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LINEAR NETWORK ANALYSIS OF SPLIT-TYPE STIRLING REFRIGERATOR

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A linear network model is developed for split-type Stirling refrigerators. We obtain an equivalent network and two transfer-function functions from which the system performance can be evaluated. Implementation of the linear network model in the system design analysis of split-type Stirling refrigerator is shown satisfactory.

INTRODUCTION

Stirling refrigerator operates at a cyclic state and the physical process is thus transient. The system design using conventional heat transfer analysis requires a sophisticated computing equipment and suffers from numerical instability and high computing cost. A *linear network model* is thus developed in the present study for the system design analysis of split-type Stirling refrigerator (Figure 1).

EQUIVALENT NETWORK OF STIRLING REFRIGERATOR

The linearly-perturbed models of the components can be derived from the governing equations and linearization method. Connecting the equivalent circuits of the components according to the process of Stirling refrigerator, we obtain the equivalent network as shown in Figure 2. Only block diagram can be drawn in Figure 2 for the connecting tube and regenerator since the distributed models are derived for them.

TRANSFER FUNCTIONS OF STIRLING REFRIGERATOR

Solving the linear dynamics equations, we obtain two transfer functions for Stirling refrigerator:

$$G_{dp}(s) \equiv \frac{\tilde{X}_d(s)}{\tilde{X}_p(s)} = \left[\frac{G_3(s)G_8(s)/G_7(s) + G_2(s)}{1 - G_5(s)}\right] \times \left[\frac{\mathbf{I}_{mp}(s)\cdot Z_{ti}(s) + \mathbf{I}_{mm}(s)}{1 + Z_{ti}(s)/Z_c(s)}\right] \times \frac{sp_o A_p}{R\bar{T}_c} \tag{1}$$

$$G_{ep}(s) \equiv \frac{\tilde{p}_e(s)}{\tilde{X}_p(s)} = \left\{\left[G_4(s) + \frac{G_3(s)G_6(s)}{1 - G_5(s)}\right]\frac{G_8(s)}{G_7(s)} + \left[\mathbf{R}_{pm}(s) + \frac{G_2(s)G_6(s)}{1 - G_5(s)}\right]\right\} \times \left[\frac{\mathbf{I}_{mp}(s)\cdot Z_{ti}(s) + \mathbf{I}_{mm}(s)}{1 + Z_{ti}(s)/Z_c(s)}\right] \times \frac{sp_o A_p}{R\bar{T}_c} \tag{2}$$

where

$$Z_{ti}(s) = \frac{G_8(s) + G_7(s)\,Z_{ct}(s)\tanh[\Gamma_t(s)\,L_t]}{G_7(s) + [G_8(s)/Z_{ct}(s)]\tanh[\Gamma_t(s)\,L_t]},$$

$G_1(s) = \mathbf{D}_{dw}(s) + \mathbf{D}_{de}(s)\cdot\mathbf{R}_{pp}(s); G_2(s) = \mathbf{D}_{de}(s)\cdot\mathbf{R}_{pm}(s); G_3(s) = G_1(s) + G_2(s)\cdot\mathbf{W}_{mp}(s); G_4(s) = \mathbf{R}_{pp}(s) + \mathbf{R}_{pm}(s)\cdot\mathbf{W}_{mp}(s); G_5(s) = G_2(s)\cdot\mathbf{W}_{md}(s); G_6(s) = \mathbf{R}_{pm}(s)\mathbf{W}_{md}(s); G_7(s)\tilde{p}_{to}(s) = G_8(s)\tilde{\dot{m}}_{to}(s); \mathbf{W}_{mp} = -sV_{wo}/(R\bar{T}_w); \mathbf{W}_{md} = -sp_o A_{dw}/(R\bar{T}_w); Z_c(s) = R\bar{T}_c/(sV_{co});$

$$G_8(s) = \mathbf{R}_{mm}(s)\left[1 + \frac{\mathbf{W}_{md}(s)G_2(s)}{1 - G_5(s)}\right] - \mathbf{E}_{me}(s)\left[\mathbf{R}_{pm}(s) + \frac{G_2(s)G_6(s)}{1 - G_5(s)}\right] - \frac{\mathbf{E}_{md}(s)G_2(s)}{1 - G_5(s)};$$

$$\tilde{\dot{m}}_{ti}(s) = \frac{sp_o A_p/(R\bar{T}_c)}{1 + Z_{ti}(s)/Z_c(s)}\tilde{X}_p(s).$$

It is found that the piston displacement X_p is the system input of a split-type Stirling refrigerator. The system outputs are the displacer displacement X_d and the expansion space pressure p_e. The Stirling refrigerator thus belongs to a single-input-multiple-output system (SIMO) in linear systems.

STIRLING REFRIGERATOR PERFORMANCE CALCULATION

Maximum Available Cooling Capacity

The maximum available cooling capacity can be evaluated by integrating the pressure $p_e(t)$ and volume $V_e(t)$ of the expansion space, where $V_e(t)$ is related to $X_d(t)$. Since the piston motion as well as the associated pressure waves approaches sinusoidal, $X_d(t)$ and $p_e(t)$ can be computed simply from the gain and phase of the frequency response functions, $G_{dp}(j\omega)$ and $G_{ep}(j\omega)$. Assuming that a_{dp} is the gain of $G_{dp}(\omega)$ with phase ϕ_d leading the piston; a_{ep} is the gain of $G_{ep}(\omega)$ with phase ϕ_e leading the piston. Then, the maximum available cooling capacity is

$$Q_{max} = f \oint_0^{2\pi} p_e dV_e = f\,\pi\,a_{ep}\,a_{dp}\,X_{po}^2\,A_{de}\,\sin(\phi_d - \phi_e) \tag{3}$$

where X_{po} is the amplitude of the piston; f is the operating frequency; θ is the piston angle; p_{eo} and X_{do} are the amplitudes of $\tilde{p}_e(t)$ and $\tilde{X}_d(t)$, respectively.

Net Cooling Capacity

The net cooling capacity Q_{net} can be evaluated by substracting the heat losses from the maximum available cooling capacity Q_{max}. There are four types of heat losses: namely, heat conduction loss of regenerator Q_{cond}, enthalpy flow loss of regenerator Q_{enth}, shuttle heat loss of displacer $Q_{shuttle}$, and hysteresis loss of spring W_{ir} [2]. The heat loss due to the gas leakage through the clearance between the displacer and the cylinder wall is related to the seal design, manufacturing process, and material used. It is ignored in the present analysis. The effect of gas leakage at the piston is hardly estimated accurately and is also ignored.

SYSTEM PERFORMANCE ANALYSIS AND IMPLEMENTATION

System Performance Analysis

A PC-based computer program is developed and packaged (called "STCS 2.0") for the system performance analysis of split-type Stirling refrigerator. A typical PV diagram is shown in Figure 3.

The frequency response of $G_{dp}(s)$ and $G_{ep}(s)$ are shown in Figure 4 and 5. The net cooling capacity at various loss coefficient C_d and cold-end temperature T_L is shown in Figure 6. Q_{net} at various operating frequency f and C_d is shown in Figure 7. The above results is consistent with the previous studies [3 – 5]. However, it takes only several seconds for the computation on PC 486.

It is also found that $G_{ep}(j\omega)$ has a peak gain (Figure 5). This indicates that an optimum operating frequency exists at the peak gain since Q_{max} is proportional to the gain $a_{ep}(= |G_{ep}(j\omega)|)$. The performance at this peak gain corresponds to the optimum value of Q_{net} in Figure 7 for a given C_d.

Testing Results and Application of "STCS 2.0"

A split-type Stirling refrigerator is designed and built in the laboratory for experiments. The lowest temperature obtained is around $100K$, with $0.25W$ at $150K$. The loss coefficient C_d can be estimated from the package STCS 2.0 by using the test results. An empirical correlation for C_d is further obtained: $C_d = 224.9 + 1.026\ T_L,\ N\ s/m$, as shown in Figure 8.

It was found experimentally that there is gas leakage between the displacer and the cylinder due to seal design problem and manufacturing defects. C_d obtained from the experiments thus includes the effect of gas leakage in addition to the frictional loss.

CONCLUSION

A linear network model is developed for the system performance analysis of split-type Stirling refrigerators. It is shown that the analytical results are consistent with the conventional heat transfer analysis. However, the computational speed is increased tremendously and numerical problems is completely avoided. A PC-based software package "STCS 2.0" was also developed according to the linear network model. STCS 2.0 can be further used to derive an empirical correlation of C_d by using the test results. The system design of Stirling refrigerator can then be simplified a great deal.

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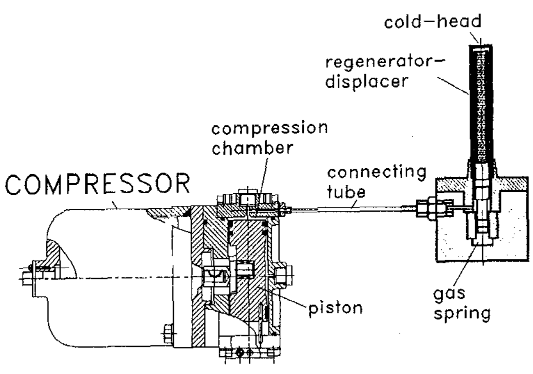

Figure 1. Split-type Stirling refrigerator.

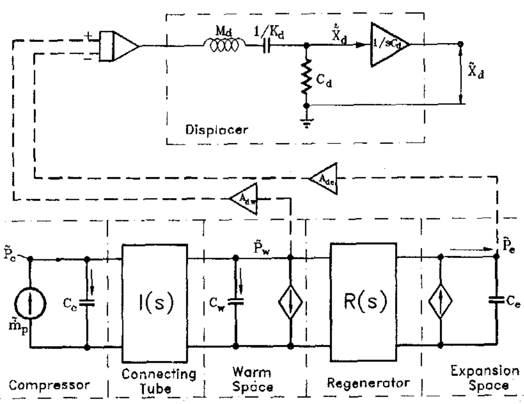

Figure 2. Equivalent network of Stirling refrigerator.

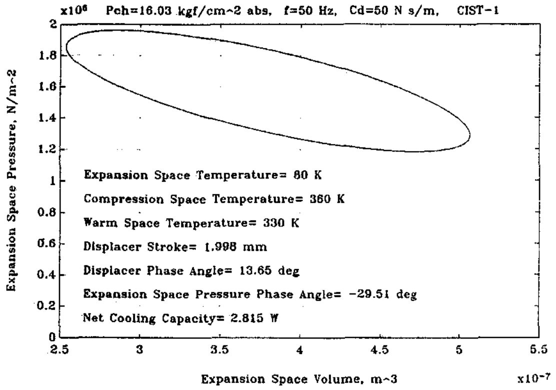

Figure 3. PV diagram

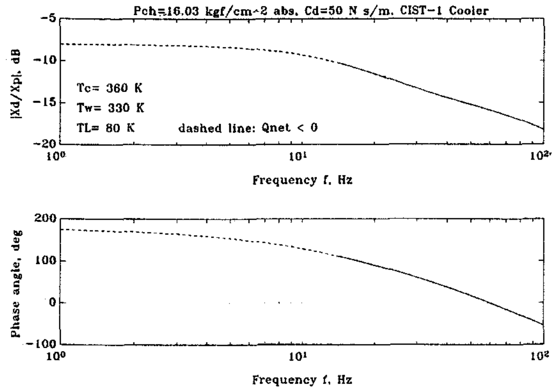

Figure 4. Frequency response of $G_{dp}(s)$

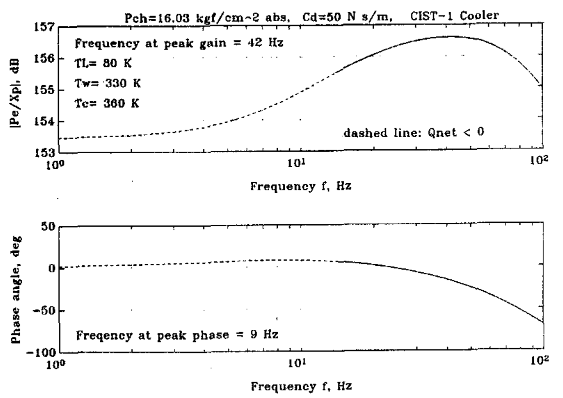

Figure 5. Frequency response of $G_{ep}(s)$

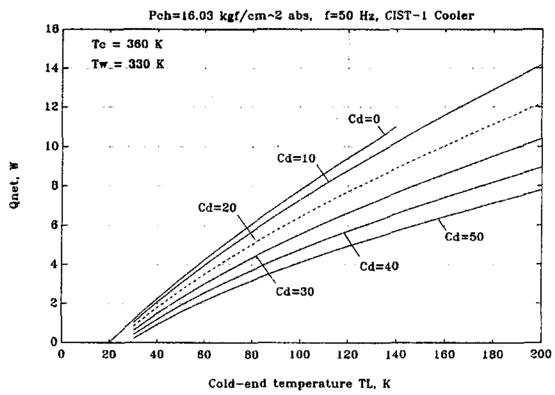

Figure 6. Q_{net} at various C_d and T_L.

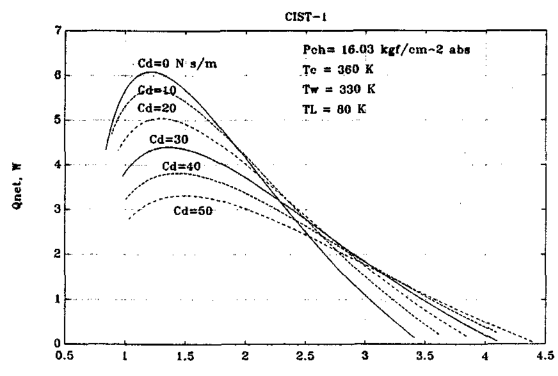

Figure 7. Q_{net} at various C_d and f.

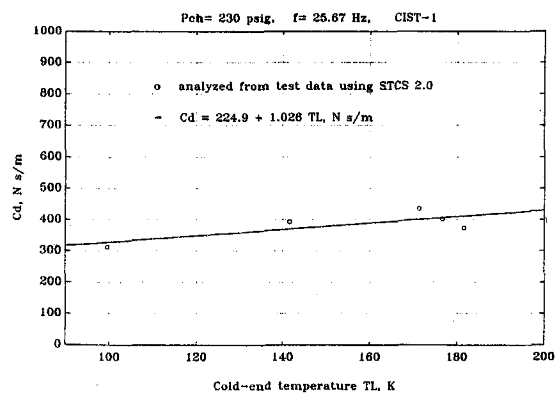

Figure 8. Experimental results of C_d.