三材料之熱應力分析

計劃編號:NSC 88-2212 E-002-008
執行期限: 87/08/01 -- 88/07/31

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一、中文摘要:

關鍵字：熱彈性，應力奇異性，三材料，異向性。

本計畫乃針對二維之三材料接合材界面，作熱彈性之應力奇異性研究分析。相較於文獻中的一般研究，在此將引進 the stroh formalism 在熱彈性問題上之架構及應用，以分析異向性之三材料接合界面問題(圖一)。

界面接合之應力奇異性研究，在以往之文獻中，經常只考慮等向性及異向性雙材料接合之彈性問題，甚少涉及熱彈性之分析，原用可能在熱彈性之理論架構較複雜之故，本計畫乃利用 the stroh formalism 在熱彈性問題上之架構，由於其所有之簡單性，將可利用於異向性三材料接合界面之熱應力奇異性之研究。

本計畫所提出之異向性三材料接合界面，由於架構簡單，其物理表現（如位移量）可利用 ESPI 實驗作即時性測量，亦可透過電腦模作分析，最後可將兩者之結果與理論解作一比較，以討論理論解之正確性，本計畫之重點在於

利用 the stroh formalism 在熱彈性問題上數理架構，研究三材料界面應力奇異性問題。

1. ESPI 實驗之架構及操作技術
   - a)量測所用材料之各種材料係數。
   - b)利用實驗與電腦模擬結果印證理論解之正確性。
2. 數值模擬三材料問題。

英文摘要:Abstract:

Keywords: thermoelasticity, stress singularity, trimaterials, anisotropic

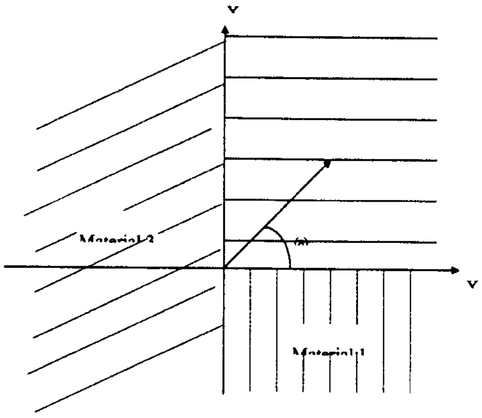
We consider the two-dimensional thermoelastic problem involving three anisotropic materials bonded together (trimaterials) as shown in Fig.1. The stress singularity will be investigated. In here, we employ the extended version of the Stroh formalism which includes the thermal effect.

The stress singularity of elastic bimaterials has been extensively studied in the literature. However, only a few consider the thermal effect because of its complexity. The simplicity of the Stroh formalism is shown again by studying the thermoelastic problem of trimaterials.

Due to the simple configuration of the trimaterials, experiments will be conducted by means of ESPI analysis and numerical modeling, solutions obtained can be employed to make comparison with the analytical one. Hence, the correctness of the asymptotic theory will be discussed.

The work deals chiefly with:
the application of the extended version of the Stroh formalism in thermo-anisotropic elasticity.

1. Set up and conduct of the ESPI experiment.
   - a). Measurement of material constants.
   - b). the comparsion of the asymptotic solution with experimental and numerical ones.
2. Numerical modeling using commercial software.



圖一、三材料接介界面圖

二、計劃緣由與目的:

The stress singularity around singular point contributes important information about the fracture failure mechanism of composite materials. Extensive studies had been made in isotropic and anisotropic bimaterials problems (Williams, 1959; Erdgan, 1963; Rice and Sih, 1965; England, 1965; Shih and Asaro, 1988; Hutchinson, 1992; Gotoh, 1967; Clements, 1971; Willis, 1971; Ting, 1986, Suo, 1990, etc). The simplicity of the Stroh formalism furnishes significant progress (Ting, 1986; Qu and Bassani, 1989; Ting, 1990). There are fewer studies concerning the stress singularity involving more than two elastic materials (Zwiers and Ting, 1983). The reason may lie on the complexity of the problem and the solution obtained does not directly apply on crack failure problems. Nevertheless, such solution contains information about the onset criteria of interfacial crack problem in real life situation. Furthermore, thermal failure is a big concern of composites application today. This worsens the situation because of the intricacy of anisotropic thermoelasticity (Yuuki, Xu and Kayama, 1993). Again the elegant of the Stroh formalism provides a possible answer. For example, Yan (1993) had considered a bimaterials problem involving thermoelastic materials.

In here, the stress singularity of an anisotropic thermoelastic trimaterials is studied (Fig. 1). Its configuration represents a general real life practice. The solution obtained provides insights for the thermal stress failure mechanism. Experiments will be conducted by using ESPI or Moire analysis. Numerical modeling will also be considered by using commercial software.

The extended version of the Stroh formalism is employed (Clements, 1973; Wu, 1984; Hwu, 1990). In general, the asymptotic solutions can be expressed as follows.

$$T^{(k)} = C(\delta+1)r^{\delta}(\xi^{(k)}\zeta_r^{(k)\delta} + \eta^{(k)}\overline{\zeta_r^{(k)}}^{\delta}),$$
$$\mathbf{u}^{(k)} = Cr^{\delta+1}(\mathbf{A}^{(k)}\square^{(k)\delta+1}\mathbf{q}^{(k)} + \overline{\mathbf{A}^{(k)}\square^{(k)}}^{\delta+1}\mathbf{y}^{(k)} + \mathbf{c}^{(k)}\xi^{(k)}\zeta_r^{(k)\delta+1} + \overline{\mathbf{c}^{(k)}}\eta^{(k)}\overline{\zeta_r^{(k)}}^{\delta+1}),$$
$$h_i^{(k)} = -C\delta(\delta+1)r^{\delta-1}[(k_{i1}^{(k)} + \tau^{(k)}k_{i2}^{(k)})\xi^{(k)}\zeta_r^{(k)\delta-1} + (k_{i1}^{(k)} + \overline{\tau^{(k)}}k_{i2}^{(k)})\eta^{(k)}\overline{\zeta_r^{(k)}}^{\delta-1}],$$
$$\mathbf{f}^{(k)} = Cr^{\delta+1}(\mathbf{B}^{(k)}\square^{(k)\delta+1}\mathbf{q}^{(k)} + \overline{\mathbf{B}^{(k)}\square^{(k)}}^{\delta+1}\mathbf{y}^{(k)} + \mathbf{d}^{(k)}\xi^{(k)}\zeta_r^{(k)\delta+1} + \overline{\mathbf{d}^{(k)}}\eta^{(k)}\overline{\zeta_r^{(k)}}^{\delta+1}).$$

The stress singularity is expressed in δ with real part in between -1 and 0.

三、研究方法及進行步驟

理論解部份

採用 the stroh formalism 在熱彈性問題上之數學架構。

實驗量測部份

將採用 ESPI 實驗做實驗量測。

此實驗可簡單且直接量測三材料接合界面上之位移量。

數值模擬部份

將採用商業用 cosmos 軟體作模擬分析。

進行步驟：

1)理論解之推導
2)三材料接合試片之製作
3)實驗量測
4)量測所需材料之各種材料係數
5)數值模擬
6)理論解之印證

四、結果與討論

By means of the extended version of the Stroh formalism on thermoelasticity , the asymptotic temperature and displacements solution around the singularity point is expected to be

$$T^{(k)} = C(\delta+1) r^{\delta} (\xi^{(k)} \zeta_r^{(k)\delta} + \eta^{(k)} \overline{\zeta_r^{(k)}}^{\delta}),$$

$$\mathrm{u}^{(k)} = C r^{\delta+1} (\mathbf{A}^{(k)} \square^{(k)\delta+1} \mathrm{q}^{(k)} + \overline{\mathbf{A}^{(k)} \square^{(k)}}^{\delta+1} \mathrm{y}^{(k)} + \mathrm{c}^{(k)} \xi^{(k)} \zeta_r^{(k)\delta+1} + \overline{\mathrm{c}^{(k)}} \eta^{(k)} \overline{\zeta_r^{(k)}}^{\delta+1})$$

$$h_i^{(k)} = -C\delta(\delta+1) r^{\delta-1} [(k_{i1}^{(k)} + \tau^{(k)} k_{i2}^{(k)}) \xi^{(k)} \zeta_r^{(k)\delta-1} + (k_{i1}^{(k)} + \overline{\tau^{(k)}} k_{i2}^{(k)}) \eta^{(k)} \overline{\zeta_r^{(k)}}^{\delta-1}]$$

$$\mathbf{f}^{(k)} = C r^{\delta+1} (\mathrm{B}^{(k)} \square^{(k)\delta+1} \mathrm{q}^{(k)} + \overline{\mathrm{B}^{(k)} \square^{(k)}}^{\delta+1} \mathrm{y}^{(k)} + \mathrm{d}^{(k)} \xi^{(k)} \zeta_r^{(k)\delta+1} + \overline{\mathrm{d}^{(k)}} \eta^{(k)} \overline{\zeta_r^{(k)}}^{\delta+1})$$

with (k)= 1，2，3 represents the 3 different materials considered   represents the stress singularity with $-1 < \mathrm{Re}\{\delta\} < 0$.

製作二維三材料接合界面試片，(圖一)並利用 ESPI (圖二) 實驗，進行一定溫差之位移量(圖三)量測。為確實掌握所用材料之各種材料係數，將直接對試片材料作量測。

數值模擬將採用商業用軟體 cosmos 作分析，並引入使用量材料之各種材料係數，以準確模擬三材料接合問題，計算其在一定溫度下之位移量。

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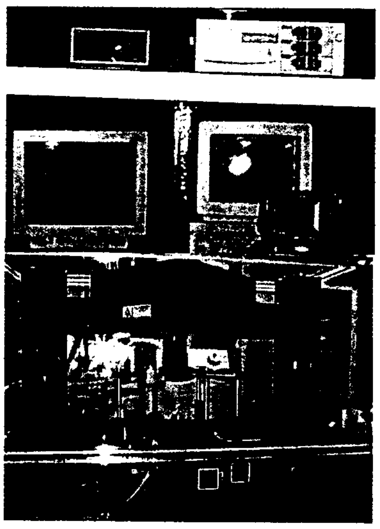

電子光斑干涉儀

圖 二

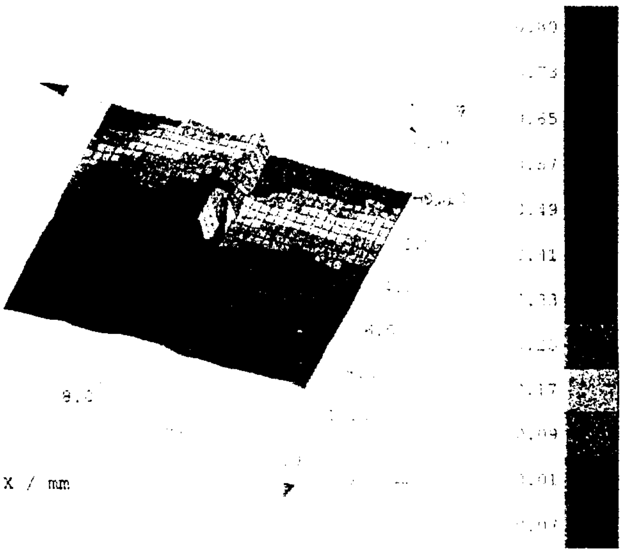

圖 三