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Micromechanics and apparent viscosities of non-Newtonian fluid suspensions

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Abstract

The influence on the overall shear-stress versus shear-rate relation of different volume-fraction solid-filled non-Newtonian fluid suspensions is investigated at the level of dilute concentration. The method developed is based on the approximate mean-field theory for plasticity of two-phase composites. Using the approximation approach of Berveiller and Zaoui [Berveiller, M., Zaoui, A., 1979. An extension of the self-consistent to plastically-flowing polycrystals. *J. Mech. Phys. Solids*, 26, 325–344.], the constraint due to the matrix phase is characterized by the apparent viscosity of the matrix, while the interaction of the solid-filled suspensions is accounted for by the Mori–Tanaka mean-field theory. It is shown that this simple, but approximate theory is capable of predicting the volume fraction dependence of the nonlinear shear-stress versus shear-rate relation. An analytic solution and a master curve of the shear-stress versus shear-rate relation are obtained. This theory of overall viscosity of solid-filled non-Newtonian fluid suspensions was checked by performing the capillary rheometer test to measure the overall viscosity of a glass bead-reinforced polyetheretherketone (PEEK) suspension. The experimental results are in very good agreement with the theoretical predictions. © 1998 Elsevier Science Ltd.

1. Introduction

For most polymer composite products, their manufacture processes, such as injection molding, compression molding, calendaring, colding forming and spinning of fibers, etc., involve flow processes. In application of polymer technology, when suspensions contain rigid fillers, rheology is necessary. Therefore, the rheology of the polymer composite is extremely important. A great variety of molding powders contain either particles or glass fibers. Most rubber composites contain carbon black, and the polyvinyl chloride in tiles and wire coating also

contains various rigid fillers. In those cases where the suspension exhibits Newtonian behavior, the presence of filler produces profound effects on the rheological behavior of suspension. If the liquid phase is a non-Newtonian fluid suspension, its rheology will become even more complex.

Einstein (1906, 1911) first predicted the effect of filler on the viscosity of a Newtonian fluid. Although the Einstein equation is valid only for very low concentrations of particles, it is surprisingly simple. The equation implies that the relative viscosity is independent of the size and nature of the particles. Many equations have been proposed for estimating the viscosity of a liquid containing spherical particles at moderate concentrations. Only a few of the better

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equations have been considered for predicting the viscosity of suspensions containing particles of spherical or other shapes. Nielsen (1974) has modified a general mixture equation of the type derived by Kerner (1956) for the elastic modulus of materials filled with spherical particles. This modified equation has very successfully predicted thermal and electrical conductivities, dielectric constants, and viscosities, in addition to the elastic moduli of solid-filled systems. Considerable work has been devoted to extending the first order solution to higher orders in the particle volume fraction, thereby bringing in the effects of particle interactions (Batchelor and Green, 1972; and others).

The above-mentioned cases are all confined in Newtonian suspensions; in other words, their relations of shear-stress versus shear-rate are linear. However, there have been very few theoretical articles related to solid-filled non-Newtonian suspensions. However, most measurements of the viscosity of suspensions of particles seem to be consistent with the one parameter empirical equation of Kataoka et al. (1978). For non-Newtonian fluid suspensions even in a very small shear-rate range, the shear-stress versus shear-rate relations is nonlinear. The suspension's constraint capability of the particulate's deformation weakens as the shear rate increases. This weakening phenomenon is similar to plastic deformation of polycrystals. The weakening phenomenon of polycrystals was discovered by Hill (1965), as he studied the plastic deformation of polycrystals. He proposed the self-consistent scheme and the relation of tangential elastic moduli for each incremental step to predict the plastic deformation of polycrystals. Due to the plastic deformation, the tangential moduli is keep weakening; therefore, the constraint of the inclusion's deformation is also weakened as the loading is increased. The elastic moduli and Eshelby's transformation tensors are all tangential tensors; therefore, they are anisotropic. Their computation is cumbersome. Berveiller and Zaoui (1979) took the undeformed state of the material as the reference point, using the relation of the secant modulus of the total stress versus strain. Via the secant modulus approach, the elastic modulus now becomes isotropic; therefore, computation becomes easier. Weng (1982) adopted the approach of the secant modulus of Berveiller and Zaoui to evaluate the plastic deforma-

tion of polycrystals. His results were very close to Hill's model. Tandon and Weng (1988) proposed an approach of the Mori-Tanaka formulation which incorporated with the secant moduli weakening effect. The secant modulus of the matrix is the linear stress-strain relation of a comparison material under the external loading $\bar{\sigma}$. To reflect the current state of the matrix more accurately, Weng (1990) and Qiu and Weng (1992) suggested that the secant modulus of the matrix should be at the stress level $\bar{\sigma} \pm \hat{\sigma}$, where $\hat{\sigma}$ is the average perturbed stress due to the presence of the inclusion. It is well known that the problem of determining the elastic stress in a 'suspension' of incompressible elastic particles embedded in an incompressible elastic matrix is mathematically identical in certain respects to that considered in this paper (Hashin, 1964). The rate of strain in the fluid suspension is the analogue of the total strain in the elastic suspension. Under the same given geometrical configurations and loading conditions, the effective shear modulus of a solid-filled incompressible material and the effective viscosities of particulate suspensions would have analogous results only when the shear modulus replaced by the shear rate. Therefore, if the same theory is used, effective viscosities can be determined as well as the effective shear modulus. In this paper, we have adopted the eigen-strain of Eshelby (1957) and the mean stress idea from Mori and Tanaka (1973) to predict the shear-stress versus shear-rate relation of particulate suspensions. Since the suspension behavior of the polyetheretherketone (PEEK) 150P is non-Newtonian, we employed the apparent viscosity idea to capture the shear-rate status at every instant. By means of this apparent viscosity approach, we are able to predict completely the shear-stress versus shear-rate relation of particulate PEEK suspensions. The theoretical predictions were verified by viscosity measurement using the capillary rheometer test.

2. Non-Newtonian suspension constitutive relations and the associated apparent viscosity

An obvious characteristic of polymeric melts is their non-Newtonian fluid behavior where the apparent (secant) viscosity η_a decreases as the shear rate increases. This viscosity decrease extends over many

decades of change in shear rate $\dot{\gamma}$, and the viscosity at a high shear rate may be several orders of magnitude smaller than the viscosity at a low shear rate. This non-Newtonian behavior is important in the processing and fabrication of plastics and elastomers.

Typical $\log \zeta^{\wedge} \sim \log \dot{\gamma}$ curves of PEEK 150P are shown in Fig. 2. The melt has a Newtonian viscosity that is high at very low rates of shear. However, over much of the usual accessible shear-rate range, the viscosity decreases nearly linearly with $\dot{\gamma}$ on the $\log \sim \log$ plot. In this linear range, the so called power law equation holds, in which

$$\bar{\tau} = d\dot{\gamma}^n, \quad (2.1)$$

where d and n are material constants, and $\dot{\gamma}$ and $\bar{\tau}$ are the shear rate and shear stress, respectively. The material constants d and n can be found by the aid of Eq. (2.1) and the method of least squares with data obtained from the capillary rheometer experiment. One can define the apparent viscosity ζ^{\wedge} as

$$\zeta^{\wedge} = \frac{\bar{\tau}}{\dot{\gamma}}. \quad (2.2)$$

From Eqs. (2.1) and (2.2), the apparent viscosity can be written as follows:

$$\zeta^{\wedge} = d\dot{\gamma}^{n-1} = d\left(\frac{\bar{\tau}}{d}\right)^{(n-1)/n}. \quad (2.3)$$

Eq. (2.3) shows that the apparent viscosity is a function of the shear stress. If n is less than one, then the bigger the shear stress the smaller is the apparent viscosity.

3. Apparent viscosity of glass bead-reinforced PEEK suspensions

Under the same given geometrical structure and loading conditions, the effective shear modulus of solid-filled incompressible material and the effective viscosities of fluid suspensions of particulate composites would have analogous results only when the shear strain was replaced by the shear rate. Therefore, if the same theory is used, the effective viscosities as well as the shear modulus can be determined.

In this section, we will adopt the eigen strain of Eshelby (1957) and the mean stress idea from Mori

and Tanaka (1973) to analyze the stress–strain relation of solid particle-reinforced nonlinear matrix composites. These results then can be applied to rigid suspensions. More details and further developments of the Mori–Tanaka averaging scheme and comments can be found in Nemat-Nasser and Hori (1993).

Consider a representative volume of a glass bead-reinforced PEEK resin that is assumed to be statistically homogeneous and macroscopically isotropic. The PEEK resin is referred to as the matrix, and the volume fraction of the glass bead is denoted by f_1 . Following the exposition of Tandon and Weng (1988), we introduce a comparison material of identical shape that is made of PEEK resin. Let the composite and the pure PEEK resin comparison sample both be subjected to the boundary traction that would give a uniform generalized shear state:

$$\bar{\sigma} = (\bar{\tau}). \quad (3.1)$$

The strain ϵ^0 in the comparison sample is related to the above by

$$\bar{\sigma} = L_0^{\wedge} \epsilon^0, \quad (3.2)$$

in which

$$\kappa_0^{\wedge} = \kappa_0^{\wedge}(\bar{\tau}), \quad \mu_0^{\wedge} = \mu_0^{\wedge}(\bar{\tau}), \quad (3.3)$$

where κ_0^{\wedge} and μ_0^{\wedge} are function of $\bar{\tau}$. The average strain and stress, $\epsilon^{(0)}$ and $\sigma^{(0)}$ in the matrix of the composite cannot be the same as ϵ^0 and $\bar{\sigma}$, and may be represented by

$$\sigma^{(0)} = \bar{\sigma} + \tilde{\sigma} = L_0^{\wedge} \epsilon^{(0)} = L_0^{\wedge} (\epsilon^0 + \tilde{\epsilon}), \quad (3.4)$$

The average stress $\sigma^{(1)}$ and strain $\epsilon^{(1)}$ of the glass beads differ from Eq. (3.4) by additional perturbations. The introduction of an equivalent transformation strain ϵ^* , combined with the Mori–Tanaka idea, leads to the identity

$$\begin{aligned} \sigma^{(1)} &= L_1(\epsilon^0 + \tilde{\epsilon} + \epsilon^p) \\ &= L_0^{\wedge}(\epsilon^0 + \tilde{\epsilon} + \epsilon^p - \epsilon^*), \end{aligned} \quad (3.5)$$

where ϵ^p and ϵ^* are related by

$$\epsilon^p = S_0^{\wedge} \epsilon^*, \quad (3.6)$$

in which S_0^{\wedge} is the Eshelby tensor associated with spherical inclusions and L_0^{\wedge} . It has the components

$$S_{\alpha_{ijmn}}^{\circ} = \frac{\eta_{(o)}}{2} (\delta_{im} \delta_{jn} + \delta_{in} \delta_{jm}) + \frac{\xi_{(o)} - \eta_{(o)}}{3} \delta_{ij} \delta_{mn}, \quad (3.7)$$

where

$$\eta_{(o)} = \frac{2(4 - 5\nu_0^*)}{15(1 - \nu_0^*)}, \quad \xi_{(o)} = \frac{1 + \nu_0^*}{3(1 - \nu_0^*)}, \quad (3.8)$$

Finally, since $\bar{\sigma}$ of Eq. (3.1) is also the uniform stress of the composite, we have

$$\bar{\sigma} = f_1 \sigma^{(1)} + (1 - f_1) \sigma^{(o)}, \quad (3.9)$$

where f_1 is the volume fraction of the glass bead. It follows from the above, Eqs. (3.4), (3.5), (3.6) and (3.9), that

$$\epsilon^* = (A^{-1} - (1 - f_1)S_0^* - f_1I)^{-1} \cdot \epsilon^o, \quad (3.10)$$

where

$$A^{-1} = I - L_0^{-1} \cdot L_1. \quad (3.11)$$

The total strain of the composite is given by the weighted average of those of the matrix and inclusion, i.e.,

$$\bar{\epsilon} = (1 - f_1) \epsilon^{(o)} + f_1 \epsilon^{(1)} = f_1 \epsilon^* + \epsilon^o \quad (3.12)$$

Substituting Eq. (3.10) into the above and using Eq. (3.2), we finally conclude that

$$\bar{\sigma} = \bar{L} \cdot \bar{\epsilon}, \quad \bar{L} = (3\bar{K}^s, 2\bar{\mu}^s), \quad (3.13)$$

where

$$\bar{L} = L_0 \left[I + f_1 (A^{-1} - (1 - f_1)S_0^* - f_1I)^{-1} \right]^{-1} \quad (3.14)$$

and \bar{L} is but the secant elasticity tensor for the composite at the stress level Eq. (3.1).

From Eq. (3.4), we can get the stress distribution of the matrix, $\sigma^{(o)}$, under a given stress, $\bar{\sigma}$,

$$\sigma^{(o)} = L_0^* (A^{-1} - S_0^*) \cdot (A^{-1} - f_0 S_0^* - f_1 I)^{-1} \cdot L_0^{-1} \cdot \sigma_0. \quad (3.15)$$

Using the decomposition

$$\gamma_{ij} = \gamma'_{ij} + \frac{1}{3} \gamma_{kk} f_{ij} \quad (3.16)$$

where γ'_{ij} is the deviatoric part and γ_{kk} is the hydrostatic part, from Eqs. (3.14) and (3.15), we can obtain

$$\frac{\bar{K}^s}{K_0^*} = 1 + \frac{f_1}{(1 - f) \epsilon_{(o)} + K_0^*/(K_1 - K_0^*)}, \quad (3.17)$$

$$\frac{\bar{\mu}^s}{\mu_0^*} = 1 + \frac{f_1}{(1 - f_1) \eta_{(o)} + \mu_0^*/(\mu_1 - \mu_0^*)}, \quad (3.18)$$

$$\sigma_{kk}^{(o)} = \frac{K_0^* + \epsilon_{(o)}(K_1 - K_0^*)}{K_0^* + (f_1 + f_0 \epsilon_{(o)})(K_1 - K_0^*)} (\bar{\gamma})_{kk}, \quad (3.19)$$

$$\sigma_{ij}^{(o)'} = \frac{\mu_0^* + \eta_{(o)}(\mu_1 - \mu_0^*)}{\mu_0^* + (f_1 + f_0 \eta_{(o)})(\mu_1 - \mu_0^*)} (\bar{\sigma})'_{ij}. \quad (3.20)$$

Consider the loading condition as Eq. (3.1); then

$$\bar{\sigma} = \begin{bmatrix} 0 & \bar{\tau} & 0 \\ \bar{\tau} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.21)$$

and

$$\bar{\sigma}_{kk} = 0, \quad (3.22)$$

$$\bar{\sigma}' = \begin{bmatrix} 0 & \bar{\tau} & 0 \\ \bar{\tau} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (3.23)$$

so Eq. (3.13) leads to

$$\bar{\epsilon}_{kk} = \frac{\bar{\sigma}_{kk}}{3\bar{K}^s} = 0, \quad (3.24)$$

$$\bar{\epsilon}' = \frac{\bar{\sigma}'}{2\bar{\mu}^s}. \quad (3.25)$$

The overall stress-strain relation of the composite material then can be derived as

$$\bar{\epsilon}_{12} = \frac{\bar{\tau}}{2\bar{\mu}^s} \quad \text{or} \quad \bar{\gamma} = \frac{\bar{\tau}}{\bar{\mu}^s}, \quad (3.26)$$

where $\bar{\gamma} = 2\bar{\epsilon}_{12}$, and $\bar{\mu}^s$ is defined as Eq. (3.18). Other strain components are zero. If we replace the shear strain with the shear rate and replace the shear modulus with the viscosity in Eqs. (3.18), (3.20) and (3.26), we arrive at the following constitutive relations:

$$\sigma^{(o)} = \begin{bmatrix} 0 & \tau^{(o)} & 0 \\ \tau^{(o)} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (3.27)$$

$$\tau^{(o)} = b_{(o)} \bar{\tau} = \frac{\eta_{(o)} + \zeta_o^{\wedge} / (\zeta_1 - \zeta_o^{\wedge})}{f_1 + (1 - f_1) \eta_{(o)} + \zeta_o^{\wedge} / (\zeta_1 - \zeta_o^{\wedge})} \bar{\tau}, \tag{3.28}$$

$$\frac{\bar{\zeta}^{\wedge}}{\zeta_o^{\wedge}} = 1 + \frac{f_1}{(1 - f_1) \eta_{(o)} + \zeta_o^{\wedge} / (\zeta_1 - \zeta_o^{\wedge})}, \tag{3.29}$$

and

$$\bar{\gamma} = \frac{\bar{\tau}}{\bar{\zeta}^{\wedge}}, \tag{3.30}$$

where

$$\eta_{(o)} = \frac{2(4 - 5v_o^{\wedge})}{15(1 - v_o^{\wedge})}. \tag{3.31}$$

For an incompressible fluid suspension, $v_o^{\wedge} = 0.5$ and $\eta_{(o)} = 0.4$, and the relation of Eq. (2.3) can be rewritten as

$$\zeta_o^{\wedge} = d \left(\frac{\tau^{(o)}}{d} \right)^{(n-1)/n}. \tag{3.32}$$

The final product of this section are the composite shear-stress versus shear-rate relations (Eq. (3.13) or Eq. (3.32)). The apparent viscosity $\bar{\zeta}^{\wedge}$, however, depends on $\bar{\tau}$ via $\tau^{(o)}$, the stress of the matrix of the composite. Thus, an iteration procedure must be improvised to facilitate the computation.

4. Shear-stress versus shear-rate master curve of a glass bead-reinforced PEEK suspension

The stiffness of glass bead is usually much harder than PEEK suspension, in the sense that

$$\zeta_1 \gg \zeta_o^{\wedge}. \tag{4.1}$$

Setting $\zeta_o^{\wedge} / \zeta_1$ to zero in Eqs. (3.28), (3.29) and (3.30), we obtain

$$\tau^{(o)} = b \bar{\tau}, \tag{4.2}$$

$$\frac{\bar{\zeta}^{\wedge}}{\zeta_o^{\wedge}} = 1 + \frac{f_1}{(1 - f_1) \eta} = \frac{1}{(1 - f_1) b}, \tag{4.3}$$

where

$$b = \frac{\eta}{(1 - f_1) \eta + f_1} = \frac{1}{(1 - f_1) + 2.5 f_1} = \frac{1}{1 + 1.5 f_1}. \tag{4.4}$$

Substituting Eqs. (4.2) and (4.3) into Eq. (3.30), leads to the following relations

$$\begin{aligned} \bar{\gamma} &= \frac{\tau}{\zeta_o^{\wedge} / (1 - f_1) b} = \frac{\tau}{(d / (1 - f_1) b) (b \tau / d)^{(n-1)/n}} \\ &= \left(\frac{\bar{\tau} b}{d} \right)^{1/n} (1 - f_1). \end{aligned} \tag{4.5}$$

By some manipulation we can have the shear-stress

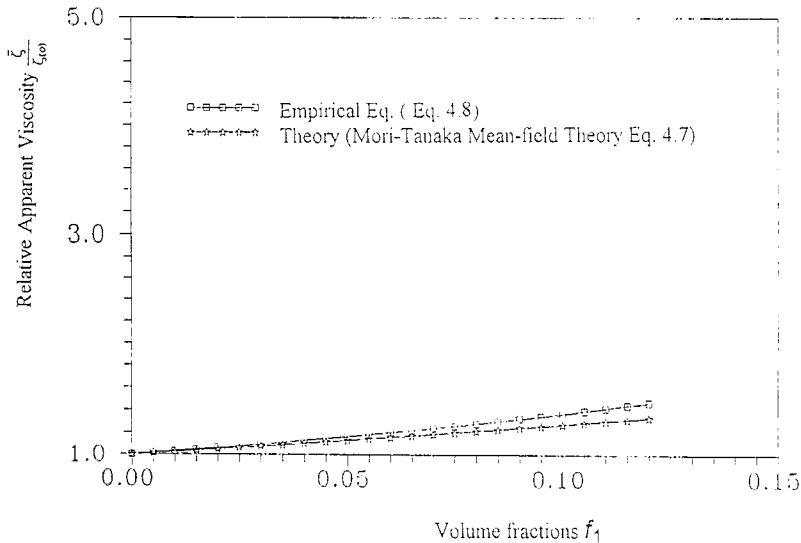


Fig. 1. The secant viscosity ζ^{\wedge} versus volume fractions f_1 curves of Eqs. (4.7) and (4.8).

versus shear-rate relation of a glass bead-reinforced PEEK suspension relation as

$$\bar{\tau} = \frac{d}{b} \left(\frac{\bar{\dot{\gamma}}}{1-f_1} \right)^n \quad (4.6)$$

By the following parameter definitions, we have the shear-stress versus shear-rate master curve as

$$\tau_m = d(\dot{\gamma}_m)^n, \quad \log \tau_m = n \log \dot{\gamma}_m + \log d, \quad (4.7)$$

where $\tau_m = b\bar{\tau}$, $\dot{\gamma}_m = \bar{\dot{\gamma}}/(1-f_1)$ and $b = 1/(1+1.5f_1)$. For pure fluid suspension $f_1 = 0$, and $b = 1$, Eq. (4.7) reduce to the constitutive equation of a pure PEEK fluid suspension. Therefore, Eq. (4.7) can be represented as the shear-stress versus shear-rate master curve of different volume fractions of hardbead-reinforced PEEK fluid suspensions. In other words, once the rheology of a non-Newtonian fluid is known the bulk rheology properties of a suspension of particles in a non-Newtonian fluid can be obtained through this master curve (τ_m vs. $\dot{\gamma}$).

Since most measurements of the viscosity of suspensions of particles seem to be consistent with the one parameter empirical equation of Kataoka et al. (1978),

$$\frac{\bar{\zeta}^s}{\zeta_0^s} = \frac{1}{(1-f_1/A)^2} \quad (4.8)$$

(using $A = 0.68$) and so its use is recommended generally, albeit with caution.

Fig. 1 shows the comparison between our theoretical model (Eq. (4.7)) and the empirical Eq. (4.8). It indicates that our theory can correlate the experimental dates up to moderate ($\sim 10\%$) concentrated suspensions.

5. Experiments

The basic polymer composites used in this work were glass bead-reinforced PEEK 150P resins sup-

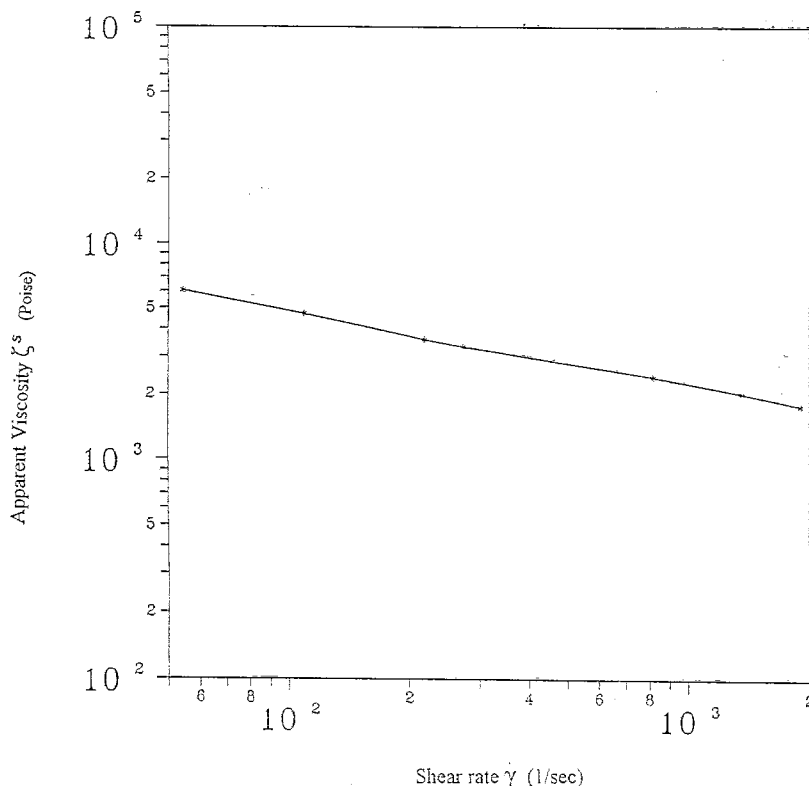


Fig. 2. The secant viscosity ζ_s^s versus shear-rate $\dot{\gamma}$ of PEEK suspensions.

plied by ICI. The materials were supplied as powders in shape. Both pure and glass bead-reinforced resins were obtained. The glass beads, 66 to 88 μm in diameter (from Toshiba-Ballotini Inc.), had been compounded into the PEEK 150P resin at concentrations of 5 and 10% volume fraction. Each set of the 5 and 10% volume fraction materials was compounded using the same base PEEK resin. The concentrations of the filled glass beads and PEEK 150P powder were determined by weight after drying in a vacuum oven at 120°C for several hours.

The viscosities of the polymeric melts at high shear were determined using a Galaxy III Capillary Rheometer (model D9052). The feed materials for the capillary rheometer were used in their powder form as supplied by ICI. The glass bead-reinforced PEEK mixtures were hand packed into the rheometer barrel at the desired temperature by alternately pouring in powders and tamping them with a brass rod. The orifice diameter D and length L are 0.03 and 1.0 in, respectively. Test results were obtained at a

temperature of 370°C for both filled and unfilled mixtures.

The first correction is for entrance and end effects and was suggested by Bagley (1957). Due to the end effects one has to use an effective flow length L_{eff} :

$$\bar{\tau} = \frac{r \Delta p}{2 L_{\text{eff}}} = \frac{\Delta p}{2(L/r + e)}, \quad (5.1)$$

where r is the capillary radius, Δp is the pressure drop and e a correction factor which can be determined by plotting (at constant $\dot{\gamma}$) Δp versus L/r . Since our melt is a non-Newtonian suspension and $L/D = 33$ is large enough, the corrections can be ignored. The second correction is for the non-Newtonian character as such. It is the so-called correction of Rabinowitsch (1929)

$$\dot{\gamma} = \frac{4\Phi}{\pi r^3} \frac{3n+1}{4n}, \quad (5.2)$$

where

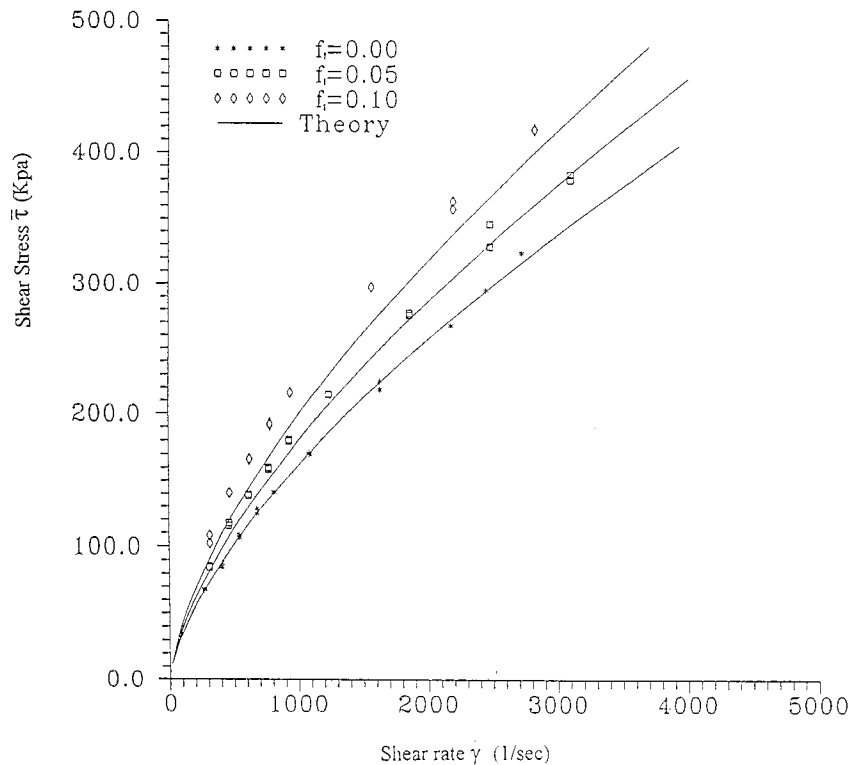


Fig. 3. The shear stress $\bar{\tau}$ versus shear-rate $\dot{\gamma}$ curve for different volume fractions of glassbeads-reinforced PEEK suspensions.

$$n = \lim_{L/r \rightarrow \infty} \frac{d \ln(\frac{1}{2} r dp/dL)}{d \ln(4\Phi/\pi r^3)} \quad (5.3)$$

(n is the Ostwald-de Waele constant). n may be determined by plotting $\ln(\frac{1}{2} r dp/dL)$ versus $\ln(4\Phi/\pi r^3)$ for different L/r ratios and extrapolating to $L/r = \infty$. Φ is the volume rate. The real viscosity is then obtained as $\zeta' = \bar{\tau}/\dot{\gamma}$.

6. Results and discussion

It is now of interest to examine the influence of different volume-fraction glass beads on the shear-stress versus shear-rate relation as predicted by theory. To this end we assume in our calculations that the inclusions and the matrix take the properties of the glass beads and the PEEK 150P resin, respectively. The Young's modulus of the glass bead is 68500 MPa and the Poisson's ratio of the glass bead is 0.24. The viscosity constants d and n of the PEEK resin can then be found to be 1.56 kPa · s and

0.67, respectively, by the aid of Eq. (2.3) and the method of least squares with the data obtained from the capillary rheometer test. A typical ζ - $\dot{\gamma}$ curve is shown in Fig. 2. Fig. 3 shows that the resulting shear-stress versus shear-rate solution (Eq. (4.6)) is in good agreement with the experimental results.

By introducing a shear-rate parameter, $\dot{\gamma}_m$, and a shear-stress parameter, τ_m , the shear-stress versus shear-rate curve of different volume fractions of glass beads regresses to the pure PEEK suspension curve. The shear-stress versus shear-rate master curve of the glass bead-filled PEEK resin can be found by the aid of Eq. (4.7). The results are shown in Fig. 4.

7. Conclusions

We have developed a micromechanically based composite model for apparent viscosities of particulate non-Newtonian suspensions which accounts for the coexistence of particulate and non-Newtonian

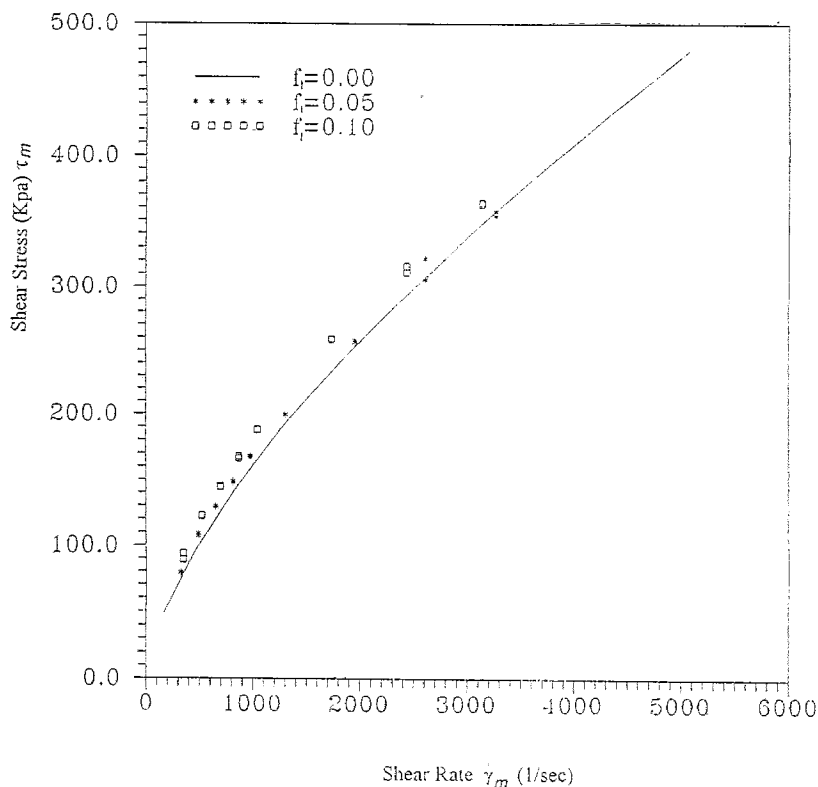


Fig. 4. The generalized shear-stress versus shear-rate master curve of glass bead-reinforced PEEK suspensions.

polymeric melt phases. Power-law constitutive equations accounting for the non-Newtonian behavior are given for the polymeric melt phase. An aggregate of two-phase composite inclusions is used in modeling the particulate-filled polymers. The Mori–Tanaka mean field theory is proposed as interaction law for the composite inclusions. As an example of the applicability of the proposed model, we consider the shear stress-shear rate behavior of particles reinforced PEEK suspensions. The mechanical behavior of a solid-filled PEEK fluid suspension is found to be non-Newtonian, depending partly on the content of the solid-fillers. To describe such Non-Newtonian characteristics, a nonlinear shear-rate parameter, $\dot{\gamma}_m$, and a shear-stress parameter, τ_m , which describe the non-Newtonian behavior of hard bead-reinforced PEEK suspensions verified by the capillary rheometer test, have been proposed, to the authors' knowledge, for the first time. The predicted results are in generally good agreement with experimental observations. The predicted macroscopic shear stress-shear rate responses of the particles-reinforced PEEK suspensions subject to different rate of shearing are shown in Fig. 4 for a given set of different volume fractions of particle reinforced.

The main reason for the success of the proposed model is that we clearly identify the basic element (mechanically coupled composite inclusion) which constitutes the spherulite of the particulate-filled polymeric melts.

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