行政院國家科學委員會專題研究計畫成果報告

邊界頻譜法在聲學問題的研究

Boundary spectral method for acoustic waves

計畫編號：NSC 88-2611-E-002-021
執行期限：87 **年** 8 **月** 1 **日至** 88 **年** 7 **月** 31 **日**

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一、中文摘要

本文應用邊界頻譜法求解邊界積分方程式。利用某些數學恆等式，將 Helmholtz **積分方程的其異性解除，所以頻譜法所用的基礎函數與非奇異核函數乘積積分能有效且精確完成。與傳統方法作比較，本方法並不直接求解節點之物理量，而是求解廣義** Fourier **係數，這對未知數數目和儲存矩陣大小的降低都有很大的幫助。本文中以圓球的散射及輻射問題為例作為本方法的驗證。**

關鍵詞：邊界頻譜法、Helmholtz **積分方程、聲波**

Abstract

A boundary spectral method is developed to solve acoustical problems with arbitrary boundary conditions. A formulation, originally derived by Burton and Miller, is used to overcome the non-uniqueness problem in the high wave number range. This formulation is further modified into a globally nonsingular form to simplify the procedure of numerical quadrature when spectral methods are applied. In the present approach, generalized Fourier coefficients are determined instead of local variables at nodes as in conventional methods. The convergence of solutions is estimated through the decay of magnitude of the generalized Fourier coefficients. Several scattering and radiation problems from a sphere are demonstrated with high wave numbers in the present paper.

Keywords: boundary integral equation; boundary spectral method; hypersingular integral; acoustics

二、緣由與目的

Boundary integral methods based on the Helmholtz integral equation have been widely used in solving exterior acoustical problems for a number of years. For applications with an infinite domain, boundary element methods offer the important advantage that the computational dimensions of the problem can be reduced by one. However, such boundary integral methods retain one serious deficit, i.e., the non-uniqueness of the solution at certain discrete wave numbers.

To overcome the nonuniqueness problem, several boundary integral formulations have been proposed in the literature. The CHIEF (combined Helmholtz integral equation formulation) method, proposed by Schenck,[1] is a simple and popular method for most applications. In the CHIEF representation, the system of linear equations is combined with a few additional equations produced from the Helmholtz equation in the interior domain. However, when those chosen interior points fall on the nodal surfaces of the corresponding interior problem, the additional equations by the CHIEF method fail to provide enough linearly independent constraints. It turns out that this method may not be completely reliable and efficient in practice, especially at high wave numbers.

Another commonly used method, which has been shown to be valid for all wave numbers, was proposed by Burton and Miller.[2] In their formulation, a linear combination of the Helmholtz equation and its normal derivative with a complex factor was presented. Although this approach is robust for numerical implementation, it suffers the drawback of hypersingular integrals, which are extremely complicated in computation.

Several other schemes were recently developed to reduce the hypersingular integral into different forms of integrals which were less singular and more suitable for computation. Krishnasamy et al.[8] showed that the hypersingular integral could be converted into regular line and surface integrals locally through an extension of Stokes' theorem. Guiggiani et al.[9] transformed the hypersingular integral in local elements into a sum of a double and a one-dimensional regular integrals in the parameter planes of local elements. Chien et al.,[10] Liu and Rizzo,[11] Cruse and Richardson,[12] and Hwang[13] proposed different kinds of regularization procedures by using certain known identities associated with the Laplace equation or elasticity. Some well-developed techniques to evaluate hypersingular integrals are discussed in solid mechanics such as Okada et al.,[14] Chien et al.,[15] and Richardson et al.[16] In the above mentioned regularization schemes, all of them were based on local elements except Hwang's formulation. In the papers of Chien et al. and Liu and Rizzo, they only lowered the order of kernel singularity but the kernels were still weakly singular. In Hwang's formulation, the hypersingular integral is successfully transformed into a sum of global regular integrals.

Traditional boundary element methods only used low-order interpolation functions for body shapes and unknown variables. The disadvantage of low-order polynomials is less accurate. In order to maintain accuracy of results, the number of elements cannot be greatly reduced. Beside that, the structure of matrices from boundary element methods is always full or dense. Such dense matrices are much more expensive to solve than banded matrices of finite element methods. Because of the dense matrices, lower-order polynomials or local approximations are inefficient. Hwang[20,21] developed similar ideas for solving acoustic and potential problems in three dimensions. It has been proven that the global approach is very successful for all these problems.

Up to now, boundary spectral methods are only developed to solve the integral equations with weakly singular kernels in all the mentioned cases. An integral equation with higher singularities has not been discussed. The reason is that the numerical treatment of a hypersingular kernel is tedious, especially when a high-order interpolation function is applied. However, the difficulty of the hypersingularity from the normal derivative of the Helmholtz integral equation has been removed by a regularization procedure[13] as mentioned before. In this paper, the author successfully extends the spectral approach to the hypersingular integral which frequently occurs in acoustical problems with high wave numbers.

三、結果與討論

The first example is a pulsating sphere of unit radius, vibrating with a uniform radial velocity. The analytical solution of this problem can be described by an acoustical source:

$$\Phi(P) = \frac{e^{ikr}}{r}$$

where the source center is at the origin. By using the axisymmetric property of the sphere, ten collocation points are distributed on a semicircle to evaluate the unknown Fourier coefficients. From the above euation, both real and imaginary parts of velocity potential are constant along the sphere surface. It is easy to check the accuracy of numerical implementation of the present method. The case, ka = 50, is calculated with several different numbers of Gaussian integration points. In Fig. 1, the velocity potentials with maximum errors on the sphere surface are shown for different numbers of numerical integration points. If the number of integration points is sufficient, convergence of the generalized Fourier coefficients is very fast. The absolute value of the generalized Fourier coefficients with 200 integration points is plotted in Fig. 2. As shown, the first term of Chebyshev polynomials has a peak value and the other terms die out immediately.

The second example is the radiation problem of an oscillating sphere of radius a with a radial velocity cosθ. The exact solution of this problem is given by[18]

$$\Phi(P)=(\frac{a}{r})^2\cos\theta\,\frac{a(1-ikr)(k^2a^2-2-2ika)}{k^4a^4+4}\cdot e^{ik(r-a)}$$

Again, ten collocation points are distributed on a semicircle to evaluate those unknown Fourier coefficients with 150 integration points. From the above equation, we know that the real part of velocity potential almost disappears, and the imaginary part of velocity potential is completely dominated by the second term of Chebyshev polynomials for the case ka = 50. Therefore, the variation of coefficients is not plotted in this case. Only the numerical results are compared with the exact solution in Fig. 3, and both solutions match excellently.

The third example is the scattering wave from a sphere of radius a by a unit plane wave. Assume the incoming unit plane wave travels to the right along x axis and is described as Φ_{in} = exp(ikx). For the scattered wave from a sphere of radius a, the scattered velocity potential at a distance r from the center of sphere and an angle θ from the x axis is given by[19]

$$\Phi_S(r,\theta)=\sum_{n=1}^{\infty}-\frac{i^n(2n+1)j_n'(ka)}{h_n'(ka)}P_n(\cos\theta)h_n(kr)$$

where $P_n(x)$ is the Legendre function of the first kind, $h_n(x)$ the spherical Hankel function of the first kind, and $j_n(x)$ the spherical Bessel function of the first kind. By using the property of axisymmetry, two cases with wave numbers, ka = 16π and 32π, are calculated. Figure 4 shows the variation of magnitude of c_j for both cases. For the case where ka = 16π, two pieces of elements are used, in which 96 Gaussian points and 48 collocation points are distributed in each element. As shown in Fig. 4, at least sixty terms of Chebyshev polynomials are required for convergence. In Fig. 5, the numerical result already matches the analytic solution very well for n = 60. Actually, the accuracy of c_j depends on not only quadrature points but also the collocation points. Another case, with a higher wave number ka = 30π, is tested. Four hundred and fifty integration points and 180 collocation points are distributed on a semicircle. As shown in Fig. 6, when the absolute value of scattering potential is compared, the result well matches the analytic solution for n = 120.

In the literature, there are only few results of exterior acoustical problems reported for very high wave numbers. In this paper, we demonstrate that the boundary spectral method works well in several examples with non-dimensional wave number ka = 50 and higher.

The advantages of spectral methods are that not only they are very accurate but also they can depict the decay of coefficients, which indicates the convergence of the series. When this method is applied in the boundary integral equation, the overall accuracy depends on both numbers of collocation points and integration points. The error of coefficients is primarily determined by the number of numerical integration points. If the number of integration points is enough, the final accuracy depends on the total terms of approximated functions, or equivalently the number of collocation points in the present approach.

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五、成果自評

本研究內容與原計畫相符、並達成預期目標、研究成果兼具學術及應用價值、適合在學術期刊發表。

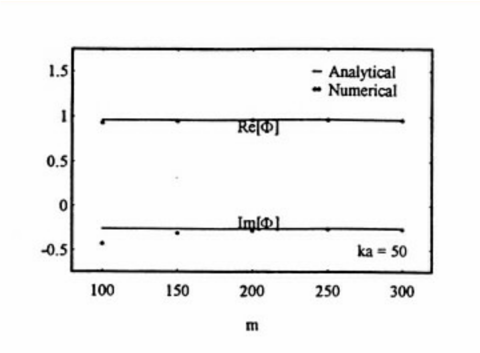

Fig. 1 Maximum errors on the surface of a pulsating sphere

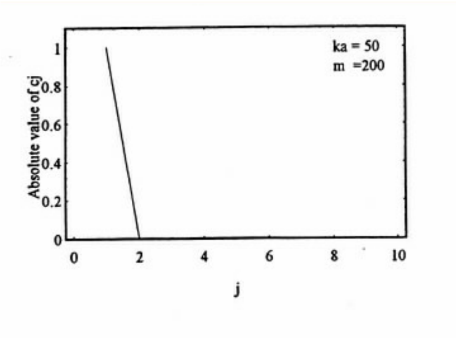

Fig. 2 Generalized Fourier coefficients for radiation from a pulsating sphere when ka = 50.

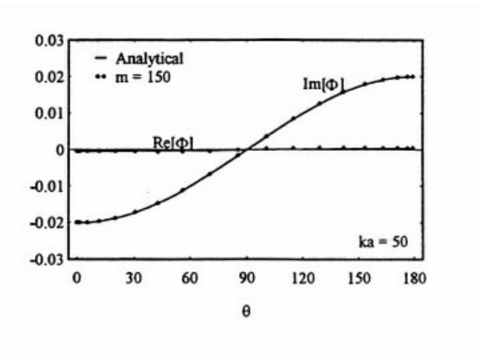

Fig. 3 Velocity potential on the surface of an oscillating sphere when ka = 50.

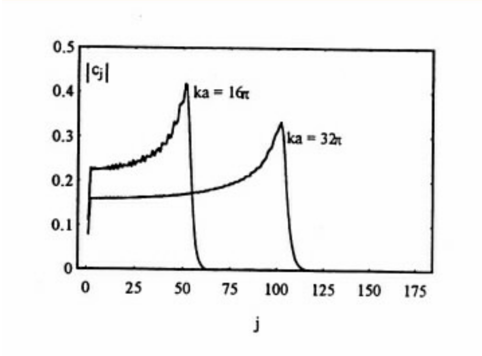

Fig. 4 Generalized Fourier coefficients for scattering from a sphere when ka = 16π and 32π

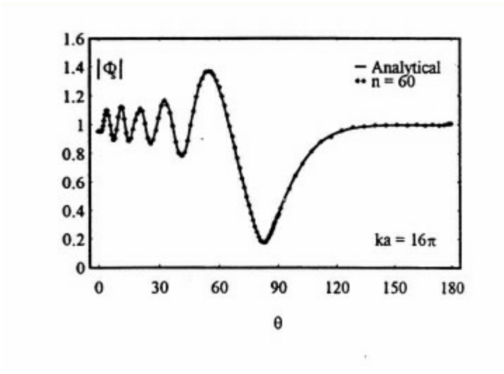

Fig. 5 Absolute values of scattered velocity potential on a sphere surface when ka = 16π.

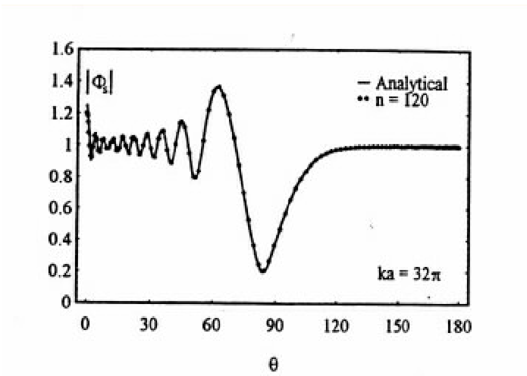

Fig. 6 Absolute values of scattered velocity potential on a sphere surface when ka = 32π.