EVALUATION OF BRIDGE SAFETY BASED ON CONCRETE NONDESTRUCTIVE TEST

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ABSTRACT

This paper develops a framework for the safety assessment of bridges based on the nondestructive test (NDT) results of concrete. The transient elastic wave test is performed to measure the P wave velocity of concrete. The Bayesian approach is adopted to construct the posterior distribution of concrete strength. Then, reliability analysis is carried out to evaluate the safety of the bridge using the modified distribution. The limit-state function is formulated according to the AASHTO specifications. Finally, the reliability index is compared to the target reliability to see if repair or reconstruction is required. A numerical example is presented to illustrate the safety assessment of a solid-slab bridge using the proposed method.

Keywords : Bridge, Bayesian statistics, Reliability analysis, AASHTO, NDT.

1. INTRODUCTION

Bridges play an indispensable role in land transportation. The collapse of a bridge often results in tremendous losses. Therefore, the safety assessment of bridges is an important and challenging task for civil engineers. The safety of a bridge can be evaluated by means of reliability analysis. It provides a reasonable estimate on the failure probability of the bridge if the distributions of the uncertainties in the system are known. However, as the concrete of the bridge is damaged, the distribution of its compressive strength changes as well. Apparently, the distribution should be modified and the reliability of the bridge should be re-evaluated.

Since the quality of concrete is a key factor to the safety of concrete structures, many methods have been developed to examine the degradation of concrete [1~3]. Among these methods, the transient elastic wave method is effective in the in-situ measurement of the wave velocity of concrete. According to the results of lab and in-situ tests, the P wave velocity of concrete can reflect the compressive strength of concrete to a certain degree [3]. However, how to apply the inspection results in the safety assessment of bridge is still a new area.

Liu and Chen [4] have adopted the Bayesian approach in the safety assessment of structures based on the results of system identification. The idea is extended in this paper to develop a method for evaluating the safety of existing bridges. The proposed method is based on reliability analysis, AASHTO specifications, nondestructive tests of concrete, and Bayesian statistics. The details are given in the following.

2. RELIABILITY ANALYSIS

The structural reliability analysis is formulated based on two fundamental assumptions: (1) the state of the structure is defined in the outcome space of a vector of basic random variables, $\mathbf{X}$; (2) the structure can be in one of two states, the safe state or the failure state. The state of the structure is determined by the value of a limit-state function $g(\mathbf{x})$, which is formulated such that when $g(\mathbf{x}) > 0$, the structure is safe, and when $g(\mathbf{x}) \le 0$, the structure fails. The boundary between the two states, $g(\mathbf{x}) = 0$, is known as the limit-state surface.

The failure probability of the structure associated with the specific failure criterion is as follows:

$$P_f = \int_{g(\mathbf{x})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \qquad (1)$$

It is usually difficult to perform the above integral directly. Hence, the first-order reliability method (FORM) is often adopted to estimate the failure probability.

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In FORM, the basic variables are transformed into a set of statistically independent, standard normal variables. The limit-state surface in the standard normal space is then replaced by the tangent hyperplane at the nearest point on the limit-state surface to the origin. The nearest point, y^*, is denoted the design point. Then, the first-order estimate of the probability of failure is given by $P_{fl} = \Phi(-\beta)$, where Φ is the standard normal cumulative probability, and the reliability index β is the distance from y^* to the origin.

FORM has a very useful feature that it can provide the sensitivity of β with respect to the distribution parameters. Let θ be a set of distribution parameters, for example, means and standard deviations. The sensitivity of β with respect to θ is as follows [5]:

$$\frac{\partial \beta}{\partial \theta} = \frac{y^{*T}}{\beta} \frac{\partial y^*(x^*, \theta)}{\partial \theta}\Bigg|_{x^*} \qquad (2)$$

where x^* is the design point in the original space.

The sensitivity measures can be used to compare the influence of the variables on the structural reliability. If the failure probability is very sensitive to the mean of a variable, shift of the mean would greatly change the reliability of the structure.

The reliability analysis can be used to assess the safety of a structure. However, two issues need to be addressed in the safety assessment of bridges. Firstly, the failure criterion should be selected properly. Secondly, the distributions of material properties change when the bridge is damaged. Hence, the distributions should be modified to reflect the degradation of materials.

3. LIMIT-STATE FUNCTION

Since bridges are designed in compliance with specifications, it is natural to define the limit-state function according to the same criterion. In the current AASHTO code, the load and resistance factor design (LRFD) formula is

$$\phi R_n > \sum \gamma_i W_i \qquad (3)$$

where ϕ = the resistance factor, R_n = the design resistance, W_i = the ith nominal load effect, and γ_i = the corresponding load factor. If only dead load and live load are considered, the formula becomes

$$\phi R_n > 1.25D + 1.5D_w + 1.75(L + I) \qquad (4)$$

where D and D_w are the dead load effects due to concrete and wearing surface, respectively. L and I are respectively the live and dynamic load effects due to moving vehicles. The load and resistance factors in Eq. (4) give a target reliability of $\beta_T = 3.5$ [6].

The formula in Eq. (3) simply states that the capacity of a bridge must exceed the load effects. Hence, the limit-state function for the flexural failure of a bridge can be written as

$$g = M_R - M_{DC} - M_{DW} - M_{L+I} \qquad (5)$$

where M_R = bending carrying capacity, M_{DC} = bending moment due to the weight of reinforced concrete, M_{DW} = bending moment due to the weight of wearing surface, and M_{L+I} = bending moment due to dynamic load.

Such limit-state function has the advantage that it is consistent with the design code. Therefore, the reliability index of the damaged bridge is comparable to the reliability index at the design stage.

4. DISTRIBUTION MODELS

To perform reliability analysis for a bridge, one has to establish the distribution models for the capacity and load effects in the limit-state function.

4.1 Dead Load Model

The distributions of dead load effects are bridge dependent. In the following, a simply supported, solid slab bridge is considered in the derivation of the distributions. The load effects are derived for a unit width of the bridge.

Suppose the weight of the reinforced concrete slab per unit area is w_{DC}. Then, the maximum bending moment in the bridge is $w_{DC}l^2/8$, where l is the span of the bridge. Following Norwak, et al. [6], assume that w_{DC} is normally distributed. Its mean to nominal ratio is λ_{DC}, and its coefficient of variation is v_{DC}. It is easily derived that

$$M_{DC} \sim N(\mu_{DC}, \sigma_{DC}) \qquad (6)$$

where $\mu_{DC} = \lambda_{DC} w_{DC} l^2/8$, and $\sigma_{DC} = v_{DC} \mu_{DC}$.

Next, consider the load effect of the wearing surface. Let the mass density of the wearing surface be a constant ρ_{DW} and the thickness be normally distributed with mean μ_H and coefficient of variation v_H. It follows that

$$M_{DW} \sim N(\mu_{DW}, \sigma_{DW}) \qquad (7)$$

in which $\mu_{DW} = \mu_H \rho_{DW} g l^2/8$, $\sigma_{DW} = v_H \mu_{DW}$, where g is the gravitational acceleration.

4.2 Live Load Model

The live load effect also depends on the layout of the bridge. If the bridge has a single lane, and its span is less than 60m, the live load effect is the maximum moment due to a HS-20 truck, as shown in Fig. 1 [7]. For this particular case, the maximum moment is

$$M_{LL} = \frac{1}{2b}\left[145(l - 4.3) + +35\left(\frac{l}{2} - 4.3\right)\right] \text{ (kN-m/m)} \qquad (8)$$
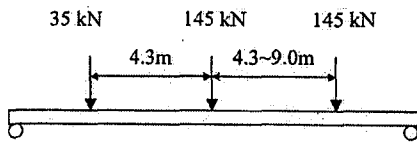
Fig. 1 Loads due to the HS-20 truck

in which b is the width of bridge.

Since the truck is moving, additional dynamic effect is induced. In ASSHTO code, such effect is included by introducing the dynamic load factor $IM = 0.33$, and the total live load is $M_{L+I} = M_{LL}(1 + IM)$.

Suppose M_{LL} is normally distributed with a mean to nominal ratio λ_{LL} and coefficient of variation v_{LL}. Then,

$$M_{L+I} \sim \mathrm{N}(\mu_{L+I}, \sigma_{L+I}) \qquad (9)$$

where $\mu_{L+I} = (1 + IM)\lambda_{LL}\mu_{LL}$, $\sigma_{L+I} = v_{LL}\mu_{L+I}$, and μ_{LL} is the maximum moment due to the live load. According to Das [8] and Tabsh and Nowak [9], $\lambda_{LL} = 1.6 \sim 2.1$, and $v_{LL} = 0.14$ for the bridges in America.

4.3 Resistance Model

Figure 2 shows a reinforced concrete slab as flexural strength is reached. It is assumed that the steel bars have yielded and the maximum strain of concrete is 0.003. For a unit width of slab, the tensile force in the steel is $T_n = A_s f_y$, and the compressive force in the concrete is $C_n = 0.85 f_c' a = 0.85 f_c' \beta_1 c$, where

$$\beta_1 = \begin{cases} 0.85 & f_c' \leq 28\mathrm{MPa} \\ 0.65 & f_c' \geq 56\mathrm{MPa} \\ 0.85 - 0.05\dfrac{f_c' - 28}{7} & 28\mathrm{MPa} \leq f_c' \leq 56\mathrm{MPa} \end{cases} \qquad (10)$$

The depth of the neural axis, c, can be solved by setting $C_n = T_n$.

Hence, the resistance of the slab is

$$M_R = T_n\left(d_s - \frac{a}{2}\right) = A_s f_y\left(d_s - \frac{A_s f_y}{1.7 f_c'}\right) \qquad (11)$$

There are four parameters in Eq. (11). In this study, A_s and d_s are considered deterministic, while f_y and f_c' are assumed normally distributed.
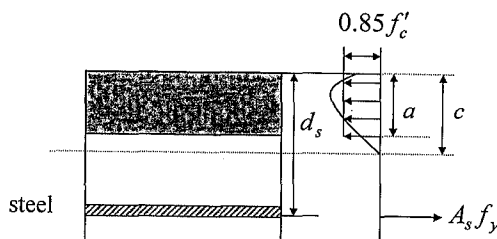

Fig. 2 Flexural strength of reinforced concrete slab

One may proceed with the reliability analysis of a bridge using Eqs. (6), (7), (9), and (11). However, if the concrete of the bridge is damaged, its compressive strength may decrease. Obviously, if reliability analysis is performed using the original distribution of f_c', one obtains the same reliability index as in the design stage. This is not reasonable because the degradation of concrete must have some influence on the safety of bridge. Hence, the damage of concrete must be measured and the distribution of f_c' must be modified before reliability analysis is carried out.

5. DISTRIBUTION OF f_c'

5.1 Nondestructive Test of Concrete

In order to estimate the compressive strength of concrete, tests must be performed on the concrete. There are various methods to evaluate the concrete strength, for example, the probe penetration method, the break-off method, the pullout method, the rebound hammer, the ultrasonic pulse velocity method, and the transient elastic wave method. Among these methods, the rebound hammer, the ultrasonic pulse velocity method, and the transient elastic wave method are truly nondestructive. Researchers have tried to establish the correlation between f_c' and the test results. It is found that the rebound hammer does not always give consistent results, especially in the field test. This is because the rebound number is governed by many factors, to name a few, the surface roughness and the moisture content of concrete.

The correlation between the ultrasonic wave velocity and f_c' is good. However, in order to yield a high signal to noise ratio, the ultrasonic transducers must be aligned and placed on two opposite sides of the concrete member. That makes the method impractical in the field.

The transient elastic wave test does not have such limitation because the test is performed on only one side of the concrete. Furthermore, the sensor has a conical shape. The contact area between the sensor and the concrete is very small. Consequently, the test result is not influenced by the surface roughness of concrete. Therefore, it is adopted in this study to evaluate the compressive strength of concrete.

In the transient elastic wave test, a tiny steel ball is dropped on the concrete surface to generate elastic waves in the concrete. Two displacement sensors are mounted on the surface of the concrete to measure the horizontal displacement. Then, the signals are processed to determine the P wave velocity, C_p, of the concrete.

Wu, et al. [3] has done a series of tests to establish the correlation between C_p and f_c'. Surprisingly, the correlation for the field data is as good as the lab data. That means the transient elastic wave test is suitable for field tests. Figure 3 depict the correlation between C_p and f_c'. It is obvious that as the P wave velocity of concrete increases, f_c' also tends to increase.
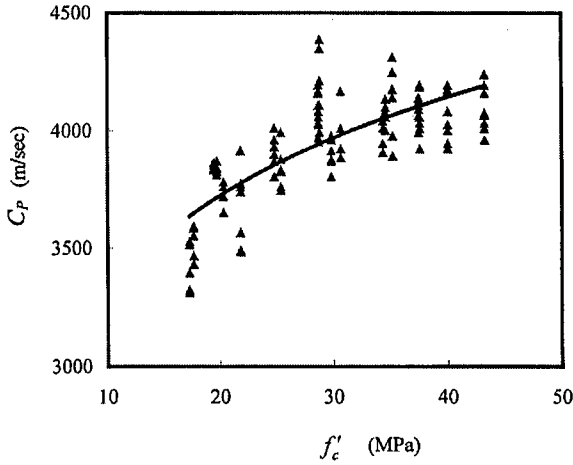
Fig. 3 Correlation of f_c' and C_p

5.2 Modification of The Distribution of f_c'

After the transient elastic wave test is performed on the concrete, one obtains the P wave velocity of the concrete. It seems that one may perform regression analysis on the data in Fig. 3 to construct the conditional probability distribution of f_c' for the measured velocity. Then, reliability analysis of the bridge can be carried out using the distribution. However, this is risky because the correlation is not universal. Therefore, neither the original distribution nor the NDT distribution should be used directly in the reliability analysis.

In principle, the modified distribution of f_c' should be constructed such that it reflects the original material as well as the damage condition of the concrete. To meet this end, the Bayesian approach is suggested here to modify the distribution of f_c' using NDT data.

For simplicity, let $f_c' = x$, and $C_p = v$. The Bayesian modification formula is as follows:

$$f_x''(x) = \frac{f_{v|x}(v \mid x)f_x'(x)}{\int_{-\infty}^{\infty} f_{v|y}(v \mid y)f_y'(y)dy} \qquad (12)$$

where $f_x'(x)$ = original distribution of f_c', $f_{v|x}(v|x)$ = conditional distribution of C_p given $f_c' = x$, and $f_x''(x)$ = modified distribution of f_c'.

In Eq. (12), the conditional distribution $f_{v|x}(v|x)$ can be determined by regression analysis of the test data. Assume that $f_{v|x}(v|x) \sim N(\mu_{v|x}, \sigma_{v|x})$, where $\mu_{v|x}$ is dependent on f_c', and $\sigma_{v|x}$ is a constant. In the regression analysis, one selects a formula for the $\mu_{v|x} \sim x$ curve. Then, use the least-squares method to determine the optimal parameters. Regression analysis on the data in Fig. 3 yields:

$$\mu_{v|x} = 606.95 \ln x + 1904.3 \quad \text{(m/sec)} \qquad (13)$$

$$\sigma_{v|x} = 138.7 \quad \text{(m/sec)} \qquad (14)$$

The unit of x is MPa, and the units of $\mu_{v|x}$ and $\sigma_{v|x}$ are m/sec.

Suppose the original distribution of f_c' is

$$f_x' = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left[-\frac{1}{2\sigma_c^2}(x - \mu_c)^2\right] \qquad (15)$$

Then, the posterior (or modified) distribution becomes

$$f_x'' = \frac{k}{2\pi\sigma_{v|x}\sigma_c} \exp\left\{-\frac{1}{2}\left[\left(\frac{v - 606.95\ln x - 1904.3}{\sigma_{v|x}}\right)^2 + \left(\frac{x - \mu_c}{\sigma_c}\right)^2\right]\right\} \qquad (16)$$

where k is a constant. Notice that posterior distribution is no longer normal. Hence, the posterior mean μ_c'' and posterior standard deviation σ_c'' need to be computed numerically.

Suppose n identification processes are conducted, and n results are obtained, namely, $C_p = v_1, v_2, \cdots v_n$. Then, the conditional probability density of observing $v_1, v_2, \cdots v_n$ as $f_c' = x$ is

$$f_{v_1, v_2 \ldots v_n|x}(v_1, v_2 \cdots v_n | x) = \prod_{i=1}^{n} f_{v|x}(v_i \mid x)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_{v|x}} \exp\left[-\frac{1}{2\sigma_{v|x}^2}(v_i - \mu_{v|x})^2\right]$$

$$= \frac{k^*}{\sqrt{2\pi}\sigma^*} \exp\left(-\frac{1}{2(\sigma^*)^2}(v^* - \mu_{v|x})^2\right) \qquad (17)$$

where k^* is a constant, and

$$v^* = \frac{1}{n}\sum_{i=1}^{n} v_i \qquad\qquad \sigma^* = \frac{\sigma_{v|x}}{\sqrt{n}} \qquad (18)$$

It is seen that the conditional distribution in Eq. (17) is still normal, and its standard deviation decreases as the number of tests increases.

Once the conditional distribution is determined, one may substitute Eqs. (15) and (17) into Eq. (12) to obtain the posterior distribution. The posterior distribution has the same form as given in Eq. (16) except that v and $\sigma_{v|x}$ are replaced by v^* and σ^*, respectively.

Once the modified distribution of f_c' is available, one may perform reliability analysis on the target bridge. In the following section, a numerical example is presented to illustrate the proposed method.

6. EXAMPLE

Consider a single span, single lane, solid slab bridge. The bridge span $l = 35$m, the width $b = 3.05$m. The thickness of concrete slab $h = 1.6$m, the depth of steel bars $d = 1.49$m, and the area of steel bars per unit width

of bridge $A_s = 0.025\text{m}^2/\text{m}$. The original distribution of $f_c' \sim N(28, 4)$ MPa, and the distribution of $f_y \sim N(420, 42)$ MPa.

Let the weight of concrete slab per unit area $w_{DC} = 37.6\text{kN/m}^2$, the mean to nominal ratio $\lambda_{DC} = 1.05$, and the coefficient of variation $v_{DC} = 0.1$. According to Eq. (6), $M_{DC} \sim N(6050, 605)$ kN-m/m^2.

The density of wearing surface $\rho_{DW} = 2250\text{kg/m}^3$. The mean of its thickness $\mu_H = 75\text{mm}$, and the coefficient of variation $v_H = 0.15$. According to Eq. (7), $M_{DW} \sim N(253, 38)$ kN-m/m^2.

Now, consider the live load. Assume that the mean to nominal ratio of M_{LL} is $\lambda_{LL} = 2$, and the coefficient of variation $v_{LL} = 0.14$. According to Eqs. (8) and (9), $M_{L+I} \sim N(2110, 296)$ kN-m/m^2.

The limit-state function for flexural failure is the same as in Eq. (5), i.e.,

$$g = M_R - M_{DC} - M_{DW} - M_{L+I} \qquad (18)$$

First order reliability analysis was performed for the above limit-state function using the aforementioned distribution models. It turned out that the failure probability is 0.00015, and the reliability index is 3.61, which is very close to the target reliability of the AASHTO specifications.

Table 1 shows the results of sensitivity analysis. The sensitivity measures in the table are dimensionless because they are normalized by the corresponding standard deviations. It is seen that sensitivity of β with respect to the standard deviation is negative for all the variables. That means, the failure probability increases with the standard deviation. Hence, the higher the uncertainty is in the system, the less reliable the bridge is.

Table 1   Results of sensitivity analysis – original bridge

Basic variable	$\sigma\dfrac{\partial \beta}{\partial \mu}$	$\sigma\dfrac{\partial \beta}{\partial \sigma}$
f_c'	0.1262	-0.0576
f_y	0.8692	-2.7311
M_{DC}	-0.4287	-0.6643
M_{DW}	-0.0269	-0.0026
M_{L+I}	-0.2097	-0.1590

Examine the sensitivity of β with respect to the means of the load effects. It is seen that all the sensitivities are negative. This is expected because the bridge is more likely to fail as the loads increase. For this linear limit-state function, one can show that these sensitivities are proportional to the standard deviation of the corresponding variable. Hence, the sensitivity of β with respect to M_{DC} is the greatest among the three sensitivities.

Different from the sensitivity of β with respect to the means of the load effects, the sensitivity of β with respect to the means of f_c' and f_y are positive. In other words, the reliability of the bridge increases with the

means of f_c' and f_y. This is reasonable because the strength of the bridge increases with f_c' and f_y.

Next, consider that the concrete is damaged after a period of service. From Eq. (13), the mean value of C_p is 3926m/sec as $f_c' = 28$MPa. Assume that the transient elastic wave test was performed on the concrete, and the P wave velocity goes down below 3926m/sec.

The results of Bayesian modification for various wave velocities are listed in Table 2. It is seen that the reduction of P wave velocity indeed leads to reduction of the modified mean value of f_c', i.e., μ_c''. Furthermore, the lower C_p is, the lower μ_c'' is.

Table 2   Results of Bayesian modification and reliability analysis

C_P (m/sec)	μ_c'' (MPa)	σ_c'' (MPa)	P_{f1} (10^{-4})	β
3700	25.07	3.38	2.17	3.51
3600	23.56	3.39	2.83	3.44
3500	21.95	3.38	4.03	3.35
3400	20.22	3.37	6.87	3.20

Table 2 also reveals that the modified standard deviations are less than the original standard deviation 4MPa. This is because extra test data can reduce the uncertainty of f_c'.

Reliability analysis was carried out using the modified distributions of f_c'. The results are also listed in Table 2. One can see that as the P wave velocity decreases to 3700m/sec, the reliability index becomes 3.51, which is still somewhat greater than the target value. However, if C_p goes down below 3600m/sec, the reliability index is no longer greater than 3.5. That means the bridge may need repair. If the reliability index is much less than the target reliability, reconstruction of the bridge may even become necessary.

Table 3 shows the influence of the P wave velocity on the sensitivity of β with respect to the means of f_c' and f_y. Notice that as the P wave velocity decreases, $\sigma\partial\beta/\partial\mu$ of f_c' increases while $\sigma\partial\beta/\partial\mu$ of f_y decreases. Furthermore, as $C_p = 3400$ m/sec, $\sigma\partial\beta/\partial\mu$ of f_y is only 13% less than that of the original bridge, but $\sigma\partial\beta/\partial\mu$ of f_c' is increased by more than 200%. In other words, f_c' becomes more and more dominant as P wave velocity decreases.

Table 3   Influence of C_p on sensitivities

C_P (m/sec)	$\sigma\dfrac{\partial \beta}{\partial \mu}$	
	f_c'	f_y
original	0.1262	0.8692
3700	0.1427	0.8592
3600	0.1776	0.8468
3500	0.2371	0.8237
3400	0.3872	0.7524

Finally, the influence of multiple tests is investigated. Suppose f_c' is reduced to 21MPa. According to Eq. (13), $f_{v|x}(v|x = 21) \sim N(3752.2, 138.7)$ m/sec. Hence, 10 C_p's are generated using this distribution. Namely, $C_p = 3770, 3792, 3593, 3917, 3917, 3747, 3797, 3776, 3726$, and 3670m/sec. The results of Bayesian modification and reliability analysis using 1 to 10 test data are listed in Table 4.

Table 4 Influence of multiple tests

n	μ_c'' (MPa)	σ_c'' (MPa)	β
1	26.07	3.38	3.55
2	25.19	2.92	3.53
3	22.92	2.50	3.43
4	23.73	2.30	3.48
5	24.29	2.15	3.5
6	23.81	1.98	3.48
7	23.67	1.85	3.48
8	23.48	1.74	3.47
9	23.14	1.64	3.46
10	22.67	1.54	3.44

It is seen that as the number of tests n increases, σ_c'' decreases monotonically, and μ_c'' approaches 21MPa. Therefore, if $f_{v|x}(v|x)$ describes the correlation between f_c' and C_p faithfully, a better modified distribution can be obtained by performing more tests.

Notice that β goes down from 3.55 ($> \beta_T = 3.5$) as $n = 1$ to 3.44 ($< \beta_T$) as $n = 10$. Obviously, the safety of the bridge may be overestimated if insufficient tests are performed. Hence, it is advisory to perform more tests to improve the results of evaluation.

CONCLUSIONS

This paper develops a framework for the safety assessment of the upper structure of bridges based on the NDT result of concrete. The Bayesian approach is adopted to modify the distribution model of concrete strength. Then, reliability analysis is performed using the modified distribution. Finally, the reliability index is compared to the target reliability to see if repair is required. Since the limit-state function is formulated according to the AASHTO specifications, the reliability index of the damaged bridge is comparable to the reliability index at the design stage.

Although only the flexural failure of a very simple bridge is considered in this paper, the evaluation method can be applied to other types of bridges and failure, even other types of structures. To extend the proposed approach to other types of structures or failure, one only needs to re-formulate the limit-state function according to the associated design code. The method can also be extended to include other type of material degradation, for example, the corrosion of reinforcing steel. Therefore, the proposed method is useful as long as there are suitable nondestructive tests that one can use to detect structural degradation.

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