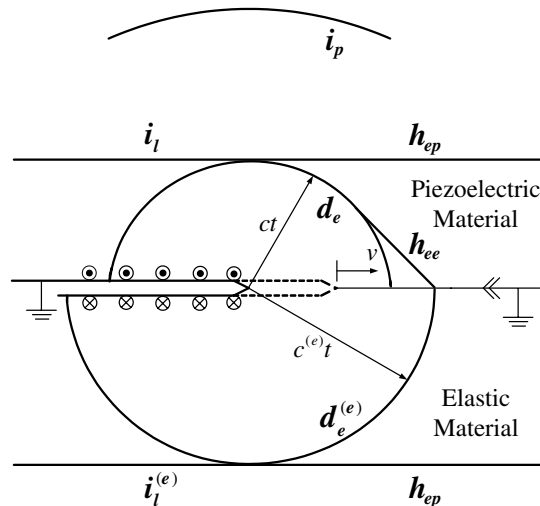


Fig. 8. The pattern of wave-fronts for  $b > b^{(e)}$  subjected to uniformly distributed dynamic anti-plane loadings on the crack faces.

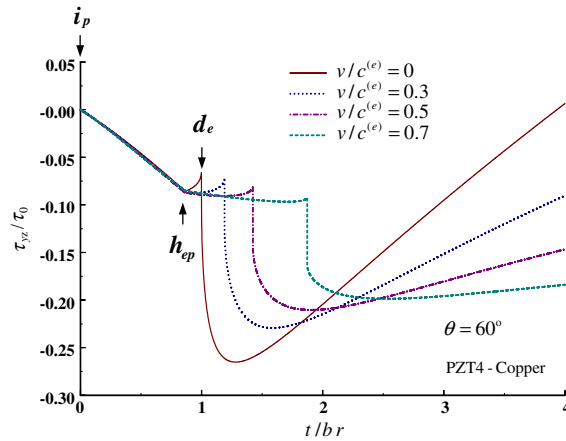


Fig. 9. Normalized piezoelectric shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Copper bi-materials.

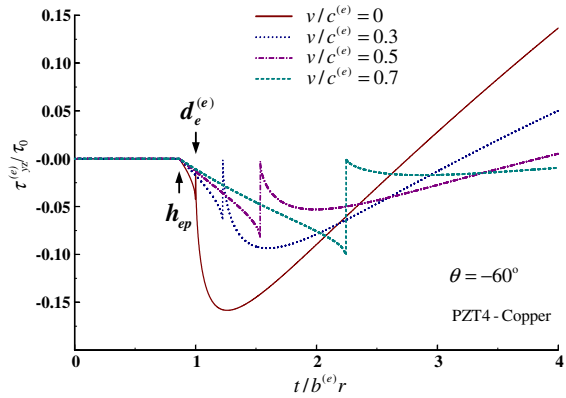


Fig. 10. Normalized elastic shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Copper bi-materials.

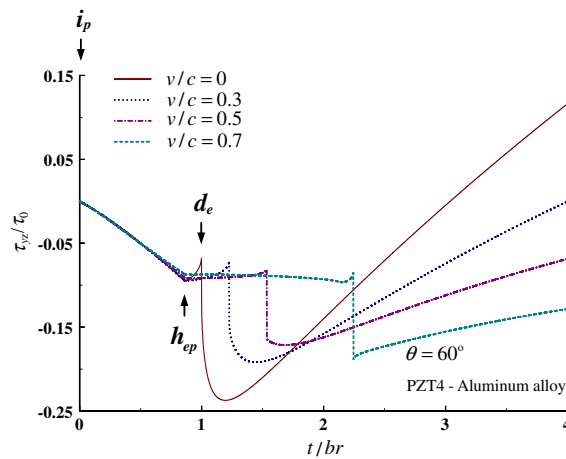


Fig. 11. Normalized piezoelectric shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Aluminum bi-materials.

materials. Figs. 13 and 14 represent the result of case (3) for PZT4-Brass bi-materials. Figs. 15 and 16 represent the result of case (4) for PZT4-Steel bi-materials. The arrival time for each wave front for the stationary

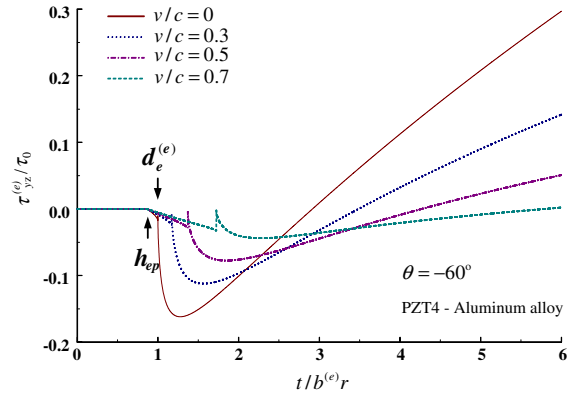


Fig. 12. Normalized elastic shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Aluminum bi-materials.

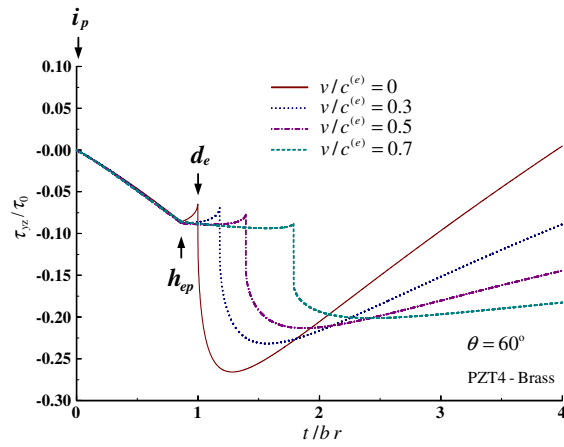


Fig. 13. Normalized piezoelectric shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Brass bi-materials.

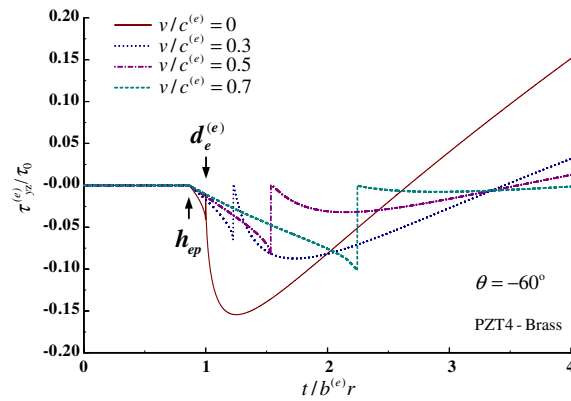


Fig. 14. Normalized elastic shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Brass bi-materials.

crack (i.e.,  $v = 0$ ) is indicated in these figures. The transient responses of material points to be evaluated are located at the  $60^\circ$  from the moving crack tip.

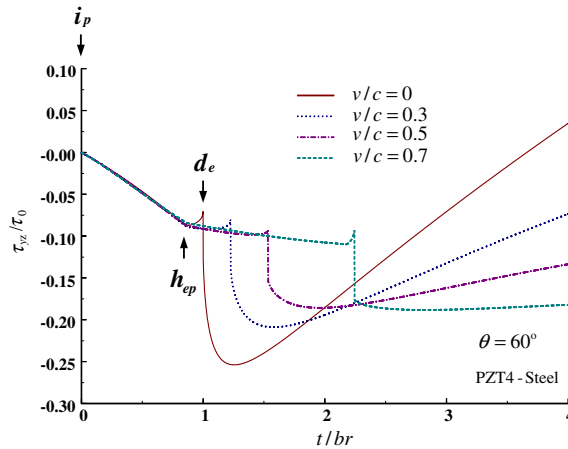


Fig. 15. Normalized piezoelectric shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Steel bi-materials.

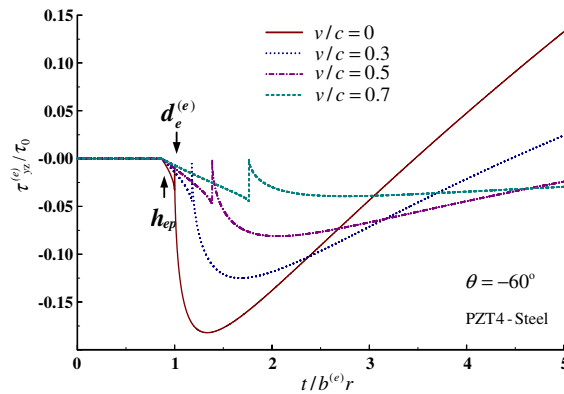


Fig. 16. Normalized elastic shear stress versus normalized time for different values of crack speed  $v$  in PZT4-Steel bi-materials.

The  $i_p$  wave, which propagates with the electromagnetic wave velocity, is the first wave arrival at the material point at the normalized time equal to zero for piezoelectric material. In general, the transient response of shear stress (Figs. 9, 11, 13, and 15) for piezoelectric material with negative value increases after the electromagnetic  $i_p$  wave arrives and it decreases after the head wave arrives. Note that the wave front of the head wave (i.e., the  $h_{ep}$  wave in Figs. 7 and 8) is represented by a horizontal straight line which is induced by the electromagnetic wave and the piezoelectric shear wave. Finally, the diffracted  $d_e$  wave which is generated from the crack tip arrives at the material point at the normalized time equal to 1. For  $[1/(c - v)]|\cos \theta| > 1/(c^{(e)} - v)$ , the head wave (i.e., the  $h_{ee}$  wave in Fig. 8) will be generated in the piezoelectric medium and the elastic shear wave and its wave front is an inclined straight line. In addition, the shear stress in the piezoelectric medium is small when the magnitude of the crack propagation velocity  $v$  is large.

The transient response of shear stress (Figs. 10, 12, 14, and 16) in the elastic medium keeps zero before the head wave (i.e., the  $h_{ep}$  wave in Figs. 7 and 8) arrives and this wave front is represented by a horizontal straight line induced by the electromagnetic wave and the piezoelectric shear wave. The shear stress has negative value after the arrival of the head wave  $h_{ep}$ . For  $[1/(c^{(e)} - v)]|\cos \theta| > 1/(c - v)$ , the head wave (i.e., the  $h_{ee}$  wave in Fig. 7) will be generated in the elastic medium and its wave front is an inclined straight line. Furthermore, the shear stress in the elastic medium is small when the magnitude of the crack propagation velocity  $v$  is large. Figs. 17–20 represent the normalized dynamic stress intensity factors versus normalized crack speed for four cases. It shows clearly that the dynamic stress intensity factor is a positive value and decreases with increase crack propagation velocity.

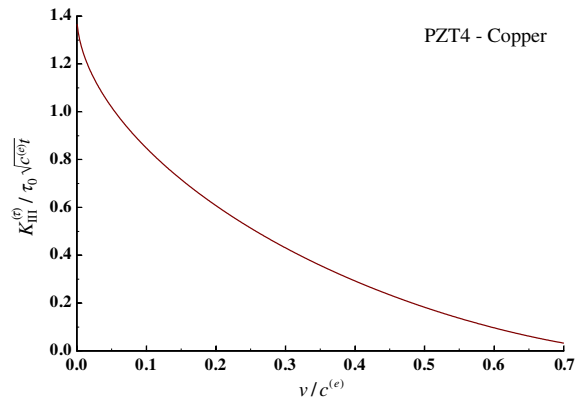


Fig. 17. Normalized dynamic stress intensity factor versus normalized crack speed in PZT4-Copper bi-materials.

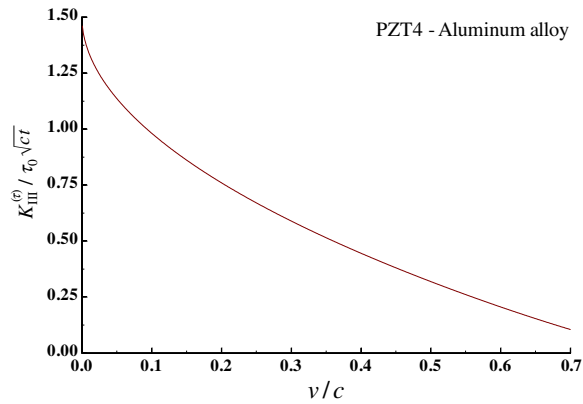


Fig. 18. Normalized dynamic stress intensity factor versus normalized crack speed in PZT4-Aluminum bi-materials.

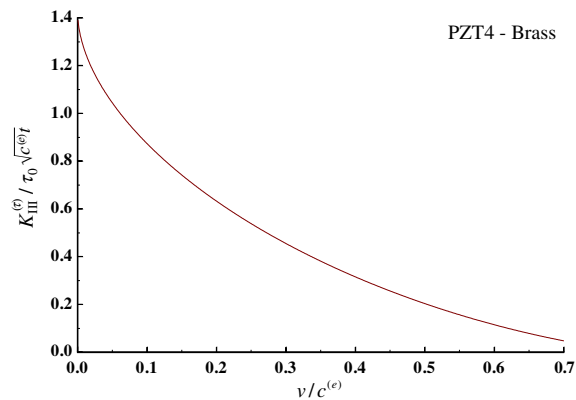


Fig. 19. Normalized dynamic stress intensity factor versus normalized crack speed in PZT4-Brass bi-materials.

#### 4. Conclusions

The phenomena of crack propagation, arrest and branching are important subjects in the areas of dynamic fracture analysis, it is very important to have the analytical results for these problems. The problem of a propagating crack subjected to uniformly distributed dynamic anti-plane loadings on the crack faces in elastic-pie-

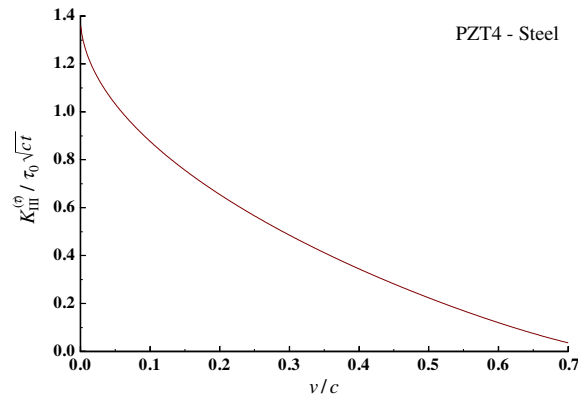


Fig. 20. Normalized dynamic stress intensity factor versus normalized crack speed in PZT4-Steel bi-materials.

zoelectric bi-materials is analyzed in this paper. The analysis is conducted on the grounded condition that the propagating crack surfaces are perfectly covered with an infinitesimally thin conducting electrode, such that the electrostatic potential vanishes over the entire crack surfaces. A coordinate transformation is introduced and the integral transform approach is used to analyze the dynamic fracture problem. Under taking into account both the well-known BG as well as the MT surface waves propagating along the interface in elastic–piezoelectric bi-materials, this problem is much more complicated than the corresponding elasto-dynamic fracture investigated in the literature. It is noted that the existence conditions of the MT surface wave for different material combinations are also provided in this study. Finally, the analytical transient full-field solutions in time domain are constructed by using the Wiener–Hopf technique and the Cagniard–de Hoop method. Furthermore, the dynamic stress intensity factor and the dynamic electric displacement intensity factor are also obtained in explicit expressions. The results obtained in this investigation provide important analytical solutions for dynamic fracture problem especially on the crack propagation event.

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