An Analysis of Flow Around Ventilated Hydrofoils

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Abstract

The present proposal considers the free-surface effect on the high-speed hydrofoils. Both cavitation and surface ventilation may occur in the near fields of high-speed hydrofoils. First, we studied the fully and partially cavitating bubbles when the bubbles are near the free surface and discuss the change of hydrofoil lift due to cavitating bubbles and free surface. The lifting force on such a composite model is calculated under the free-surface assumption. All the computations are based on nonlinear equations instead of the conventional linear equations. In this project a two-dimensional wedge entering a free surface is considered.

Keywords: Ventilation, wedge

Two-dimensional cavitating flows have been addressed for a long time by using the hodograph technique in details see Birkhoff & Zarantonello (1957). However, due to the difficulty of extending this technique to three-dimensions, the linear theory was, somehow, preferred to analyze cavitating flows even for two-dimensional hydrofoils. Many references of such applications based on the linear theory could be found in Tulin & Hsu (1980). Uhlman (1987) first developed a nonlinear numerical method to investigate the effects of hydrofoil thickness and camber on cavity volume. After that, the nonlinear boundary integral method has become the main tool to solve the flow problems due to a partial or super cavitation. Different formulations based on the boundary integral equations [4] - [8] were derived by many researchers in the last few years.

As for the ventilated foil or propeller, the discussion is even less. A conformal mapping technique was applied in solving water entry and exit problems in the two-dimensional flows [9], [10]. A boundary integral method was used to investigate partially submerged ventilated propellers by Furuya [11]. A periodic solution was solved in this paper. Although the boundary integral method was widely used to simulate the wave problems and fluid-structure interaction problems such as impact problems, the water entry or exit problem was caught less attention in the society. In this paper we try to analysis the flow of water entry by the boundary element method.

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Consider the unsteady, irrotational flow field of an inviscid, incompressible liquid penetrated by a two-dimensional wedge near a free surface. The flow is a potential flow and has a potential function, c, which satisfies the Laplace equation

$$\nabla^2 c = 0 . \tag{1}$$

Let u and v be the flow velocities in X and Y directions, respectively. Then, by the definition,

$$\frac{\hat{o}¤}{\hat{o}¶} = ³ , \frac{\hat{o}¤}{\hat{o}·} = ´ \tag{2}$$

In addition, the following boundary conditions on c are needed. There are four types of boundaries on the problem: a body surface boundary Γ_w, a rigid bottom Γ_b, and a free surface boundary Γ_f, and a radiation condition. On the free surface, Γ_f, there are two boundary conditions required, dynamic and kinematic conditions. Let $\overset{\rho}{R}$ be the position vector of a particle on the free surface. The kinematic boundary condition on Γ_f is denoted as

$$\frac{a\overset{\rho}{o}}{a^2} = \left(\frac{\hat{o}}{\hat{o}^2} + \overset{\rho}{r} \cdot \nabla \right) \overset{\rho}{o} = \overset{\rho}{r} \text{ on } \Gamma_{¤} \tag{3}$$

in which $\overset{\rho}{U}$ is the velocity vector of the particle, and

$$\frac{D}{Dt} = \frac{\hat{o}}{\hat{o}t} + \frac{\hat{o}¤}{\hat{o}x}\frac{\hat{o}}{\hat{o}x} + \frac{\hat{o}¤}{\hat{o}y}\frac{\hat{o}}{\hat{o}y} \tag{4}$$

represents the time derivative in the Lagrangian description. Hence, the velocity of free surface particles is,

$$\frac{a\overset{\rho}{o}}{a^2} = \nabla¤ \text{ on } \Gamma_{¤} . \tag{5}$$

Let E¶ls l¦F be the position vector of a particle on the free surface. Then

$$³ = \frac{a¶}{a^2} , \quad ´ = \frac{as}{a^2} \quad \text{on } \Gamma_{¤} \tag{6}$$

The other boundary condition of the free surface, the dynamic boundary condition, is the Bernoulli equation:

$$\frac{a¤}{a^2} = -¥¦ + \frac{N}{0}\left|\nabla¤\right|^{0} \quad \text{on } \Gamma_{¤} \tag{7}$$

in which g is the gravitational acceleration, ¦ is the altitude of free surface, and º is the density of fluid. The atmospheric pressure p_a is taken to be zero. In this paper, the bottom is fixed, so the normal velocity of the bottom boundary vanishes.

$$\frac{\hat{o}¤}{\hat{o}¬} = M \text{ on } \Gamma \ , \tag{8}$$

Equation (8) is the boundary condition on the bottom boundary Γ , and n represents the unit normal vector. In the same way, the body surface boundary condition can be assigned as:

$$\frac{\hat{o}¤}{\hat{o}¬} = s_{¬}\left(¶l \cdot l_{¸} l^2 \right) , \text{ on } \Gamma_w , \tag{9}$$

where $s_{¬}$ E¶l · l¸ l² F is the outward normal velocity of the body surface boundary.

Theoretically speaking, the radiation condition means that the local disturbance cannot propagate into infinity. In practice a virtue radiation boundary is frequently imposed for numerical purpose such that

$$\frac{\hat{o}¤}{\hat{o}n} = c\frac{\hat{o}¤}{\hat{o}t} , \tag{10}$$

where c is the speed of energy propagation.

The boundary value problems mentioned in the above section can be transformed into a boundary integral equation along the boundary Γ. Let the source point S locate at (x_s, y_s, z_s) on the boundary, and the field point q locate at (X_q, Y_q, Z_q) inside the domain. Denoting $G(S,q)$ as the two-dimensional free-space Green's function of the Laplace Equation, we can write

$$G(S,q)=\frac{\ln r}{2®}, \qquad (11)$$

$$r(S,q)=\sqrt{(x_S-X_q)^2+(y_S-Y_q)^2} \qquad (12)$$

We substitute equation (11) and (12) into Green's second identity

$$\int_v\left(¤\nabla^2 G-G\nabla^2 ¤\right)dv=$$

$$\int_\Gamma\left(¤\frac{ôG}{ôn}-G\frac{ô¤}{ôn}\right)d\Gamma \qquad (13)$$

Moving the field point q toward the boundary, we have the boundary integral equation:

$$c(Q)¤(Q)+\int_\Gamma ¤(S)\frac{ôG(S,Q)}{ôn}d\Gamma$$

$$=\int_\Gamma G(S,Q)\frac{ô¤(S)}{ôn}d\Gamma, \qquad (14)$$

where c is a coefficient from the geometry defined as

$$c(Q)=-\int_\Gamma\frac{ôG(S,Q)}{ôn}d\Gamma. \qquad (15)$$

In this study, the isoparametric linear element is utilized. For discretization, the geometrical or physical function f on the boundary can be expressed as

$$f(¸_1)=\sum_{k=1}^{2}¨_k(¸_1)f_k, \qquad (16)$$

where $¸_1$ is the local element coordinates, $¨_k(¸_1)$ is the linear element shape function and f_k is the relative function value on the node in each element. Then the derivative of function f on the boundary element is defined as

$$\frac{ôf}{ô¸_i}=\sum_{k=1}^{2}\frac{ô¨_k(¸_1)}{ô¸_1}f_k \qquad (17)$$

After the boundaries being discretized, equation (14) is treated as the Boundary Element Method.

In the following, the Lagrangian description is applied at the free surface only. When the liquid tank is subjected to a forced ground motion, the free surface position and the velocity potential change with time. We use the Taylor series expansion with Lagrangian description to catch the new position of the free surface and to calculate its relevant boundary properties, when the time marches. Between the time step t and $t+\Delta t$, in which Δt is a small time increment, a particle ($¶,s$) on the free surface at time t will move to its new position ($¶',s'$) at time $t+\Delta t$. We trace this particle and its velocity potential by the Taylor series expansion, such as

$$¶_i'=¶_i+\Delta t\frac{D¶_i}{Dt}+\frac{\Delta t^2}{2}\frac{D^2¶_i}{Dt^2}+$$

$$O\left[(\Delta t)^3\right]\ \text{i=1,2} \qquad (18)$$

$$¤'=¤+\Delta t\frac{D¤}{Dt}+\frac{\Delta t^2}{2}\frac{D^2¤}{Dt^2}+$$

$$O\left[(\Delta t)^3\right] \qquad (19)$$

where $¶_1=¶$, and $¶_2=s$.

At each time step, we first solve the velocity potential or its normal derivative from the boundary integral equation described in the last section. Then, we apply the Lagrangian description in succession to predict the new position of the free surface by Eqs. (18) and (19) to update the velocity potential at the next time step. In this study, the second-order expansion is adopted, and the higher order terms are dropped.

For calculating the time derivative terms in the above Taylor series expansion equations, we must know the normal and tangential vectors at each node on the free surface elements. Therefore, a few intermediate steps are needed, which are described in the following. First, we have to establish a local orthogonal coordinate of each free surface node, in which $\vec{s}_1$ is a unit tangential vector to the local element coordinate $¸_1$ at one node, and $\vec{n}$ is the outward unit normal vector at the same node. By the above definition, the tangential derivatives of a function f can been written as:

$$\frac{ôf}{ôs_1}=\frac{ô¸_1}{ôs_1}\frac{ôf}{ô¸_1}, \qquad (20)$$

Hence the derivative terms between global and local coordinates are related as

$$\frac{\partial}{\partial x_i}=N_i\frac{\partial}{\partial n}+S_{1i}\frac{\partial}{\partial s_1}, \qquad (22)$$

in which N_i and S_{1i} are the components of unit normal and unit tangential vectors in X and Y directions, respectively. From Eqs. (6)

and (8), we have the first-order Lagrangian time derivative of position (i.e. velocity components) at every surface node in global coordinates:

$$\frac{D¶_i}{Dt} = U_i = N_i \frac{\hat{o}¤}{\hat{o}n} + S_{1i}\frac{\hat{o}¤}{\hat{o}s_1} \qquad (23)$$

$$\frac{D¤}{Dt} = -g¦ + \frac{1}{2}U_i U_i \,. \qquad (24)$$

Next, the second order time derivatives of positions and ¤ on the free surface can be written as

$$\frac{D^2¶_i}{Dt^2} = \frac{\partial^2¤}{\partial t \partial x_i} + U_j \frac{\partial ¤,_j}{\partial x_i}, \qquad (25)$$

$$\frac{D^2¤}{Dt^2} = U_i \frac{DU_i}{Dt} - g\frac{D¦}{Dt}\,. \qquad (26)$$

Since ¤ is continue everywhere in the tank, the term $\partial^2¤/\partial t\partial x_i$ in equation (25) can be expressed as $\partial ¤_t/\partial x_i$ and

$$\frac{\partial ¤_t}{\partial x_i} = N_i \frac{\partial ¤_t}{\partial n} + S_{1i}\frac{\partial ¤_t}{\partial s_1}, \qquad (27)$$

where $¤_t$ on the free surface is calculated from equation (7). Because $¤_t$ in the computing domain satisfies the Laplace equation, we can use the same idea mentioned in the last section to solve the boundary value problems such as

$$\nabla^2 ¤_t = 0\,,$$

$¤_t$ or $\dfrac{\partial ¤_t}{\partial n}$ is specified on free surface,

body surface, or bottom (28)

Then the unknown value $\partial ¤_t/\partial n$ on the free surface will be solved. In this kind of problems, it is unnecessary to do any more integration for solving $\partial ¤_t/\partial n$ because the coefficient matrix for Eq. (14) is exactly the same as that for Eq. (28) in the same time step. The other terms in Eq. (25) such as, $\partial ¤,_j/\partial x_i$, can be represented as

$$\frac{\partial ¤,_j}{\partial x_i} = N_i \frac{\partial ¤,_j}{\partial n} + S_{1i}\frac{\partial ¤,_j}{\partial s_1}\,. \qquad (29)$$

Further expanding the above equation, we have

$$\begin{aligned}\frac{\partial ¤,_j}{\partial n} &= \frac{\partial}{\partial n}\left[\frac{\hat{o}¤}{\hat{o}n}N_j + \frac{\hat{o}¤}{\hat{o}s_1}S_{1j}\right] \\ &= N_j \frac{\partial}{\partial n}\frac{\partial ¤}{\partial n} + S_{1j}\frac{\partial}{\partial s_1}\frac{\partial ¤}{\partial n}\end{aligned} \qquad (30)$$

$$\begin{aligned}\frac{\partial ¤,_j}{\partial s_1} &= \frac{\partial}{\partial s_1}\left[\frac{\hat{o}¤}{\hat{o}n}N_j + \frac{\hat{o}¤}{\hat{o}s_1}S_{1j}\right] \\ &= N_j \frac{\partial}{\partial s_1}\frac{\partial ¤}{\partial n} + S_{1j}\frac{\partial}{\partial s_1}\frac{\partial ¤}{\partial s_1}\end{aligned} \qquad (31)$$

With previous calculation, all the terms except $\partial^2¤/\partial n^2$ in Eqs. (30)-(31) can be evaluated one by one on local elements. Finally, the term $\partial^2¤/\partial n^2$ can be evaluated through the Laplace equation such that

$$\frac{\partial^2¤}{\partial n^2} = -\frac{\partial^2¤}{\partial {s_1}^2}\,. \qquad (32)$$

The computing results are shown in Figs. 1 and 2 for flows without and with air buble, respectively.

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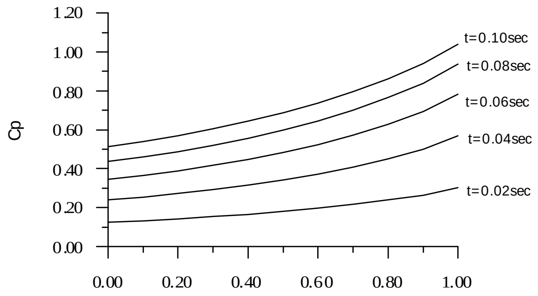

Fig. 1 The pressure distribution on the wedge without air bubble.

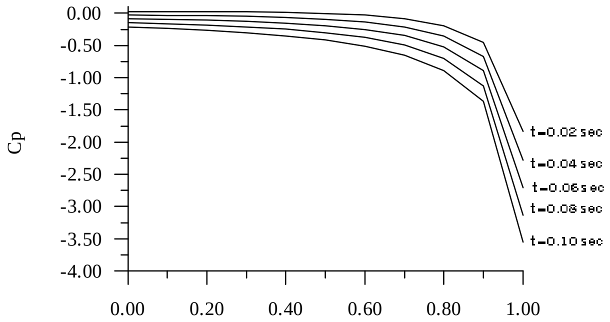

Fig. 2 The pressure distribution on the wedge with air bubble.