

System performance analysis of Gifford–McMahon cooler

B.J. Huang and S.C. Chang

Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan

Received 12 July 1994; revised 12 September 1994

System performance analysis of a Gifford–McMahon (GM) cooler was carried out using the half-cycle mean model of a regenerator and instantaneous solutions of the mass and momentum equations. A PC-based simulation package GMSYS was also developed for system design analysis of a single-stage GM cryocooler. It is shown experimentally that the predicted values agree very well with test results.

Keywords: Gifford–McMahon cooler; system performance analysis; computer simulation

Nomenclature

A_r	Free flow area of regenerator (m^2)
C_p	Constant-pressure heat capacity ($\text{J kg}^{-1} \text{K}^{-1}$)
C_v	Constant-volume heat capacity ($\text{J kg}^{-1} \text{K}^{-1}$)
C_s	Heat capacity of regenerator matrix ($\text{J kg}^{-1} \text{K}^{-1}$)
f	Reciprocating frequency of displacer (Hz)
L_r	Length of regenerator (m)
\dot{m}_{in}	Mass flowrate of gas from valve to compression space (kg s^{-1})
\dot{m}_{out}	Mass flowrate of gas from compression space to valve (kg s^{-1})
\dot{m}_r	Mass flowrate through regenerator (kg s^{-1})
\dot{m}_v	Mass flowrate through valve (kg s^{-1})
p_c	Gas pressure in compression space (N m^{-2})
p_e	Gas pressure in expansion space (N m^{-2})
p_H	Gas pressure supplied by compressor (N m^{-2})
p_h	Gas pressure in expansion space at state 2 (N m^{-2})
p_L	Compressor return pressure (N m^{-2})
p_o	Gas pressure in expansion space at state 4 (N m^{-2})

Q_L	Net cooling capacity (W)
R	Gas constant
T_c	Compression space temperature (K)
T_e	Expansion space temperature (K)
T_H	Compressor supplied gas temperature (K)
t	Time (s)
V_c	Compression space volume (m^3)
V_e	Expansion space volume (m^3)
V_r	Regenerator volume ($= A_o L_r$) (m^3)
V_{swept}	Swept volume of displacer (m^3)

Greek letters

α, β	Coefficients of Darcy's Equation (8)
γ	C_p/C_v
ϵ	Porosity of regenerator
η	Effectiveness of regenerator
ρ_c	Gas density in compression space
ρ_e	Gas density in expansion space
ρ_r	Gas density in regenerator
τ	Period of GM cycle, $\tau = 1/f$

The Gifford–McMahon (GM) cooler (*Figure 1*) belongs to a class of cooling systems utilizing a gas-compression refrigeration cycle. The refrigeration effect of the GM cooler results from a series of thermodynamic processes acting on the gas, including charging and compression, displacement and heat exchange with the regenerator, expansion and heat absorption (cooling effect), as shown in *Figure 2*.

It seems that progress in the development of system design tools for GM coolers is still limited since only a few researchers have carried out system performance analysis. Gifford¹ assumed that the GM cycle is composed of a cycle

similar to the Brayton cycle but with an intermediate expansion pressure which varies with the displacer motion and the gas intake/exhaust process. In addition, the two isenthalpic processes of gas moving through the intake and exhaust valves were considered. Gifford¹ derived an expression for *COP* in terms of pressure and temperature ratios based on the ideal analysis.

Thirumaleshwar and Subramanyam² further modified the Gifford model¹ by considering the thermal losses in the cooler and derived an expression for cooler performance. It was, however, shown experimentally that their analysis over-estimates actual values in practice³.

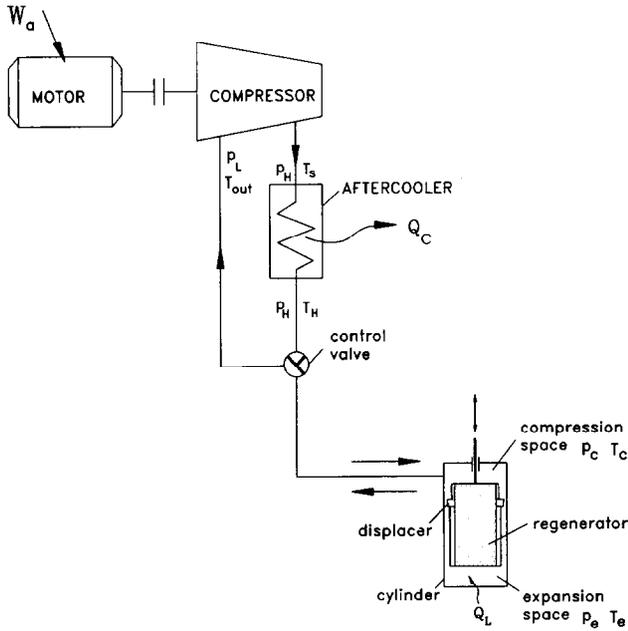


Figure 1 Schematic diagram of GM cryocooler

Instead of using the aforementioned thermodynamic approach^{1,2}, Ackermann and Gifford⁴ derived a transient heat transfer model for the regenerator of a GM cooler according to a parallel flow analogy to account for the regenerator loss. The model can only be used, however, to evaluate the regenerator heat transfer property *NTU* using the test results of a GM cooler.

Minas and Hualde⁵ derived a dynamic model for a GM cooler using a pneumatically-driven displacer. The model includes the dynamic, fluid dynamic and thermodynamic effects and consists of a non-linear time-varying system of differential equations which are solved numerically. The predicted system performance is shown to coincide with experimental results only qualitatively.

To overcome the aforementioned difficulties in the desired analysis, a method of system performance analysis based on a half-cycle mean model is developed here for a GM cooler with a mechanically-driven displacer.

Performance model

Thermal process of Gifford-McMahon cooler

For a single-stage GM cooler, the working gas is compressed by the compressor from p_L to p_H and then cooled from T_s to T_H by the aftercooler of the compressor. The

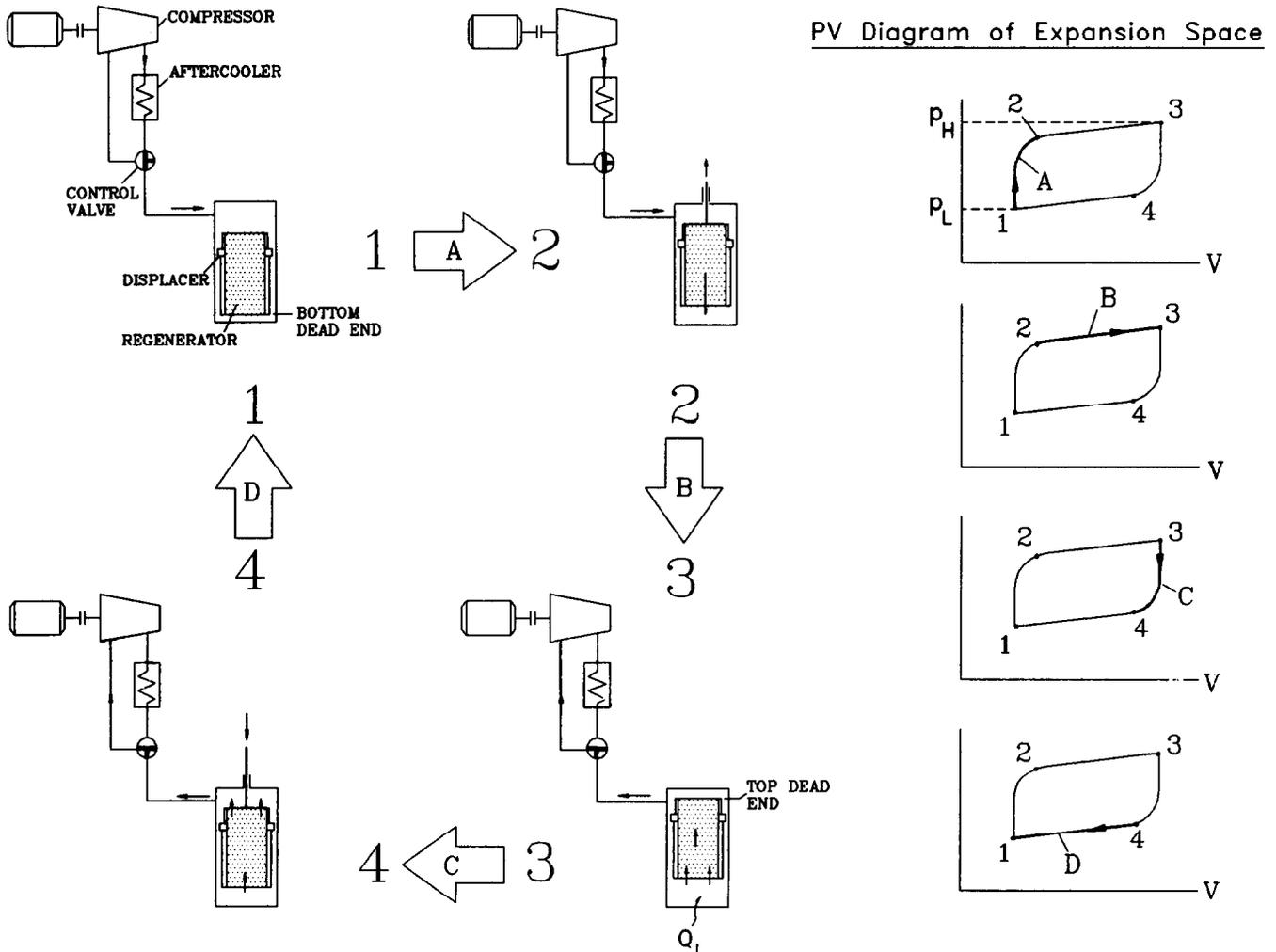


Figure 2 Refrigeration cycle of GM cryocooler

thermodynamic processes of the gas in a GM cooler can be divided into four stages (see Figure 2):

- 1 Process A (1 → 2): intake and compression
- 2 Process B (2 → 3): displacement and heat exchange (gas cooled)
- 3 Process C (3 → 4): expansion and refrigeration
- 4 Process D (4 → 1): displacement and heat exchange (gas heated)

In process A (1 → 2), the control valve is switched to the intake loop and the regenerator/displacer starts to move upwards from the bottom dead end. The volume of the expansion space is V_{eD} at the beginning (state 1). High pressure gas is then purged into the compression chamber and expels or compresses the residue gas in the expansion space, the connecting tube and the regenerator from a low pressure p_1 to a higher pressure p_2 .

In process B (2 → 3), the displacer keeps moving upwards and expels the gas in the compression space to the expansion space. During this process, the expelled gas is cooled by the regenerator matrix and mixed with the incoming gas from the compressor. The volume of the compression space becomes V_{cD} and the pressure in the expansion space becomes p_3 as the displacer reaches the top dead end, at the end of process B (state 3). During process A and B (1 → 3), the control valve is switched to the intake loop.

In process C (3 → 4), the control valve is switched to the exhaust loop and the displacer starts to move downwards from the top dead end. The gas starts to expand outwards from the expansion space to give rise to the refrigeration effect and flows back to the compressor. The regenerator matrix is meanwhile cooled by the expanded cold gas (or the gas is heated by the regenerator). The gas pressure in the expansion space drops from p_3 to p_4 in process C.

In process D (4 → 1), the displacer keeps moving downwards with the valve still in the exhaust loop. The cold gas in the expansion space is expelled to the compression space and cools the regenerator matrix. Meanwhile, part of the gas will flow back to the compressor. The volume of the expansion space changes to V_{eD} ($= V_1$) as the displacer reaches the bottom dead end. The temperature distribution in the GM cooler is shown in Figure 3.

Since process A and process B form the gas intake process, the residue gas inside the cooler will be mixed with the gas supplied from the compressor. Hence, the working fluid cannot be clearly defined for the thermodynamics

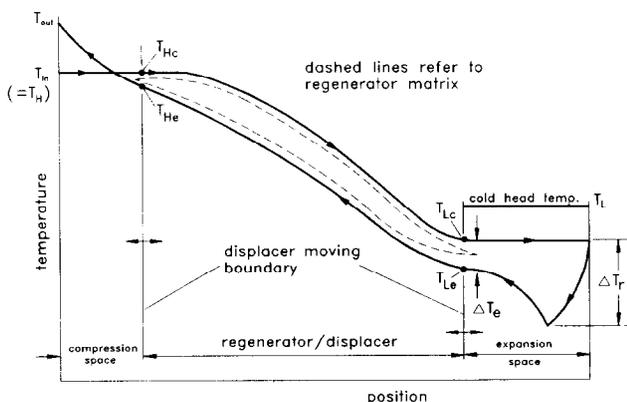


Figure 3 Temperature distribution inside the GM cooler

analysis^{1,2}. In the present study, mass and momentum equations of the GM cooler will be derived and solved numerically in conjunction with the regenerator energy equation which is derived based on a half-cycle mean concept.

Half-cycle mean model

The thermal performance of the regenerator can be evaluated by numerically solving the transient heat transfer equations as derived by Ackermann and Gifford⁴. The numerical computation is, however, quite tedious and is subject to numerical problems. To simplify the heat transfer analysis of the regenerator, a so-called 'half-cycle mean model' is developed here.

It is seen that processes A and B involve gas intake and processes C and D involve gas exhaust. Therefore, the process taking place in a GM cycle can be divided into two half-cycle periods: namely, the intake period and exhaust period. The gas undergoes a compression process during the intake half-cycle and an expansion/refrigeration process during the exhaust half-cycle. Variables such as temperature, pressure and mass flowrate can be converted into half-cycle mean variables through the following definition

$$\bar{x} = \frac{1}{\tau/2} \int_0^{\tau/2} x(t) dt \quad (1)$$

where x represents the system variable and τ is the period of the displacer motion ($= 1/f$).

Governing equations

Half-cycle mean energy equation of regenerator. The regenerator is an important cold storage component in the GM cycle. The regenerator is used to precool the gas during the intake period (process A and B). In the exhaust period (process C and D), the regenerator matrix is cooled by the cold gas. The heat exchange process of the regenerator can be expressed in terms of the effectiveness $\eta = Q_{\text{actual}}/Q_{\text{max}}$, where Q_{max} is the maximum heat exchange rate and Q_{actual} is the actual heat exchange rate.

For the regenerator in a GM cooler

$$\eta = \frac{H_{h,\text{in}} - H_{c,\text{in}}}{H_{h,\text{in}} - H_{c,\text{out}}} \quad (2)$$

where $H_{c,\text{out}}$ and $H_{c,\text{in}}$ are the total enthalpy flows at the regenerator cold end during the exhaust and the intake periods, respectively, and $H_{h,\text{in}}$ is the total enthalpy flow at the regenerator hot end during the intake period. Here, the total enthalpy flow H is defined as the integration of the rate of enthalpy flow over a half-cycle, i.e.

$$H \equiv \int_{\text{half cycle}} \dot{m}(t) C_p T(t) dt$$

Since the operating frequency of a GM cooler is relatively low (around 50–80 rev min⁻¹), the phase shift between the mass flowrate and temperature or pressure is small⁶. Therefore, the half-cycle mean of the product of two variables xy will approximate the product of the half-cycle mean of each product⁷, i.e. $\overline{xy} = \bar{x}\bar{y}$.

According to the above assumptions, Equation (2) can thus be approximated by

$$\eta \approx \frac{\dot{m}_r C_p (\bar{T}_{He} - \bar{T}_{Le})}{\dot{m}_r C_p (\bar{T}_{Hc} - \bar{T}_{Le})} = \frac{\bar{T}_{He} - \bar{T}_{Le}}{\bar{T}_{Hc} - \bar{T}_{Le}} \quad (3)$$

where \bar{T}_{Hc} and \bar{T}_{He} are the half-cycle mean temperatures at the regenerator hot end during the intake and exhaust periods, respectively, and \bar{T}_{Lc} and \bar{T}_{Le} are the half-cycle mean temperatures at the regenerator cold end during the intake and exhaust periods, respectively.

If the thermal capacity flow of the regenerator matrix C_r is much larger than the thermal capacity flow of gas C_g , i.e. $C_r/C_g = \infty$, the effectiveness of a regenerator for oscillating flow is, according to Kays and London⁸

$$\eta = \frac{NTU}{NTU + 2} \quad (4)$$

Combining Equation (3) with Equation (4), we obtain

$$\frac{\bar{T}_{He} - \bar{T}_{Le}}{\bar{T}_{Hc} - \bar{T}_{Le}} = \frac{NTU}{NTU + 2} \quad (5)$$

where NTU (number of transfer units) is defined as

$$NTU = \frac{\bar{h}A_T}{\bar{m}C_p}$$

where: \bar{h} is the mean heat convection coefficient in a half cycle; A_T is the surface area of the regenerator; \bar{m} is the half-cycle mean mass flowrate through the regenerator; and C_p is the heat capacity of the gas.

Equation (5) represents the half-cycle mean energy equation of the regenerator operated in oscillating flow. It is seen that the governing equation is greatly simplified. Solution of the non-linear time-variant differential equations for the regenerator can thus be avoided by using this half-cycle mean model.

For a regenerator made from a metal wire screen, \bar{h} can be evaluated by the empirical correlation of Tanaka *et al.*⁹: $Nu_h = 0.33Re_h^{0.67}$. Therefore

$$\bar{h} = 0.33 \left(\frac{k_g}{d_h} \right) \left(\frac{\bar{m}d_h}{\mu_g A_r} \right)^{0.67} \quad (6)$$

where: $A_T = (V_r/\epsilon)4(1 - \epsilon)/d_m$; d_h is the hydraulic diameter of screen = $\epsilon d_m/(1 - \epsilon)$; d_m is the screen wire diameter; and ϵ is the regenerator porosity. The thermal conductivity k_g and the viscosity μ_g of helium gas are corrected according to the operating temperature¹⁰.

Instantaneous governing equations of Gifford-McMahon cooler. Three control volumes (Figure 4), namely the expansion space (CV1), whole cylinder (CV2) and compression space (CV3), are used in the derivation of the instantaneous governing equations. For low frequency operation of the GM cooler, the quasi-steady assumption

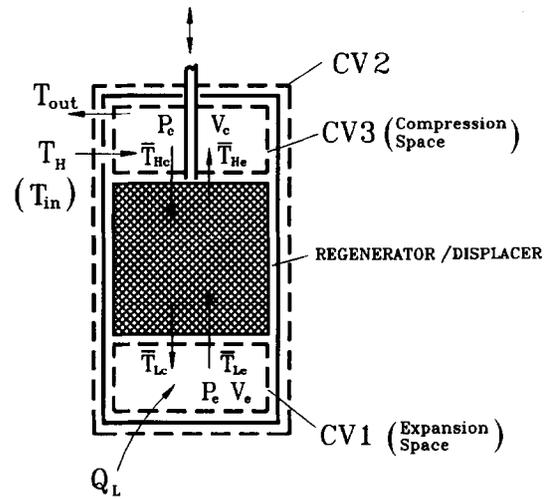


Figure 4 Control volume definition in the system analysis

can hold for the relation between the mass flowrate and the pressure drop across the valve and the regenerator⁶.

In the intake period, the momentum equation for the flow through the control valve is

$$p_H - p_c(t) = K_{v1}\dot{m}_v(t) + K_{v2}\dot{m}_v^2(t) \quad (7)$$

The momentum equation for the regenerator at quasi-steady state is basically Darcy's law

$$p_c(t) - p_e(t) = \alpha\dot{m}_r(t) + \beta\dot{m}_r^2(t) \quad (8)$$

Taking the mass balance of the compression and expansion spaces into account leads to the following equations

$$\frac{1}{RT_c} \frac{d}{dt}(p_c V_c) + \dot{m}_r - \dot{m}_v = 0 \quad (9)$$

$$\frac{1}{RT_c} \frac{d}{dt}(p_e V_e) - \dot{m}_r = 0 \quad (10)$$

Similarly, the mass and momentum equations for the exhaust period are

$$p_c(t) - p_L = K_{v1}\dot{m}_v(t) + K_{v2}\dot{m}_v^2(t) \quad (11)$$

$$p_e(t) - p_c(t) = \alpha\dot{m}_r(t) + \beta\dot{m}_r^2(t) \quad (12)$$

$$\frac{1}{RT_c} \frac{d}{dt}(p_c V_c) + \dot{m}_v - \dot{m}_r = 0 \quad (13)$$

$$\frac{1}{RT_c} \frac{d}{dt}(p_e V_e) + \dot{m}_r = 0 \quad (14)$$

The volumes of the compression and expansion spaces, $V_c(t)$ and $V_e(t)$, for a mechanically-driven displacer can be expressed as

$$V_c(t) = V_{cD} + eA_d[1 + \cos(2\pi ft)] \quad (15)$$

$$V_e(t) = V_{eD} + eA_d[1 - \cos(2\pi ft)] \quad (16)$$

where e is the eccentric length of the cam drive and A_d is the cross-sectional area of the displacer. The half-cycle mean energy equation of the regenerator is represented by Equation (5).

Equations (7)–(16) are the governing equations of the GM cooler at transient states.

System performance analysis

Net cooling capacity

Applying the laws of energy balance to the expansion space (CV1), we obtain the maximum available cooling capacity Q_{max}

$$Q_{max} = f \oint p_c \left(\frac{dV_c}{dt} \right) dt - \bar{m}_r C_p (\bar{T}_{Lc} - \bar{T}_{Lc}) \quad (17)$$

Combining the energy equations of the cylinder (CV2) and compression space (CV3), we obtain

$$Q_{max} + \bar{m}_r C_p (\bar{T}_{Hc} - \bar{T}_{He}) + f \oint p_c \left(\frac{dV_c}{dt} \right) dt = 0 \quad (18)$$

Thirumaleshwar and Subramanyam¹⁰ found experimentally that the major heat loss in a GM cooler is the motional heat loss or shuttle loss of the displacer Q_M , which can be evaluated by the empirical relation^{3,10}

$$Q_M = k_g \pi D S^2 C_T \frac{T_H - T_L}{2Lc} \quad (19)$$

where: D is the displacer outside diameter; S is the displacer stroke; C_T is the time fraction of heat transfer between the displacer and cylinder (for a displacer with harmonic motion $C_T = 0.2$); L is the displacer length; and c is the clearance between the displacer and the cylinder.

The net cooling capacity of a GM cooler is

$$Q_L = Q_{max} - Q_M \quad (20)$$

Performance index of GM cooler

Coefficient of performance (COP) is used in the performance evaluation of a GM cooler; it is defined as

$$COP = \frac{Q_L}{W_c} \quad (21)$$

where W_c is the input power which is taken as the adiabatic compression work, i.e.

$$W_c = \bar{m}_v C_p T_H \left[\left(\frac{p_H}{p_L} \right)^{(\gamma-1)/\gamma} - 1 \right] \quad (22)$$

Another performance index for a GM cooler is the Carnot efficiency (% Carnot), which is defined as

$$\% \text{ Carnot} = 100 \times \frac{\text{actual COP}}{\text{Carnot COP}} \quad (23)$$

where Carnot COP is the COP of the ideal Carnot refrigerator defined as

$$\text{Carnot COP} = \frac{T_L}{T_H - T_L} \quad (24)$$

% Carnot is used to judge how close the cooler performance is to that of an ideal Carnot cooler.

It should be noted that T_H in Equation (24) is taken as the compressed gas temperature at the inlet of the cylinder, i.e. $T_H = T_{in}$. COP thus does not include the efficiency of the gas compressor.

System analysis procedure

To simplify the system analysis, the following assumptions are made:

- 1 The heat loss in the connecting pipe between the compressor aftercooler and the cylinder is negligible. Therefore, the half-cycle mean temperature at the regenerator hot end in the intake period \bar{T}_{Hc} approximates T_H , i.e.

$$\bar{T}_{Hc} \approx T_H \quad (25)$$

- 2 In the intake period, the half-cycle mean gas temperature at the regenerator cold end \bar{T}_{Lc} approximates the cold head temperature T_L , i.e.

$$\bar{T}_{Lc} \approx T_L \quad (26)$$

The system inputs in the analysis of a GM cooler include the operating conditions p_H , p_L , T_H , T_L and the geometric design specifications. For a given geometric design, a system performance simulation can be carried out. The analytical procedure is shown in Figure 5. The governing equa-

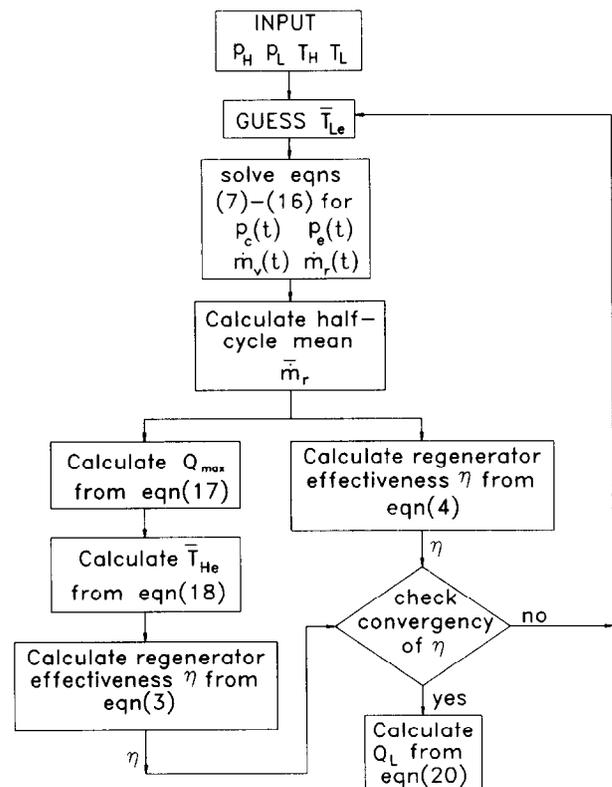


Figure 5 Simulation flow chart

tions, Equations (6)–(17), for the instantaneous pressure and mass flowrate in the compression and expansion spaces are first solved numerically for a guessed \bar{T}_{Lc} . The half-cycle mean mass flow rate in the regenerator \bar{m}_r is then evaluated. The calculated regenerator effectiveness η , determined using Equation (3) in conjunction with Equations (17) and (18), is compared with that calculated from Equation (4). An iteration process is then followed until convergence of η . Physical properties such as density, viscosity and thermal conductivity of the gas are also corrected iteratively according to the half-cycle mean temperature and pressure and the ideal gas law.

It is noted that the performance analysis can be divided into two parts. The first part includes the solution of transient Equations (6)–(17); the second part includes an algebraic calculation using the regenerator half-cycle mean energy equations, Equations (3) and (4). Since the transient equations (7)–(16) are solved numerically using the IMSL DGEAR subroutine and numerical solution of the regenerator transient equations is avoided, the performance analysis is simplified a great deal. A PC-based package GMSYS is developed in the present study and the entire computation for an analysis takes a few minutes with a PC 486.

Experimental verification

Experimental set-up

A single-stage GM cooler is designed and tested to verify the system performance analysis. The regenerator design is integrated with the displacer to form a displacer/regenerator component. The regenerator is 34.4 mm in diameter, 70 mm in length and 176 g in weight. The regenerator is made from 652 200-mesh phosphor bronze wire screen discs with a wire diameter of 0.042 mm. The regenerator case is made from a phenolic fibre material with an outside diameter of 51.6 mm, an inside diameter of 34.4 mm and weight of 197 g.

The cold head (cylinder) is 135.6 mm in length, 52.5 mm in inside diameter and 0.35 mm in wall thickness. The stroke of the displacer/regenerator is 25.4 mm. The dead volumes of the compression and the expansion spaces are 3.46 and 5.09 cm³, respectively. The clearance between the displacer and the cylinder is 0.2 mm and the swept volume of the displacer is 55 cm³.

Helium gas of 99.999% purity is used as the working fluid. A CTI Model SC helium compressor is used in the present experiment. The charged pressure is 250 psig. During normal operation, the supplied pressure $p_H = 275$ psig and the return pressure $p_L = 110$ psig*. The vacuum chamber is evacuated to below 10⁻⁶ torr† by an Osaka OV-TH162 turbopump. A thermocouple made from KPVsAu7Fe material and a YEW7563 digital thermometer are used to measure the cold end temperature.

Determination of valve constants

Darcy's law used to describe the pressure drop across the valve [Equations (7) and (11)] has two empirical constants, K_{v1} and K_{v2} , which have to be determined experimentally. Since the valve is simply a small hole, we measure the

pressure drop at various mass flowrates and obtain the following correlation for the frictional coefficient

$$f = \frac{95398}{Re} + 0.329 \quad (27)$$

where Re is the Reynolds number defined with respect to the hole diameter D of the valve, which is 5 mm. That is, $Re = \rho u_v D / \mu = \dot{m}_v D / (\frac{1}{4} \pi D^2 \mu)$.

Converting the above relation into the pressure drop of helium gas flow across the valve, we obtain

$$\Delta p = -2.5378 \times 10^9 \dot{m}_v^2 + 1.1129 \times 10^8 \dot{m}_v - 9.973 \times 10^4 \text{ Pa} \quad (28)$$

Therefore, the valve constants K_{v1} and K_{v2} are determined as

$$K_{v1} = 1.1129 \times 10^8 \text{ Pa kg}^{-1} \text{ s}^{-1}$$

$$K_{v2} = -2.5378 \times 10^9 \text{ Pa kg}^{-2} \text{ s}^{-2}$$

Experimental results

It can be seen from Figure 6 that the predicted Q_L agrees with the simulation results to within $\pm 15\%$ for the cold head temperature $T_L < 120$ K (or $Q_L < 50$ W). It is also shown that for $T_L > 120$ K (or $Q_L > 50$ W), the predicted Q_L is smaller than the test results and the deviation increases with increasing T_L . This is due to the approximation $\bar{T}_{Lc} \approx T_L$ [Equation (26)] used in the simulation for all cases. Equation (26) implies that the convective heat transfer between the gas in the expansion space and the cold head wall is assumed to be very large. This may be true only for cases of low Q_L or T_L .

The half-cycle mean temperature at the regenerator cold end in the intake period \bar{T}_{Lc} is expected to be equal to the cold head temperature T_L only for low T_L (< 120 K) with lower cooling capacity ($Q_L < 50$ W). The deviation between \bar{T}_{Lc} and T_L thus increases and causes a larger error in Q_L when T_L is high.

To verify this, a thermocouple with 0.075 mm diameter wire was installed inside the expansion space to measure the instantaneous gas temperature using a fast digital recorder YEW 3655E. The test results show that $T_L - \bar{T}_{Lc} = 10$ K at $T_L = 80$ K and $Q_L = 35$ W; $T_L - \bar{T}_{Lc} = 20$ K at $T_L = 100$ K and $Q_L = 47$ W; and $T_L - \bar{T}_{Lc} = 75$ K at $T_L = 200$ K and $Q_L = 82$ W, as shown in Figure 7a. The convective heat transfer coefficient of the helium gas in the expansion space evaluated from the test data is also seen to be very high at low T_L as shown in Figure 7b.

The discrepancy between the predicted Q_L and the test results may also be caused by the following factors:

- 1 The half-cycle mean energy equation of the regenerator [Equation (4)] is accurate if $C_r/C_g \rightarrow \infty$. However, the thermal capacity flow of the regenerator matrix C_r may decrease dramatically at an operating temperature < 80 K such that the assumption of $C_r/C_g = \infty$ becomes invalid.
- 2 The displacer shuttle loss Q_M is probably underestimated, since the clearance c between the cylinder and the displacer may be smaller at higher temperature due to differential shrinkage of the cylinder and the displacer.

* 1 psi = 6894.76 N m⁻²

† 1 torr = 133.322 N m⁻²

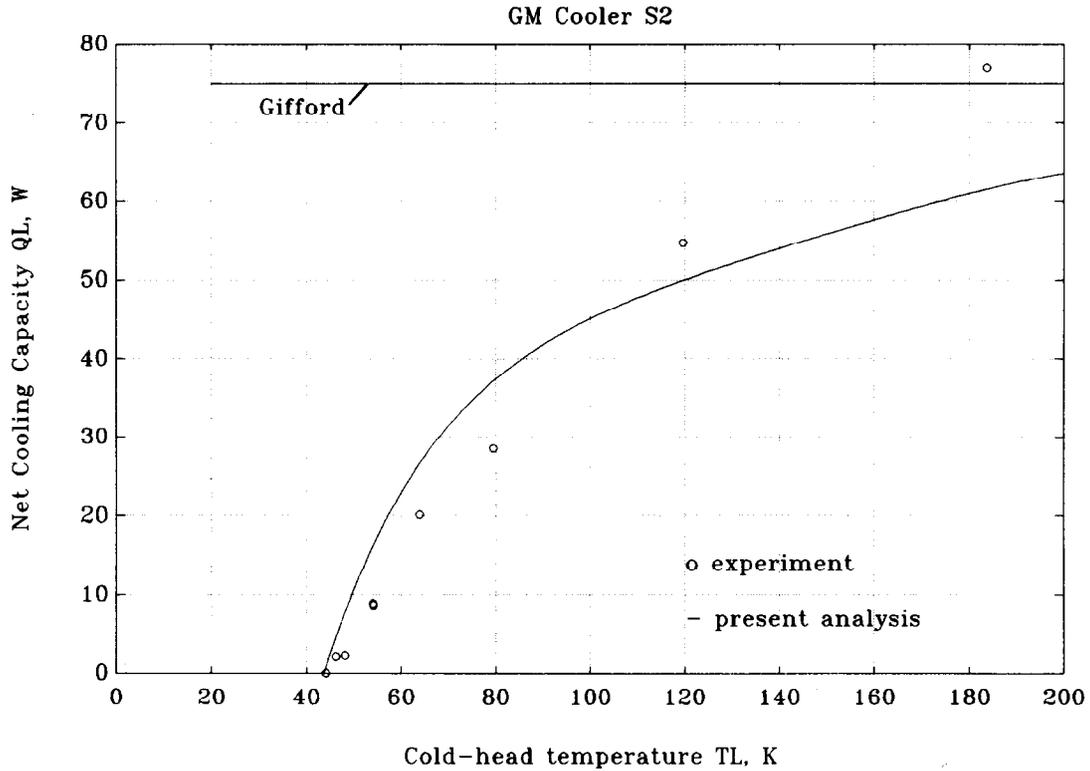


Figure 6 Comparison of test results with simulation

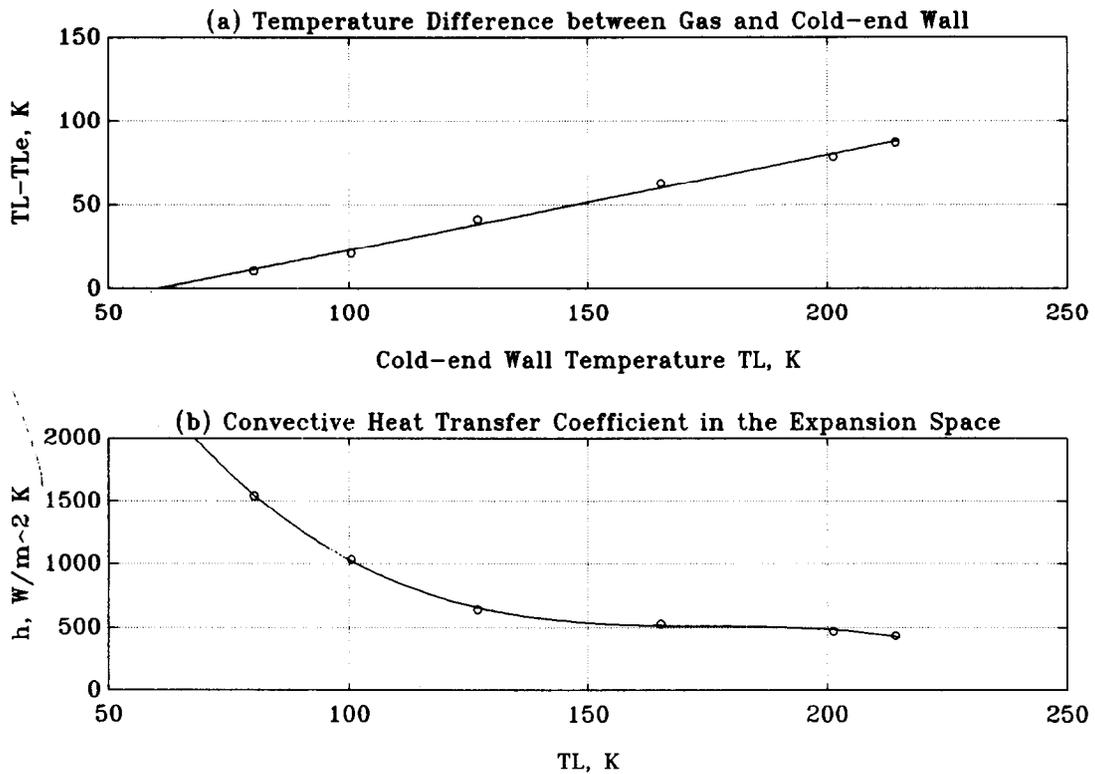


Figure 7 (a) Deviation of cold end temperature; (b) convective heat transfer coefficient in the expansion space

3 The conduction loss of the regenerator and the gas leakage loss through the displacer clearance are not taken into account.

To assess the energy efficiency of the GM cooler, the energy efficiency ratio (*EER*) can be defined as the ratio of the cooling capacity to the total power input, including power consumed by the compressor, fans, motor, etc.

$$EER = \frac{Q_L}{W_{total}} \tag{29}$$

The overall Carnot efficiency % Carnot0 can also be defined as the ratio of *EER* to Carnot *COP* defined in Equation (23), i.e.

$$\% \text{ Carnot0} = \frac{EER}{\text{Carnot } COP} \tag{30}$$

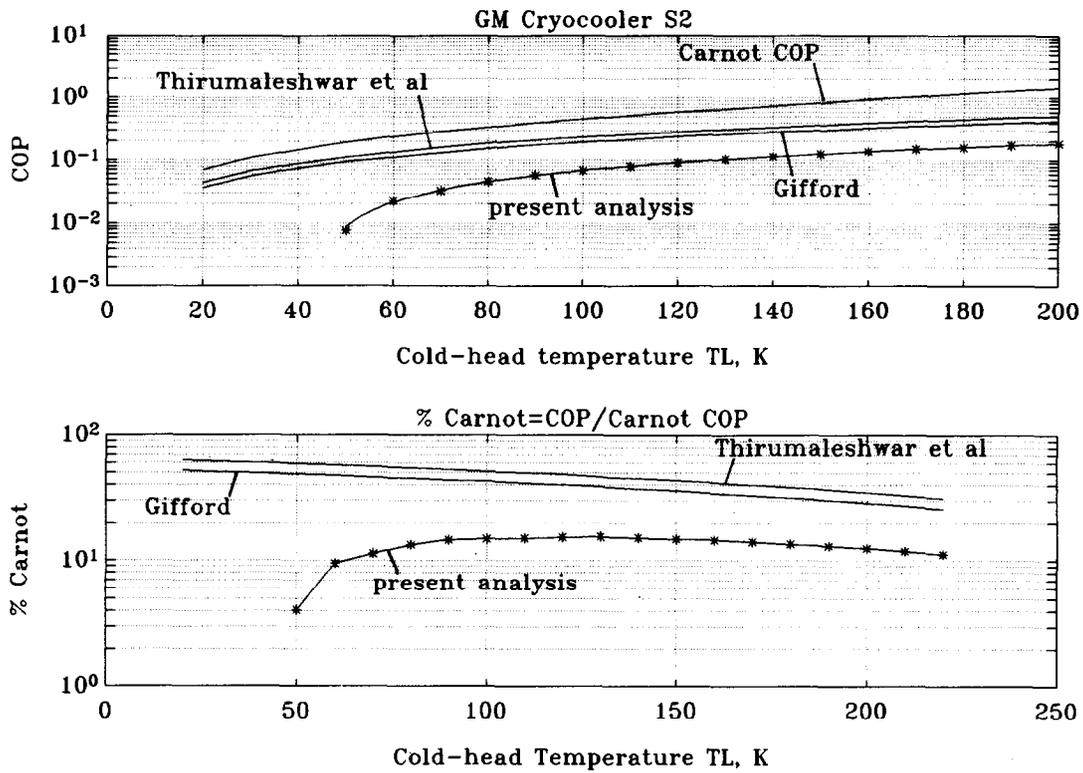


Figure 8 Comparison of analytical results using different models

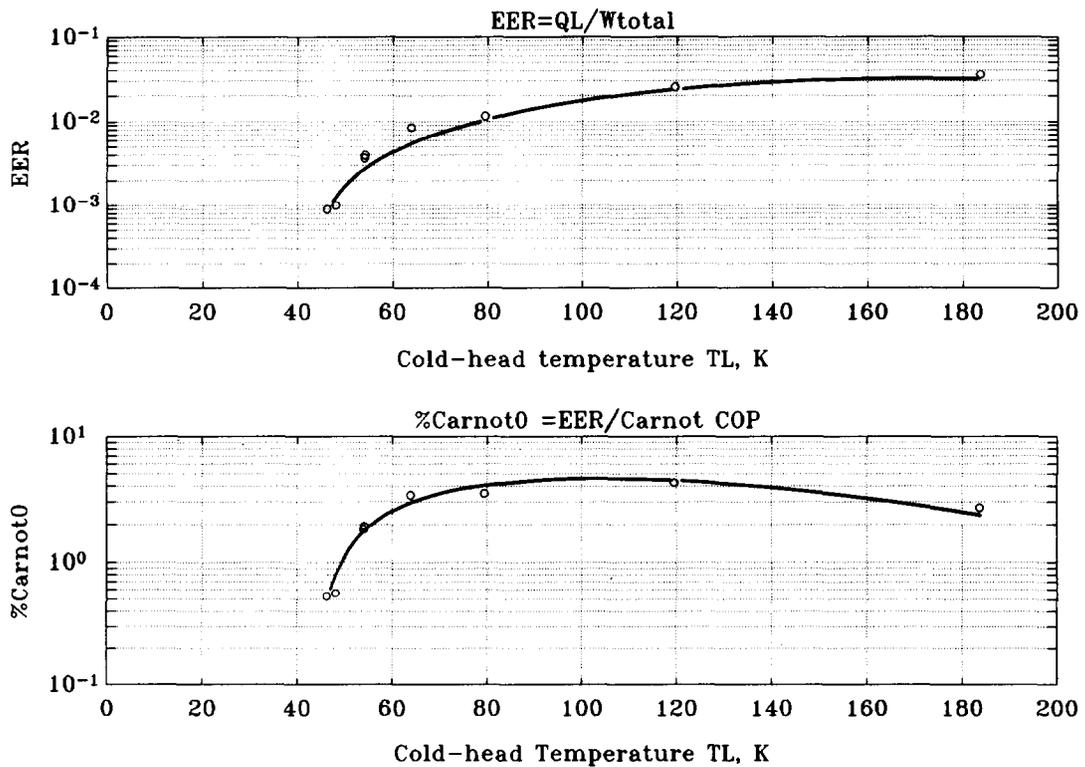


Figure 9 Test results for EER and % Carnot 0

The test results in *Figure 9* show that the measured *EER* increases with T_L and the optimum % Carnot θ is found around $T_L = 100$ K.

The cold head temperature T_L of the single-stage GM cooler designed in the laboratory can reach a lowest temperature of 39 K at zero cooling capacity. A cooling capacity of 29 W is obtained at $T_L = 80$ K with *EER* = 0.0116. It was also found that the test results of *EER versus* T_L do not alter even if a different displacer/regenerator is used during the tests. This is caused by the fact that most of the total power input of the GM cooler is consumed by the helium compressor which is designed with a special gas purification and oil separation device.

The *COP* and % Carnot of the GM cooler can be evaluated from Equations (20)–(24). Gifford¹¹ assumed that the compressor undergoes an adiabatic process and derived equations for Q_L , *COP* and % Carnot

$$Q_L = fV_{\text{swept}}(p_H - p_L) \quad (31)$$

$$COP = \frac{R(r_p - 1)}{C_p(r_p r_T - 1)[r_p^{(\gamma-1)/\gamma} - 1]} \quad (32)$$

$$\% \text{ Carnot} = 100 \times \frac{(T_H - T_L)/T_L}{C_p(r_p r_T - 1)[r_p^{(\gamma-1)/\gamma} - 1]/R(r_p - 1)}$$

where $r_p = p_H/p_L$; $r_T = T_H/T_L$.

Thirumaleshwar *et al.*² assumed that the compressor undergoes an isothermal compression process and derived the following results

$$COP = \frac{r_p - 1}{(r_p r_T - 1) \ln r_p} \quad (34)$$

$$\% \text{ Carnot} = 100 \times \frac{(T_H - T_L)/T_L}{[(r_p r_T - 1) \ln r_p]/(r_p - 1)} \quad (35)$$

The analytical results using the above models are also shown in *Figure 8*. It is seen that *COP* values determined from the thermodynamic analysis of Gifford¹¹ and Thirumaleshwar *et al.*² are much higher than those from the present analysis. % Carnot is shown to decrease monotonically with increasing T_L without an optimum value for the thermodynamic analysis^{2,11}. However, % Carnot is shown first to increase with increasing T_L , reach a maximum value and then decrease in the present analysis. An optimum % Carnot around 100 K exists. This can also be verified exper-

imentally by the measured *EER* as shown in *Figure 9*, since *EER* is approximately proportional to % Carnot.

Conclusions

A regenerator half-cycle mean energy model is derived in the present study to avoid transient heat transfer analysis of the regenerator. The dynamic equations of the mass and momentum balances of a GM cooler are first derived and solved numerically for the time variables $p_e(t)$, $p_c(t)$, $\dot{m}_v(t)$, $\dot{m}_r(t)$. The half-cycle mean variables are then calculated and used to compute the regenerator effectiveness using the regenerator half-cycle mean energy equation. Using an iteration scheme, the final solutions can be obtained, from which the system performance of the GM cooler can be evaluated. A PC-based simulation package GMSYS was also developed to carry out system analysis of a single-stage GM cryocooler. Finally it was shown experimentally that the performance prediction agrees with the test results very well.

Acknowledgement

The present study was supported by The Energy Commission, Ministry of Economic Affairs, Taiwan.

References

- 1 Gifford, W.E. The Gifford–McMahon cycle *Adv Cryog Eng* (1966) **11** 152–159
- 2 Thirumaleshwar, M. and Subramanyam, S.V. Gifford–McMahon cycle – a theoretical analysis *Cryogenics* (1986) **26** 177–188
- 3 Thirumaleshwar, M. and Subramanyam, S.V. Heat balance analysis of single stage Gifford–McMahon cycle cryocooler *Cryogenics* (1986) **26** 189–195
- 4 Ackermann, R.A. and Gifford, W.E. Small cryogenic regenerator performance *ASME J: Eng for Industry* (Feb 1969) 273–281
- 5 Minas, C. and Hualde, P.M. Dynamic modelling of a Gifford–McMahon cryorefrigerator *Cryogenics* (1992) **32** 634–639
- 6 Huang, B.J. and Lu, C.W. Dynamic response of regenerator in cyclic flow system *Cryogenics* (1993) **33** 1046–1052
- 7 Qvale, E.B. and Smith, J.L. Jr An approximate solution for the thermal performance of a Stirling-engine regenerator *Trans ASME J: Eng. for Power* (April 1969) 109–112
- 8 Kays, W.M. and London, A.L. *Compact Heat Exchangers* 2nd Edn. McGraw–Hill, New York, USA (1984)
- 9 Makoto, T., Iwao, Y. and Fumitake, C. Flow and heat transfer characteristics of the Stirling engine regenerator in an oscillating flow *JSME Int J: Series II* (1990) **33**(2) 283–289
- 10 Thirumaleshwar, M. and Subramanyam, S.V. Energy analysis of a Gifford–McMahon cycle cryorefrigerator *Cryogenics* (1986) **26** 248–251
- 11 Gifford, W.E. The Gifford–McMahon cycle *Adv Cryog Eng* (1965) **11** 152–159