[illegible] M.E.K [illegible]

Application of Subspace Algorithm and State Space Realization Technique to Identify the Equivalent M,E,K Matrices of Structures

[illegible] NSC 88-2611-E-002-008

[illegible]

[illegible] e: [illegible]

[illegible]

[illegible]

[illegible] (subspace algorithm method) [illegible] (M) [illegible] (E) [illegible] (K) [illegible] 20 [illegible]

[illegible]

Abstract

In this project we developed a procedure to identify the equivalent mass, damping, and stiffness matrices from the measured I/O data using the subspace algorithm, and the vibration behaviors of structures cab be improved by the identified system matrices. To verify the effectiveness of the subspace algorithm, a 2-dimensional frame structure with 20 DOF is provided to illustrate the identification process.

Keywords: Subspace algorithm, Accelerance Frequency Response Function (AFRF), Modal model, Physical model

[illegible]

[illegible] (modal model) [illegible] (physical model) [illegible] (mode frequencies) [illegible] (mode damping ratios) [illegible] mode shapes) [illegible] (non-proportional damping) [illegible] (complex mode) [illegible] [1-6] [illegible] Samir R. Ibrahim [7] [illegible]

ó ß æ ệ ặ å ậ ě ă ĕ ð ă Rayleigh â å ð ß ` ä ã ¼ ì đ ĝ «[8-10] â ß æ ặ ǒ ç Ð â å Ð ø ú Ð ß ß ð ä ù è ặ å ķ â ß î ð æ ģ Ą Ð ^ â å ű Ą ß ğ ű Ą ß ệ ẹ ậ ã ç ỹ â ß ä ä ÷ ã ë Ç ę ī û Ą Ð â å ű Ą ß Ç Ŏ Ą Ð â å ò ÷ ě Ő å ặ ý Ð (normal mode) æ ě ã Č ä ä å þ ã Č ç ę â Ĝ å ¶ â Ă ã § ò ŷ ŷ ¼ ß ` ä ÷ ã ß ф Ĩ ã ý Ð â å â ỹ ű Ě ê û ă ǒ ó ð æ ² ð Ă å Ã Ð ß `

î Ç ã â û ã đ ã Ê ß ã ë ® ó ï § ő ` ò ĸ ä ñ ÷ ý ŋ ý ŭ Ĩ Ą ű ă ǒ ó ð æ ² ð Ĕ Ã ħ ú Ð ß å ø ð ¹ å ặ æ ệ Ħ ê û ă ő ò ç Ð è ø ý ŋ ý ŭ Ĩ Ą Ğ ŏ æ ² ð Ć Ā ŏ ð ê è ă ß â å Ą ą Ð ß ã ê æ ð Č ¶ ß ý ŋ ¶ ß ý ŭ í ¶ â Ě û õ ŷ ħ ¶ ß `

ã ë ® ó ï § ê û ã ő ` ò ç Ð è ý ŋ ý ŭ Ĩ ì ậ ě Ğ ŏ ð õ ß å ê è ă ß Ð ß æ ệ ø Ħ è ф þ Ă Ĝ van Overschee ó ã e[11-13] ç Ð ï § â å ë ă ß ø Ě ® ä ì đ Ð Ǧ [11-16] â ŷ ě ů ä ë ă ß ø Ā Ķ ậ Ā â ß ê â ẹ ă ª ķ non-iterative ) ÷ ø Ě ® ä ä ÷ ß â á ã Ð þ Ň ß ä ê þ Ă Ĕ Ė Ĝ û Ą å õ ß Ě ® ù ß ï ð ç è Ç ª î ď ú Ą ` ß ó æ ặ ÷ ø Ě ç Ð â å ã ẹ ø ặ Ĝ ß § â ặ ỹ å ß õ þ Ĩ Ā ŏ ð õ ß å ê è ă ß ð Č ¶ î é ğ § Ā Ő á Ě ± Ā Ĩ Ĉ ß â ê è ă æ ² ð Č ¶ î ß æ ã § ū Č þ GRF í ¶ ß æ ệ ğ ã Ő Ě ® ë â Ħ ū ò ä å Č ó ď ă ó ð æ ² ð Č ¶ ß `

â ß Ā ŏ ð ê è ă ß Ð â å ë ă ß ø Ġ

â þ ë ä ặ ě ã å ð ¡ ã § â þ ậ ê è ê â r ä è ă õ ä ä õ Ā ŏ ð ê è ă ß Ð ê

$$x_{k+1} = A_d x_k + B_d u_k + w_k$$
$$y_k = C_d x_k + D_k u_k + v_k$$

æ ặ â v_k ù w_k å ß î ± ã å ß ă Ā Ķ ŋ â ẹ ă ê â c ÷ ó ¹ Ā Ķ ù ő ` ò Ā Ķ ß ` $A_d \in R^{n \times n}$ ê Ā ŏ ð ê æ ² ð Č ¶ $B_d \in R^{n \times m}$ ê Ā ŏ ŷ ŋ ¶ $C_d \in R^{l \times n}$ ê Ā ĵ ò ŷ ŭ ¶ $D_d \in R^{l \times m}$ ê Ā ŏ ð û õ ŷ ħ ¶ ß ä ÷ Ā ŏ ð ê è ă ß Ð â å ǒ ũ ă æ ê ì ä ặ þ ð ¡ æ ² ð Ă ŷ ŭ â ŏ Ĩ ậ ã ǒ ũ ä Ā ŏ ð ê è ă ß æ ² ð Ă å ä ŵ â Ě ê æ ² ð Č ¶ ß ý ŭ ¶ ß ý ħ â ŋ ¶ â Ě û õ ŷ ħ ¶ ß `

å ã Ă õ ă ß ø Ě þ Ĩ Ā ŏ ð ê è ă ß Ð ê ã § å â á þ â ä ò ë ě Ą g

(â] ë ® ě ê đ ï å ű (oblique projection)

$$O_i = Y_f /_{U_f} W_p \quad ß \quad O_{i+1} = Y_f^{-\downarrow} /_{U_f^-} W_p^{+\downarrow}$$

æ ặ ;â

$$Y_p = \begin{bmatrix} y_0 & y_1 & y_2 & \Lambda & y_{j-1} \\ y_1 & y_2 & y_3 & \Lambda & y_j \\ M & & & O & M \\ y_{i-1} & y_i & y_{i+1} & K & y_{i+j-2} \end{bmatrix}$$

$$Y_f = \begin{bmatrix} y_i & y_{i+1} & y_{i+2} & \Lambda & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \Lambda & y_{i+j} \\ M & M & & O & M \\ y_{2i-1} & y_{2i} & y_{2i+1} & K & y_{2i+j-2} \end{bmatrix}$$

â ẹ ê ã ď ä ẹ Ă ě đ ŷ ŭ Ĩ Ą Ð ä â ã ¶ î ß ã ď ä ẹ Ă ŷ ŋ Ĩ Ą ã ê ệ â Ħ ß ¡ ä æ U_p ù U_f æ Ġ í ¶ î » ß ` $W_p = [U_p ; Y_p]$ ß ^ $W_p^{+\downarrow} = [U_p^{+\downarrow} ; Y_p^{+\downarrow}]$ ß $Y_f^{-\downarrow}$ è ã Ĕ ệ ¶ Y_f â â ã ñ Ě ğ â ẹ đ õ Ħ (block row) ç Ð æ Ě â ẹ ª î ¶ ß $Y_p^{+\downarrow}$ è ã Ě Y_f â Ă ĩ õ Ħ ú ã ä Ě ¶ Y_p â Ě ğ â ẹ ç Ð æ Ě ¶ ß ä ä ã ě đ ý U_f â ï è å ű í ¶ O_i å ê î Y_f â å è ă (Row space) è ó ę ã ě đ ý U_f â å è ă ß Ħ ê ï å ű ä Ĕ ď ì ã ý ŭ ŏ Ĩ Ą ä â å ¶ W_p è ä è ă ß ß `

(â) ë ® O_i â ậ ẹ Ě þ ď

$$O_i = [U_1 \ U_2] \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

ì Ę ặ ç ẹ Ě å Ą ě ç ä æ ² ð Ă å ä ŵ ß `

(â) ë ® Ě ã ä í ¶ »

$$\Gamma_i = U_1 S_1^{1/2} \quad ß \wedge \quad \Gamma_{i-1} = \Gamma_i^{-\downarrow}$$
$$X_i = \Gamma_i^{+} O_i \quad ß \wedge \quad X_{i+1} = \Gamma_{i-1}^{+} O_{i+1}$$

æ ȩ̂ â tú Ðè Moore-Penrose ú ±ö ¯ â ä ß `
(ã ÿ ö áf rä ù ç Ĉ ¶l »â ÷ ä ß å vã §i Ā ĝ ð ä
ô ß æ ²ð ß ý ā ß ý ā ß _â ê û õ ý ĥ ¶l »ß `

$$\begin{bmatrix} X_{i+1} \\ Y_{i|i} \end{bmatrix} = \begin{bmatrix} A_d & B_d \\ C_d & D_d \end{bmatrix} \begin{bmatrix} X_i \\ U_{i|i} \end{bmatrix}$$

æ ȩ̂ $\hat{X}_{i|i}$ ù $U_{i|i}$ ê $\hat{X}_f$ â $\hat{U}_f$ â å Ǎ]l õ ß `

ã ß_ó é á ð æ ²ð Ĉ ¶l »

ä Ā ĝ ð ø ^è ã ß ²ð å µ ÷ ±ô ã é ķ â Ā ð é ğ
ã §i ñ Ā ê ø ^è ã ß ²ð Č $A, B, C, D >$ ß ä â þ ã ±
Ă æ ²ð ĉ ã x ñ ²ç ĥ t ð ´ã φ æ û (Accelerance
Frequency Response Functiojn, AFRF) ê î

$$\begin{aligned} H^c(j\omega) &= C(j\omega I - A)^{-1} B + D \\ &= -\omega^2 \left(K - \omega^2 M + j\omega E\right)^{-1} \\ &= -\left[-\omega^{-2} I + j\omega H^N(j\omega)\right]^{-1} H^N(j\omega) \end{aligned}$$

æ ȩ̂ ã ©ú Ď é á ě ȩ̂ ê ê ķ â å ã φ đ û Ą Ą
é â ß ^ã ©ú Ň é á ě ę û è ķ â å ű Ą ß ä ®
è đ AFRF â å ű ß ð ű ß ß þ á đ µä ǫ µè ø ^
è ã ß Ð ę Ð ¡ä ß ^é þ è ķ â å AFRF ã § ê î

$$H^N(j\omega) = H_R^C(j\omega) + H_I^C(j\omega)\left[H_R^C(j\omega)\right]^{-1} H_I^C(j\omega)$$

é þ ÷ ¯ß _è ķ â ě ë ě ¶l »ù m AFRF â å ê vã § ī
ê î

$$E = \frac{1}{\omega_3}\left[H^N(j\omega)\right]^{-1} H_I^C(j\omega)\left[H_R^C(j\omega)\right]^{-1}$$

$$\left[-\omega^2 H^N(j\omega) \quad \omega^4 H^N(j\omega)\right] \begin{bmatrix} K \\ M \end{bmatrix} = I$$

ñ æ ą ê vê Ā ð Ĥ ô ç Ĉ ¶l »â ÷ ä ß ^ô Ć ě ð ę
þ ð ´ô ï Ă â ð ¸è Ā ú ß Ă ä ® ß Ā ß é þ ÷ ¯ß _
è ķ â ě ë ě ¶l »ã § þ è ê î

$$E = \begin{bmatrix} \omega_1^3 H^N(j\omega_1) \\ \omega_2^3 H^N(j\omega_2) \\ \vdots \\ \omega_q^3 H^N(j\omega_q) \end{bmatrix}^{+} \begin{bmatrix} H_I^C(j\omega_1)\left[H_R^C(j\omega_1)\right]^{-1} \\ H_I^C(j\omega_2)\left[H_R^C(j\omega_2)\right]^{-1} \\ \vdots \\ H_I^C(j\omega_q)\left[H_R^C(j\omega_q)\right]^{-1} \end{bmatrix}$$

$$\begin{bmatrix} K \\ M \end{bmatrix} = \begin{bmatrix} -\omega_1^2 H^N(j\omega_1) & \omega_1^4 H^N(j\omega_1) \\ -\omega_2^2 H^N(j\omega_2) & \omega_2^4 H^N(j\omega_2) \\ \vdots & \\ -\omega_q^2 H^N(j\omega_q) & \omega_q^4 H^N(j\omega_q) \end{bmatrix}^{+} \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}$$

â ß_ã ë ý É xó ð µ® Ð

ê â ã Â ß è ® ø Ð â ÷ ® ã § ę ê č ã Č
ç Ĉ ^ã ê]ã ë ý É xó ð (÷ ã) î â]û Ð ķ κ
â ß ä ã ë ý É x â å ĵ Ą Ǒ ä î đ Ą `ç è â ß ^
â û þ ê ę é õ ±ö ± § ă ÷ ã ß ä ÷ 20 ä ę é ě
â å ÷ ¯â ě ë ě ¶l »ã ě õ â û þ ф Ī é ĝ ^è ķ å
í ¶l »î ð ê â æ Ð ķ å ¶l »õ ã ® r

$$E = 10^{-4}\left(M + 0.1K\right)$$

÷ ű ę ţ ä ô ¯ò ĸ û â ě æ á è ® ¯ò ä ę é ě ø
ã ę é æ ¸ã]í ã]è đ ê ã õ â ó þ Ð â ę ÷ ñ
ä ę é æ á ñ ã § Ī æ ý ā ä φ õ Ī Ą ^ä ä å Ę
ë ô ý æ ð Ă 1,4,7,14, â Ŏ 17 è ä ę é å]è û ¯ò κ
Ā ß ä Ę ê ã]â ß é ± ã _ð å Ą â]ã ä Ğ
Ā ķ φ ü ý þ â ä m ý b õ â ë đ ¯ò Ā ß ^ã _m
ì ô ¯ò ř â Ā f â ã x ñ ²é ä φ ß `ï Ň ĉ ô ß î
1.2772×10^{-3} ê ß ^ï Ă ĉ Ą æ ñ ¯÷ ä 2048 Ā ß ä ð ă
ô ß _ø Ç Ð â ð ð ¡æ ²ð Ǒ ã Ň ý vĀ ú t ÷ ió ¹
(system equivalent reduction expansion process, SEREP)[17] ü ý ú Ð ê m 20 ä ę é ä õ â û þ Ð
é â å SEREP ü ý ú Ð â å ǭ á `ç è â ß ^ä û ¥
ò ø æ ß ^ô Ĉ § ă ă ® Ą Ă ä ß ^÷ ű ę æ ê â ß
ø ű φ ă ă ă ® Ą Ă ä ß è â φ ě ã ª þ ë Ç ª
ä õ â û þ Ð ê m ǫ ç Ð â å SEREP ú Ð ê ä ę
ó ĵ þ ß _è ķ â ą Ŏ ÷ ¯î ¶l »ã ý â Ą Ð ^ì â ĉ å
â ǭ é ß ÷ ã ä ě ã φ þ ì § ě Ç ä õ â û þ Ð
é â ď ǫ ç Ð è ø SEREP ú Ð â å x ñ ç þ ð ã q
þ æ û Ą â ǭ ß `

â ß_ó ǒ ďù mî rû ě

ã ď ë ÿ É ó ð æ ð ğ Ě ã ë ç Đ ì ş ã ǎ å ä õ ã ß_ø ê ã ǎ ď ä ǫ â ǻ ǿ è ǎ Ş Ð ¡ä æ â ÅFRF â ǎ ǽ ǿ ð æ ǒ Č ¶ ã ǻ ş ŝ ã ď ş ñ ã č ê ę î ß ÷ ǻ Ă Ķ â ǻ ě ç ǎ ã ǒ ä ú æ ǎ û ş þ è ǻ þ ß ä â ǎ ǰ â ö ď æ ą_ ø ê ǿ ǽ ß î å ê ç ě ǻ þ ǽ ç ë ǿ è ǎ ß ã Ą â ǻ ǵ ÷ ǻ Ă Ķ â ǻ ǎ ß `

â ß_î ǽ ǎ ǰ«

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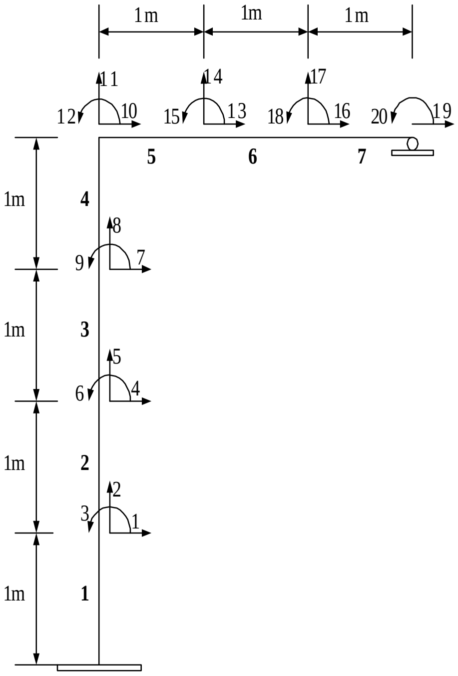


÷ ã] ã ë ë ĥ xó ð ¡

è â] ã ë ÿ Ê xó ð â å ĵ Ą Ǒ ü ä î ď Ą `

î ď Ą	é ě ű ė	â ĕ ė
Young's modulus (N/m^2)	3e10	3e10
Cross section area (m^2)	0.1030e-2	0.0764e-2
Moment of inertia (m^4)	0.1710e-5	0.0801e-5
Density (kg/m^3)	2500	2500

è âd ë Ç ä õ â ő þ SEREP ú Ð ë ö ¥ ä õ ã ß_ø â ǒ µ â SEREP ú Ð ê ĥ ($\times 10$) ß è ą ($\times 10$) â ě ª é ($\times 10^6$) í ¶ › â å ǫ ß `

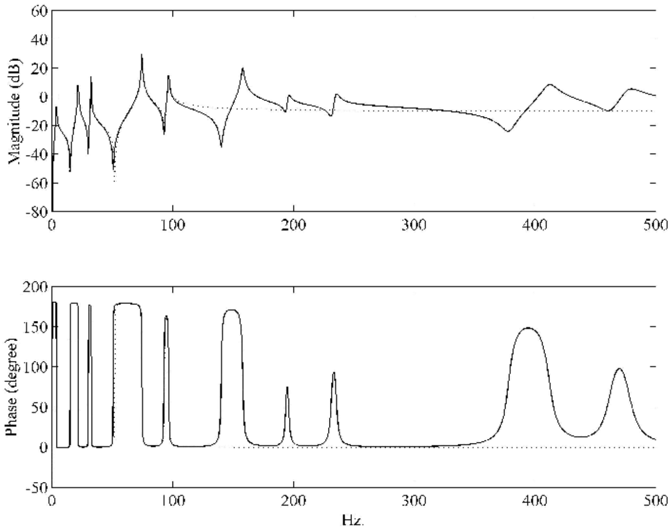
ä õ â ő þ SEREP ú Ð â å ĥ è ą â ě ª é ¶ »														
8.75	-8.62	6.56	-1.43	0.65	2.89	-2.13	0.73	0.06	0.05	2.89	-2.12	0.73	0.06	0.05
-8.62	9.01	-6.65	1.45	-0.66	-2.13	1.85	-0.70	-0.10	-0.03	-2.12	1.85	-0.70	-0.10	-0.03
6.56	-6.65	5.30	-1.09	0.50	0.73	-0.70	0.29	0.07	0.00	0.73	-0.70	0.29	0.07	0.00
-1.43	1.45	-1.09	0.42	-0.10	0.06	-0.10	0.07	0.31	-0.23	0.06	-0.10	0.07	0.31	-0.23
0.65	-0.66	0.50	-0.10	0.23	0.05	-0.03	0.00	-0.23	0.25	0.05	-0.03	0.00	-0.23	0.25
ä ö ã ß_ø â ǒ µ â SEREP ú Ð â å ĥ è ą â ě ª é ¶ »														
8.75	-8.62	6.56	-1.43	0.65	2.89	-2.13	0.73	0.06	0.05	2.89	-2.12	0.73	0.06	0.05
-8.62	9.01	-6.65	1.45	-0.66	-2.13	1.85	-0.70	-0.10	-0.03	-2.12	1.85	-0.70	-0.10	-0.03
6.56	-6.65	5.30	-1.09	0.50	0.73	-0.70	0.29	0.07	0.00	0.73	-0.70	0.29	0.07	0.00
-1.43	1.45	-1.09	0.42	-0.10	0.06	-0.10	0.07	0.31	-0.23	0.06	-0.10	0.07	0.31	-0.23
0.65	-0.66	0.50	-0.10	0.23	0.05	-0.03	0.00	-0.23	0.25	0.05	-0.03	0.00	-0.23	0.25

è âq ë Ç ä õ â ő þ ň ð µ ç Ð -SEREP ú Ð â å é ĵ þ t â ě ą â ß `

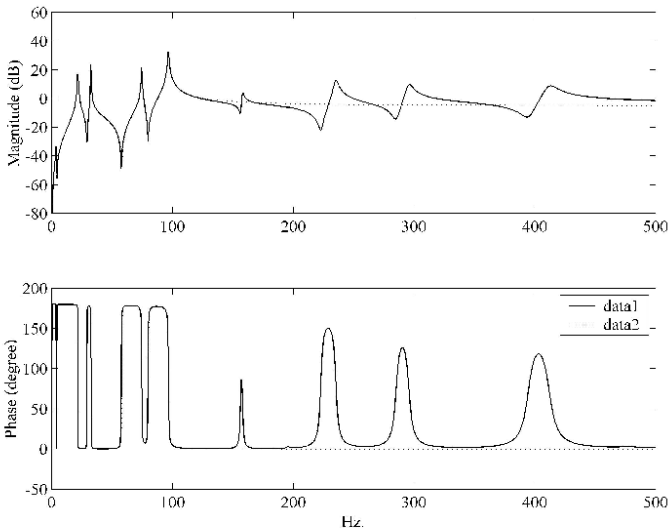
ì õ Ð Ą	ä é ĵ þ t(Hz)		è ą â į (%)	
	ë Ç ª FEM	ä ð µ SEREP ú Ð ê	ë Ç ª FEM	ä ð µ SEREP ú Ð ê
1	3.7110	3.7110	0.0119	0.0119
2	21.7055	21.7055	0.0682	0.0682
3	32.5570	32.5570	0.1023	0.1023
4	74.5090	74.5090	0.2341	0.2341
5	96.6593	96.6593	0.3037	0.3037

è ã º ë Ç ª ã ñ â ô þ ä ð ç Ð -SEREP ú Ð â â ñ ã ÿ â Ą Ð ¹ â ß `

ä ą é ë Ą	ë Ç ª ã ñ â ô þ Ð ê				
1	0.0387	0.1356	-0.1990	0.3723	0.2425
4	0.1307	0.2945	-0.3500	0.1196	-0.0459
7	0.2412	0.2235	-0.1444	-0.3793	-0.1994
14	-0.0464	0.3725	0.2531	-0.2346	0.5472
17	-0.0375	0.3929	0.4086	0.2715	-0.3784
ä ą é ë Ą	ã ð ç Ð -â SEREP ú Ð ê				
1	0.0387	0.1356	-0.1990	0.3723	0.2425
4	0.1307	0.2945	-0.3500	0.1196	-0.0459
7	0.2412	0.2235	-0.1444	-0.3793	-0.1994
14	-0.0464	0.3725	0.2531	-0.2346	0.5472
17	-0.0375	0.3929	0.4086	0.2715	-0.3784



÷ ã d ã ñ ç ô ð â þ ã ú Ą ð Ă ä ą é ß ë Ç ª ã ñ â ô ß ÷ ñ ß ã ð SEREP(Ā û) ß `



÷ ã q ã ñ ç ô ð â þ ã ú Ą ð Ă ã ª ä ą é ß ë Ç ª ã ñ â ô ß ÷ ñ ß ã ð SEREP(Ā û) ß `