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Model-Based Approach for Fault Diagnosis. 2. Extension to Interval Systems

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Since chemical processes are often operated over a range of operating conditions and some of the system parameters are only known to a certain degree, uncertainties exist in the process model. Interval types of process models offer an attractive alternative for process description in an operating environment. In terms of fault diagnosis, an interval process model based diagnostic system is robust as compared to conventional quantitative model-based systems. In this work, an interval model is incorporated into the deep model algorithm (DMA) for fault diagnosis. A design procedure is given, and characteristics of interval DMA are also discussed. One unique property is that the interval parity equations generally give better diagnostic resolution than the crisp ones under the DMA framework. A CSTR example with interval coefficients is used to illustrate the design and effectiveness of the interval DMA. Results show that the proposed method is not only successful in handling wide range of operating conditions but also capable of identifying correct fault origins accurately.

1. Introduction

The proliferation of process control computer makes on-line fault diagnosis a reality from the hardware point of view. Over the past decade, many attempts have been made for automatic fault diagnosis using models (Himmelblau, 1978; Watanane, and Himmelblau, 1982; Mah and Tamhane, 1982; Willsky, 1984; Iri *et al.*, 1985; Kramer, 1987; Petti *et al.*, 1990; Gertler and Anderson, 1992; Yu and Lee, 1992). The models employed for diagnosis ranging from qualitative, semiquantitative, to quantitative model. Qualitative models include the signed directed graph of Iri *et al.* (1985), Kramer and Palowitch (1987), and Chang and Yu (1990); linguistic variables of Eshraph and Mamdani (1979), Kichert (1979), and Forbus (1984); qualitative simulation of de Kleer and Brown (1984), Rich and Venkatasubramanian (1987), and Kuipers (1989); SR1 algebra of Williams (1991); and dimensional analysis of Bhaskar and Nigam (1990). Semiquantatitive models include the following: digraph + fault tree of Ulerich and Powers (1988); order-of-magnitude (O[M]) analysis of Mavrovouniotis and Stephanopoulos (1988); fuzzy set modeling of Zadeh (1983), Yu and Lee (1992), Petti *et al.* (1992), and Sugeno and Yasukawa (1993); interval analysis of Moore (1979), Struss (1988), Parsons (1993), and Chang *et al.* (1993). Quantitative models are also utilized for fault diagnosis. These include the following: certainty factors of Kramer (1987); non-Boolean analysis diagnostic model processor (DMP) of Petti *et al.* (1990); deep model algorithm (DMA) of Chang *et al.* (1994); statistical analysis of Gertler and Singer (1985); Kalman filter approach of Isermann (1984) and Fathi *et al.* (1993).

As pointed out in part 1 (Chang *et al.*, 1994), despite the fact that a quantitative process model is employed in DMP or DMA for fault diagnosis, the resolution of this type of non-Boolean reasoning is limited to the qualitative level. Furthermore, since uncertainties always exist, it is extremely difficult to obtain a precise qualitative model over a range of operating conditions. Therefore, a more likely solution is to find a somewhat *relaxed* quantitative model: a semiquantitative model. A systematic methodology using qualitative/quantitative process knowledge based on fuzzy set has been proposed by Yu and Lee (1992). However the fuzzy operations make the computation complicated. The simplest way to describe uncertainties is the "interval number" (Moore, 1979). For example, the statement "the heat transfer coefficient ranges from 7 to 10" can easily be described using an interval number, *e.g.*, $U = [7, 10]$. The interval arithmetics are discussed by Moore (1979), Krawczyk (1986), and Fichtner *et al.* (1990) in detail. Interval numbers are also employed in process design, operation, and analysis (Friedman and Reklaitis, 1975; Himmelblau, 1987; Grossmann and Floudas, 1987; Fichtner *et al.*, 1990) by chemical engineers. Chang *et al.* (1993) use interval mathematics to model uncertain process sytems. The purpose of this work is to extend the deep model algorithm (Chang *et al.*, 1994) to handle uncertain systems involving modeling errors, measurement noises, and different operating ranges. Interval arithmetic is employed for uncertainty description.

This paper has the following structure. The definitions and basic operations of interval numbers are outlined in the next section. The architectural features, *i.e.*, the formation of interval models, knowledge representation schemes, and parity equations, are also described in this section. In section 3, the DMA—definitions and algorithms—is extended to the interval systems and characteristics of interval DMA are also explored. Comparisons are also made between interval and fuzzy DMA. A CSTR example is used to illustrate the effectiveness of the purposed method in section 4, followed by the conclusions.

2. Interval Systems

It is well-known that uncertainties exist invariably in describing physical systems. Fuzzy set theory is one of the most celebrated tools in describing systems with uncertainties (Zedeh, 1965, 1983; Kaufmann, 1984; Yu and Lee, 1992; Sugeno and Yasukawa, 1993). Despite the apparent successes of fuzzy set theory in control applications, specialized hardware and complex software are often required to carry out complicated fuzzy operations. More importantly, bell-shaped (or trapezoid-shaped) membership functions of fuzzy set system are less straightforward to process engineers. For example,

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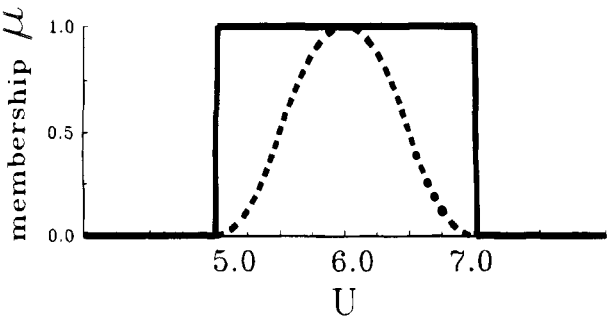

Figure 1. Description of heat transfer coefficient close to 6 (dashed line) and between 5 and 7 (solid line) using fuzzy set.

the description of "the heat transfer coefficient (U) is close to 6" is shown in Figure 1. The bell-shaped membership function shows that fuzziness of U which in many occasions are difficult to construct. Interval system is an attractive alternative in describing uncertain systems. As shown by Chang *et al.* (1993), the interval model is equivalent to the quantitative (crisp) model at one extreme and can be relaxed to the qualitative model at the other extreme. By interval model we mean a mathematical model with interval numbers as coefficients. Conceptually, an interval model is easier to formulate and much more familiar to process engineers. Here, the same description for "the heat transfer coefficient falls between 5 and 7" is quite familiar to most engineers. Figure 1 shows this description in terms of membership function. In this example, no discrimination is made between 5 and 7 (an interval number). In addition to the familarity to most engineers, the interval system is computationally less extensive than the fuzzy system. Therefore, interval models are employed in this work.

2.1. Definitions. A detailed treatment of interval numbers is given in our previous paper (Chang *et al.*, 1993). Some important definitions are given here for subsequent development. An interval number a is defined as

$$a = [\underline{a}, \bar{a}] = \{x | \underline{a} \le x \le \bar{a}, x \in \mathrm{R}\} \tag{1}$$

where $\underline{a}$ and $\bar{a}$ are the lower and upper bounds of the interval, respectively, and R is the field of real number. For a crisp number a of the value 3, it can be expressed as

$$a = [3, 3] \tag{2}$$

For a qualitative description of a positive number a, it becomes

$$a = (0, \infty) \tag{3}$$

Therefore, it becomes obvious that the interval number is a generalization of qualitative/quantitative algebra. Several relevant definitions are

(1) width

$$w(a) = \bar{a} - \underline{a} \tag{4}$$

The width is an indication of the fuzziness of the quantity, and it ranges from 0 to ∞.

(2) absolute value

$$|a| = [\min(|\underline{a}|, |\bar{a}|), \max(|\underline{a}|, |\bar{a}|)] \tag{5}$$

where $\max(|\underline{a}|, |\bar{a}|)$ and $\min(|\underline{a}|, |\bar{a}|)$ are denoted the maximum and minimum values of $|\underline{a}|$ and $|\bar{a}|$, respectively.

(3) sign

$$\mathrm{sgn}(a) = \begin{cases} +1 & \text{if } \underline{a} \ge 0, \bar{a} > 0 \\ 0 & \text{if } \underline{a} = \bar{a} = 0 \\ -1 & \text{if } \bar{a} \le 0, \underline{a} < 0 \\ \sim & \text{if } \underline{a} < 0, \bar{a} > 0 \end{cases} \tag{6}$$

The sign function poses little problem for a crisp number. Unfortunately, an interval number can cross over zero, e.g., $a = [-1, 2]$. In this case, a is neither positive nor negative; therefore, "$\sim$" is defined for the case of interval number crossing zero.

(4) negation

$$-a = [-\bar{a}, -\underline{a}] \tag{7}$$

2.2. Operations. Following the definitions of interval numbers, several basic operations of interval numbers are defined. Segment interval arithmetic (SIA) of Chang *et al.* (1993) is employed in interval operations. It should be emphasized that SIA differs from the familiar interval arithmetic (IA) (Moore, 1979) for the following reasons. From a physical point of view, any real system is governed by strict *quantitative* laws of conservation (*e.g.*, conservations of energy and mass) regardless of the ambiguities of the system parameters. This implies that, to the end, the equations (qualitative, semiquantitative, or quantitative) are constrained by the physical laws of conservation, a *crisp* relationship. A crisp zero on the right-hand side of an equation is thus obtained. The arithmetic SIA can achieve this goal with any degree of ambiguity in system parameters. Considering two interval numbers $a = [\underline{a}, \bar{a}]$ and $b = [\underline{b}, \bar{b}]$, the basic operations are

(1) addition

$$a + b = [\underline{a}+\underline{b}, \bar{a}+\bar{b}] \tag{8}$$

(2) multiplication

$$a \cdot b = [\min(\underline{a}\cdot\underline{b}, \bar{a}\cdot\bar{b}), \max(\underline{a}\cdot\underline{b}, \bar{a}\cdot\bar{b})] \tag{9}$$

(3) subtraction

$$\begin{aligned} a - b &= \mathrm{minmax}(\underline{a}-\underline{b}, \bar{a}-\bar{b}) \\ &= [\min(\underline{a}-\underline{b}, \bar{a}-\bar{b}), \max(\underline{a}-\underline{b}, \bar{a}-\bar{b})] \end{aligned} \tag{10}$$

where $\mathrm{minmax}(x, y)$ is an operator that rearranges the crisp quantities x and y into the lower and upper bounds of an interval.

(4) division

$$\begin{aligned} a/b &= \mathrm{minmax}(\underline{a}/\underline{b}, \bar{a}/\bar{b}) \\ &= [\min(\underline{a}/\underline{b}, \bar{a}/\bar{b}), \max(\underline{a}/\underline{b}, \bar{a}/\bar{b})] \quad 0 \notin b \end{aligned} \tag{11}$$

In addition to these four operations, the following definitions are relevant for the development of interval type of DMA.

(5) ranking two intervals

$$\text{Let} \quad a - b = c = [\underline{c}, \bar{c}] \tag{12}$$

(a) $a > b$

$$\text{if} \quad \text{sgn}(c) = 1 \quad \text{then} \quad a > b \tag{13a}$$

$$\text{if} \quad \text{sgn}(c) = \sim \text{and} \ |\bar{c}| > |\underline{c}| \quad \text{then} \ a > b \tag{13b}$$

$$\text{if} \quad \text{sgn}(c) = \sim, \ |\bar{c}| = |\underline{c}|, \text{and} \ a \supset b \quad \text{then} \quad a > b \tag{13c}$$

(b) $a < b$

$$\text{if} \quad \text{sgn}(c) = -1 \quad \text{then} \quad a < b \tag{13d}$$

$$\text{if} \quad \text{sgn}(c) = \sim \text{and} \ |\bar{c}| < |\underline{c}| \quad \text{then} \quad a < b \tag{13e}$$

$$\text{if} \quad \text{sgn}(c) = \sim, \ |\bar{c}| = |\underline{c}|, \text{and} \ a \subset b \quad \text{then} \quad a < b \tag{13f}$$

(c) $a = b$

$$\text{if} \quad \text{sgn}(c) = 0 \quad \text{then} \quad a = b \tag{13g}$$

(6) maximum of two intervals

$$\text{MAX}(a,b) = \begin{cases} a & \text{if } a > b \\ a \text{ or } b & \text{if } a = b \\ b & \text{if } a < b \end{cases} \tag{14}$$

(7) minimum of two intervals

$$\text{MIN}(a,b) = \begin{cases} b & \text{if } a > b \\ a \text{ or } b & \text{if } a = b \\ a & \text{if } a < b \end{cases} \tag{15}$$

where MAX(a,b) and MIN(a,b) are denoted as the maximum and minimum values of two intervals. Following these definitions one is ready to perform interval operations on system equations.

Notice that, similar to fuzzy set systems, basic properties of the interval system are quite different from those of crisp systems. Therefore, several precautions in the sequence of operations should be taken. From the definitions of negation, it is clear that

$$\begin{aligned} a + (-1)a &= [\underline{a}, \bar{a}] + (-1)[\underline{a}, \bar{a}] = [\underline{a}, \bar{a}] + [-\bar{a}, -\underline{a}] \\ &= \text{minmax}[\underline{a} - \bar{a}, \bar{a} - \underline{a}] \\ &\neq 0 \quad \text{for} \quad \underline{a} \neq \bar{a} \end{aligned} \tag{16}$$

However, from the definition of subtraction, we have

$$a - a = 0 \tag{17}$$

for $a = [\underline{a}, \bar{a}]$. This example clearly illustrates the difference between crisp and interval operations. The reason is obvious that "negation" is different from "subtraction" in SIA arithmetic (*e.g.*, interval operation is lack of an additive inverse, as discussed in Chang *et al.* (1993)). In other words, generally,

$$[\underline{a}, \bar{a}] - (-\lambda)[\underline{b}, \bar{b}] \neq [\underline{a}, \bar{a}] + \lambda[\underline{b}, \bar{b}] \tag{18}$$

except for the case of crisp a or b. Therefore, one should always perform the multiplication or division between the crisp and interval numbers first and, furthermore, the negation and subtraction are not interchangeable.

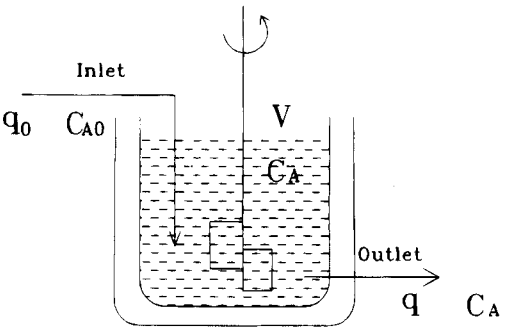

Figure 2. Chemical reactor example.

2.3. Interval Models. Following the introduction of interval arithmetic, the derivation of interval models can be carried out in a straightforward manner. In terms of model structure, the interval model is exactly the same as the "crisp" model. In other words, the model structure of an interval model is no different from the conventional one. Consider the reaction system in Figure 2. The mass balance for the species A is

$$\frac{d(VC_A)}{dt} = q_0 C_{A_0} - qC_A - kC_A V \tag{19}$$

where q_0 and q are inlet and outlet flow rate, C_{A_0} and C_A are inlet and outlet concentration of A, V is the volume of the reactor, and k is reaction rate, with nominal condition $q_0^s = 3$, $C_{A_0}^s = 0.5$, $q^s = 2.5$, $C_A^s = 0.2$, $V = 2$, and $k = 2.5$. This model structure is the same for both the crisp and interval models. The difference, however, lies in the fact that for the interval model, if the rate constant k, for example, is known to a certain degree (*i.e.*, k is an interval number), we can still utilized this model to find the unknowns (*e.g.*, outlet concentration of A). Therefore, the only difference between these two models is that the coefficients of the interval model can be interval numbers. Obviously, this is a very useful relaxation, since, in practice, the accuracy of the model parameters is only known to a certain degree. More importantly, one is able to utilize such information for fault diagnosis using the interval model.

3. Interval DMA

3.1. Structure. The structure of interval deep model algorithm (DMA) for fault diagnosis is exactly the same as that of DMA in part 1 (Chang *et al.*, 1994). Firstly, the residuals are generated once the measurements are available and the satisfaction factors (sf's) are computed for different parity equations with respect to each fault assumption. With sf's available, fault discimination is processed with the indication of degree of fault provided with consistency factor. Therefore, it is important to point out that the model equations involved can be fuzzier than the crisp parity equations. That is, interval types of parity equations are allowed. Obviously, this may lead to very different characteristics in comparison with crisp DMA.

3.2. Interval Parity Equations. Similar to parity equations for crisp DMA, the interval parity equations come directly from material and energy balances of the system. The parity equations can be formulated with respect to fault of interest. Consider a system described by parity equations and n faults $\mathbf{a} = [a_1, a_2, ..., a_n]^T$ to be diagnosed with k process measurements $\mathbf{m} = [m_1,$

$m_2, ..., m_k]^T$. In a vector form, it becomes

$$\mathbf{c}(\mathbf{a}, \mathbf{m}) = \mathbf{e} \tag{20}$$

where $\mathbf{e}$ is the vector of residuals. Here, interval coefficients are allowed in this set of non-linear equations. Obviously, we can utilize eq 20 to generate residuals. For the sake of clarity, this set of model equation is linearized at nominal operating condition (*e.g.*, $\mathbf{a}^s = [a_1^s, a_2^s, ..., a_n^s]^T$ and $\mathbf{m}^s = [m_1^s, m_2^s, ..., m_k^s]^T$, where the superscript s denotes the steady-state condition) and the linear interval parity equations become

$$\begin{aligned}
&p_{11}a_1 + p_{12}a_2 + ... + p_{1i}a_i + ... + p_{1n}a_n + k_1 = e_1\\
&p_{21}a_1 + p_{22}a_2 + ... + p_{2i}a_i + ... + p_{2n}a_n + k_2 = e_2\\
&\vdots\\
&p_{j1}a_1 + p_{j2}a_2 + ... + p_{ji}a_i + ... + p_{jn}a_n + k_j = e_j\\
&\vdots\\
&p_{m1}a_1 + p_{m2}a_2 + ... + p_{mi}a_i + ... + p_{mn}a_n + k_m = e_m
\end{aligned} \tag{21}$$

where p_{ji} is the parameter which can be an interval number. The width of that interval number depends a great deal on the degree of the understanding of system by engineers. If the parameters of the system are known precisely, then p_{ji}'s are simply crisp numbers. In a more realistic case, very often the parameters are only known to a certain degree (or the plant is operated in a given range) and the p_{ji}'s are interval numbers. Under these circumstances, the residuals of the parity equations cannot be zero even at nominal operating condition. That is,

$$\begin{aligned}
e_1^s &= p_{11}a_1^s + p_{12}a_2^s + ... + p_{1i}a_i^s + ... + p_{1n}a_n^s + k_1\\
e_2^s &= p_{21}a_1^s + p_{22}a_2^s + ... + p_{2i}a_i^s + ... + p_{2n}a_n^s + k_2\\
&\vdots\\
e_j^s &= p_{j1}a_1^s + p_{j2}a_2^s + ... + p_{ji}a_i^s + ... + p_{jn}a_n^s + k_j\\
&\vdots\\
e_m^s &= p_{m1}a_1^s + p_{m2}a_2^s + ... + p_{mi}a_i^s + ... + p_{mn}a_n^s + k_m
\end{aligned} \tag{22}$$

or,

$$\mathbf{e}^s = \mathbf{P}\mathbf{a}^s + \mathbf{k} \tag{23}$$

where e_j^s is the residual for the jth parity equation at the nominal condition. Recall that, for a crisp system, the residual should be zero at the nominal condition. Unfortunately, this is no longer the case for the interval system. The approximated zero e_j^s can be an interval number. The width of this approximated zero, e_j^s, depends on the width of interval coefficients. The following example illustrates the relationship between e_j^s and the fuzziness of the interval coefficients. Consider the parity equation with $a_1^s = a_2^s = 1$.

$$e_1 = p_{11}a_1 + p_{12}a_2 + k \tag{24}$$

For an exactly known (crisp) system with $p_{11} = 3, p_{12} = -2$, and $k = -1$, the residual e_1^s is exactly zero. This is the DMA discussed in part 1.

$$e_1^s = 3a_1 + (-2)a_2 + (-1) = 0 \tag{25}$$

However, if the abstraction of p_{11} is only to ±10% of the exact value, (*i.e.*, $p_{11} = [2.7, 3.3]$), then we have

$$\begin{aligned}
e_1^s &= [2.7, 3.3]a_1^s + (-2)a_2^s + (-1)\\
&= [-0.3, 0.3]
\end{aligned} \tag{26}$$

Here, the interval number $e_1^s = [-0.3, 0.3]$ is an approximated zero, since the nonzero residual is the result of the fuzzy information involved in the parity equation. If p_{11} is only known to ±50% of its true value (*i.e.*, $p_{11} = [1.5, 4.5]$) then the approximated zero becomes

$$e_1^s = [1.5, 4.5]a_1^s + (-2)a_2^s + (-1) = [-1.5, 1.5] \tag{27}$$

If the abstraction of the process knowledge is to an even fuzzier degree, (*e.g.*, p_{11} is known to be a positive number between 0 and 6), then e_1^s becomes

$$\begin{aligned}
e_1^s &= [0^+, 6]a_1^s + (-2)a_2^s + (-1)\\
&= [-3.0, 3.0]
\end{aligned} \tag{28}$$

where 0^+ denotes a small positive number with the value of 10^{-4} used for this example. From these four different degrees of abstraction, it can be seen clearly that the width of the approximated zero (e_1^s) becomes wider as the process knowledge becomes fuzzier. However, all these different levels of abstraction can all be utilized for the purpose of diagnosis.

3.3. Tolerances. Similar to the crisp DMA in part 1, the tolerances $\tau_{ji} = [\tau_{ji}^L, \tau_{ji}^H]$ are defined for each parity equation ($j = 1, 2, ..., m$) with respect to each defined fault origin ($i = 1, 2, ..., n$). The upper and lower bounds of the tolerance (τ_{ji}^H and τ_{ji}^L) are defined via the introduction of upper and lower bounds of tolerable perturbations ($\bar{\alpha}_i$ and $\underline{\alpha}_i$). For the set of nonlinear parity equations, the tolerances are defined as

(a) violation high

$$\tau_{ji}^H = c_j(a_1^s, a_2^s, ..., (1 + \bar{\alpha}_i)a_i^s, a_{i+1}^s, ..., a_n^s, \mathbf{m}) - 0 \tag{29a}$$

(b) violation low

$$\tau_{ji}^L = 0 - c_j(a_1^s, a_2^s, ..., (1 - \underline{\alpha}_i)a_i^s, a_{i+1}^s, ..., a_n^s, \mathbf{m}) \tag{29b}$$

For the linearized version of parity equations, the tolerance becomes

(a) violation high

$$\tau_{ji}^H = p_{ji}\,\bar{\alpha}_i\,a_i^s \tag{30a}$$

(b) violation low

$$\tau_{ji}^L = p_{ji}\,\underline{\alpha}_i a_i^s \tag{30b}$$

Notice that since the coefficients p_{ji} can be an interval numbers, the tolerances (τ_{ji}^H and τ_{ji}^L) can also be interval numbers. For the sake of clarity, let us assume that the upper and lower tolerance bounds are the same

($\underline{\alpha}_i = \bar{\alpha}_i$). Under this circumstance, we have

$$\tau_{ji} = \tau_{ji}^{L} = \tau_{ji}^{H} \tag{31}$$

and τ_{ji} can take the form of an interval number

$$\tau_{ji} = [\underline{\tau}_{ji}, \bar{\tau}_{ji}] \tag{32}$$

where $\underline{\tau}_{ji}$ and $\bar{\tau}_{ji}$ are the lower bound and upper bound of the interval.

3.4. Satisfaction Factor. When a fault occurs (*e.g.*, $a_i^* = a_i^s + \delta a_i$), the DMA generates residuals from interval parity equations followed by the computing of the satisfaction factors (from residuals). Under faulty conditions, the process measurements ($\mathbf{m}^*$) and the assumed nominal conditions ($\mathbf{a}^s$) are substituted into the interval parity equations and the interval residuals are generated.

$$\mathbf{e} = [\underline{\mathbf{e}}, \bar{\mathbf{e}}] = \mathbf{P}(\mathbf{m}^*)\mathbf{a}^s + \mathbf{k}(\mathbf{m}^*) \tag{33}$$

As pointed out earlier, for interval systems, the residuals are not exactly zero even at the nominal condition ($e_j^s \neq 0$). The errors ($\mathbf{e}$) generated here are the contribution of the nominal residuals (e_j^s) and the faulty situation. Therefore, the *net* residual as the result of process fault is

$$e_{\text{net},j} = [\underline{e}_{\text{net},j}, \bar{e}_{\text{net},j}] = e_j - e_j^s \tag{34}$$

Notice that this is an extra step in obtaining the net residual for interval systems. Since for conventional crisp systems the nominal residual e^s is zero, the subtraction in eq 34 is not necessary ($e_j^s = 0$).

Since the net residual $e_{\text{net},j}$ comes from the subtraction of two interval numbers (eq 34), in theory, the resulting $e_{\text{net},j}$ may take any possible form, *e.g.*, with the sign of +, −, or ∼. Here, the net residual comes from the repeated applications of the same parity equations, in general, it has a definite sign, *e.g.*, + or −. The following example illustrates this property. Consider the parity equation in eq 26. Nominally, the approximated error is (with $a_1^s = 1$)

$$e_1^s = [-0.3, 0.3] \tag{35}$$

which has the sign of "∼". If a positive deviation of 10% occurs in a_1, then we have

$$e_1 = [2.7, 3.3]1.1 + (-2)1 + (-1) = [-0.03, 0.63] \tag{36}$$

and, again, with the sign of "∼". However the net residual becomes

$$e_{\text{net},1} = e_1 - e_1^s = [-0.03, 0.63] - [-0.3, 0.3] = [0.27, 0.33] \tag{37}$$

which has a sign of "+". Actually, the net residual is simply

$$e_{\text{net},1} = p_{11}(1.1 - 1.0) = [2.7, 3.3]0.1 = [0.27, 0.33] \tag{38}$$

Therefore, as long as p_{11} has a definite sign, the net residual also has a definite sign.

Once the net residual is calculated, one can proceed to compute satisfaction factors sf_{ji}. The residuals arise not only from the fault itself but also from process noise and modeling errors. A non-Boolean measure is employed to evaluate the degree of satisfaction to each parity equation with respect to a different fault (Kramer, 1987; Chang *et al.*, 1994). It should be noticed that since the net residual $e_{\text{net},j}$ and the tolerance (τ_{ji}) can take the form of interval number, the satisfaction factor (sf_{ji}) is also an interval number. Similar to the crisp DMA, two cases are considered.

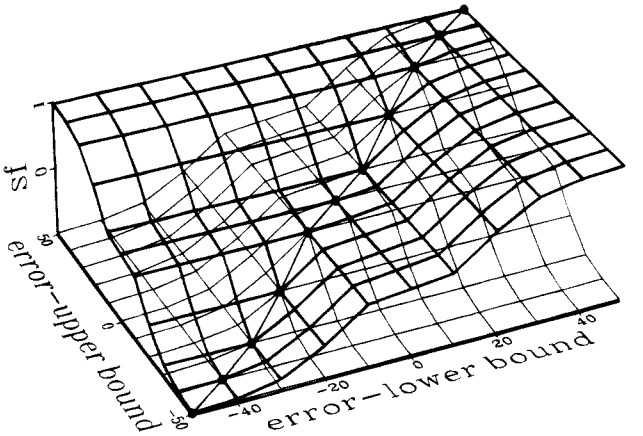

Figure 3. Interval satisfaction factor with different ranges of errors.

(1) $p_{ji} \neq 0$

$$\text{sf}_{ji} = [\underline{\text{sf}}_{ji}, \bar{\text{sf}}_{ji}] \equiv \text{sgn}(e_{\text{net},j}/\tau_{ji}) \frac{(e_{\text{net},j}/\tau_{ji})^4}{1+(e_{\text{net},j}/\tau_{ji})^4} \tag{39}$$

After some algebraic manipulation, eq 39 becomes

$$\text{sf}_{ji} = \text{minmax}\left(\text{sgn}(\underline{e}_{\text{net},j}/\underline{\tau}_{ji}) \frac{(\underline{e}_{\text{net},j}/\underline{\tau}_{ji})^4}{1 + (\underline{e}_{\text{net},j}/\underline{\tau}_{ji})^4}, \text{sgn}(\bar{e}_{\text{net},j}/\bar{\tau}_{ji}) \frac{(\bar{e}_{\text{net},j}/\bar{\tau}_{ji})^4}{1 + (\bar{e}_{\text{net},j}/\bar{\tau}_{ji})^4} \right) \tag{40}$$

Provided with $e_{\text{net},j}$ and τ_{ji}, eq 40 gives the basis for the computation of the interval sf_{ji}.

As pointed out in part 1, sf_{ji} measures the validity of ith fault assumption based on the jth parity equation. For the crisp DMA, the crisp sf_{ji} gives a clearer indication for the degree of satisfaction. The solid line in the diagonal of Figure 3 is the belief function for crisp systems. For interval systems, sf_{ji} is an interval number and the width of sf_{ji} depends on the width of the error ($e_{\text{net},j}/\tau_{ji}$ to be exact) as shown in Figure 3. The width (of sf_{ji}) is an indication of the ambiguity in the resultant belief function. For example, if the error is [−50, 50], then sf is an interval number with extremely ambiguous interpretation (ranging from possible negative deviation sf = [−1, 1]) (Figure 3). If, however, the error is a crisp number [50, 50], the resultant sf gives a clear indication of positive deviation sf = [1, 1]. Since fault isolation will be discussed in the next section, more of the ambiguous nature in interval sf will be discussed.

The resultant interval sf's may not be as troublesome as it looks (Figure 3). Let us take the example in eqs 25−28 to illustrate this fact. Equations 25−28 describe a parity equation with different degrees of abstraction.

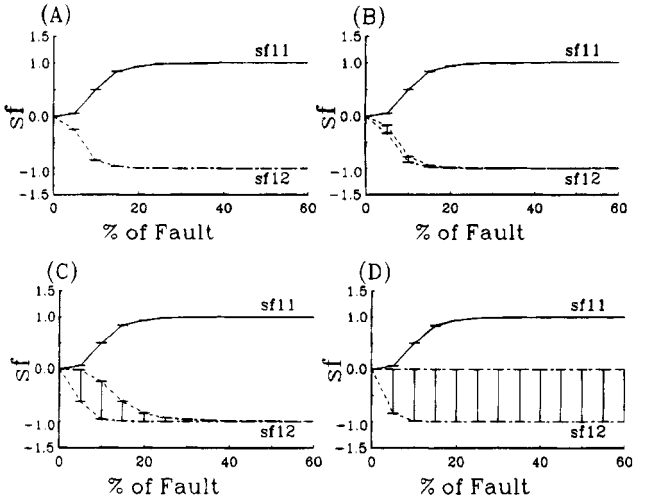

Figure 4. Satisfaction factor for a range of ambiguities in p_{11}. (A) p_{11} exact ($p_{11} = 3$), (B) ±10% error in p_{11} ($p_{11} = [2.7, 3.38]$), (C) ±50% error in p_{11} ($p_{11} = [1.5, 4.5]$), (D) ±100% error in p_{11} ($p_{11} = [10^{-4}, 10^{10}]$).

The knowledge about the parameter p_{11} starts from exactly known (eq 25) to known to ±50% (eq 27) and ±100% (eq 28). Consider that faults occur in a_1 and a range of faults are considered (0% to +100% deviations). Figure 4 shows that, regardless of the degree of abstraction, the satisfaction factors for the true fault origin (sf_{11}) are always a *crisp number* and sf_{11} increases as the fault becomes more severe. However, the degree of abstraction has significant impact in the sf for the other fault assumption a_2 (sf_{12}). The more ambiguous the knowledge about p_{11} is the wider sf_{12} will be, as shown in Figure 4. Furthermore, as the degree of fault increases, the width of sf_{12} decreases. Obviously, this is a reasonable result, since significant deviation (larger error) reduces the ambiguity in the belief function (sf_{12}).

Since the coefficient of parity equations with a value of zero (*i.e.*, $p_{ji} = 0$) is also utilized for fault isolation in DMA, the satifaction factor for this case is defined as

(2) $p_{ji} = 0$

$$sf_{ji} = sgn^*\left[1 - \frac{(e_{net,j}/\tau_{j,min})^4}{1 + (e_{net,j}/\tau_{j,min})^4}\right] \quad (41)$$

where $\tau_{j,min}$ is the smallest (in the absolute sense) nonzero tolerance in the jth parity equation, as

$$\tau_{j,min} = MIN(|\tau_{jr}|) \quad \text{for all } r \text{ with } p_{jr} \neq 0 \quad (42)$$

The sign of sf_{ji}, sgn*, is redefined as

$$sgn^* = \begin{cases} sgn(sf_i^*) & \text{if } sgn(sf_i^*) = \pm 1 \text{ and } |sf_i^*| \geq 0.1 \\ sgn(\overline{sf}_i^*) & \text{if } sgn(sf_i^*) = \sim \text{ and } |sf_i^*| \geq 0.1 \text{ and } |\overline{sf}_i^*| \geq |\underline{sf}_i^*| \\ sgn(\underline{sf}_i^*) & \text{if } sgn(sf_i^*) = \sim \text{ and } |sf_i^*| \geq 0.1 \text{ and } |\overline{sf}_i^*| < |\underline{sf}_i^*| \\ +1 & \text{if } |sf_i^*| < 0.1 \end{cases} \quad (43)$$

where $sf_i^* = [\underline{sf}_i^*, \overline{sf}_i^*]$ is the maximum sf (in the absolute sense) for the ith fault in different parity equations with nonzero coefficient. This definition indicates the sign of sf with $p_{ji} = 0$ follows the sign of the largest sf (for the same fault assumption, sf_i^*) if its (absolute) value exceeds 0.1 and it has a definite sign. If the sign of sf_i^* is undetermined ("~"), then sgn* takes the sign of centroid of sf_i^*. Despite the fact that the definition of sgn* appears complicated, the computing procedure is rather straightforward.

3.5. Fault Isolation. Two measures d_i, degree of fault, and cf_i, consistency factor, are utilized to discriminate faults once sf's are available. d_i's indicate the direction and degree of the deviations in the parity equations as the result of a fault, and cf_i represents the consistency among the parity equations as a fault occurs. For the interval systems, similar approach can be taken.

The degree of fault (d_i) for the ith fault assumption is

$$d_i = \left(\sum_{j=1}^{m} sf_{ji}\right) \Big/ m \quad (44)$$

From the definition, it is clear that d_i is simply the arithmetic mean of sf_{ji} in the ith column. However, unlike crisp DMA, d_i can be an interval number (since sf_{ji}'s can be interval numbers) with the lower and upper bounds falling between −1 and 1. A positive (or negative) d_i indicates that a_i deviates from its nominal value in a positive (or negative) direction. In theory, the true fault origin (a_i) under *perfect modeling* and noise-free conditions gives a zero width (crisp) d_i as implied by the example in Figure 4 (crisp sf's give crisp d_i).

In the crisp DMA, cf_i measures the similarity between the sf_{ji}'s ($j = 1, 2, ..., m$). The range where *crisp* sf_{ji} distributed is normalized and cf_i can be calculated subsequently (Chang *et al.*, 1994). For the interval systems, comparison has to be made between *interval* sf_{ji}'s, and the distribution of these intervals can, then, be utilized for finding cf_i. A simple way for the consistency checking is to use the center point (centroid) of an interval, instead of the interval itself, for comparison. Therefore, the computation of cf_i for the interval systems is exactly the same as the crisp one except that centroid of sf_{ji} is employed. The center point, a^c, of an interval a is $a^c = (1/2)(\underline{a} + \overline{a})$. Following this modification the computation of cf_i is exactly the same as the crisp DMA.

$$cf_i = 1 - \left[\frac{sf^c_{max,i} - sf^c_{min,i}}{\max(|sf^c_{1i}|, |sf^c_{2i}|, ..., |sf^c_{mi}|)}\right] \quad (45)$$

in which

$$sf^c_{max,i} = \max_i(sf^c_{1i}, sf^c_{2i}, ..., sf^c_{mi}) \quad (46)$$

$$sf^c_{min,i} = \min_i(sf^c_{1i}, sf^c_{2i}, ..., sf^c_{mi}) \quad (47)$$

Notice that cf_i is a *crisp* number and ranges from +1 to −1 (consistency to inconsistency). Once the consistency factor cf_i and degree of fault d_i are calculated, the complete measure of fault in a_i is

$$(d_i)_{cf_i} \quad (48)$$

This is a combined index where d_i indicates the degree and direction of the assumed fault and cf_i measures the consistency of the assumption from the system (the set of the parity equations) point of view.

3.6. Results. Based on different knowledge abstraction, process information can be expressed in terms of crisp, interval, or fuzzy numbers. Examples of Chang *et al.* (1994) are used to illustrate the similarities and differences between the crisp, interval, and fuzzy information employed.

3.6.1. Crisp vs Interval. Assume that system parameters are known to ±10% of the exact value for the diagnostic procedure. Consider the examples of part 1 (Chang *et al.*, 1994). For the crisp information employed, we have

$$e_1 = 2a_1 + 2a_2 + 0a_3 - 4 \tag{49}$$

$$e_2 = 2a_1 - 2a_2 + 2a_3 - 2 \tag{50}$$

For the interval information employed, if p_{11} and p_{21} are known to ±10% of the exact values, then the parity equations becomes

$$e_1 = [1.8, 2.2]a_1 + 2a_2 + 0a_3 - 4$$

$$e_2 = [1.8, 2.2]a_1 - 2a_2 + 2a_3 - 2 \tag{51}$$

The difference between these two systems lies in the fact that the system is known to different degrees. Figure 5A shows the parameter p_{11} (2 in eq 50 and [1.8, 2.2] in eq 51) with different levels of abstraction. Assume that the tolerances are taken as 10% of the nominal system parameters as

$$\tau_{ji} = p_{ji} \cdot 10\% \tag{52}$$

Following the design procedure, the DMA is constructed and the diagnostic results for 20% error in a_1 for the crisp and interval are

	$(d_1)_{cf_1}$	$(d_2)_{cf_2}$	$(d_3)_{cf_3}$
crisp	$(0.94)_1$	$(0)_{-1}$	$(0.46)_0$
interval	$(0.94)_1$	$([-0.02, 0.02])_{-0.95}$	$([0.45, 0.47])_0$

(53)

Figure 5B illustrates the degree of fault (d_1, d_2, and d_3) for these two systems. The results clearly indicate that, despite the fact that fuzzier information is employed in the DMA, the diagnostic resolution of DMA is as good as that of the crisp DMA (the one requires more precise process information). Similar results can be obtained for the other two examples as shown in Table 1. One important point from these examples (Table 1) is that, contrary to one's intuition, fuzzier information of an interval (or fuzzy) parity equation does not deteriorate diagnostic resolution. Actually, relatively speaking, the diagnostic resolution is improved since the true fault origin gives noninterval (crisp) d_i. The reason for that is 2-fold. The "SIA" arithmetics are involved in DMA, and this algorithm tends to produce crisp d_i in the true fault origin for a system with an interval model under "perfect" process conditions (the process parameters remain the same under nominal and faulty conditions).

3.6.2. Interval vs Fuzzy. The interval DMA is compared to a fuzzy set modeled DMA. The bell-shaped fuzzy number (Dubois and Prade, 1978) is used to describe uncertain process information. Here the fuzzy number is characterized by two numbers β and γ which indicate the peak point and bandwidth of the fuzzy set, respectively (Figure 15). In terms of equations we have $\{\beta, \gamma\}$. Therefore, in the case of fuzzy system, the parity equations are defined as

$$e_1 = \{2, 0.2\}a_1 + 2a_2 + 0a_3 - 4$$

$$e_2 = \{2, 0.2\}a_1 - 2a_2 + 2a_3 - 2 \tag{54}$$

Figure 5A compares the fuzziness of the process information employed. The Appendix gives the computation procedure for the fuzzy set, and its corresponding arithmetic is only an approximation to the true fuzzy operation. The diagnostic results are

	$(d_1)_{cf_1}$	$(d_2)_{cf_2}$	$(d_3)_{cf_3}$
interval	$(0.94)_1$	$([-0.02, 0.02])_{-0.95}$	$([0.45, 0.47])_0$
fuzzy	$(\{0.94, 0.35\})_1$	$(\{0, 0.23\})_{-0.95}$	$(\{0.46, 0.23\})_0$

(55)

Figure 5B shows the degree of faults for these two systems. The results indicate that the interval DMA gives better diagnostic resolution than the fuzzy DMA despite the fact that more involved process information is employed (Figure 5A). One reason for that is different arithmetics are used in the fuzzy operation (Chang *et*

Table 1. Examples (Chang *et al.*, 1994) and Diagnostic Results by Using DMA

(A) Crisp Information

example 1
$e_1 = 2a_1 + 2a_2 + 0a_3 - 4$
$e_2 = 2a_1 - 2a_2 + 2a_3 - 2$

example 2
$e_1 = 1a_1 + 10a_2 + 0a_3 - 11$
$e_2 = 10a_1 - 1a_2 + 10a_3 - 19$

example 3
$e_1 = 1a_1 + 10a_2 + 10a_3 - 21$
$e_2 = 1a_1 - 11a_2 + 1a_3 - 13$

(B) Interval Information

example 1
$e_1 = [1.8, 2.2]a_1 + 2a_2 + 0a_3 - 4$
$e_2 = [1.8, 2.2]a_1 - 2a_2 + 2a_3 - 2$

example 2
$e_1 = [0.9, 1.1]a_1 + 10a_2 + 0a_3 - 11$
$e_2 = [9, 11]a_1 - 1a_2 + 10a_3 - 19$

example 3
$e_1 = [0.9, 1.1]a_1 + 10a_2 + 10a_3 - 21$
$e_2 = [0.9, 1.1]a_1 - 11a_2 + 1a_3 - 13$

(C) Fuzzy Information

example 1
$\tilde{e} = \{2, 0.2\}a_1 + 2a_2 + 0a_3 - 4$
$\tilde{e}_2 = \{2, 0.2\}a_1 - 2a_2 + 2a_3 - 2$

example 2
$\tilde{e}_1 = \{1, 0.1\}a_1 + 10a_2 + 0a_3 - 11$
$\tilde{e}_2 = \{10, 1\}a_1 - 1a_2 + 10a_3 - 19$

example 3
$\tilde{e}_1 = \{1, 0.1\}a_1 + 10a_2 + 10a_3 - 21$
$\tilde{e}_2 = \{1, 0.1\}a_1 - 11a_2 + 1a_3 - 13$

(D) Diagnostic Results (20% Faults in a_1)

	$(d_1)_{cf_1}$	$(d_2)_{cf_2}$	$(d_3)_{cf_3}$
example 1			
crisp	$(0.94)_{1.0}$	$(0)_{-1.0}$	$(0.46)_0$
interval	$(0.94)_{1.0}$	$[-0.02, 0.02]_{-0.95}$	$[0.45, 0.47]_0$
fuzzy	$\{0.94, 0.35\}_{1.0}$	$\{0, 0.23\}_{-0.95}$	$\{0.46, 0.23\}_0$
example 2			
crisp	$(0.94)_{1.0}$	$(-0.5)_0$	$(0.5)_{0.062}$
interval	$(0.94)_{1.0}$	$(-0.5)_0$	$[0.477, 0.523]_{0.0675}$
fuzzy	$\{0.94, 0.386\}_{1.0}$	$\{-0.5, 0.5\}_0$	$\{0.5, 0.012\}_{0.06}$
example 3			
crisp	$(0.94)_{1.0}$	$(0.0005)_{0.68}$	$(0.47)_0$
interval	$(0.94)_{1.0}$	$[0.0002, 0.0009]_{0.68}$	$[0.457, 0.48]_{0.0013}$
fuzzy	$\{0.94, 0.364\}_{1.0}$	$\{0.0005, 0.001\}_{0.69}$	$\{0.48, 0.121\}_{0.001}$

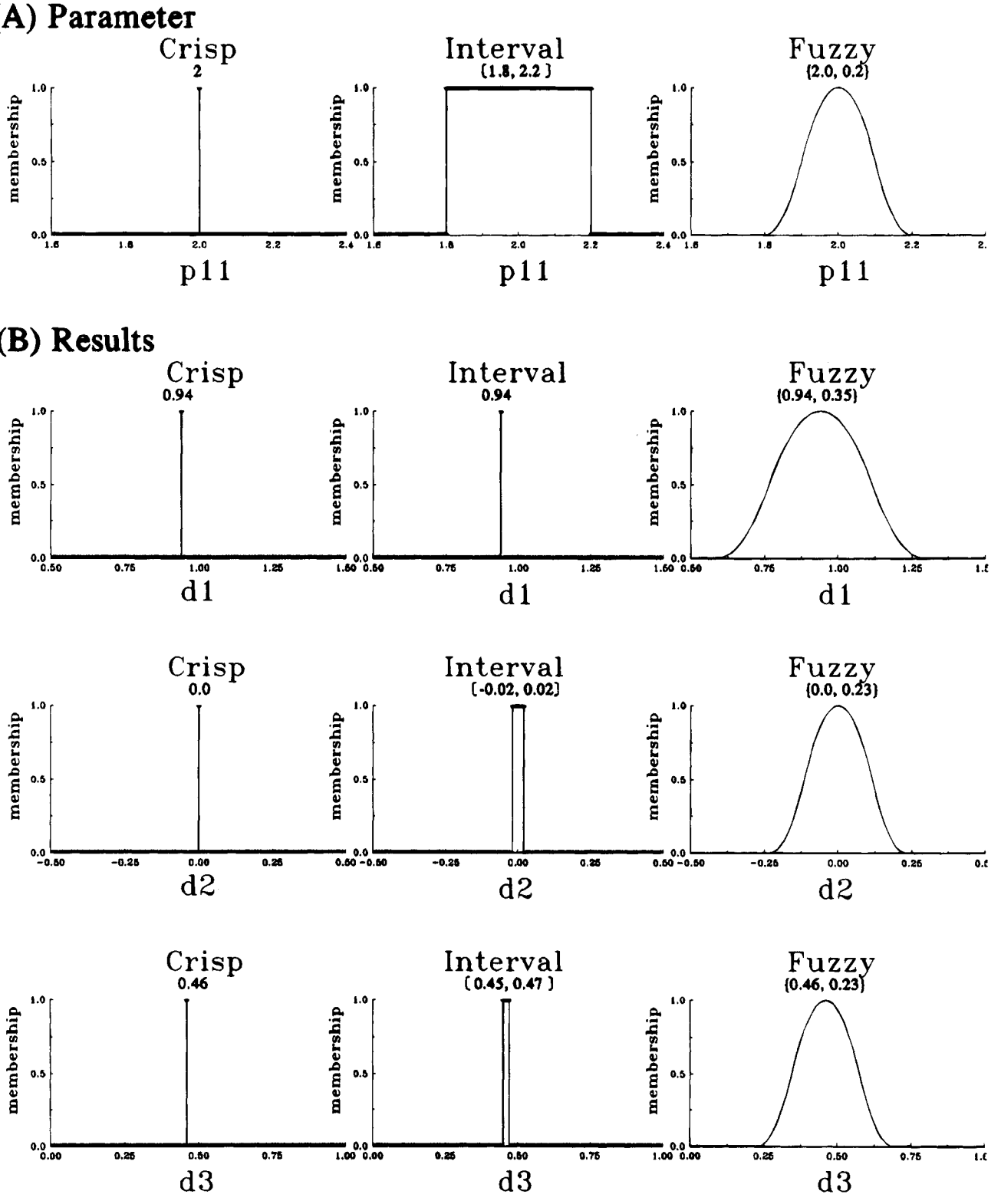

Figure 5. Diagnosis results of example 1 with different information employed.

al., 1993). Nonetheless, the results clearly indicate that the proposed interval DMA is very effective in fault diagnosis as compared to the crisp or fuzzy DMA.

3.7. Characteristics. As mentioned earlier, ambiguous information does not necessarily deteriorate diagnostic resolution in interval DMA. A system (example of Table 1A) with three different levels of abstraction (ranging from crisp to purely qualitative systems) is used to illustrate the relationship between ambiguity and resolution. In all the systems investigated tolerances (τ_{ji}) are taken as 10% of the nominal value of coefficients (p_{ji}).

(1) All coefficients are crisp:

$$e_1 = 1a_1 + 10a_2 + 0a_3 - 11$$

$$e_2 = 10a_1 - 1a_2 + 10a_3 - 19 \qquad (56)$$

A range of faults in a_i is used to test the resolution of interval DMA. The results (Figure 6) show that DMA

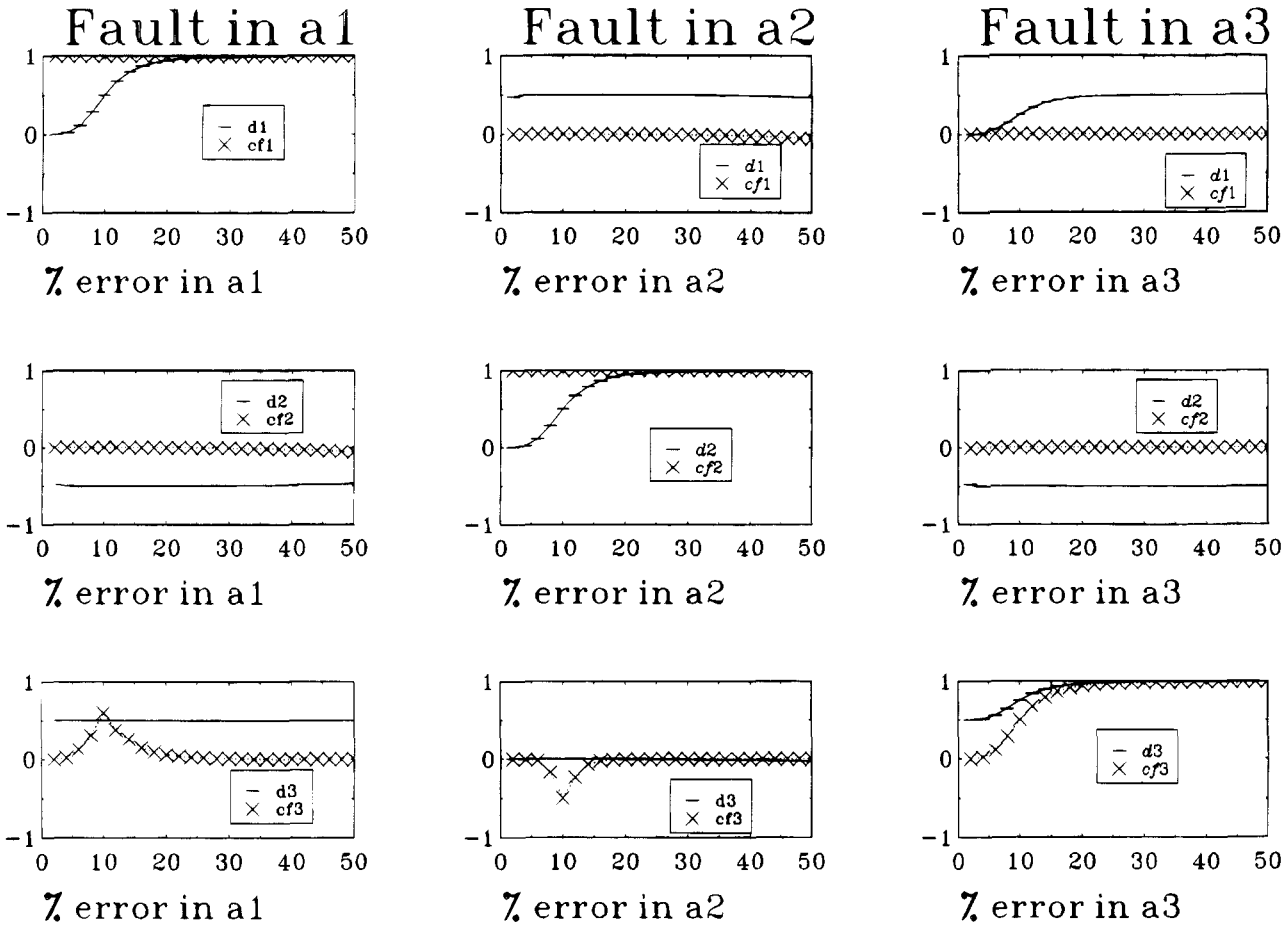

Figure 6. Diagnosis results of example 2 (eq 56) with crisp (0% uncertainty) p_{11} and p_{21} under different degree of fault.

can correctly identify the fault origin with the crisp pair of $(d_i)_{cf_i}$ and as the magnitude of the fault increases clearer indication of fault results.

(2) The coefficients p_{11} and p_{21} are known to ±50% of exact values:

$$e_1 = [0.5, 2.0]a_1 + 10a_2 + 0a_3 - 11$$

$$e_2 = [5, 20]a_1 - 1a_2 + 10a_3 - 19 \qquad (57)$$

In this case, p_{11} is known to fall in [0.5, 2] and p_{21} is [5, 20]. Again, faults in a_i (ranging from 0 to 50%) are used to test DMA with interval parity equations. Figure 7 shows that, similar to the results of the crisp system (Figure 6), interval DMA, $(d_i)_{cf_i}$ can correctly find the fault origin. The results of $(d_i)_{cf_i}$ for faults in a_i are exactly the same as that of the crisp system (graphs in the diagonal of Figures 6 and 7). However, interval d_i's result when a fault occurs in a_j $(j \neq i)$ as shown in Figure 7. The results clearly show that, as opposed to performance deterioration using ambiguous information, interval DMA differentiates the true fault origin from others utilizing the fuzzy and nonfuzzy (crisp) nature of process knowledge. That is in part due to the employed SIA (Chang *et al.*, 1993) as will be explained in greater detail later.

(3) The coefficients p_{11} and p_{12} approach pure qualitative values:

$$e_1 = [10^{-4}, 10^{10}]a_1 + 10a_2 + 0a_3 - 11$$

$$e_2 = [10^{-4}, 10^{10}]a_1 - 1a_2 + 10a_3 - 19 \qquad (58)$$

Since numerical values are needed for interval DMA, qualitative values are represented using exactly small (*e.g.*, 10^{-4}) and large (*e.g.*, 10^{10}) values. Again, interval DMA using eq 58 is tested for a range of failure in a_i. Similar to the results of 50% uncertainties in p_{11} and p_{21} (Figure 7), $(d_i)_{cf_i}$ for a fault in a_i are exactly the same as that of the previous two cases. However, the width of the interval of d_j grows for incorrect interpretation (Figure 8).

The results of these three different levels of abstraction show the advantages of interval DMA when fuzzier information is employed. The reason for that is actually quite simple. Consider the set of two parity equations (crisp or interval type); if a_1 is fault ($a_1 = (1 + \alpha_1)a_1^s$, $a_2 = a_2^s$, and $a_3 = a_3^s$) the residuals are

$$e_{net,1} = e_1 - e_1^s = p_{11}(1 + \alpha_1)a_1^s + p_{12}a_2^s + p_{13}a_3^s + k_1 - (p_{11}a_1^s + p_{12}a_2^s + p_{13}a_3^s + k_1)$$

$$= p_{11}\alpha_1 a_1^s \qquad (59a)$$

$$e_{net,2} = e_2 - e_2^s = p_{21}(1 + \alpha_1)a_1^s + p_{22}a_2^s + p_{23}a_3^s + k_2 - (p_{21}a_1^s + p_{22}a_2^s + p_{23}a_3^s + k_2)$$

$$= p_{12}\alpha_1 a_1^s \qquad (59b)$$

and the tolerances are taken as 10% of their nominal values.

$$\tau_{ji} = 0.1p_{ji} \qquad (60)$$

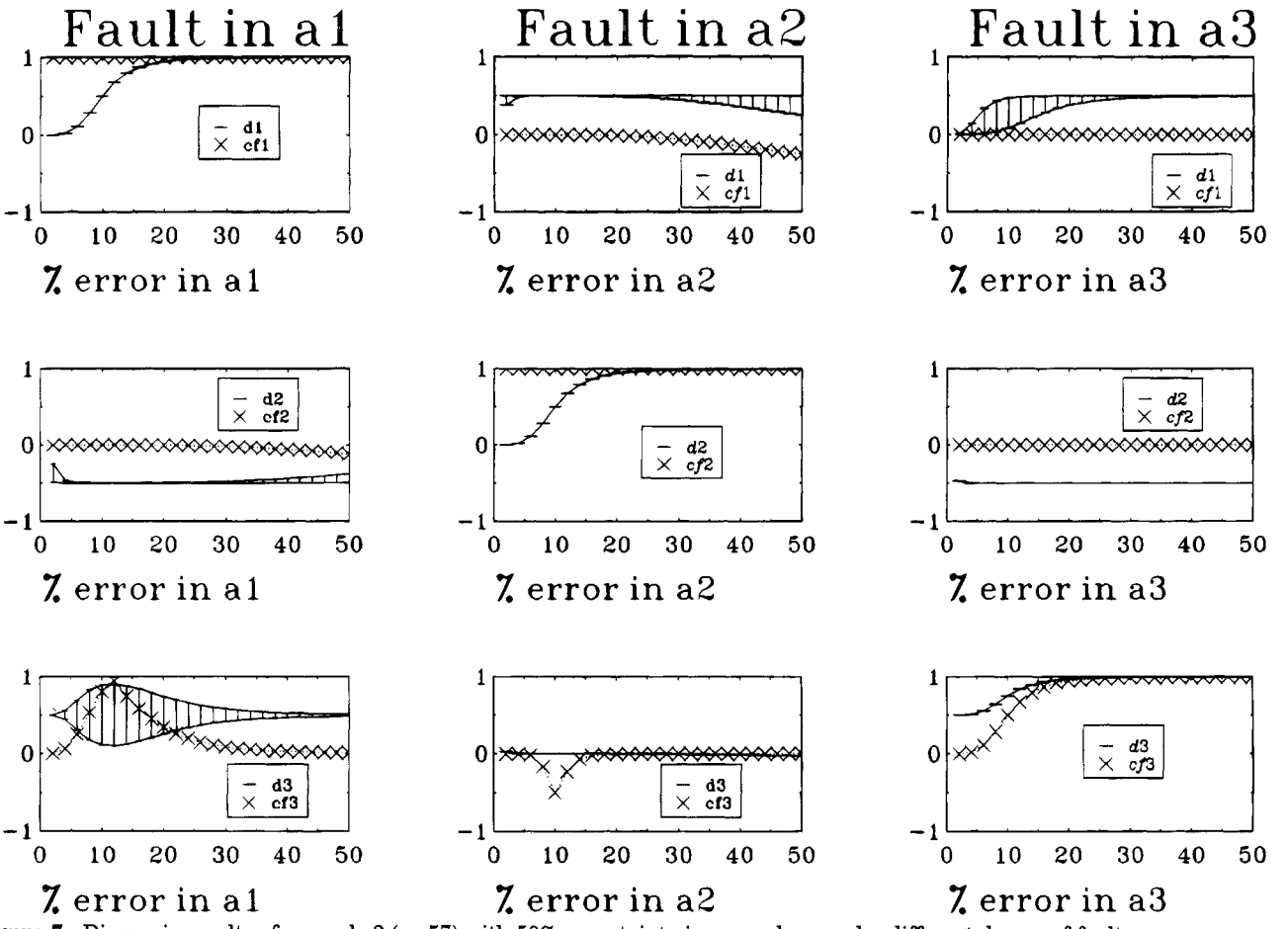

Figure 7. Diagnosis results of example 2 (eq 57) with 50% uncertainty in p_{11} and p_{21} under different degree of fault.

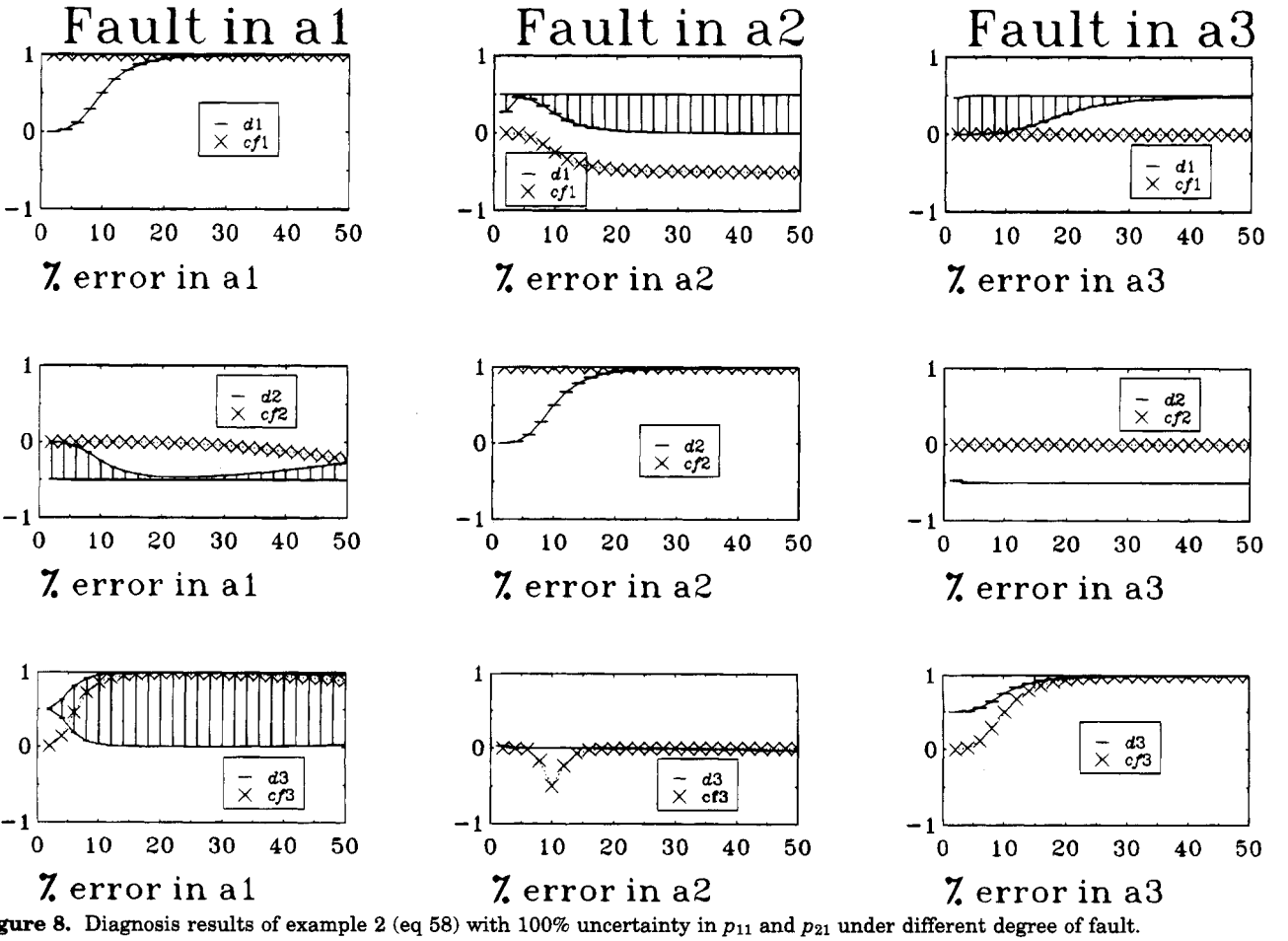

Figure 8. Diagnosis results of example 2 (eq 58) with 100% uncertainty in p_{11} and p_{21} under different degree of fault.

Table 2. Steady-State Operating Conditions for CSTR

$F = 40$ ft^3/h	$U = 150$ BTU/(h·ft^2·°R)
$V = 48$ ft^3	$A = 250$ ft^2
$C_{A_0} = 0.50$ mol/ft^3	$T_{j0} = 530$ °R
$C_A = 0.245$ mol/ft^3	$\Delta H = -30\,000$ BTU/mol
$T = 600$ °R	$c_p = 0.75$ BTU/(lb$_m$ °R)
$T_j = 594.6$ °R	$c_j = 1.0$ BTU/(lb$_m$ °R)
$F_j = 49.9$ ft^3/h	$\varrho = 50$ lb$_m$/ft^3
$V_j = 3.85$ ft^3	$\varrho_j = 62.3$ lb$_m$/ft^3
$k_0 = 7.08 \times 10^{10}$ h^{-1}	$T^{set} = 600$ °R
$E = 30\,000$ BTU/mol	$k_{c1} = 32$
$R = 1.987$ BTU/(mol °R)	$k_{c2} = 10$
$A_h = 19.6$ ft^2	$\tau_{I^1} = 0.9$ h
$F_{max} = 96$ ft^3/h	$\tau_{I^2} = 0.6$ h
bias$_1$ = 12 psi	$L^{set} = 0.192$ ft
bias$_2$ = 9 psi	

Therefore, based on the definition of sf_{ji} (eq 39), sf_{11} and sf_{21} are

$$sf_{11} = \frac{(e_{net,1}/\tau_{11})^4}{1 + (e_{net,1}/\tau_{11})^4} = \frac{\left(\frac{\alpha_1 a_1^s p_{11}}{0.1 p_{11}}\right)^4}{1 + \left(\frac{\alpha_1 a_1^s p_{11}}{0.1 p_{11}}\right)^4} \quad (61a)$$

$$sf_{21} = \frac{(e_{net,2}/\tau_{21})^4}{1 + (e_{net,2}/\tau_{21})^4} = \frac{\left(\frac{\alpha_1 a_1^s p_{21}}{0.1 p_{21}}\right)^4}{1 + \left(\frac{\alpha_1 a_1^s p_{21}}{0.1 p_{21}}\right)^4} \quad (61b)$$

In general, both the numerator $e_{net,i}$ and denominator τ_{ji} can be interval numbers. However, according to SIA (Chang *et al.*, 1993), the ratio of these two interval number is simply a crisp number. Therefore, we have

$$sf_{11} = \frac{\left(\frac{\alpha_1 a_1^s}{0.1}\right)^4}{1 + \left(\frac{\alpha_1 a_1^s}{0.1}\right)^4} \quad (62a)$$

$$sf_{21} = \frac{\left(\frac{\alpha_1 a_1^s}{0.1}\right)^4}{1 + \left(\frac{\alpha_1 a_1^s}{0.1}\right)^4} \quad (62b)$$

The degree of fault d_i, then, becomes

$$d_i = \frac{sf_{11} + sf_{21}}{2} = \frac{(10\alpha_1 a_1^s)^4}{1 + (10\alpha_1 a_1^s)^4} \quad (63)$$

which is again a crisp number. Generally, the ratio of two different interval coefficients, *e.g.*, p_{11}/p_{21}, will not result in crisp sf and d. The unique property shown here is the result of interval DMA and the employment of SIA in interval arithmetics.

4. Application

A CSTR example (Chang *et al.*, 1994) is used to illustrate the design and diagnosis of the interval DMA. It should be noticed that at a given instance any physical system is always described by a "crisp" process model. In this example, a set of interval parity equations is used to describe the "crisp" physical system. Following the linearization procedure with respect to the fault origins and the known process measurements, the linearized parity equations become

Table 3. Cases of Different Uncertainty

	case 1	case 2	case 3
U, BTU/(h·ft^2·°R)	[135, 165]	[135, 165]	[120, 180]
ΔH, BTU/mol	−30 000	[−33 000, −27 000]	[−57 000, −3000]

Table 4. Fault Origins of CSTR Example

symbol	fault origin
F_0	changes in the feed flow rate
C_{A_0}	changes in the feed concentration
k_0	changes in the preexponential factor of the rate constant
U	changes in the overall heat transfer coefficient
T_j	sensor failure in cooling water outlet temperature

$$0 = F_0 - F \quad (64)$$

$$0 = \left(T_0^s - \frac{\Delta H}{\varrho c_p}\frac{C_{A_0}^s k^s}{F^s + k^s V^s} V^s\right) F_0 + F_0^s T_0 + \left(-F^s - \frac{U^s A}{\varrho c_p} - \frac{\Delta H}{\varrho c_p}\frac{F_0^s F^s C_{A_0}^s V^s}{(F^s + k^s V^s)^2}\frac{E k^s}{R T^{s2}}\right) T - \left(T^s - \frac{\Delta H}{\varrho c_p}\frac{F_0^s K^s C_{A_0}^s V^s}{(F^s + k^s V^s)^2}\right) F + \left(\frac{U^s A}{\varrho c_p}\right) T_j + \left(-\frac{A}{\varrho c_p} T^s + \frac{A}{\varrho c_p} T_j^s\right) U + \left(-\frac{\Delta H}{\varrho c_p}\frac{F_0^s F^s C_{A_0}^s V^s}{(F^s + k^s V^s)^2} e^{-E/RT^s}\right) k_0 + \left(-\frac{\Delta H}{\varrho c_p}\frac{F_0^s k^s V^s}{F^s + K^s V^s}\right) C_{A_0} + \left(-\frac{\Delta H}{\varrho c_p}\frac{C_{A_0}^s k^s F_0^s}{F^s + k^s V^s}\right) V + \left(\frac{\Delta H}{\varrho c_p}\frac{F_0^s F^s C_{A_0}^s V^s}{(F^s + k^s V^s)^2}\frac{E k^s}{R T^s} + \frac{\Delta H}{\varrho c_p}\frac{F_0^s V^s C_{A_0}^s k^s}{(F^s + k^s V^s)}\right) \quad (65)$$

$$0 = F_j^s T_{j0} + \frac{A}{\varrho_j c_{pj}}(T^s - T_j^s) U + \frac{A}{\varrho_j c_{pj}} U^s T - (T_j^s - T_{j0}^s) F_j - \left(F_j^s + \frac{A U^s}{\varrho_j c_{pj}}\right) T_j \quad (66)$$

Steady-state conditions are listed in Table 2. Up to this point, the interval DMA design procedure is exactly the same as the crisp one. In any real system, such as this CSTR process, some parameters are only known to a certain degree. For example, the heat of reaction ΔH or activation energy E is generally not exactly known. Even if the parameter is exactly known at a given instance, it may change or degrade over time. The overall heat transfer coefficient U is a typical example. Therefore, an interval process model is an ideal choice for realistic process description over a range of operating conditions. In this example, ΔH and U are assumed to be known in a given range and three cases with different levels of abstraction are studied (Table 3). Five possible fault origins are to be diagnosed which include load changes, performance degradation, and sensor failure as shown in Table 4.

4.1. Case 1: Heat transfer coefficient is an interval number (±10%). If the overall heat transfer coefficient (U) is known to be within the range $U = [135, 165]$, the parity equations can be obtained by substitut-

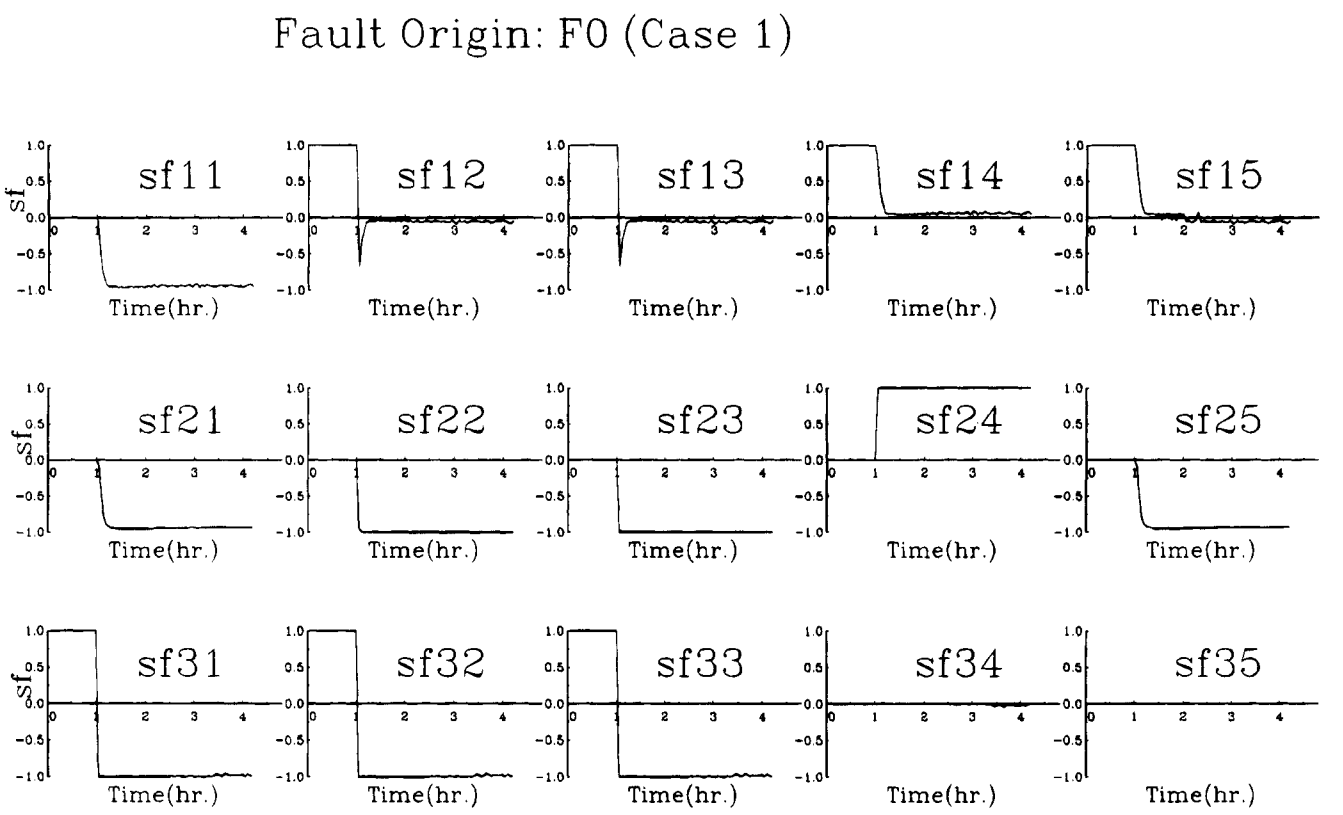

Figure 9. Satisfaction factors of case 1 in CSTR example.

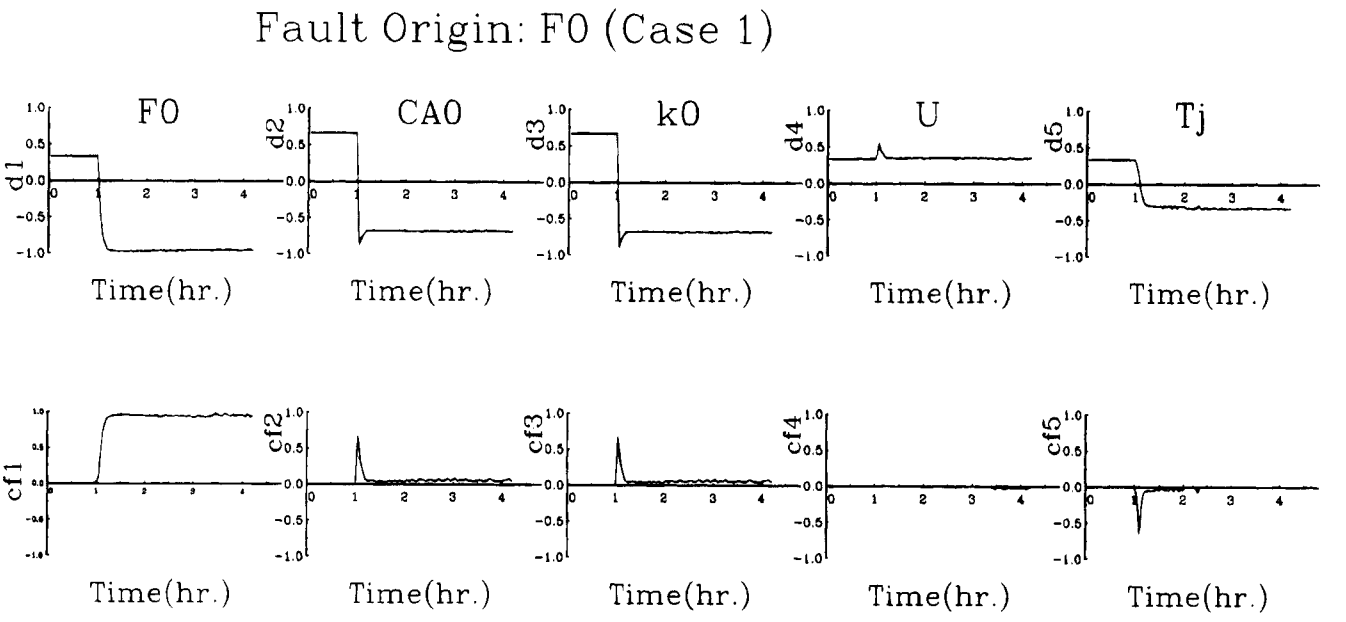

Figure 10. The diagnostic resolutions $(d_i)_{cf_j}$ of case 1 in CSTR example.

ing steady-state values of Table 2 into eqs 63–65. The vector form of the parity equations is

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 7.395 & 2.03 & 1.0 & -1.357 & [135.5, 165.6] \\ 0 & 0 & 0 & 1.0 & -[109.1, 131.3] \end{bmatrix} \times \begin{bmatrix} \check{F}_0 \\ \check{C}_{A_0} \\ \check{k}_0 \\ \check{U} \\ \check{T}_j \end{bmatrix} + \begin{bmatrix} -\check{F} \\ [-147.7, -117.3]\check{T} + 5.367\check{T}_0 - 7.089\check{F} + 2.03\check{V} + (-27.51) \\ [100.78, 123.17]\check{T} + (-1)\check{F}_j + 8.201\check{T}_{j0} \end{bmatrix} \quad (67)$$

Following the interval DMA procedure (section 3), the tolerances are

$$\begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} & \tau_{14} & \tau_{15} \\ \tau_{21} & \tau_{22} & \tau_{23} & \tau_{24} & \tau_{25} \\ \tau_{31} & \tau_{32} & \tau_{33} & \tau_{34} & \tau_{35} \end{bmatrix} = \begin{bmatrix} 0.1 & \tau_{1,\min} & \tau_{1,\min} & \tau_{1,\min} & \tau_{1,\min} \\ 0.739 & 0.2 & 0.1 & -0.135 & [13.5, 16.5] \\ \tau_{3,\min} & \tau_{3,\min} & \tau_{3,\min} & 0.1 & -[10.9, 13.1] \end{bmatrix} \quad (68)$$

Here, $\tau_{j,\min}$'s indicate the positions where the coefficients

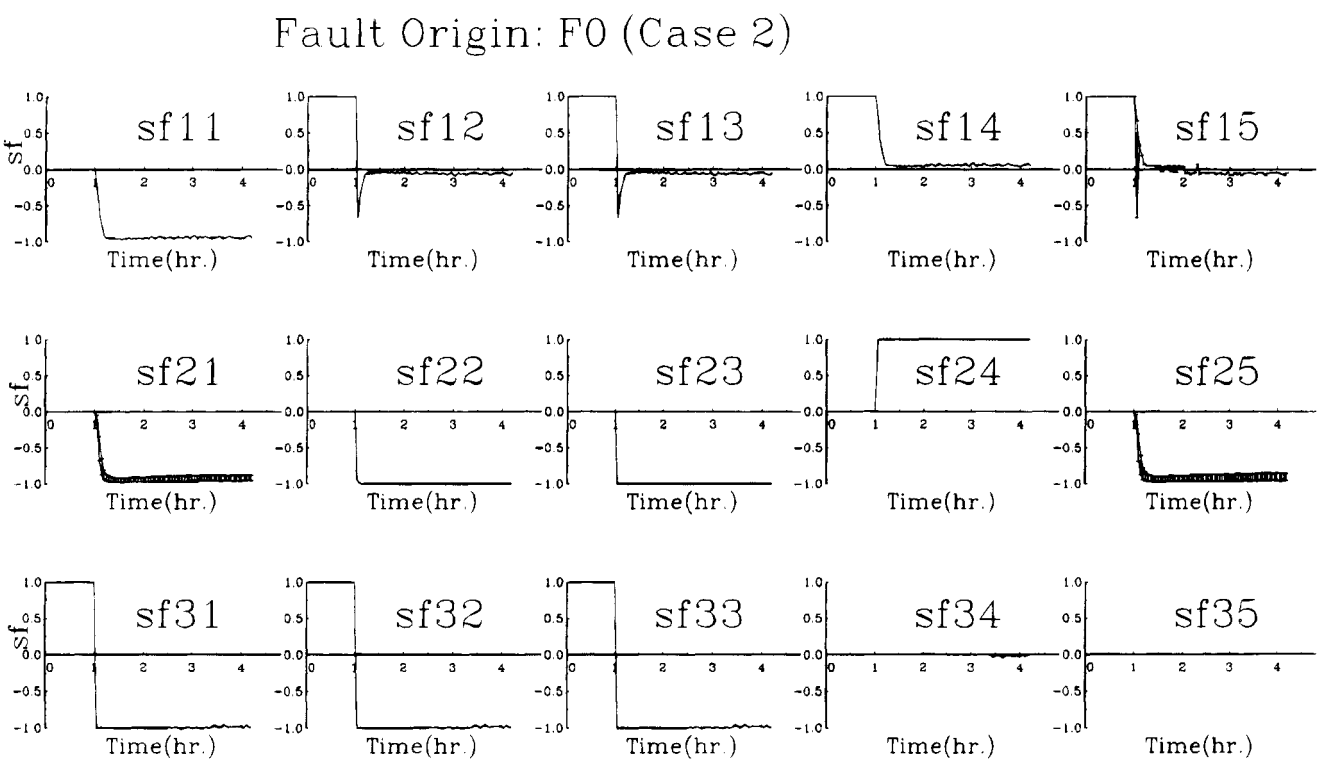

Figure 11. Satisfaction factors of case 2 in CSTR example.

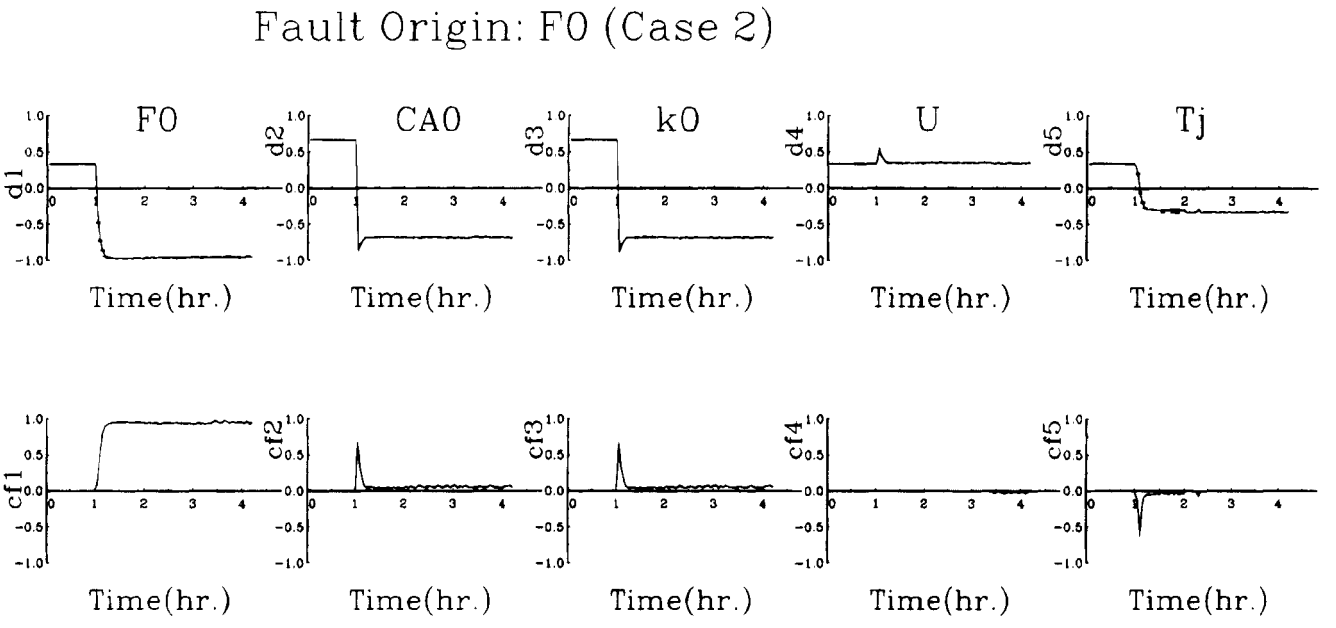

Figure 12. The diagnostic resolutions $(d_i)_{cf_j}$ of case 2 in CSTR example.

are zero. According to eq 42, $\tau_{1,min}$ and $\tau_{3,min}$ take the value of 0.1. For a 20% decrease in F_0 ($t = 1$ h), the interval DMA computes the satisfaction factors as shown in Figure 9. Results (Figure 9) show that sf_{11}, sf_{21}, and sf_{31} give a rather consistent indication of a F_0 decrease and the proposed measure $(d_1)_{cf_1}$ also confirms this (Figure 10). Despite the fact that interval residuals result from the fault, the diagnosis results (sf_{ji}'s and d_i's) remain rather crisp as the results of relatively small residuals as compared to the tolerances.

4.2. Case 2: Heat transfer coefficient and heat of reaction are interval numbers (both ±10%). In this case, both the heat transfer coefficient and heat of reaction are known to ±10% of their exact value, *e.g.*, $U = [135, 165]$ and $\Delta H = [-33\,000, -27\,000]$. Substituting known data and the interval numbers (*i.e.*, U and ΔH) into the parity equations, we have

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ [6.9, 7.9] & 2.03 & 1.0 & -1.357 & [135.5, 165.6] \\ 0 & 0 & 0 & 1.0 & -[109.1, 131.3] \end{bmatrix} \times \begin{bmatrix} \check{F}_0 \\ \check{C}_{A_0} \\ \check{k}_0 \\ \check{U} \\ \check{T}_j \end{bmatrix} + \begin{bmatrix} -\check{F} \\ [-161.3, -133.2]\check{T} + 5.367\check{T}_0 - [7.76, 7.99]\check{F} + [2.02, 2.48]\check{V} + [-30.26, -24.76] \\ [100.78, 123.17]\check{T} + (-1)\check{F}_j + 8.201\check{T}_{j0} \end{bmatrix} \quad (69)$$

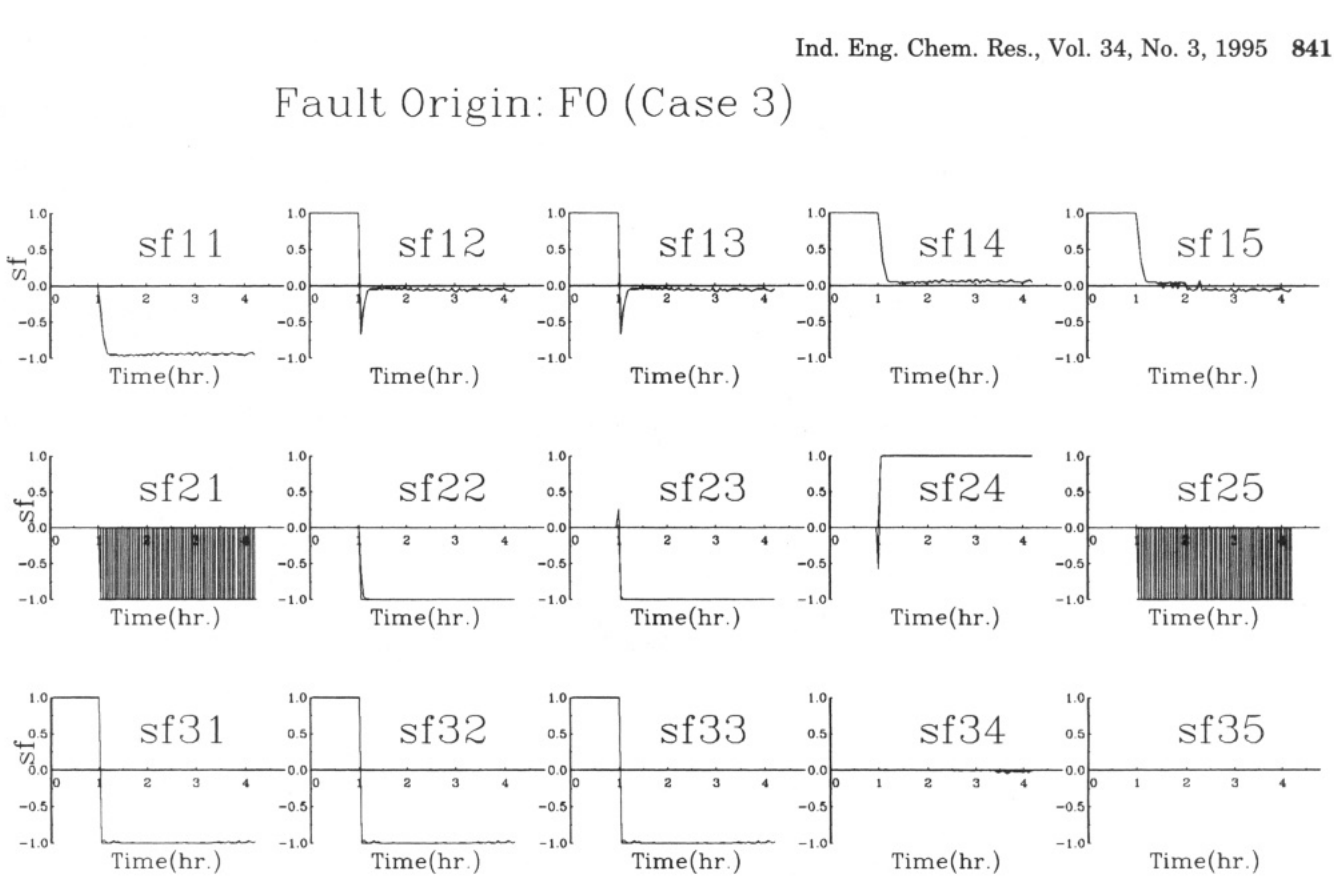

Figure 13. Satisfaction factors of case 3 in CSTR example.

Again a 20% increase in F_0 (at $t = 1$ h) is used to evaluate interval DMA. Here, the ambiguities (in U and ΔH) lead to satisfaction factors with more significant interval nature as shown in Figure 11. However, the sf's still give a consistent indication of the fault origin (F_0) as shown in Figure 12 using $(d_i)_{cf_i}$. Notice that the consistency factor (cf_i) gives a clear indication of fault regardless of the ambiguities involved in the parity equations.

4.3. Case 3: Heat transfer coefficient and the heat of reaction are interval numbers (±20% and ±90%, respectively). In this case, a still more ambiguous knowledge is employed with $U = [120, 180]$ and $\Delta H = [-57\,000, -3000]$. The parity equations become

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ [4.85, 55.7] & 2.03 & 1.0 & -1.357 & [120.44, 180.66] \\ 0 & 0 & 0 & 1.0 & -[97.98, 142.57] \end{bmatrix} \begin{bmatrix} \check{F}_0 \\ \check{C}_{A_0} \\ \check{k}_0 \\ \check{U} \\ \check{T}_j \end{bmatrix} +$$

$$\begin{bmatrix} -\check{F} \\ [-139.9, -125.05]\check{T} + 5.367\check{T}_0 + [-8.05, -6.17]\check{F} + [2.03, 38.5]\check{V} + [-52.27, -2.75] \\ [89.58, 134.37]\check{T} - (1)\check{F}_j + 8.201\check{T}_{j0} \end{bmatrix} \tag{70}$$

Again, a 20% increase in F_0 is used to test the interval DMA. Similar to the results of case 2, a wider interval coefficient leads to sf's with wider intervals, as shown in Figure 13. However, $(d_i)_{cf_i}$ still gives a clear indication of the fault origin (F_0) as shown in Figure 14. Similar results can be obtained for all five different fault origins (Chang, 1994).

The results from different levels of abstraction (cases 1−3) clearly show that the interval DMA is able to utilize ambiguous information in isolating fault origin. More importantly, diagnostic resolution is maintained while a wide range of operating conditions is included.

5. Conclusions

Uncertainties arise naturally in chemical processes as a result of different operating conditions or lack of precise knowledge about system phenomena. In this work an interval model based diagnosis system was proposed. The interval DMA is a direct extension of DMA to handle a wider range of operating conditions. Following the development of DMA, the design procedure of interval DMA was given and the characteristics were also discussed. Different levels of abstraction in an interval model were investigated, and results showed that the increased ambiguity in the parity equations does not degrade the diagnostic resolution. A CSTR example was used to illustrate the design and implementation of the interval DMA. Results showed that the proposed method is not only handling a wider of operating condition but also showing a good diagnostic resolution. Furthermore, it reduces to crisp DMA (part 1) when *all* the system parameters are known exactly.

Nomenclature

A = heat transfer area of CSTR, ft^2
a = vector of fault assumption

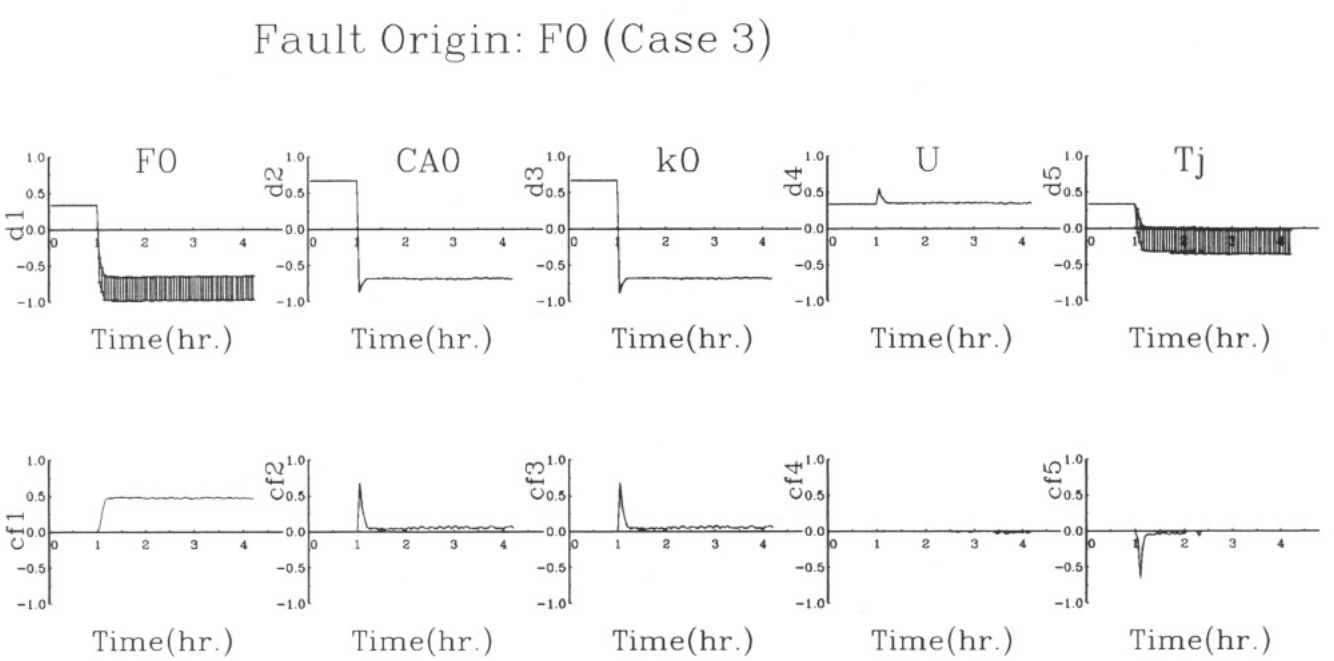

Figure 14. Diagnostic resolutions $(d_i)_{cf_i}$ of case 3 in CSTR example.

a_i = ith fault
$c_j(.)$ = jth confluence of system model
c_p = heat capacity of process liquid, BTU/(lb_m °R)
c_{pj} = heat capacity of cooling water, BTU/(lb_m °R)
C_A = concentration of reactant A, mol/ft^3
C_{A_0} = feed concentration of reactant A, mol/ft^3
cf_i = consistency factor for ith fault
d_i = fault degree of ith fault
E = activation energy, BTU/mol
e = residual vector
e_j = residual value of jth parity equation
e_j^s = approximated zero of jth parity equation
$e_{net,j}$ = net residual value of jth parity equation
F = reactor outlet flow rate, ft^3/h
F_j = cooling water flow rate, ft^3/h
F_0 = reactor inlet flow rate, ft^3/h
ΔH = heat of reaction, BTU/mol
k = constant vector term of parity equations
k_j = constant term of jth parity equation
k_0 = Arrhenius constant, h^{-1}
m = vector of process measurements
m = number of parity equations
n = number of fault origins
P = matrix of p_{ji}
p_{ji} = coefficient for ith fault on jth parity equation
sf_{ji} = satisfaction factor for ith fault of jth parity equation
sf_i^* = the maximum sf (in absolute sense) for the ith fault
sgn* = sign of sf_{ji}, defined by eq 43
T = reactor temperature, °R
T_j = cooling water temperature, °R
T_{j0} = cooling water inlet temperature, °R
T_0 = reactor inlet temperature, °R
U = overall heat transfer coefficient, BTU/(h·ft^2·°R)
V = reactor volume, ft^3
V_j = heat transfer area of jacket, ft^2

Greek Letters

α_i = percent of deviation in a_i
β = peak point of bell-shaped fuzzy number
γ = bandwidth of bell-shaped fuzzy number
μ = membership of fuzzy set
ϱ = density of process liquid, lb_m/ft^3
ϱ_j = density of cooling water, lb_m/ft^3
τ_{Ii} = reset time of ith loop of PI controller
τ_{ij} = tolerance for ith fault of jth parity equation

Subscripts

− = lower bound
max = maximum value
min = minimum value

Superscripts

− = upper bound
~ = fuzzy set
ˇ = normalized fault with respect to steady-state value
c = centroid of interval value
s = steady-state value
set = set point

Appendix. Fuzzy DMA

As the fuzzy information is used to represent system parameter $\tilde{\chi}$, fuzzy number $\tilde{\chi} = \{\beta, \gamma\}$ can be defined by a bell-shaped membership function (Figure 15).

$$\begin{aligned}
\mu_\chi(x) &= 0 && \text{for } x \le \beta - \gamma \\
&= \frac{2}{\gamma^2}(x - \beta + \gamma)^2 && \text{for } \beta - \gamma \le x \le \beta - \frac{\gamma}{2} \\
&= 1 - \frac{2}{\gamma^2}(x - \beta)^2 && \text{for } \beta - \frac{\gamma}{2} \le x \le \beta + \frac{\gamma}{2} \\
&= \frac{2}{\gamma^2}(x - \beta - \gamma)^2 && \text{for } \beta + \frac{\gamma}{2} \le x \le \beta + \gamma \\
&= 0 && \text{for } x \ge \beta + \gamma
\end{aligned} \tag{A1}$$

where x is in the universe $\mathbb{R}$, *i.e.*, $x \in (-\infty, \infty)$. The membership function is characterized by two parameters: (a) the peak point, β, *i.e.*, the point at which $\mu = 1$, and (b) the bandwidth, γ, which is defined as the distance between the crossover points, *i.e.*, the point at which $\mu = 0.5$. Thus, the fuzzy information, $\tilde{\chi}$, is expressed as $\{\beta, \gamma\}$, where β is the peak point and γ is the bandwidth. This short exposition of the properties of fuzzy numbers follows the presentation of Dubois and Prade (1978).

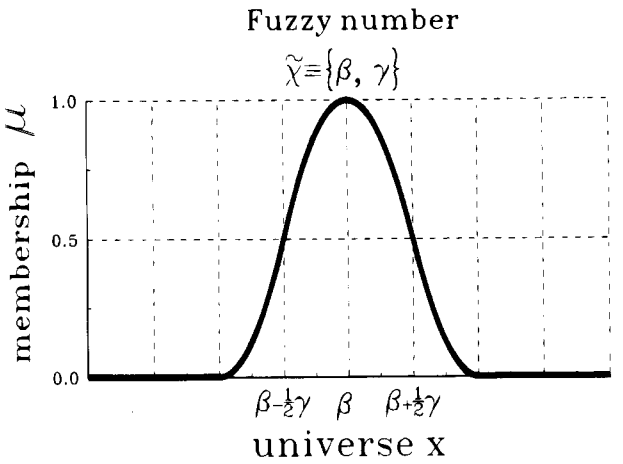

Figure 15. Bell-shaped fuzzy number $\tilde{\chi} = \{\beta, \gamma\}$.

The operations of fuzzy numbers $\{\beta_1, \gamma_1\}$ and $\{\beta_2, \gamma_2\}$ can readily be established by extension principle:

$$\{\beta_1, \gamma_1\} + \{\beta_2, \gamma_2\} = \{\beta_1 + \beta_2, \gamma_1 + \gamma_2\} \quad \text{(A2)}$$

$$\{\beta_1, \gamma_1\} - \{\beta_2, \gamma_2\} = \{\beta_1 - \beta_2, \gamma_1 + \gamma_2\} \quad \text{(A3)}$$

In the case of multiplication and division, the approximations become

$$\{\beta_1, \gamma_1\}\cdot\{\beta_2, \gamma_2\} = \{\beta_1\cdot\beta_2, \gamma_1 + \gamma_2\} \quad \text{(A4)}$$

$$\{\beta_1, \gamma_1\}/\{\beta_2, \gamma_2\} = \left\{\frac{\beta_1}{\beta_2}, \left(\frac{\gamma_1}{\beta_1} + \frac{\gamma_2}{\beta_2}\right)\right\}$$
$$\beta_1 \neq 0 \text{ and } \beta_2 \neq 0 \quad \text{(A5)}$$

Assume that the fuzzy residual and tolerance are formed as $\tilde{e}_j = \{\beta_e, \gamma_e\}$ and $\tilde{\tau}_{ji} = \{\beta_t, \gamma_t\}$, respectively. For DMA diagnostic procedure, the satisfaction factors, $\tilde{sf}_{ji}$, degree of fault, $\tilde{d}_i$, and consistency factor, cf_i, are redefined as

$$\tilde{sf}_{ji} = \{\beta_{ji}, \gamma_{ji}\} = \text{sgn}(\tilde{e}_j/\tilde{\tau}_{ji})\frac{(\tilde{e}_j/\tilde{\tau}_{ji})^4}{1 + (\tilde{e}_j/\tilde{\tau}_{ji})^4}$$

$$= \text{sgn}(\tilde{e}_j/\tilde{\tau}_{ji})\frac{\left\{\left(\frac{\beta_e}{\beta_t}\right), \left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)\right\}^4}{1 + \left\{\left(\frac{\beta_e}{\beta_t}\right), \left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)\right\}^4}$$

$$= \text{sgn}\left(\left\{\frac{\beta_e}{\beta_t}, \left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)\right\}\right)\frac{\left\{\left(\frac{\beta_e}{\beta_t}\right)^4, 4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)\right\}}{1 + \left\{\left(\frac{\beta_e}{\beta_t}\right)^4, 4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)\right\}}$$

$$= \text{sgn}\left(\frac{\beta_e}{\beta_t}\right)\left\{\frac{\left(\frac{\beta_e}{\beta_t}\right)^4}{1 + \left(\frac{\beta_e}{\beta_t}\right)^4}, \left|\frac{4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)}{\left(\frac{\beta_e}{\beta_t}\right)^4} + \frac{4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_t}\right)}{1 + \left(\frac{\beta_e}{\beta_t}\right)^4}\right|\right\} \quad \text{(A6)}$$

where $\text{sgn}(\tilde{\chi})$ is the signature function of $\tilde{\chi}$, given the value of +1, 0, −1. The signature function is taking the sign of peak point that is defined as

$$\text{sgn}(\tilde{\chi}) = \text{sgn}(\beta) = \begin{cases} +1 & \text{for } \beta > 0 \\ 0 & \text{for } \beta = 0 \\ -1 & \text{for } \beta < 0 \end{cases} \quad \text{(A7)}$$

For the case of $\tilde{p}_{ji} = 0$, the satisfaction factor is rewritten as

$$\tilde{sf}_{ji} = \text{sgn}^*\left[1 - \frac{(\tilde{e}_j/\tilde{\tau}_{j,\min})^4}{1 + (\tilde{e}_j/\tilde{\tau}_{j,\min})^4}\right]$$

$$= \text{sgn}^*\left[\left\{1 - \frac{\left(\frac{\beta_e}{\beta_{t,\min}}\right)^4}{1 + \left(\frac{\beta_e}{\beta_{t,\min}}\right)^4}, \left|\frac{4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_{t,\min}}{\beta_{t,\min}}\right)}{\left(\frac{\beta_e}{\beta_{t,\min}}\right)^4} + \frac{4\left(\frac{\gamma_e}{\beta_e} + \frac{\gamma_t}{\beta_{t,\min}}\right)}{1 + \left(\frac{\beta_e}{\beta_{t,\min}}\right)^4}\right|\right\}\right] \quad \text{(A8)}$$

where $\tilde{\tau}_{j,\min} = \{\beta_{t,\min}, \gamma_{t,\min}\}$ is fuzzy expression and sgn* is defined similarly as eq 43. Therefore, the degree of fault and consistency factor become

$$\tilde{d}_i = \left(\sum_{j=1}^{m} \tilde{sf}_{ji}\right)\Big/ m \quad \text{(A9)}$$

and

$$cf_i = 1 - \left[\frac{sf^c_{\max,i} - sf^c_{\min,i}}{\max(|sf^c_{1i}|, |sf^c_{2i}|, \ldots, |sf^c_{mi}|)}\right] \quad \text{(A10)}$$

in which

$$sf^c_{\max,i} = \max_i(sf^c_{1i}, sf^c_{2i}, \ldots, sf^c_{mi}) = \max_i(\beta_{1i}, \beta_{2i}, \ldots, \beta_{mi}) \quad \text{(A11)}$$

$$sf^c_{\min,i} = \min_i(sf^c_{1i}, sf^c_{2i}, \ldots, sf^c_{mi}) = \min_i(\beta_{1i}, \beta_{2i}, \ldots, \beta_{mi}) \quad \text{(A12)}$$

where sf^c_{ji} is the center point of $\tilde{sf}_{ji} = \{\beta_{ji}, \gamma_{ji}\}$.

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