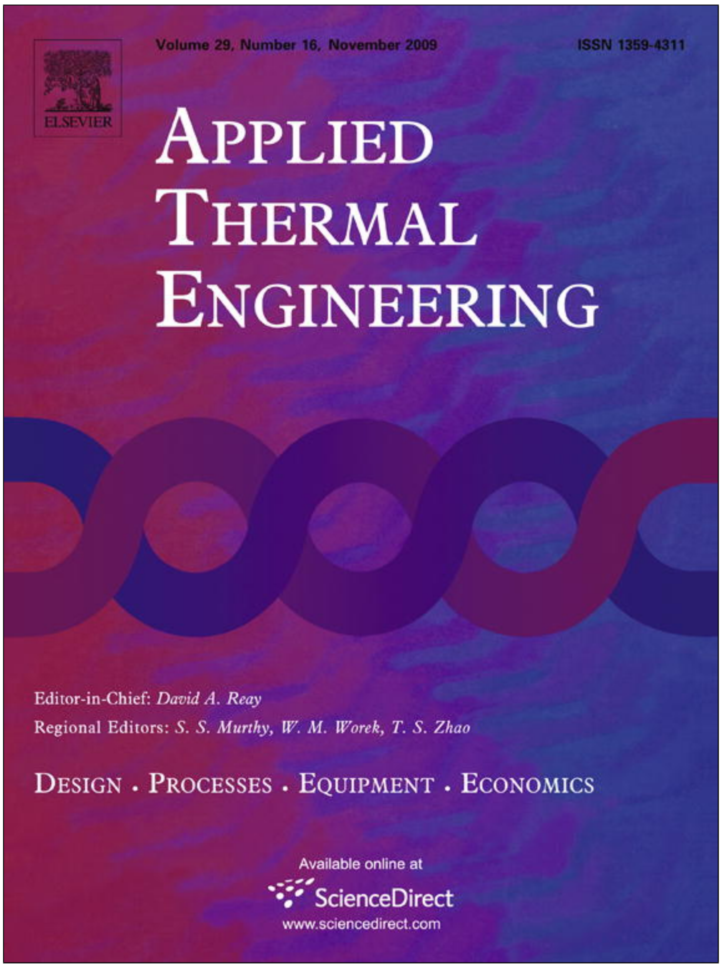
Volume 29, Number 16, November 2009
ISSN 1359-4311
ELSEVIER
APPLIED THERMAL ENGINEERING
Editor-in-Chief: David A. Reay
Regional Editors: S. S. Murthy, W. M. Worek, T. S. Zhao
DESIGN . PROCESSES . EQUIPMENT . ECONOMICS
Available online at
ScienceDirect
www.sciencedirect.com

Applied Thermal Engineering 29 (2009) 3366–3373

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Thermal–electrical–luminous model of multi-chip polychromatic LED luminaire

Bin-Juine Huang *, Chun-Wen Tang

New Energy Center, Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan

ARTICLE INFO

Article history:
Received 3 June 2008
Accepted 15 May 2009
Available online 6 June 2009

Keywords:
RGB LEDs
Thermal–electrical–luminous model
LED junction temperature
LED luminous intensity
LED system identification

ABSTRACT

This paper proposed a thermal–electrical–luminous dynamic model of red–green–blue (RGB) light-emitting diode (LED) luminaire for lighting control. The thermal–electrical–luminous model consists of three parts, namely, electrical–thermal (E–T), electrical–luminous (E–L), and thermal–luminous (T–L) models. Using step response method, the electrical–thermal (E–T) model $G(s)$ is derived as a first-order bi-proper system. The electrical–luminous (E–L) and thermal–luminous (T–L) models are zeroth order model with a constant gain since the luminous response to electric or thermal input is much faster. The thermal–electrical–luminous model shows that the luminous intensity is proportional to input power and inversely proportional to junction temperature. The dynamic response of luminous intensity is dominated by the electrical–thermal model $G(s)$.

The whole thermal–electrical–luminous model can be further divided into a constant gain and a first-order bi-proper system. The constant gain causes the instantaneous response at power switch on; the first-order system represents the luminous variation due to junction temperature change which is mainly related to the heat sink design. The complete model can accurately describe luminous dynamic behavior and be used in control system design of RGB LED lighting luminaire.

1. Introduction

High brightness light-emitting diode (LED) is a promising technology for lighting. Due to the rapid technology improvement, the illuminating efficiency of LED has reached > 80 lm/W in commercial products and is proved energy saving as compared to traditional lightings, such as incandescent (<20 lm/W), fluorescent (<50 lm/W), and mercury (<70 lm/W) lamps. In addition, red–green–blue (RGB) LEDs are the only light source which can vary color in wide chromatic range [1,2] and has been applied in architectural, commercial, and residential lighting.

The lighting performance of RGB LEDs, such as illumination intensity and light color, is determined by power inputs of color LEDs. The light color is determined by luminous ratio of three color LEDs. However, the illuminations of LEDs vary with junction temperature variation due to self-heating of LEDs and variation of ambient temperature. Hence, the thermal effect will affect both illumination intensity and output color of LED. A lighting control of RGB LEDs is thus needed. In order to develop the control system, a system dynamics model of RGB LEDs taking into account the effect of thermal, electrical, and luminous properties must be derived first.

The RGB LED lighting can be controlled using feedback system design [3–7]. The derivation of a system dynamic model of LED luminaire is thus important for the feedback system design. However, very few researchers investigate the system dynamics model of a LED luminaire. Masana [8] derived a RC thermal model for a general semiconductor package. Gu et al. [9] studied optical properties of LED at steady state. Muthu et al. [10] proposed a constant luminous model which ignores the thermal effect. Farkas et al. [11] developed a thermal model for luminous output and thermal resistance in monochromatic light-emitting unit. Huang et al. [12,13] derived a system dynamics model of a luminaire to relate the energy input to LED junction temperature. In fact, the junction temperature variation will affect the light output of LEDs. Little has been studied on this subject. The present paper intends to derive a thermal–electrical–luminous model of RGB LED luminaire for the control system design of RGB LED lighting.

2. Modelling of RGB LED luminaire

From the principle of solid-state lighting, the luminance of LED is induced from two physical mechanisms: energy effect and opto-electronic effect. Both effects are related to junction temperature. The thermal–electrical–luminous model of RGB LED luminaire thus consists of three major parts: electrical–thermal (E–T) model; electrical–luminous (E–L) model; thermal–luminous (T–L) model.

The RGB LED luminaire is a lighting fixture which is made of multiple RGB LED lamps with a heat sink. The schematic diagram of RGB LED luminaire is shown in Fig. 1. The LEDs are driven by electrical input power which will raise the LED junction temperature by self-heating [14]. The dynamic behavior of junction

* Corresponding author. Tel.: +886 2 2363 4790; fax: +886 2 2364 0549.
E-mail address: bjhuang@seed.net.tw (B.-J. Huang).

1359-4311/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.applthermaleng.2009.05.024

Nomenclature

E_P	power coefficient (cd/W)
E_T	temperature coefficient (cd/°C)
G	thermal model of luminaire (°C/W)
G_0	averaged thermal model of luminaire (°C/W)
G_b	thermal model of heat sink (°C/W)
G_c	thermal model of chip (°C/W)
H_{LED}	thermal–electrical–luminous model of luminaire (°C/W)
H_P	power effect of H_{LED} (cd/W)
H_T	temperature effect of H_{LED} (cd/W)
k	gain of thermal model (°C/W)
M	temperature sensitive parameter (V/°C)
P	input power (W)
P_{LED}	input power (W)
p	pole of thermal dynamics
s	Laplace operator
T	lump temperature (°C)
T_{LED}	junction temperature (°C/W)
V_F	averaged forward voltage (V)
V_{F0}	initial forward voltage (V)
z	zero of thermal dynamics

Subscripts

a	ambient air
B	blue LEDs
G	green LEDs
i	notation of R,G or B
j	LED junction
P	input power
R	red LEDs
T	junction temperature

Greeks symbols

Φ	luminous intensity (cd)
Φ_{LED}	luminous intensity of RGB LED luminaire (cd)
$\sim$	perturbation
–	average

temperature is related to thermal design of luminaire and can be described by the electrical–thermal (E–T) model, $G(s)$. Moreover, the luminous output will be influenced by electrical input power and junction temperature simultaneously. The electrical–luminous (E–L) model, E_P, relates the electrical input power to the luminous output. The thermal–luminous (T–L) model, E_T, relates the junction temperature to the luminous output.

For lighting fixture, the luminous output of LED can be described according to three kinds of definitions: namely, luminous flux, luminous intensity, and illuminance. The luminous intensity was used in the present study to represent the luminous output. For multi-monochromatic LEDs, the thermal–electrical–luminous physical relationship can be expressed as:

$$\Phi_{LED}(s) = H_{LED}(s)P_{LED}(s) = E_P P_{LED}(s) + E_T T_{LED}(s) = [E_P + E_T G(s)]P_{LED}(s) \quad (1)$$

where Φ_{LED} is luminous intensity of luminaire, H_{LED} is thermal–electrical–luminous model of luminaire, P_{LED} is input power for RGB LEDs, T_{LED} is junction temperature for RGB LEDs. The block diagram is shown in Fig. 2.

2.1. Electrical–thermal (E–T) model, G(s)

The E–T model $G(s)$ is related to the heat transfer of LED luminaire which obeys the law of energy conservation. The heat generated from chips conducts through the heat sink to ambient.

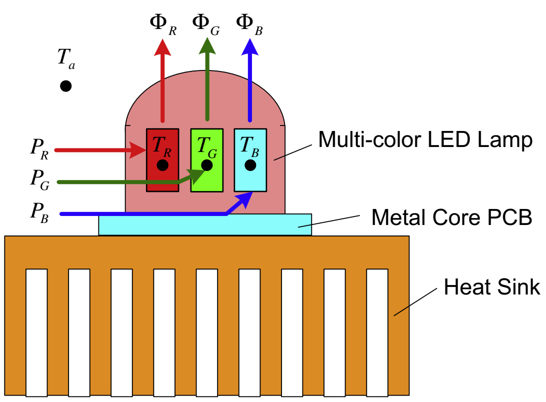

Fig. 1. Schematic diagram of RGB LED luminaire.

The electrical–thermal system dynamics can be treated as multi-variable system with three inputs (input power of red, green and blue LEDs – $P_R(s)$, $P_G(s)$, $P_B(s)$) and three outputs (junction temperature of red, green, and blue LEDs – $T_R(s)$, $T_G(s)$, $T_B(s)$). Neglecting the variation of ambient temperature, assuming the lumped condition for the temperature of LED chip, i.e. lumped assumption, and using linear perturbation concept, we obtain the thermal model $G(s)$ which is a 3 × 3 transfer function [15]:

$$G(s) = \begin{bmatrix} G_{RR}(s) & G_{GR}(s) & G_{BR}(s) \\ G_{RG}(s) & G_{GG}(s) & G_{BG}(s) \\ G_{RB}(s) & G_{GB}(s) & G_{BB}(s) \end{bmatrix} = \begin{bmatrix} \frac{\tilde{T}_R(s)}{\tilde{P}_R(s)} & \frac{\tilde{T}_R(s)}{\tilde{P}_G(s)} & \frac{\tilde{T}_R(s)}{\tilde{P}_B(s)} \\ \frac{\tilde{T}_G(s)}{\tilde{P}_R(s)} & \frac{\tilde{T}_G(s)}{\tilde{P}_G(s)} & \frac{\tilde{T}_G(s)}{\tilde{P}_B(s)} \\ \frac{\tilde{T}_B(s)}{\tilde{P}_R(s)} & \frac{\tilde{T}_B(s)}{\tilde{P}_G(s)} & \frac{\tilde{T}_B(s)}{\tilde{P}_B(s)} \end{bmatrix} \quad \text{at } \tilde{T}_a \cong 0, \quad (2)$$

The linearly-perturbed junction temperature can be derived as:

$$\tilde{T}_{LED}(s) = \begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = \begin{bmatrix} G_{RR}(s) & G_{GR}(s) & G_{BR}(s) \\ G_{RG}(s) & G_{GG}(s) & G_{BG}(s) \\ G_{RB}(s) & G_{GB}(s) & G_{BB}(s) \end{bmatrix} \cdot \begin{bmatrix} \tilde{P}_R(s) \\ \tilde{P}_G(s) \\ \tilde{P}_B(s) \end{bmatrix} = G_{LED}(s) \cdot \tilde{P}_{LED}(s), \quad (3)$$

The block diagram of the E–T model is shown in Fig. 3.

2.2. Electrical–luminous (E–L) model, E_P

The electrical–luminous (E–L) model, E_P, relates the input power to the output luminous intensity. Since the light response to the input power is very fast, the E–L model of RGB LEDs is of

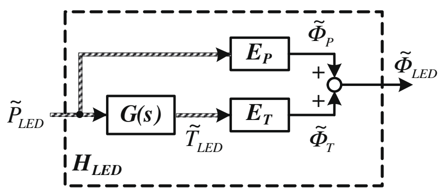

Fig. 2. The block diagram of thermal–electrical–luminous model of RGB LED luminaire.

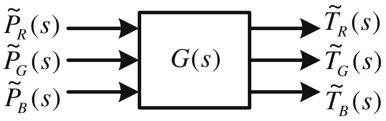

Fig. 3. Input–output block diagram of the E–T model.

zeroth order with a constant gain, defined as, in terms of linear perturbation concept:

$$E_P = [E_{PR} \quad E_{PG} \quad E_{PB}] = \left[\frac{\tilde{\Phi}_{LED}(s)}{\tilde{P}_R(s)} \quad \frac{\tilde{\Phi}_{LED}(s)}{\tilde{P}_G(s)} \quad \frac{\tilde{\Phi}_{LED}(s)}{\tilde{P}_B(s)}\right]. \tag{4}$$

Fig. 4 depicts the block diagram of the electrical–luminous (E–L) model, E_P.

2.3. Thermal–luminous (T–L) model, E_T

The thermal–luminous (T–L) model, E_T, relates the junction temperature to the output luminous intensity. Since the light response to the junction temperature variation is very fast, the T–L model of RGB LEDs is of zeroth order with a constant gain, defined as, in terms of linear perturbation concept:

$$E_T = [E_{TR} \quad E_{TG} \quad E_{TB}] = \left[\frac{\tilde{\Phi}_{LED}(s)}{\tilde{T}_R(s)} \quad \frac{\tilde{\Phi}_{LED}(s)}{\tilde{T}_G(s)} \quad \frac{\tilde{\Phi}_{LED}(s)}{\tilde{T}_B(s)}\right]. \tag{5}$$

Fig. 5 is the block diagram of the thermal–luminous (T–L) model, E_T.

3. Identification of thermal–electrical–luminous model of RGB LED luminaire

The mathematical model of RGB LED luminaire described above will be identified experimentally.

3.1. Experimental setup

An RGB LED luminaire was designed and built in the present study, as shown in Fig. 6. The lighting fixture consists of four mechanical parts: light engines, heat sink, shell, and circular baffle. The light engine is made of RGB LED lamps soldered on a metal core PCB (MCPCB). In order to measure the light spectrum of LED at the same junction temperature, a single LED lamp is soldered on a ∅34 mm MCPCB which is attached to the light engine body.

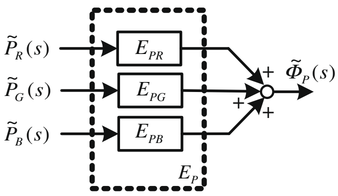

Fig. 4. Input–output block diagram of the electrical–luminous (E–L) model, E_P.

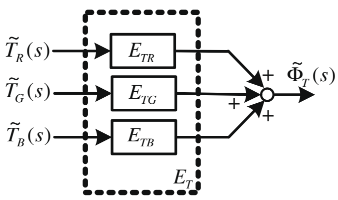

Fig. 5. Input–output block diagram of the thermal–luminous (T–L) model, E_T.

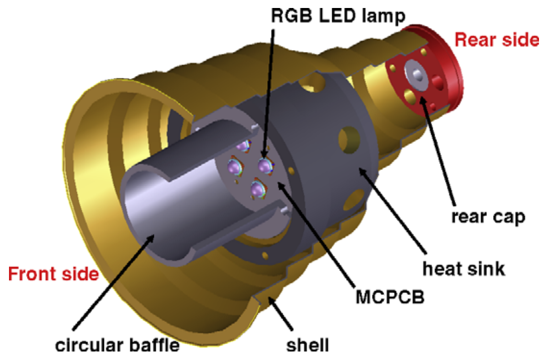

Fig. 6. Schematic diagram of RGB LED luminaire.

Table 1
Specification of LED lamp (forward current 350 mA).

Items	No. of chip	Peak wavelength (nm)	Photopic power (lm)	Power dissipation (W)
Red	1	621.2	33	1.21
Green	2	520.8	50	2.56
Blue	1	459.8	11	1.27

Annotation: four chip in single package.

This single LED lamp acts as a sensor for measuring the LED junction temperature and is called "LED sensor".

The circular baffle is used in front side to isolate the stray light during the measurement of the light spectrum. The specification of RGB LED lamp and luminaire are listed in Tables 1 and 2, respectively.

The LED is driven by a constant-current with pulse-width modulation (DC-PWM) for minimal chromaticity shifts [16]. The switching frequency is set at 120 Hz to avoid perceptible flicker [17]. The input power is controlled by the duty cycle of DC-PWM. The junction temperature is measured by pulse method [13,18].

A linear junction temperature–voltage relation at low current (1 mA) was first determined experimentally in a well-controlled environment [13]:

$$T_j = a + bV_F. \tag{6}$$

The junction temperature can then be measured at the OFF-interval of DC-PWM using the LED sensor with 1 mA current.

An experimental apparatus was built for light measurement of RGB LED luminaire. The apparatus includes integrating sphere, current meter with photopic detector, spectrometer and personal computer. The measuring system can provide color mixing and

Table 2
Specification of RGB LED luminaire.

Light engine	● Four lamps on ∅85 mm MCPCB. ● Lamps arrangement: 2 × 2. ● Lamps spacing: 20 mm. Total power dissipation: 25 W Thickness of aluminum MCPCB: 2.0 mm
Heat sink	Material: aluminum Weight: 900 g.
Shell	Material: aluminum Surface area: 950 cm^2
Circular baffle	Material: black acrylonitrile–butadiene–styrene Diameter: 125 mm Length: 65 mm

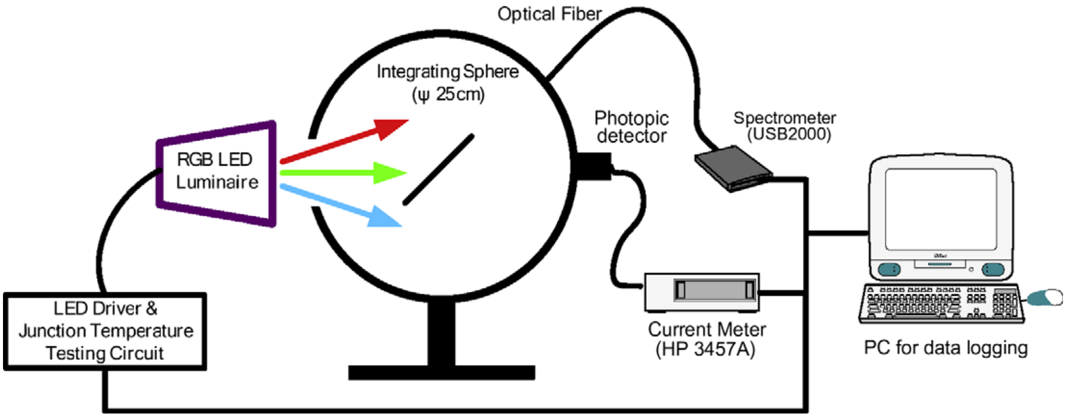

Fig. 7. Schematic diagram of luminous experimental setup.

data recording of luminous intensity and junction temperature. The experimental facility is shown in Fig. 7.

3.2. Electrical–thermal (E–T) model, G(s)

The electrical–thermal model $G(s)$ can be identified using step response method. The E–T model $G(s)$ is a 3×3 MIMO model. All the elements can be identified experimentally using step test method. First, the step input power was applied to red LEDs while keeping all other inputs constant. The time responses of R–G–B junction temperatures then can be determined and analyzed to obtain $G_{RR}(s)$, $G_{RG}(s)$, $G_{RB}(s)$, from Eq. (7):

$$\begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = \begin{bmatrix} G_{RR}(s) & G_{GR}(s) & G_{BR}(s) \\ G_{RG}(s) & G_{GG}(s) & G_{BG}(s) \\ G_{RB}(s) & G_{GB}(s) & G_{BB}(s) \end{bmatrix} \cdot \begin{bmatrix} \tilde{P}_R(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{RR}(s) \cdot \tilde{P}_R(s) \\ G_{RG}(s) \cdot \tilde{P}_R(s) \\ G_{RB}(s) \cdot \tilde{P}_R(s) \end{bmatrix}$$
$$\text{at } \tilde{P}_G(s) = 0 \text{ and } \tilde{P}_B(s) = 0, \tag{7}$$

Similarly, the step input powers were applied to green or blue LEDs, respectively, while keeping all other inputs constant. The time responses of *R–G–B* junction temperatures then can be determined and analyzed to obtain $G_{GR}(s)$, $G_{GG}(s)$, $G_{GB}(s)$, from Eq. (8) and $G_{BR}(s)$, $G_{BG}(s)$, $G_{BB}(s)$, from Eq. (9):

$$\begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = \begin{bmatrix} G_{RR}(s) & G_{GR}(s) & G_{BR}(s) \\ G_{RG}(s) & G_{GG}(s) & G_{BG}(s) \\ G_{RB}(s) & G_{GB}(s) & G_{BB}(s) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \tilde{P}_G(s) \\ 0 \end{bmatrix} = \begin{bmatrix} G_{GR}(s) \cdot \tilde{P}_G(s) \\ G_{GG}(s) \cdot \tilde{P}_G(s) \\ G_{GB}(s) \cdot \tilde{P}_G(s) \end{bmatrix}$$
$$\text{at } \tilde{P}_R(s) = 0 \text{ and } \tilde{P}_B(s) = 0, \tag{8}$$

$$\begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = \begin{bmatrix} G_{RR}(s) & G_{GR}(s) & G_{BR}(s) \\ G_{RG}(s) & G_{GG}(s) & G_{BG}(s) \\ G_{RB}(s) & G_{GB}(s) & G_{BB}(s) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \tilde{P}_B(s) \end{bmatrix} = \begin{bmatrix} G_{BR}(s) \cdot \tilde{P}_B(s) \\ G_{BG}(s) \cdot \tilde{P}_B(s) \\ G_{BB}(s) \cdot \tilde{P}_B(s) \end{bmatrix}$$
$$\text{at } \tilde{P}_R(s) = 0 \text{ and } \tilde{P}_G(s) = 0. \tag{9}$$

$G(s)$ varied with the magnitude of input power. All the nine elements of $G(s)$ were identified at different input power as first-order bi-proper system by using Rake's analysis [13,19]. Table 3 shows the poles, zeros and gains of $G(s)$ at different input power perturbation. An average model for E–T model $G(s)$ is derived as, using the mean value of each parameter:

Table 3
Poles, zeros and gains of thermal model using different input power perturbation.

	No.	G_{RR}			No.	G_{RG}			No.	G_{RB}		
		p	*z*	*k*		*p*	*z*	*k*		*p*	*z*	*k*
Step $\tilde{P}_R$ (W)												
1.8 → 3.9	1	−0.00083	−0.00153	1.0889	5	−0.00083	−0.00229	0.2097	9	−0.00082	−0.00212	0.4164
3.9 → 6.0	2	−0.00080	−0.00227	0.5253	6	−0.00074	−0.00236	0.2024	10	−0.00073	−0.00221	0.3883
6.0 → 3.9	3	−0.00088	−0.00231	0.5290	7	−0.00081	−0.00181	0.2547	11	−0.00084	−0.00189	0.4616
3.9 → 1.8	4	−0.00083	−0.00156	1.1093	8	−0.00083	−0.00224	0.2213	12	−0.00084	−0.00222	0.4096
Average model		−0.00083	−0.00192	0.8131		−0.00080	−0.00217	0.2220		−0.00081	−0.00211	0.4190
Step $\tilde{P}_G$ (W)												
3.9 → 8.3	13	−0.00087	−0.00368	0.9536	17	−0.00087	−0.00157	1.4477	21	−0.00083	−0.00167	1.8957
8.3 → 12.7	14	−0.00085	−0.00342	0.9284	18	−0.00084	−0.00124	2.2138	22	−0.00085	−0.00171	1.7378
12.7 → 8.3	15	−0.00085	−0.00316	0.9965	19	−0.00085	−0.00126	2.1829	23	−0.00086	−0.00179	1.6930
8.3 → 3.9	16	−0.00073	−0.00347	0.9081	20	−0.00073	−0.00141	1.4214	24	−0.00075	−0.00185	1.6558
Average model		−0.00083	−0.00343	0.9467		−0.00082	−0.00137	1.8164		−0.00082	−0.00175	1.7456
Step $\tilde{P}_B$ (W)												
1.9 → 4.1	25	−0.00080	−0.00297	0.6142	29	−0.00082	−0.00256	0.3127	33	−0.00075	−0.00128	1.7247
4.1 → 6.3	26	−0.00083	−0.00245	0.6714	30	−0.00078	−0.00197	0.3488	34	−0.00080	−0.00112	2.6812
6.3 → 4.1	27	−0.00085	−0.00346	0.5262	31	−0.00084	−0.00245	0.3154	35	−0.00085	−0.00121	2.6632
4.1 → 1.9	28	−0.00079	−0.00317	0.5618	32	−0.00079	−0.00208	0.3582	36	−0.00081	−0.00129	1.7869
Average model		−0.00082	−0.00301	0.5934		−0.00081	−0.00227	0.3338		−0.00080	−0.00122	2.2140

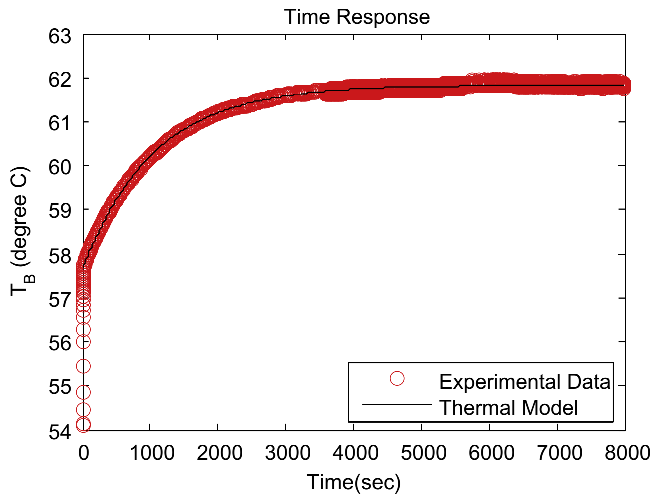

Fig. 8. The time response of $G_{BR}(s)$ using step power input of green LEDs from 8.3 W to 12.7 W.

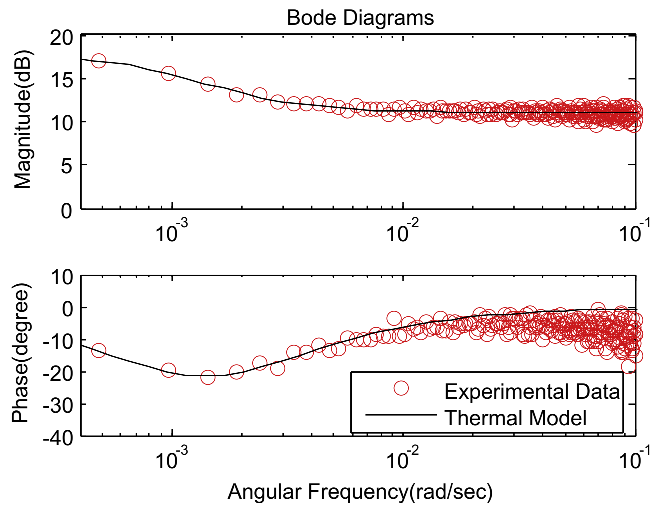

Fig. 9. The frequency response of $G_{BR}(s)$ model identified using step power input of green LEDs from 8.3 W to 12.7 W.

$$G_0(s)=\begin{bmatrix} \frac{0.8132(s+0.00192)}{(s+0.00083)} & \frac{0.2220(s+0.00343)}{(s+0.00083)} & \frac{0.4190(s+0.00301)}{(s+0.00082)} \\ \frac{0.9467(s+0.00217)}{(s+0.00080)} & \frac{1.8164(s+0.00137)}{(s+0.00082)} & \frac{1.7456(s+0.00227)}{(s+0.00081)} \\ \frac{0.5934(s+0.00211)}{(s+0.00081)} & \frac{0.3338(s+0.00175)}{(s+0.00082)} & \frac{2.2140(s+0.00122)}{(s+0.00080)} \end{bmatrix}. \tag{10}$$

The model parameters at each operating conditions are shown in Table 3. The comparison of the measured step response with the calculation from the average model, the element $G_{BR}(s)$ in $G_0(s)$, is shown in Figs. 8 and 9 shows the measured frequency response

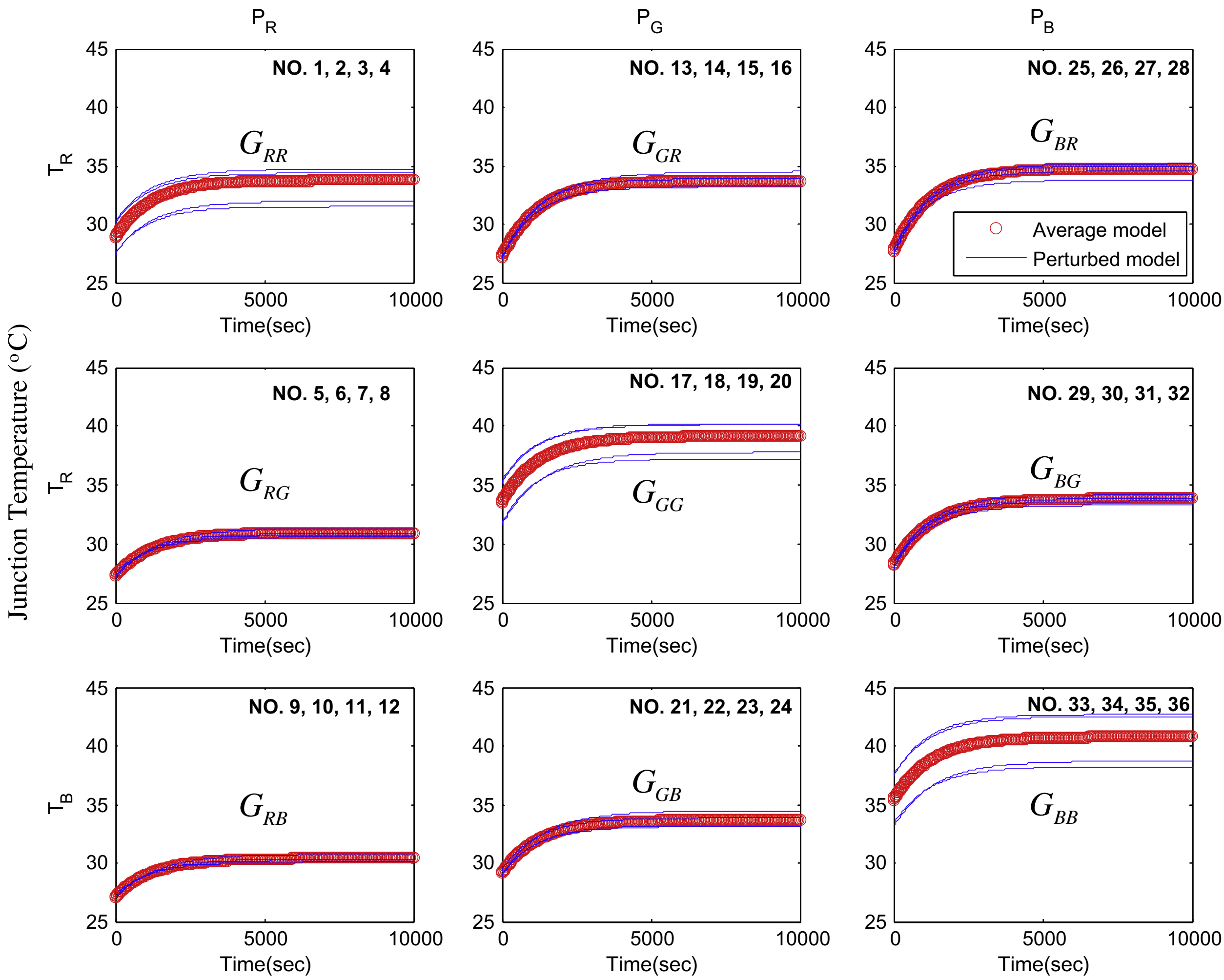

Fig. 10. The junction temperature responses of all nine elements in $G(s)$ using 4 W step power input for each LEDs.

and the calculation from the average model. Fig. 10 presents the junction temperature responses of all nine elements in the thermal model $G(s)$ for 4 W step power input to each LEDs which are calculated using the above identified model.

3.3. Electrical–luminous (E–L) model, E_P

The optoelectric response of LEDs is much faster than the thermal behavior of luminaire. The electrical–luminous (E–L) model, E_P, can thus be treated as zeroth order system. Moreover, input power and junction temperature will influence linearly the luminous intensity of LEDs [14]. The reception of eyes is also linear due to linearity laws [17]. Therefore, E_P is constant.

The luminous response of different color LEDs due to power input can be determined separately using isolating method. That is, applying step power input to red LED, while keeping the power input of other LEDs at constant, and measuring the luminous intensity of LEDs, we can determine the elements of E_P.

$$\tilde{\Phi}_P(s) = E_P \cdot \tilde{P}_{LED}(s) = [E_{PR} \quad E_{PG} \quad E_{PB}] \cdot \begin{bmatrix} \tilde{P}_R(s) \\ 0 \\ 0 \end{bmatrix} = E_{PR} \cdot \tilde{P}_R(s) \quad \text{at } \tilde{P}_G(s) = 0 \text{ and } \tilde{P}_B(s) = 0, \tag{11}$$

$$\tilde{\Phi}_P(s) = E_P \cdot \tilde{P}_{LED}(s) = [E_{PR} \quad E_{PG} \quad E_{PB}] \cdot \begin{bmatrix} 0 \\ \tilde{P}_G(s) \\ 0 \end{bmatrix} = E_{PG} \cdot \tilde{P}_G(s) \quad \text{at } \tilde{P}_R(s) = 0 \text{ and } \tilde{P}_B(s) = 0, \tag{12}$$

$$\tilde{\Phi}_P(s) = E_P \cdot \tilde{P}_{LED}(s) = [E_{PR} \quad E_{PG} \quad E_{PB}] \cdot \begin{bmatrix} 0 \\ 0 \\ \tilde{P}_B(s) \end{bmatrix} = E_{PB} \cdot \tilde{P}_B(s) \quad \text{at } \tilde{P}_R(s) = 0 \text{ and } \tilde{P}_G(s) = 0. \tag{13}$$

A procedure was designed to obtain the step input power and luminous intensity. Input power of one color LEDs firstly was applied at a fixed value and others set to 0 W. The junction temperature of powered LEDs will rise by self-heating. The luminous intensities were recorded at junction temperature 60 °C. In this study, three input power levels were used for each color LEDs (red LEDs: 5.45 W, 4.24 W and 3.03 W; green LEDs: 4.48 W, 3.20 W and 1.92 W; blue LEDs: 2.22 W, 1.33 W and 0.44 W). The results are shown in Fig. 11. E_P is then determined:

$$E_P = [E_{PR} \quad E_{PG} \quad E_{PB}] = [279.52 \quad 542.87 \quad 161.39]. \tag{14}$$

3.4. Thermal–luminous (T–L) model, E_T

The thermal–luminous (T–L) model, E_T, relates the junction temperature to the output luminous intensity. Since the light response to the junction temperature variation is very fast, the T–L model of RGB LEDs is of zeroth order with a constant gain, defined in Eq. (5).

The luminous response of different color LEDs can be calculated as:

$$\tilde{\Phi}_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \quad E_{TG} \quad E_{TB}] \cdot \begin{bmatrix} \tilde{T}_R(s) \\ 0 \\ 0 \end{bmatrix} = E_{TR} \cdot \tilde{T}_R(s) \quad \text{at } \tilde{T}_G(s) = 0 \text{ and } \tilde{T}_B(s) = 0, \tag{15}$$

$$\tilde{\Phi}_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \quad E_{TG} \quad E_{TB}] \cdot \begin{bmatrix} 0 \\ \tilde{T}_G(s) \\ 0 \end{bmatrix} = E_{TG} \cdot \tilde{T}_G(s) \quad \text{at } \tilde{T}_R(s) = 0 \text{ and } \tilde{T}_B(s) = 0, \tag{16}$$

$$\tilde{\Phi}_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \quad E_{TG} \quad E_{TB}] \cdot \begin{bmatrix} 0 \\ 0 \\ \tilde{T}_B(s) \end{bmatrix} = E_{TB} \cdot \tilde{T}_B(s) \quad \text{at } \tilde{T}_R(s) = 0 \text{ and } \tilde{T}_G(s) = 0. \tag{17}$$

Input power was applied at fixed level in one color LEDs, and set other LEDs at 0 W. The curve of luminous intensity and junction temperature were recorded continuously. In this study, input power level were 4.84 W for red, 3.33 W for green and 0.83 W for blue LEDs. The results are linear as shown in Fig. 12. E_T can be obtained by using linear regression method as

$$E_T = [E_{TR} \quad E_{TG} \quad E_{TB}] = [-12.76 \quad -4.32 \quad -0.69]. \tag{18}$$

3.5. Thermal–electrical–luminous model of RGB LED luminaire

Combining the above results, the thermal–electrical–luminous model is described as:

$$H_{LED}(s) = E_P + E_T \cdot G(s) = \left[260.97 - \frac{0.0371}{(s+0.000816)} \quad 534.69 - \frac{0.0176}{(s+0.000816)} \quad 156.04 - \frac{0.0114}{(s+0.000816)} \right]$$

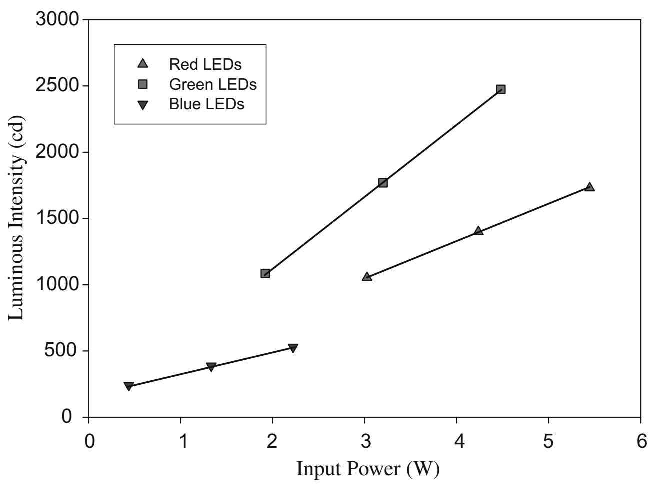

Fig. 11. The relation between luminous intensity and input power.

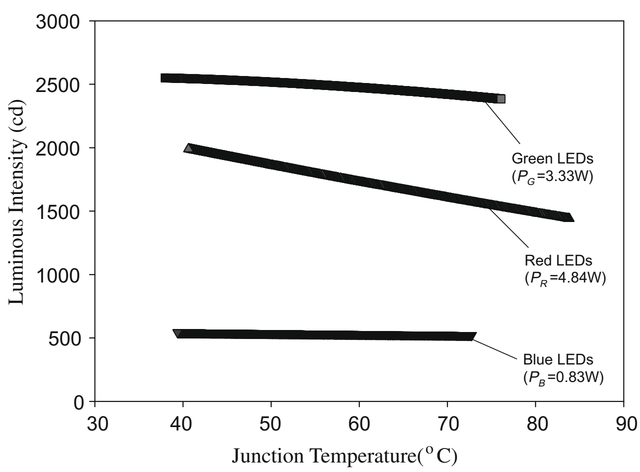

Fig. 12. The relation between luminous intensity and junction temperature.

Table 4
Step test condition for verification.

Time (s)	Perturbation of input power (W)		
	$\tilde{P}_R$	$\tilde{P}_G$	$\tilde{P}_B$
0	4.84	3.33	0.83
13,000	0	0	0.12
19,000	0	0.41	0
25,000	0.3	0	0

Annotation: correlative color temperature 4500 K.

3.6. Verification of thermal–electrical–luminous model of RGB LED luminaire

The complete thermal–electrical–luminous model of RGB LED luminaire $H_{LED}(s)$ needs experimental verification. To simulate the general lighting application, the operating condition was set at luminous intensity 3800 cd and color temperature 4500 K. The light appears as normal warm white color. Under this condition, the conversion of illuminance at 2 m distance is 950 lux which is suitable for indoor lighting application [20]. A step test procedure with four input power perturbations was employed. The perturbed inputs are listed in Table 4. The first step input is big and causes large junction temperature rise. The luminous intensity response can be calculated using the electrical–thermal model $G(s)$ and the thermal–luminous (T–L) model, E_T. For small perturbation input to individual color LEDs with small temperature variation, the luminous response can be calculated using the electrical–luminous (E–L) model E_P. Since the light color is determined by luminous ratio of individual color LEDs, the last three input power steps were set to result in equal color shift in light output.

The calculation of the total luminous intensity response using Eq. (19) can be used to compare with the measurements shown in Fig. 13a. The temperature responses of RGB LED luminaire were measured, as shown in Fig. 13b. It is seen that the response of luminous intensity can fit the experimental data well. The luminous responses during the first step coincide with the prediction of E–T and T–L model, and the luminous jumps at each step at time 13,000 s, 19,000 s, and 25,000 s, follows the E–L model as well. At the duration of the last step test, the ambient temperature (uncontrolled in experiment) changed (decreasing). Hence, the junction temperature decrease since the effect of decreasing ambient temperature is larger than input power step change. The lower junction temperature thus induces a higher luminous intensity according to T–L model. This explains why the luminous intensity responses in Fig. 13a showing opposite trend between experimental results and the thermal–electrical–luminous model during the last step transient.

4. Discussion and conclusions

The present paper has derived a thermal–electrical–luminous dynamic model of red–green–blue (RGB) light-emitting diode (LED) luminaire for lighting control. The thermal–electrical–luminous model consists of three parts, namely, electrical–thermal (E–T), electrical–luminous (E–L), and thermal–luminous (T–L) models. Using step response method, the electrical–thermal (E–T) model G(s) is identified experimentally as a first-order bi-proper system. The electrical–luminous (E–L) and thermal–luminous (T–L) models are zeroth model with a constant gain since the luminous response to electric or thermal input is much fast.

The thermal–electrical–luminous model shows that the luminous intensity is proportional to input power and inversely proportionally to junction temperature. The dynamics of luminous intensity is dominated by the electrical–thermal model $G(s)$. The complete model can describe the luminous dynamic behavior and can be used in control system design of RGB LED lighting luminaire.

The electrical–thermal (E–T) model $G(s)$, Eq. (10), can be separated into a constant model $G_c(s)$ and a first-order model $G_b(s)$, as shown in Eq. (20). The constant model $G_c(s)$ represents the instantaneous heating of LED chip due to poor cooling in chip package. The first-order model $G_b(s)$ is resulted from slow dynamic behavior of the heat sink. It is seen that all the nine poles of $G_b(s)$ are very close to each other, within ± 2%.

$$G(s)=G_c(s)+G_b(s)=\begin{bmatrix}0.8132 & 0.2220 & 0.4190\\ 0.9467 & 1.8164 & 1.7456\\ 0.5934 & 0.3338 & 2.2140\end{bmatrix}+\begin{bmatrix}\frac{0.000881}{(s+0.00083)} & \frac{0.002470}{(s+0.00083)} & \frac{0.001302}{(s+0.00082)}\\ \frac{0.000304}{(s+0.00080)} & \frac{0.000991}{(s+0.00082)} & \frac{0.000486}{(s+0.00081)}\\ \frac{0.000545}{(s+0.00081)} & \frac{0.001629}{(s+0.00082)} & \frac{0.000932}{(s+0.00080)}\end{bmatrix}. \quad (20)$$

The time response of optoelectric effect is at nano-second level and much faster than thermal effect. The electrical–luminous (E–L) model E_P is thus a zero-order system. The luminous intensity is proportional to input power. However, E_{PG} is greater than others because there are twice numbers of green chips in each LED lamp [17].

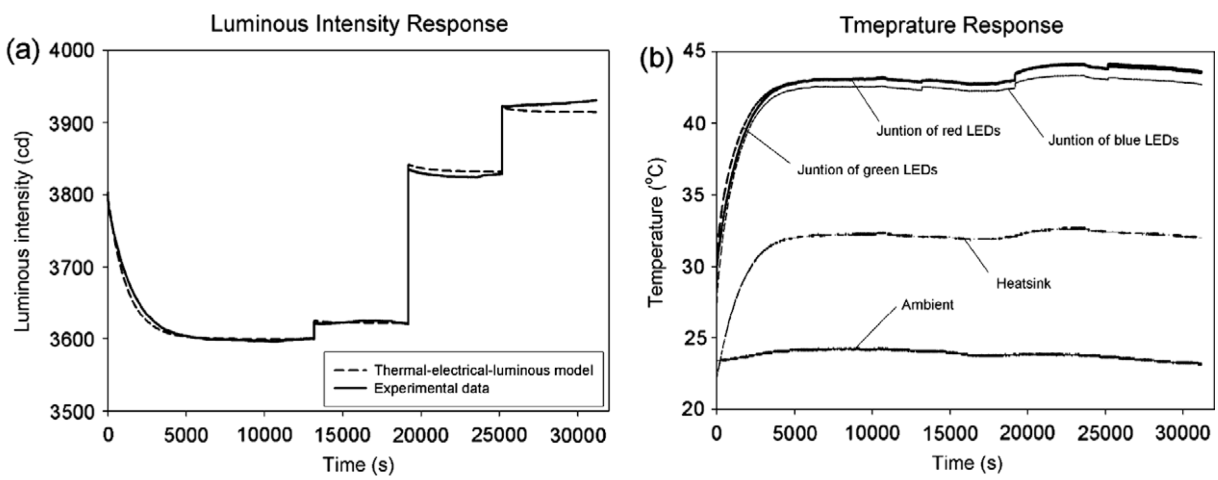

Fig. 13. The output response under sequential input power perturbation in Table 4; (a) luminous intensity comparison and (b) measured temperature responses.

The thermal–luminous (T–L) model E_T describes the luminous response of junction temperature change. It is a zero-order and constant system. The luminous intensity is inversely proportional to junction temperature. The luminous decay rate by thermal effect is thus determined by electrical–luminous (E–L) model E_P and the thermal–luminous (T–L) model E_T. Furthermore, the spectrum shift and decay of red LEDs due to junction temperature effect is much greater than other LEDs [9] since E_{TR} for red LEDs is greater than other LEDs. The prediction of E–T and T–L model coincides very well with experiment as shown in Fig. 13a.

Since the input power increase will cause a junction temperature rise and reduce luminous intensity. Therefore, both E–T and T–L model must be tested at the same time. For the verification of the E–L model, the input power step is small in order to reduce junction temperature rise. The large gain of the E–L model helps in validating the model using small step. By examining the response at the instant of applied step, the jump of predicted luminous response coincides with experimental results at 13,000 s, 19,000 s, and 25,000 s.

The present thermal–electrical–luminous model for a RGB LED luminaire is able to predict the luminous response. However, the slight deviation occurs in the last two step responses. This deviation is caused by the disturbance of ambient temperature since the effect of ambient temperature is ignored in the present model. According to the E–T model of Eq. (2), the sequential increasing input power steps in Table 4 will raise junction temperatures while the ambient temperature keeps constant. However, at the duration of the last step test from 23,000 s in Fig. 13b, the ambient temperature (uncontrolled in experiment) changed (decreasing). Hence, the junction temperature decrease since the effect of decreasing ambient temperature is larger than input power step change. The lower junction temperature thus induces a higher luminous intensity according to T–L model. This explains why the luminous intensity responses in Fig. 13a showing opposite trend between experimental results and the thermal–electrical–luminous model during the last step transient.

The whole thermal–electrical–luminous model can be further divided into two parts as:

$$\begin{aligned} H_{LED}(s) &= [E_P + E_T \cdot G(s)] = [E_P + E_T \cdot G_c(s)] + [E_T \cdot G_b(s)] = H_P(s) \\ &\quad + H_T(s) = [260.97 \;\; 534.69 \;\; 156.04] \\ &\quad + \left[-\frac{0.0371}{(s+0.000816)} \;\; -\frac{0.0176}{(s+0.000816)} \;\; -\frac{0.0114}{(s+0.000816)} \right] \end{aligned} \tag{21}$$

H_P is constant matrix which can be treated as the instantaneous response at switch on. H_T represents the luminous variation due to junction temperature change which is mainly related to the heat sink.

The thermal–electrical–luminous model can be used in the control system design for RGB LED lighting.

Acknowledgement

The present study was supported by National Science Council, Taiwan, under Grant No. NSC95-2221-E-002-244-MY2.

References

[1] M.G. Craford, LED's challenge the incandescents, IEEE Circ. Dev. Mag. 8 (1992) 24–29.
[2] D.A. Steigerwald, J.C. Bhat, D. Collins, R.M. Fletcher, M.O. Holcomb, Illumination with solid state lighting technology, IEEE J. Select. Topic. Quant. Electron. 8 (3) (2002) 310–320.
[3] S. Muthu, F. Schuurmans, M. Pashley, Red, green, and blue LEDs for white light illumination, IEEE J. Select. Topic. Quant. Electron. 8 (2) (2002) 333–338.
[4] A. Zukauskas, R. Vaicekauskas, F. Ivanauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M.S. Shur, R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, Proc. SPIE 5187 (2003) 185–198.
[5] P. Deurenberg, C. Hoelen, J. van Meurs, J. Ansems, Achieving color point stability in RGB multi-chip LED modules using various color control loops, Proc. SPIE 5941 (2005) 63–74.
[6] C. Hoelen, J. Ansems, P. Deurenberg, W. van Duijneveldt, M. Peeters, G. Steenbruggen, T. Treurniet, A. Valster, J.W. ter Weeme, Color tunable LED spot lighting, Proc. SPIE 6337 (2006) 63370Q.
[7] I. Ashdown, Neural networks for LED color control, Proc. SPIE 5187 (2003) 215–226.
[8] F.N. Masana, A new approach to the dynamic thermal modelling of semiconductor packages, Microelectron. Reliab. 41 (6) (2001) 901–912.
[9] Y. Gu, N. Narendran, T. Dong, H. Wu, Spectral and luminous efficacy change of high-power LEDs under different dimming methods, Proc. SPIE 6337 (2003) 63370J.
[10] S. Muthu, F.J. Schuurmans, M.D. Pashley, Red green blue LED based white light generation: Issues and control, in: IAS'02, Proc. IEEE 1 (2002) 327–333.
[11] G. Farkas, Q.v.V. Vader, A. Poppe, G. Bognar, Thermal investigation of high power optical devices by transient testing, IEEE Trans. Compon. Package. Technol. 28 (1) (2005) 45–50.
[12] B.-J. Huang, P.-C. Hsu, M.-S. Wu, C.-W. Tang, Study of system dynamics model and control of a high-power LED lighting luminaire, Energy 32 (11) (2007) 2187–2198.
[13] B.-J. Huang, C.-W. Tang, M.-S. Wu, System dynamics model of high-power LED luminaire, Appl. Therm. Eng. 29 (4) (2009) 609–616.
[14] E.F. Schubert, Light Emitting Diodes, Cambridge University Press, Cambridge, England, 2003.
[15] G.C. Goodwin, S.F. Graebe, M.E. Salgado, Control System Design, Prentice-Hall, Inc., Upper Saddle River, New Jersey, 2001.
[16] M. Dyble, N. Narendran, A. Bierman, T. Klein, Impact of dimming white LEDs: chromaticity shifts due to different dimming methods, Proc. SPIE 5941 (2005) 291–299.
[17] G. Wyszecki, W.S. Stiles, Color Science Concepts and Methods Quantitative Data and Formulae, second ed., John Wiely & Sons, New York, 1982.
[18] J.W. Sofia, Electrical temperature measurement using semiconductors, Electron. Cool. 3 (1) (1997) 22–25.
[19] H. Rake, Step response and frequency response methods, Automatica 16 (1980) 519–526.
[20] N. Oi, Preferred combinations between illuminance and color temperature in several settings for daily living activities, in: Proceedings of the 2nd International Symposium on Design of Artificial Environments, 2007, pp. 214–215.