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黏彈阻尼結構之振動與波傳特性之研究(2/2)

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黏彈阻尼結構之振動與波傳特性之研究(2/2)

On the Vibrations and wave propagation in viscoelastic damped structures (2/2)

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主持人：薛文証
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一、中文摘要

本計畫以 TWSF method 進行黏彈阻尼結構波動與高頻振動問題之基礎性研究。本計畫預計分兩年進行，本年度計畫將根據黏彈阻尼材料力學特性，分析結構具非傳統黏彈阻尼邊界，其應力波動傳遞特性。

關鍵詞：黏彈阻尼邊界、波動傳遞、結構動力

1. Abstract

This study presents a novel method to analyze the vibration of an viscoelastically mounted concentrated mass supported on the joint of crossed beams with viscoelastic foundation. The frequency responses of the displacement of the mounted mass and every beam are derived. Moreover, the force transmissibility from the vibrating mass to the foundation and the frequency equation are obtained. The derived results are expressed in both analytical and closed forms.

Keywords: viscoelastic structures, wave propagation, structure dynamics

2. Introduction

The vibration behavior and transmissibility of a concentrated mass mounted on single and multiple degree-of-freedom lumped isolator systems have received considerable interest [1]. Analytical and numerical methods for obtaining the fundamental frequency and mode shape of a single beam carrying a concentrated mass in free and forced vibrations have also been presented [2-4]. Although previous studies commonly considered the single beam structure, vibrating engines supported by multiple crossed beams are normally used in practical design. The vibration behavior of a system is affected by the dynamic interaction not only between the mounted mass and supported beams but also between one beam and another. Moreover, the flexibility of the foundation for each beam is combined into the dynamics of a system.[5]

In this project, forced vibration analysis of crossed beams with flexible boundary carrying an viscoelastically mounted mass is investigated using a graph method [6]. Viscoelastically foundation for the support of these beams is also considered.

3. Theory

An viscoelastical mounted concentrated mass (the primary system) supported on the joint of N symmetrically crossed beams structure is considered. As assumed herein, each beam is uniform and joined at the midpoint. The ends of each beam are supported by viscoelastic foundation, which is modeled as the combination of the linear spring and dashpot damper. For the *i*th beam, since no load acts between the midpoint and the end of each beam, the governing equation for small amplitude vibration of the beam is given by [7]

$$E_i I_i \frac{\partial^4 w_i}{\partial x_i^4}(x_i, t) + m_i \frac{\partial^2 w_i}{\partial t_i^2}(x_i, t) = 0, \quad (1)$$

where $w_i(t)$ is the beam's displacement at the cross-section x_i, E_i is Young's modulus of beam i, I_i is the moment of inertia of the beam, m_i is the mass per unit length of the beam, and b_i is the half length of the beam. The solution of Eq. (1) can be calculated by separation of variables. Thus, the amplitude of the response of the beam can be expressed in the following form:

$$W_i(x_i) = a_{i,1}\sin a_i x_i + a_{i,2}\cos a_i x_i + a_{i,3}\sinh a_i x_i + a_{i,4}\cosh a_i x_i \tag{2}$$

where W_i is the complex amplitude of the displacement response at location x_i. $a_{i,1}$, $a_{i,2}$, $a_{i,3}$, and $a_{i,4}$ depend on the boundary conditions, and a_i is a function of the forced frequency ω given by $a_i = (m_i\omega^2 / E_i I_i)^{1/4}$. Notably, the slope at the midpoint of the beam is zero since the vibration response of the beam is symmetric. In addition, only translational flexibility for both ends of the beam is considered, while the reaction moment at both ends is zero.Moreover, the shear force around the center of the beam equals half of the summation forces acted by the primary system and other beams, denoted as $f_{c,i}(t)$. If the complex amplitude of the displacement at the end of the beam $W_{b,i}$ is given, the constants $a_{i,1}$, $a_{i,2}$, $a_{i,3}$, and $a_{i,4}$ in Eq. (2) can be expressed by $W_{b,i}$ and $F_{c,i}$

$$a_{i,1} = -\frac{1}{4a_i^3 E_i I_i} F_{c,i}\,, \tag{3}$$

$$a_{i,2} = \frac{1}{2c_i} W_{b,i} + \frac{s_i}{4a_i^3 E_i I_i c_i} F_{c,i}\,, \tag{4}$$

$$a_{i,3} = \frac{1}{4a_i^3 E_i I_i} F_{c,i}\,, \tag{5}$$

$$a_{i,4} = \frac{1}{2ch_i} W_{b,i} - \frac{sh_i}{4a_i^3 E_i I_i ch_i} F_{c,i}\,, \tag{6}$$

where c_i, s_i, ch_i, and sh_i are the symbols of $\cos(a_i b_i)$, $\sin(a_i b_i)$, $\cosh(a_i b_i)$, and $\sinh(a_i b_i)$, respectively.

Substituting Eqs. (3)-(6) into Eq. (2) yields the displacement response of the beam. According to the results, the complex amplitude of the displacement response at the center of the beam can be represented by $W_{b,i}$ and $F_{c,i}$. The boundary condition at the end of the beam reveals that the shear force at the end of the beam equals the force acted by the flexible foundation $f_{b,i}(t)$. Thus, the complex amplitudes $F_{b,i}$ and $F_{c,i}$ are expressed as follows:

$$\frac{F_{c,i}}{2a_i^3 E_i I_i} = \frac{2c_i ch_i}{s_i ch_i - c_i sh_i} W_c - \frac{c_i + ch_i}{s_i ch_i - c_i sh_i} W_{b,i}\,, \tag{7}$$

$$\frac{F_{b,i}}{a_i^3 E_i I_i} = \frac{c_i + ch_i}{s_i ch_i - c_i sh_i} W_c - \frac{1 + c_i ch_i}{s_i ch_i - c_i sh_i} W_{b,i}\,. \tag{8}$$

According to Eqs. (7) and (8), the relationships between $F_{b,i}$, $F_{c,i}$, $W_{b,i}$ and W_c can be described by a two-way state-flow graph model as shown in the upper part of Fig. 1. For the flexible foundation, a combined model of massless spring and dashpot damper is assumed. The relationship between the displacement and the force response is given by

$$W_{b,i} = \frac{F_{b,i}}{j\omega c_{b,i} + k_{b,i}}\,. \tag{9}$$

Equation (9) can also be represented as a graph model as shown in the lower portion of Fig. 1.

The graph model of beam i and its supports shown in Fig. 1 indicate that the paths following the state flow form a closed-loop. There are two forward paths from W_c to $F_{c,i}$. Although one forward path touches the loop, the other one does not. Thus, the ratio of the complex amplitude of the displacement response W_c to the force response $F_{c,i}$, denoted as $H_{W_c,F_{c,i}}$, can be obtained as [6]

$$H_{W_c,F_{c,i}} = 2a_i^3 E_i I_i(-a_i^3 E_i I_i(s_i ch_i - c_i sh_i) + 2c_i ch_i(c_{b,i}\omega + k_{b,i}))/(a_i^3 E_i I_i(1 + c_i ch_i) + (c_{b,i}\omega + k_{b,i})(s_i ch_i - c_i sh_i)) \tag{10}$$

When all of N crossed beams are considered, the force acting on the spring and the damper of the primary system F_c equals the summation of the force acting on all beams. Thus, all the transfer functions $H_{W_c,F_{c,i}}$ can be combined by directly summation. The reduced graph model contains only two loops. For the combined model, there is only one forward path from F_e to W_d. This forward path touches one of the loops. Thus, the complex amplitude of the displacement response of the mass of the primary leads to

$$W_d = ((j\omega c_d + k_d) + \sum_{i=1}^{N} H_{W_c,F_{c,i}})F_e / (-m_d\omega^2(j\omega c_d + k_d) + (-m_d\omega^2 + j\omega c_d + k_d)\sum_{i=1}^{N} H_{W_c,F_{c,i}}) \quad (11)$$

In the same manner, the response of the displacement at the joint W_c can be calculated. There is only one forward path from F_e to W_c. This forward path touches both loops in the combined graph model. Thus, the complex amplitude of the displacement W_c is

$$W_c = (j\omega c_d + k_d)F_e/(-m_d\omega^2(j\omega c_d + k_d) + (-m_d\omega^2 + j\omega c_d + k_d)\sum_{i=1}^{N} H_{W_c,F_{c,i}}) \quad (12)$$

In order to calculate the response of each beam, the graph model of the total system can be rearranged by combining two cascade models: the dynamic coupling of all components of the system and the uncoupling dynamics of each beam and its constraints. From the rearranged model, $W_{b,i}$ and $F_{b,i}$ can be obtained

$$W_{b,i} = a_i^3 E_i I_i (c_i + ch_i) H_{F_e,W_c} F_e /((j\omega c_{b,i} + k_{b,i})(s_i ch_i - c_i sh_i) + a_i^3 E_i I_i (1 + c_i ch_i)) \quad (13)$$

$$F_{b,i} = a_i^3 E_i I_i (c_i + ch_i)(j\omega c_{b,i} + k_{b,i}) H_{F_e,W_c} F_e ((j\omega c_{b,i} + k_{b,i})(s_i ch_i - c_i sh_i) + a_i^3 E_i I_i (1 + c_i ch_i)) \quad (14)$$

If responses $W_{b,i}$ and $F_{b,i}$ as shown in Eqs.(20) and (14) are substituted into Eqs. (3)-(6), the coefficients of $a_{i,1}$, $a_{i,2}$, $a_{i,3}$, and $a_{i,4}$ can be calculated. Thus, the response at each location of the beam is known.

3. Examples

Two identical crossed simply supported beams carrying an elastically mounted mass subjected to force load $F_e \sin\omega t$ on the mass is considered in the example. The damping ratio and the natural frequency of the primary system are 0.05 and 100 rad/sec, respectively. The mass ratio of the mounted mass to each of beam, $m_d/(2\overline{m}\overline{b})$ is 0.5. If the parameter of each beam $\overline{b}^2\sqrt{\overline{m}/\overline{EI}}$ equals $1/\sqrt{g}$, the frequency variable $\overline{a}\overline{b}$ equals $\sqrt{\omega/\sqrt{g}}$. The complex amplitude of the displacement of concentrated mass can be calculated by equation (11). Figure 2 presents the magnitude of the non-dimensional dynamic response of the mounted mass Y, defined as $Y = \left| W_d m_d \omega^2 / F_e \right|$.

4. Conclusions

This work studies the dynamic interaction of an viscoelastically mounted mass supported on the beams with a viscoelastic foundation. Analytical and closed form results of the frequency response of the displacement of each component of the system, the force transmissibility, and the frequency equation are derived. Numerical examples reveal the ease in calculating the dynamic response using the derived formulas.

5. References

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6. Figures

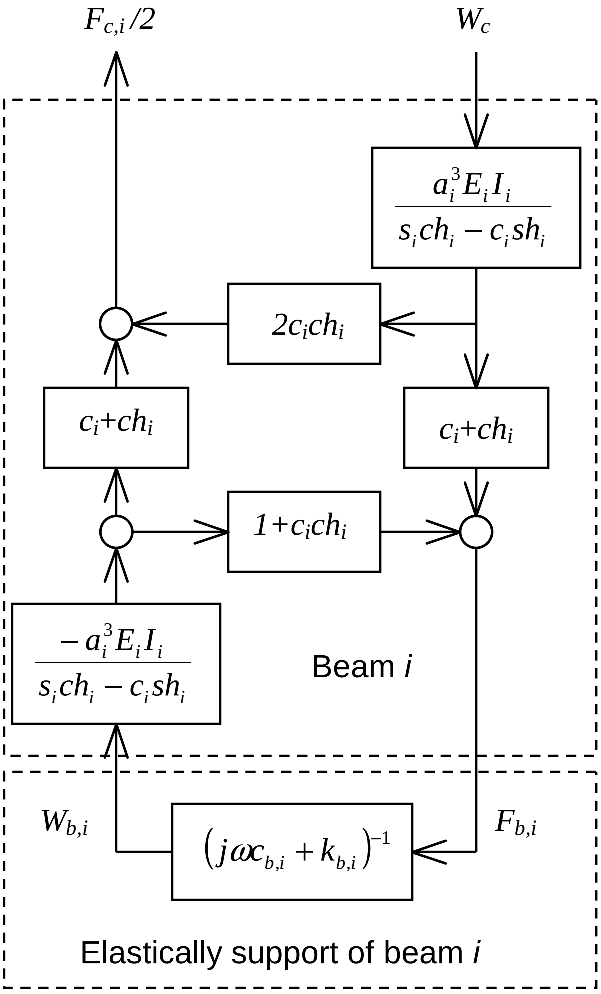

Fig. 1 Graph model for beam *i* and its viscoelastic support.

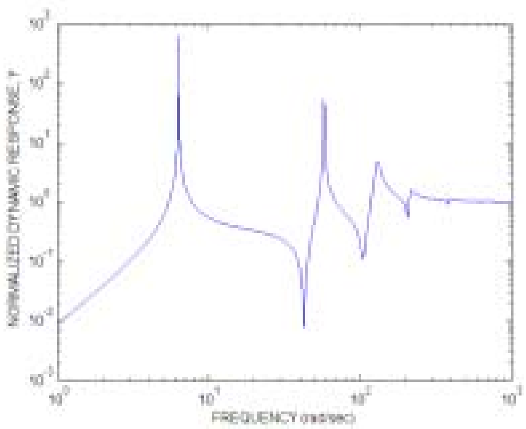

Fig. 2 Magnitude of non-dimensional dynamic response of the system.