高樓結構受地表振動之波傳研究

Wave propagation in high-rise building subjected to ground motion

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一、中文摘要

本計畫主要針對地表運動產生的振波在高層結構中傳遞行為進行研究。本計畫發展一結構中波傳 TWSF model**，此模式將考慮振波在傳遞時，遇到結構特性突變或遇到邊界處，產生部份穿越與部份反射，以及反射波多次返回時與入射波產生疊加之定量描述，依此模式推導振波運動的解析解。**

關鍵詞：應力波傳、結構動力

1. Abstract

An analytical method for the analysis of wave propagation in non-uniform shear beam structures is proposed. In this study, the structures are modeled as stepwise shear beams. For the analysis, a TWSF graph model is developed on the base of the wave equations. According to the developed model, analytical and closed-form shear stress and deformation of wave propagation in the structures are determined using the graph theory. The analytical and closed-form frequency response of the structures is also obtained. Finally, a four-segment shear structure is calculated by the derived formulas are presented as illustrations.

Keywords: stress wave, structure dynamics, wave propagation

2. Introduction

Since high frequency excitations consisting of earthquake, traffic disturbance, and machine noise are critical in design, there has been much research on elastic wave propagation in shear beam model. Shear beam structures with non-uniform structural properties and cross-section are usually modeled as multi-segment stepped shear beams for analysis. Several methods including the finite element method, transfer matrix method and dynamic stiffness matrix method have been proposed. [1-4]. However, the classical methods are suitable for numerical computation.

In this project, an analytical method for wave propagation in non-uniform shear structures is proposed. The shear structures are modeled as stepwise cantilever shear beams in the investigation. A graph model is developed to represent the dynamic characteristics of the uniform shear beam segments on base of the wave equation. From the developed graph model, the shear wave responses are derived.

3. Theory

Wave propagation in a multistory structure subjected to lateral load is considered as shown in Fig. 1. The structure includes N uniform segments, in which each segment i consists of n_i identical stories. In the uniform segment i, the height of each story is h_i, while the complex rigidity of all columns is EI_i^*. It is assumed that the floor of each story is rigid. The relation between the total story shear $f_{i,j}$ and the story-to-story displacement of the jth story in the ith segment is [1]

$$u_{i,j+1} - u_{i,j} - \frac{h_i^3 f_{i,j}}{12EI_i^*} = 0\,, \qquad (1)$$

where $u_{i,j}$ is the displacement of the floor of the jth story in the ith segment. If the mass of the floor of each story in segment i is m_i, the equilibrium equation will be

$$f_{i,j+1} - f_{i,j} = m_i \ddot{u}_{i,j}\,. \qquad (2)$$

It is reasonable to regard the system as a multi-segment uniform shear beam continuum, in which one end of the beam is free and the other end is subjected to excitations, as shown in Fig. 2. The relation between the shear force and the displacement variation of the ith segment on cross-section x_i can be expressed as

$$\frac{\partial u_i(x_i,t)}{\partial x_i} - \frac{f_i(x_i,t)}{G_i^* A_i} = 0\,, \qquad (3)$$

where $u_i(x_i,t)$ is the lateral displacement at cross-section x_i, $f_i(x_i,t)$ is the shear force at cross-section x_i, and A_i is the cross-section area of the ith segment. G_i^* is the equivalent shear modulus of the ith segment of the beam. The equation of motion is

$$\frac{\partial^2 u_i(x_i,t)}{\partial x_i^2} - \left(\frac{1}{c_i^2}\right)\frac{\partial^2 u_i(x_i,t)}{\partial t^2} = 0\,, \qquad (4)$$

If a sinusoidal excitation is considered, the displacement and the shear force of any section of the beam should also be sinusoidal with the same frequency. The relation between each complex amplitude of the shear force and displacement at both ends of segment i can be expressed as

$$F_{i-1} = F_i \cos k_i l_i + U_i A_i G_i^* k_i \sin k_i l_i\,, \qquad (5)$$

$$U_i = \frac{F_{i-1} \sin k_i l_i}{A_i G_i^* k_i} + U_{i-1} \cos k_i l_i \qquad (6)$$

where U_{i-1} and U_i are the complex amplitudes of the lateral displacement at both ends of the ith segment for $x_i=0$ and $x_i=l_i$, respectively. According to Eqs. (5) and (6), these relations can be expressed as a two-way state-flow model as shown in Fig.3. In the graph model, the two variables correspond to the shear force and the displacement with opposite directions at both the upper and the lower ends. Moreover, the left state-flows at both ends are all shear forces, whose flow directions are all downward. The right state-flows at both ends are all displacements with upward flow directions. Thus, the state-flow models for each two neighboring segments can be connected in a series to form the entire graph model of the N segment beam. Based on the state-flow model, we obtain the complex frequency response H_{U_i} [5,6]

$$U_i = \frac{U_0 \prod_{j=1}^{i} \cos k_i l_j \sum_{k=0}^{N-i} E_{i+1,N,k}}{\sum_{k=0}^{N} E_{1,N,k}}\,, \qquad (7)$$

The displacement response at location x_i of segment i can be expressed as the frequency response of the displacement at the junction of i and i-1 gives

$$U(x_i) = \frac{U_{i-1}\sin(k_i l_i - k_i x_i) + U_i \sin(k_i x_i)}{\sin(k_i l_i)}\,, \qquad (8)$$

By substituting Eq.(7) into Eq. (8), the frequency response of the displacement at location x_i of segment i can be expressed as

$$U(x_i) = U_0 (\sin k_i (l_i - x_i) \sum_{k=0}^{N-i+1} E_{i,N,k} + \cos k_i l_i \sin k_i x_i \sum_{k=0}^{N-i} E_{i+1,N,k}) \prod_{j=1}^{i-1} \cos k_i l_j / \sin k_i l_j \sum_{k=0}^{N} E_{1,N,k} \qquad (9)$$

3. Examples

A non-uniform shear structure modeled as a four-segment stepped shear beam with equal length l and identical structural properties G^* as well as wave speed c for each segment is investigated in the first example. The area of segments 1, 2, 3, and 4 is 3*A*, 2*A*, *A*, and 2*A*, respectively.

The frequency responses of displacement at junctions 1, 2, 3, and 4 can be calculated by Eq. (7) giving

$$U_1 = \frac{3\cos kl(9\cos^2 kl - 7)}{45\cos^4 kl - 47\cos^2 kl + 8} U_0,$$

$$U_2 = \frac{6(3\cos^2 kl - 2)}{45\cos^4 kl - 47\cos^2 kl + 8} U_0,$$

$$U_3 = \frac{6\cos kl}{45\cos^4 kl - 47\cos^2 kl + 8} U_0,$$

$$U_4 = \frac{6}{45\cos^4 kl - 47\cos^2 kl + 8} U_0,$$

The frequency response of the displacement at any location of each segment can also be calculated by Eq. (9) giving

$$U(x_1) = (\frac{-\cos kl(45\cos^4 kl - 74\cos^2 kl + 29)\sin kx_1}{\sin kl(45\cos^4 kl - 47\cos^2 kl + 8)} + \cos kx_1)U_0$$

$$U(x_2) = 3U_0(\cos kl(9\cos^2 kl - 7)\cos kx_2 + \sin kl (9\cos^2 kl - 4)\sin kx_2)/(45\cos^4 kl - 47\cos^2 kl + 8)$$

$$U(x_3) = 6U_0((3\cos^2 kl - 2)\cos kx_3 + 3\sin kl\cos kl\sin kx_3)/(45\cos^4 kl - 47\cos^2 kl + 8)$$

$$U(x_4) = 6U_0\cos k(l - x_4)/(45\cos^4 kl - 47\cos^2 kl + 8)$$

4. Conclusions

A graph model of a uniform shear beam has been developed for the analysis of wave propagation in shear structures. Based on the graph model, the non-uniform shear structure, modeled as a stepped cantilever shear beam, subjected to excitation at the fixed end is investigated. The deformation responses at any section of each segment are derived and expressed as a concise form. Finally, an example of four-segment shear structure has been investigated to illustrate the implementation of the derived formulae. Although only the shear beam structure has been investigated, this method can be extended to deal with the problem of wave propagation in more complex systems.

5. References

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6. Figures