无人載具之導航與控制 (II)

Navigation and Control of Unmanned Vehicles (II)

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Introduction

To perform the navigation of an unmanned vehicle, various sensors, such as GPS, INS, compass, encoder, etc, can be used. Depending on their characteristics, different sensors may have different advantages. For example, GPS may be more sensitive to low-freqency noise, while INS is more susceptible to high-frequency noises. In order to integrate these sensors, the algorithm of data fusion along with the Kalman filter may be adopted. However, there are some issues having to be tackled. First, the initial setting of the algorithm must be given. Secondly, if the assumption of independence in the Kalman filtering is not valid, it is necessary to deal with dependent processes. In this report, an algorithm of determining the initial settings, including the error covariance, the process noise covariance and the measurement noise covariance, is proposed. On the other hand, the covariance intersection algorithm is used to solve the problem regarding the dependence of the information. The combination of these strategies is then used to design the fusion INS-GPS system for the navigation of a vehicle. The experimental results showed that the algorithm is more robust comparing with classical Kalman filtering algorithm.

1 Background Material

First, some basic formula in estimation theory are reviewed. Let x, z be random vectors. From the observation Z on z, it is desired to estimate x. The MMSE(Minimum mean-square error) estimator is defined to be

$$\hat{x}^{MMSE} = \arg\min_{\hat{x}} E\left[(\hat{x}-x)^2 |Z\right].$$

It can be shown that the solution of the previous minimization problem is

$$\hat{x}^{MMSE} = E\left[x|Z\right]. \tag{1}$$

Furthermore, if x, z are jointly Gaussian with covariance matrices denoted by

$$\begin{aligned} P_{xx} &= E\left[(x-\bar{x})(x-\bar{x})^T\right], \\ P_{xz} &= E\left[(x-\bar{x})(z-\bar{z})^T\right] \\ &= P_{zx}^T, \\ P_{zz} &= E\left[(z-\bar{z})(z-\bar{z})^T\right], \end{aligned} \tag{2}$$

where $\bar{x}$, $\bar{z}$ are the mean vectors of x, z respectively, the conditional mean can be further expressed as

$$E\left[x|Z\right] = \bar{x} + P_{xz}P_{zz}^{-1}(z-\bar{z}). \tag{3}$$

The associated conditional variance is

$$P_{xx|z} = P_{xx} - P_{xz}P_{zz}^{-1}P_{zx}. \tag{4}$$

Accordingly, the MMSE estimate (1) can be found as

$$\hat{x} = \bar{x} + P_{xz}P_{zz}^{-1}(Z-\bar{z}), \tag{5}$$

and the corresponding covariance matrix is computed through (4). On the other hand, by the least-square type argument, the estimator in (5) can be also obtained for the estimation of Non-Gaussian random vectors.

Secondly, we review the Chi-square distribution. A random variable q has a Chi-Square distribution with degree of freedom n, denoted by $q \backsim \chi_n^2$, if its pdf is of the form

$$p(q) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} q^{\left(\frac{n-2}{2}\right)} e^{-\frac{q}{2}},$$

where Γ denotes the Gamma function defined through

$$\begin{aligned}\Gamma(m+1) &= m\Gamma(m), \\ \Gamma(1/2) &= \sqrt{\pi}, \\ \Gamma(1) &= 1.\end{aligned}$$

The random variable q defined by

$$q = (x-\bar{x})^T P^{-1}(x-\bar{x})$$

is chi-square distributed with degree of freedom n. Furthermore, the mean value and the variance of q are n and $2n$ respectively. For given two independent chi-Square random variables $q_1 \backsim \chi_m^2$ and $q_2 \backsim \chi_n^2$, a new random variable q_3 defined by

$$q_3 = q_1 + q_2,$$

is also Chi-Square distributed with $(m+n)$ degrees of freedom.

2 Basic Kalman Filtering Algorithm

Consider a linear system

$$x(k) = \Phi(k-1)x(k-1) + u(k-1), \qquad (6)$$

with measurement

$$z(k) = H(k)x(k) + w(k) \qquad (7)$$

where the process noise $u(k-1)$ and the measurement noise $w(k)$ are assumed to be independent with Gaussian distributions $N(0, Q(k-1))$ and $N(0, R(k))$ respectively. The problem is to estimate $\hat{x}(k)$,given measurement

$$Z^k = \{z(1), z(2), ..., z(k)\}.$$

A recursive process, termed the Kalman filter, was developed to perform the estimation, and the process can be divided into two parts. One is to predict the state at k from the observations through $(k-1)$. Next is to correct the prediction by current measurement at k. The predictions of both the states and measurement based on Z^{k-1} can be obtained from the MMSE estimator as

$$\begin{aligned}\hat{x}(k|k-1) &= E\left[x(k)|Z^{k-1}\right], \\ \hat{z}(k|k-1) &= E\left[z(k)|Z^{k-1}\right].\end{aligned}$$

Regarding the above equations as the means of x, z respectively. The correction step based on the observation $z(k)$ is then performed through (5),

$$\hat{x}(k|k) = \hat{x}(k|k-1) + P_{xz}(k|k-1)P_{zz}^{-1}(k|k-1) \\ (z(k) - \hat{z}(k|k-1)) \qquad (8)$$

and

$$P_{xx}(k|k) = P_{xx}(k|k-1) - P_{xz}(k|k-1)P_{zz}^{-1}(k|k-1) \\ P_{zx}(k|k-1) \qquad (9)$$

where the conditional covariance matrices, P_{xz} and P_{zz} are defined similar to (2). In fact, denoting

$$\tilde{x}(k|k-1) = x(k) - \hat{x}(k|k-1),$$

$$\nu = z(k) - \hat{z}(k|k-1),$$

we have

$$P_{xx}(k|k-1) = E\left[\tilde{x}(k|k-1)\tilde{x}(k|k-1)^T|Z^{k-1}\right],$$

$$P_{xz}(k|k-1) = E\left[\tilde{x}(k|k-1)\tilde{z}(k|k-1)^T|Z^{k-1}\right],$$

$$P_{zz}(k|k-1) = E\left[\tilde{z}(k|k-1)\tilde{z}(k|k-1)^T|Z^{k-1}\right].$$

From the dynamic equations(6,7), the prediction $\hat{x}(k|k-1)$ can be further expressed in terms of $\hat{x}(k-1|k-1)$ as

$$\hat{x}(k|k-1) = \Phi(k-1)\hat{x}(k-1|k-1).$$

The corresponding update rule for the covariance matrix is

$$P_{xx}(k|k-1) = \Phi(k-1) P_{xx}(k-1|k-1) \Phi^T(k-1) + Q(k-1).$$

The gain in eqs. (8) is called the Kalman gain

$$K(k) = P_{xz}(k|k-1) P_{zz}^{-1}(k|k-1).$$

The covariance of the innovation ν can be computed as

$$\begin{aligned} S(k) &= P_{zz}(k|k-1) \\ &= H(k) P_{xx}(k|k-1) H^T(k) + R(k). \end{aligned}$$

Based on these notations, the Kalman filtering algorithm can be summarized as, from $\hat{x}(k-1|k-1)$, $P_{xx}(k-1|k-1)$,

$$\hat{x}(k|k-1) = \Phi(k-1) \hat{x}(k-1|k-1),$$

$$\hat{z}(k|k-1) = H(k) \hat{x}(k|k-1),$$

$$P_{xx}(k|k-1) = \Phi(k-1) P_{xx}(k-1|k-1) \Phi^T(k-1) + Q(k-1),$$

$$S(k) = H(k) P_{xx}(k|k-1) H^T(k) + R(k),$$

$$K(k) = P_{xx}(k|k-1) H^T(k) S^{-1}(k),$$

$$\nu(k) = z(k) - \hat{z}(k|k-1),$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\nu(k),$$

$$P_{xx}(k|k) = P_{xx}(k|k-1) - K(k) S(k) K^T(k).$$

However, there are some problems associated with the applications of this method. First, it is not straightforward to determine the initial condition of this problem. Secondly, for multiple sensors in different sampling rates, it is desired to develop a synthesis method to integrate those information. These two topics are the main issues discussed in this report.

3 Determination of the Initial Setting

Assume that all initial covariance matrices are diagonal. The normalized states error $q(0)$ is

$$q(0) = \tilde{x}^T(0) P^{-1}(0|0) \tilde{x}(0).$$

then $q(0)$ should be smaller than the upper bound c_n which is dependent on the states degree of freedom n and determined by chi-Squared confidence gate as(two-side)

$$c_n' \leq q(0) \leq c_n.$$

Also, the initial states error is smaller than the maximized value of an acceptable state error bound l_i as

$$|\tilde{x}_i(0)| \leq l_i.$$

Further, we need to know how to obtain the $P(0|0)$ components in terms of the confidence c_n and the i^{th} state error bound l_i. A straightforward way is, assuming that all states are uncorrelated, given a definite positive diagonal matrix $P(0|0)$ is a scaler σ_1^2, then applying the confidence gate and desired upper bound to as

$$\frac{l_i^2}{\sigma_i^2} = c_n.$$

Thus, the maximized and minimized values of standard deviation σ_i of $\tilde{x}_i$ are

$$0 < \sigma_i \leq \frac{n l_i}{\sqrt{c_n'}}$$

As a result, we obtain an initial covariance matrix $P(0|0)$ as

$$P(0|0) = \begin{bmatrix} \frac{n^2 l_1^2}{c_n'} & 0 & ... & 0 \\ 0 & \frac{n^2 l_2^2}{c_n'} & ... & ... \\ ... & ... & ... & 0 \\ 0 & ... & ... & \frac{n^2 l_n^2}{c_n'} \end{bmatrix}_{n \times n},$$

where l_i is the i^{th} acceptable state error upper bound, for $i = 1, ..., n$.

For the process noise, we have

$$u^T(0) Q^{-1}(0) u(0) = q_n,$$

which is also a chi-square distributed with n degrees of freedom. Similarly, by the confidence area,

$$0 \le u^T(0)\,Q^{-1}(0)\,u(0) = q_n \le c_n,$$

which implies, under the assumption that $Q(0)$ is a diagonal with the elements $Q_i^2(0)$, $i = 1, ..., n$.

$$\Sigma_i^n \frac{u_i^2}{c_n Q_i^2(0)} \le 1.$$

It is desired to find $Q_1, ..., Q_n$ such that the disturbance u_i, $i = 1, .., n$ leis within the region bounded by a ellipsoid $\Sigma_i^n \frac{u_i^2}{c_n Q_i^2} = 1$. This ellipsoid may be approximated by a sphere with radius $\frac{\sqrt{c_n}}{n}\Sigma_i^n Q_i(0)$. If the averaged error on the disturbance along each axis is denoted by ρ_α, with probability $1-\alpha$, then we have

$$\frac{\sqrt{c_n}}{n}\Sigma_i^n Q_i(0) = \rho_\alpha.$$

Moreover, define the nominal value

$$\bar{Q}(0) = \frac{\rho_\alpha}{\sqrt{c_n}}.$$

The other components may be written as

$$Q_i(0) = \beta_i \bar{Q}(0), \tag{10}$$

in which β_i varies about 1 and $\Sigma_i^n \beta_i = n$. The ratios are determined through the knowledge on the disturbance about each variable.

Similar process may be used to determine the initial value of R, for the measurement noise.

Let the average error for the measurement along each axis is denoted by r_α. We have

$$\bar{R}(0) = \frac{r_\alpha}{\sqrt{c_m}}$$

and

$$R_i(0) = \xi_i \bar{R}(0), \tag{11}$$

where c_m is determined by the chi-distribution with m degrees of freedom and ξ_i corresponds to the knowledge on the sensor specifications on each axis.

After the initial condition on Q and R are determined, it is desired to adjust them during the filtering process so that the change of actual environment can be accommodated. Defines the state error

$$\varepsilon_x = \tilde{x}^T P^{-1} \tilde{x},$$

which is a random variable with $\varepsilon_x \backsim \chi_n^2$. To certain confidence, it is anticipated that ε_x must be in the range $[c_n', c_n]$ such that

$$P_r(c_n' \le \varepsilon_x \le c_n) = 1 - \alpha.$$

If ε_x lies above c_n or below c_n', the process may be good enough to model the disturbances. In order to have ε_x lies in the region, we may adjust $\gamma(k)$ in

$$P_{xx}(k|k-1) = \Phi(k-1)\,P(k-1|k-1)\,\Phi^T(k-1) + \gamma(k)\,Q(k-1).$$

For either $\varepsilon_x > c_n$ or $\varepsilon_x < c_n'$, we may choose the parameter $\gamma(k)$ be equal to ratio between ε_x and its mean value n, i.e.

$$\gamma(k) = \frac{\varepsilon_x}{n}.$$

If $\varepsilon_x > c_n, \gamma(k) > 1$, the process noise is scaled up, and the state error may be reduced in the region. On the other hand, if $\varepsilon_x < c_n', \gamma(k) < 1$, which means that the process noise is reduced. Moreover, for $\varepsilon_x \in (c_n', c_n)$,we also perform the fine tuning on the process covariance matrix through the formula

$$\gamma(k) = 0.5\left(\frac{\varepsilon_x}{n} + \frac{n}{\varepsilon_x}\right).$$

We denote $Q^*(k-1) = \gamma(k)\,Q(k-1)$. If ε_x is out of range, we should initialize the Kalman filter.

Recall that the innovation process may be used to the performance of the sensors. The covariance matrix for the innovation process may be written as

$$S(k) = H(k)\{\Phi(k-1)\,P_{xx}(k-1|k-1)\,\Phi^T(k-1) + Q^*(k-1)\}H^T(k) + R(k).$$

Define the sensor error

$$\varepsilon_\nu(k) = \nu^T S^{-1}(k)\,\nu$$

with $\varepsilon_\nu \backsim \chi^2_m$. Specifying some confidence region given by

$$P_r\left(c'_m \le \varepsilon_\nu\left(k\right) \le c_m\right) = 1-\alpha,$$

if ε_ν does not lie in $\left(c'_m, c_m\right)$, either the algorithm fails or possibly the measurement covariance matrix R is not properly given. For the later reason, we try to adjust R according to following rule.

$$R^*\left(k\right) = \eta\left(k\right) R\left(k\right),$$

where the factor $\eta\left(k\right)$ is determined as

$$\begin{cases} \eta\left(k\right) = \frac{\varepsilon_\nu}{m},\ \varepsilon_\nu \notin \left(c'_m, c_m\right) \\ \eta\left(k\right) = 0.5\left(\frac{m}{\varepsilon_\nu(k)} + \frac{\varepsilon_\nu(k)}{m}\right),\ \text{otherwise.} \end{cases}$$

The innovation covariance matrix is then updated

$$\begin{aligned} S^*\left(k\right) = H\left(k\right)\{\Phi\left(k-1\right)P_{xx}\left(k-1|k-1\right)\Phi^T\left(k-1\right) \\ + Q^*\left(k-1\right)\}H^T\left(k\right) + R^*\left(k\right). \end{aligned}$$

If the algorithm still does not work after the adjustment, the message of sensor fault should be reported.

By appropriately adjusting Q and R, the filter can be made robust in the presence of environmental changes. Such notion of adaptive Kalman filter (AKF) is necessary for long term navigation.

4 Data Fusion

To perform estimation of the states in a physical system, it is sometimes necessary to use multi-sensors. The problem of how to combine the output data from each mode of a data-log network is the main concern in the section. The notion of covariance intersection is introduced to solve the problem of dependency between these filters. The local observation information can be obtained from each sensor. If the measurements are independent, i.e. the cross covariance matrices can are zero, then it is easy to achieve a combination of information. On the other hand, if the information are correlated, which the cross covariance matrices are unknown, the information fusion becomes a much more complicated process.

Consider the case that *local* information measurements are independent. Let $\hat{x}_1$ and $\hat{x}_2$ be two estimates of x with covariance P_1 and P_2, respectively. The combined information by applying MMSE estimator can be expressed as

$$\hat{x}_c = \left[P_1^{-1} + P_2^{-1}\right]^{-1}\left[P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2\right]$$

with the resulting covariance[4, Bayerian Inference][2, Chap10]

$$P_C = \left[P_1^{-1} + P_2^{-1}\right]^{-1}.$$

In general, if the estimates are dependent and the fused system covariance matrices are[2, Chap10.3][3, Chap8]

$$\begin{bmatrix} P_1 & P_{12} \\ P_{21} & P_2 \end{bmatrix},$$

where the cross-covariance matrix is

$$P_{12} = E\left[\left(\tilde{x}_1\right)\left(\tilde{x}_2\right)^T\right].$$

By using again the MMSE estimates, we obtain the fused state estimate

$$\begin{aligned} \hat{x}_C = \hat{x}_1 + \left[P_1 - P_{12}\right]\left[P_1 + P_2 - P_{12} - \left(P_{12}\right)^T\right]^{-1} \\ \left[\hat{x}_2 - \hat{x}_1\right] \end{aligned}$$

and the corresponding covariance is

$$\begin{aligned} P_C = P_1 - \left[P_1 - P_{12}\right]\left[P_1 + P_2 - P_{12} - \left(P_{12}\right)^T\right]^{-1} \\ \left[P_1 - P_{12}\right]^T. \end{aligned} \tag{12}$$

Because the cross covariance matrices are too complicate and frequently unknown, to deal with this problem, one can modify the information fusion algorithm via *convex combination* idea of two system error covariance matrices[7, CI]. Instead of (12), we choose a parameter α, where $0 \le \alpha \le 1$, such that

$$P_C = \left[\alpha P_1^{-1} + \left(1-\alpha\right)P_2^{-1}\right]^{-1},$$

and the corresponding updated estimate is

$$\hat{x}_C = P_C\left[\alpha P_1^{-1}\hat{x}_1 + \left(1-\alpha\right)P_2^{-1}\hat{x}_2\right].$$

This method is called Covariance Intersection(CI) filtering algorithm[7]. Extending the Covariance Intersection to an $n-$subsystem sensory system[7], the fused covariance P_C and state $\hat{x}_C$ are

$$P_C = \left[\Sigma \alpha_j P_j^{-1}\right]^{-1}, \tag{13}$$

and

$$\hat{x}_C = P_C \left[\Sigma \alpha_j P_j^{-1} \hat{x}_i\right] \tag{14}$$

where

$$0 \leq \alpha_j \leq 1,$$

$$\Sigma_j \alpha_j = 1.$$

Without loss of generality, for the j^{th} subsystem at the k^{th} time step, by selecting the factor c_j for each node, we define

$$\alpha'_j = P_D^j \left(P_G^j\right)^{-1} c_j e^{-\left(\frac{1}{2}\varepsilon_{\nu j}^2(k)\right)}$$

where P_D^j is a probability of detection [2, detection] which determined by the j^{th} signal-noise ratio above some threshold, and the gate probability P_G^j is a gated threshold[2, gate] as the chi-square probability for the j^{th} node. The parameters in CI algorithm may be then chosen as

$$\alpha_j = \frac{\alpha'_j}{\Sigma_i^n \alpha'_i}.$$

5 Fused INS-GPS system

The ideas presented above are now applied to the fused-INS-GPS system. Let P_{ins}, P_{gps} be the states error covariance matrices of INS(Inertial Navigation System) and GPS(Global Positioning System) respectively. The covariance matrices from INS and GPS via Kalman filter algorithm individually, which may be termed decentralized or distributed approach[3, Chap8], can be expressed as

$$P_{ins}(k|k) = P_{ins}(k|k-1) - WS_{(ins)}W^T$$

and

$$P_{gps}(k|k) = P_{gps}(k|k-1) - WS_{(gps)}^T W^T.$$

The fused, updated covariance matrix is then

$$P_C(k|k) = \left[\alpha P_{ins}^{-1}(k|k) + (1-\alpha) P_{gps}^{-1}(k|k)\right]^{-1} \tag{15}$$

and the fused estimate vector is

$$\hat{x}_C = P_C(k|k) \left[\alpha P_{ins}^{-1}(k|k) \hat{x}_{ins} + (1-\alpha) P_{gps}^{-1}(k|k) \hat{x}_{gps}\right].$$

The problem is then to choose the coefficient α. Obviously, the role of α should provide good information with the better fusion performance and some criterion of faults rejection. In the data logging network of a multisensor system, the signals are sampled in different sampling rates. If ε_{ins}, ε_{gps} are all located within the 95% confidence area, their corresponding measurements are acceptable. Either the value of ε_{ins} or ε_{gps} is out of bound, it means this system has abnormal or the faults occurred.[5, faults detection][1, FDI] [6, FD] From the previous section, we compute

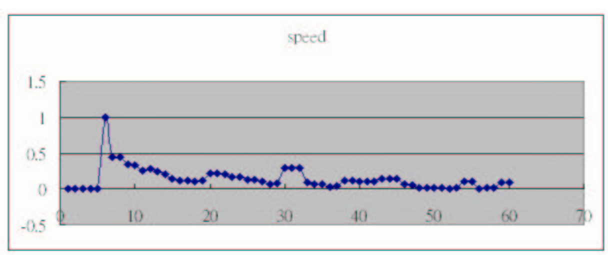
$$\alpha'_{ins} = c_{ins} P_D^{ins} \left(P_G^{ins}\right)^{-1} e^{\frac{-1}{2}\varepsilon_{ins}},$$

$$\alpha'_{gps} = c_{gps} P_D^{gps} \left(P_G^{gps}\right)^{-1} e^{-\frac{1}{2}\varepsilon_{gps}}.$$

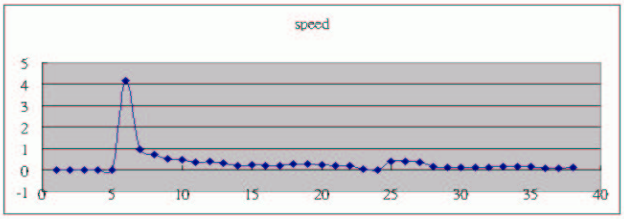
The values of c_{ins}, c_{gps} are given by 0.5, since there are no bias on each node. The probability of detection P_D^{gps}, P_D^{ins} are assumed to be 1. The estimated α is then chosen as

$$\alpha = \frac{\alpha'_{ins}}{\alpha'_{ins} + \alpha'_{gps}}.$$

Incorperating the algorithm of initialization and the covariance intersection algorithms, the fused-INS-GPS system can be then used to estimate the speed of an unmanned vehicle. Two experimental results are shown in the following figures.



Push and drive some small torque.



At 5^{th} step, push the car then free driving to zero speed.

6 Conclusion

The design of a fused-INS-GPS navigation system is presented in this report. Comparing with classical notion of Kalman filter, the ideas of the adaptive change of the noise covariances and the covariance intersection are adopted. Experimental results showed that the algorithm is applicable and more robust.

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